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ENHANCED MESH ADAPTATION USING LOCALIZED CONFORMAL QUADRILATERAL AND HEXAHEDRAL COARSENING

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Abstract

High fidelity mesh adaptation requires the ability to both conformably refine and coarsen any given triangular, quadrilateral, tetrahedral, or hexahedral mesh. Conformal refinement and coarsening algorithms are robust and well established for triangular and tetrahedral meshes and refinement algorithms for quadrilateral and hexahedral meshes are becoming available. However, little work has been completed for conformal coarsening of quadrilateral or hexahedral grids. This paper presents robust algorithms to locally coarsen both quadrilateral and hexahedral meshes. Basic to these coarsening algorithms is a procedure to adjust the connectivity of some of the elements that surround the region to be coarsened. This adjusted boundary provides an encasing ring of elements that can be subsequently removed in a manner that leaves the mesh conformal. This basic procedure is easily adapted to be applied in multiple steps such that several levels of localized coarsening can be achieved.

Introduction

In most finite element analyses, a uniform mesh is not the optimum way to model the problem. Areas of high stress gradients require a dense mesh and areas of low stress gradients allow a coarser mesh density. Mesh adaptation is the ability to modify a finite element model to meet these conditions. Mesh adaptation is a field which has received extensive study in both computational mechanics and computer graphics. Conformal refinement and coarsening algorithms are robust and well established for triangular and tetrahedral meshes [Garland M, Heckbert P (1997), Hoppe H, DeRose T, Duchamp T, McDonald J, Stuetzle W (1993), Cignoni P, Montani C, Scopigno R (1997), Silva S, Silva F, Madiera J, Santos B (2007)] and refinement algorithms for quadrilateral and hexahedral meshes are becoming available [Schneiders R (1996), Tchon K.-F, Dompierre J and Camerero R (2004), Harris N, Benzley S, and Owen S (2004)]. However, little work has been accomplished in quadrilateral and hexahedral mesh coarsening for computational mechanics needs [Benzley S, Harris N, Scott M, Borden M, and Owen S (2005)]. The work presented in this paper provides a unique solution to allow conformal coarsening of both structured and unstructured quadrilateral and hexahedral finite element meshes.

Conformal Quadrilateral Coarsening

The method presented here is based on dual chord operations and dual chord removal. The dual operations have been described previously by Murdock *et al.* (1997). A chord is constructed by connecting the centroid of a quadrilateral with the centroids of its neighboring quadrilaterals. The lines connecting a successive set of quadrilaterals through opposite edges form a single chord of the mesh. Figure 1 shows a chord in the left panel as a dotted line running through the mesh. As is shown in this figure, the mesh can be conformably coarsened by collapsing all elements that are defined by the chord.

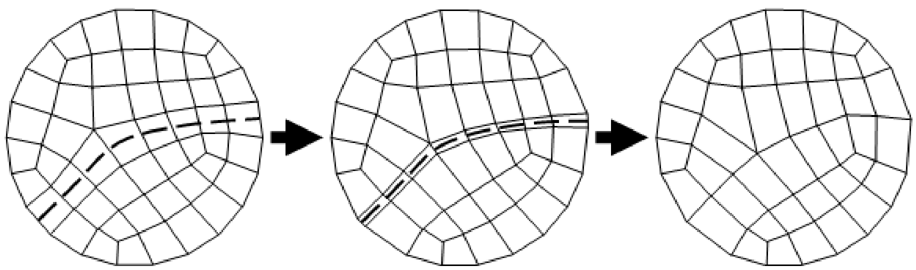

Figure 1. Removing a chord from a mesh.

Figure 2 shows three connectivity modifications that can be used to change the structure of the chords. These operations are an edge swap, a face close, and a doublet insertion. The effect of these operations is to redirect the bearing of a chord.

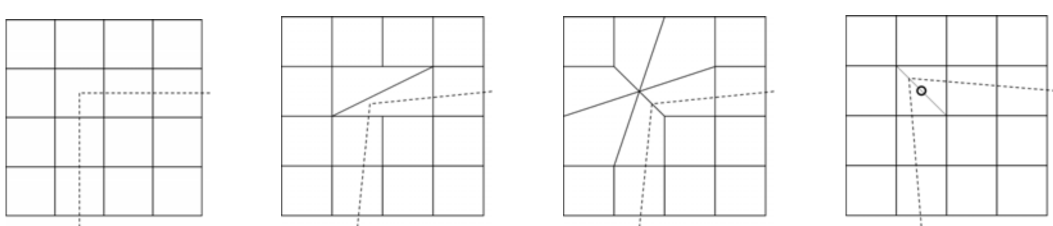

Figure 2. Original mesh, edge swap, faces close and doublet insertion operations.

Figure 3 shows how applying these connectivity operations on a simple mesh can create a closed chord surrounding a region and, by subsequently collapsing this closed chord, the mesh can be conformably coarsened. Note however, that this coarsening process results in a degradation of the quality of the mesh.

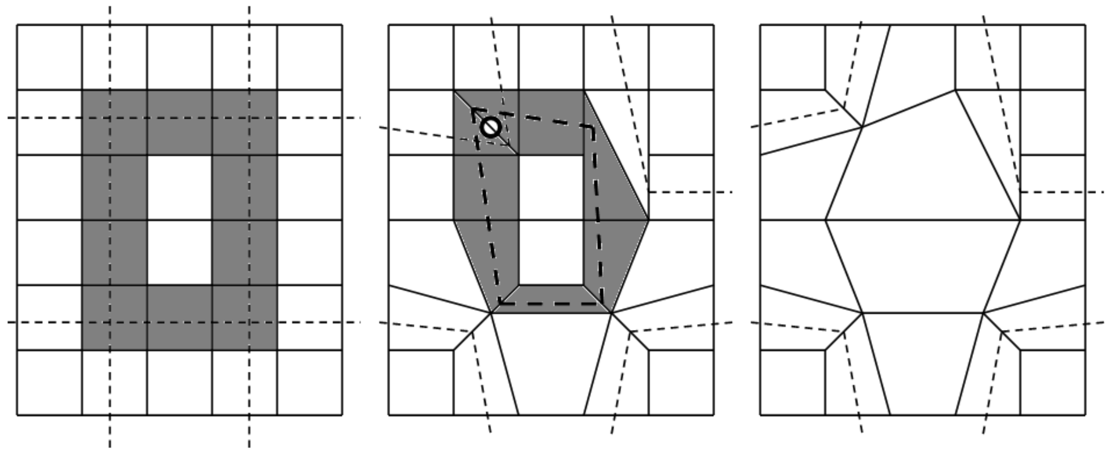

Figure 3. Chord operations and removal.

Dewey (2008) has automated this procedure to provide a powerful all-quadrilateral coarsening tool. His procedure consists of the following six steps. An example of how these steps are accomplished is given in Figure 4.

1. A contiguous coarsening region is defined and a removal parameter is specified which estimates how coarse the end mesh should be.
2. One or more coarsening rings are selected within the coarsening region which will remove a number of quads less than or equal to a goal number of quads.
3. The bounding partial chord intersections may be altered with chord operations to increase the final quality of the mesh or to prevent merging nodes illegally.
4. The coarsening rings identified are collapsed from the mesh and the mesh is reconnected in a manner that retains its conformal properties.
5. The mesh is checked, cleaned up, and smoothed to ensure that elements have optimal quality.
6. Steps two through five are repeated if insufficient coarsening has taken place in a given iteration of the algorithm.

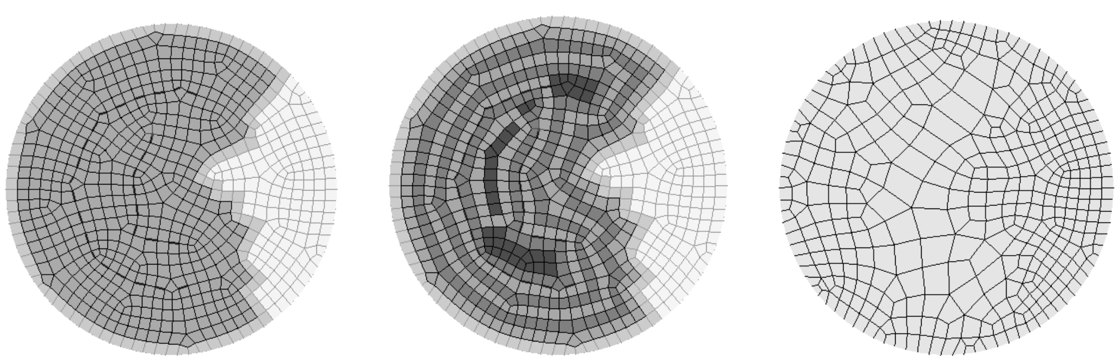

Figure 4. Coarsening region selection, automatic generation of coarsening rings and resulting coarsened mesh.

Conformal Hexahedral Coarsening

The concepts of dual chord modification and extraction extend directly to three dimensions as shown by Staten M *et al.* (2008). For this case, hexahedral sheets or twist planes as defined by Murdock *et al.* (1997) are used. Like a dual chord in a quadrilateral mesh, a dual sheet can be extracted from a hexahedral mesh. Figure 5 shows a simple hexahedral mesh with a single sheet highlighted. This sheet can be extracted as shown in the figure to coarsen the mesh. However, hexahedral sheets usually extend beyond a local region that has been selected for coarsening. Removing these sheets from the mesh would produce changes in areas where coarsening is not desired. Therefore, it is necessary to modify the mesh in such a way that produces sheets that are confined to the coarsening region.

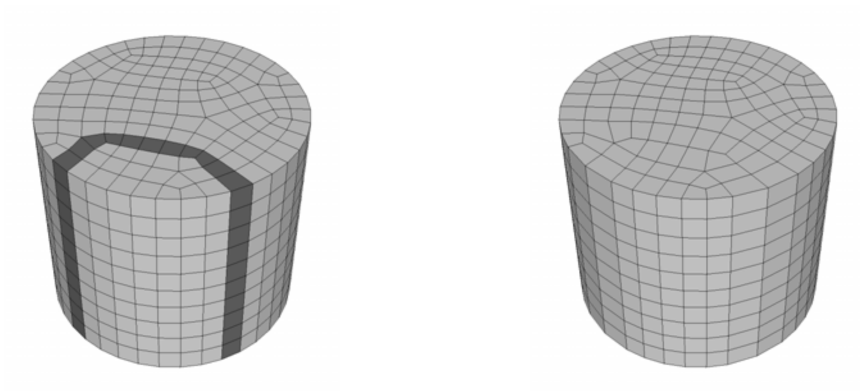

Figure 5. Coarsening a hexahedral mesh by sheet extraction.

The quadrilateral dual operations of element collapse, element open, doublet insertion, and edge swap are extensible into three dimensions as the column collapse, column open, doublet column insertion, and face swap operations as shown in Figure 6.

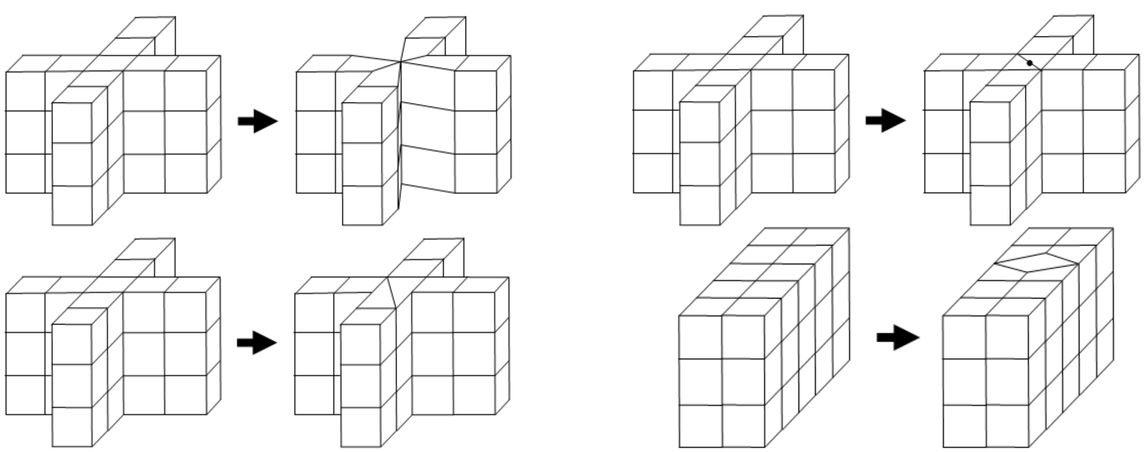

Figure 6. Hexahedral connectivity modification to adapt element sheets.

Figure 7 shows how the column collapse operation can be used to form a sheet that is confined to a specific region.

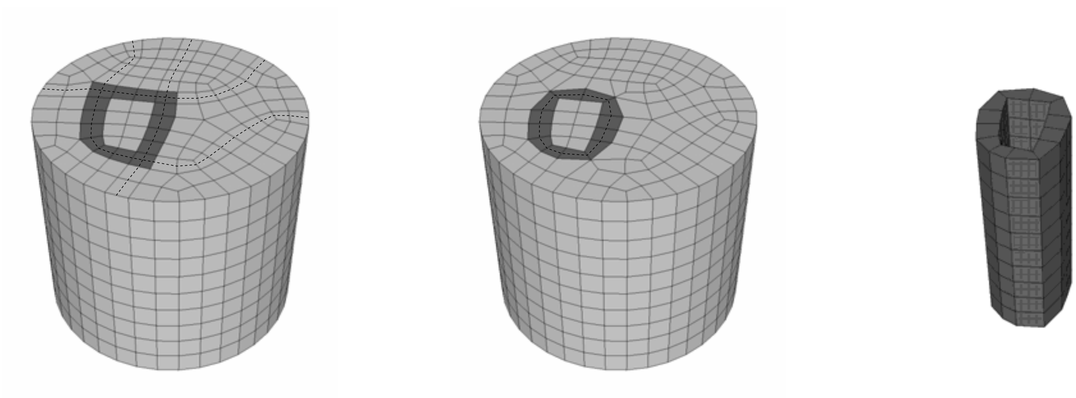

Figure 7. Column collapse operation to form a local sheet.

Just like sheets, columns often extend beyond the boundaries of a limited coarsening region. In many cases, collapsing a column of hexes would produce global rather than local changes. To eliminate this problem another step must be added to the coarsening process. Before any elements are removed, a sheet must be inserted around the boundary of the region using a technique known as pillowing [Mitchell, S.A., Tautges, T.J. (1995)]. Figure 8 shows a coarsening region and the sheet that is formed when the region is pillowed.

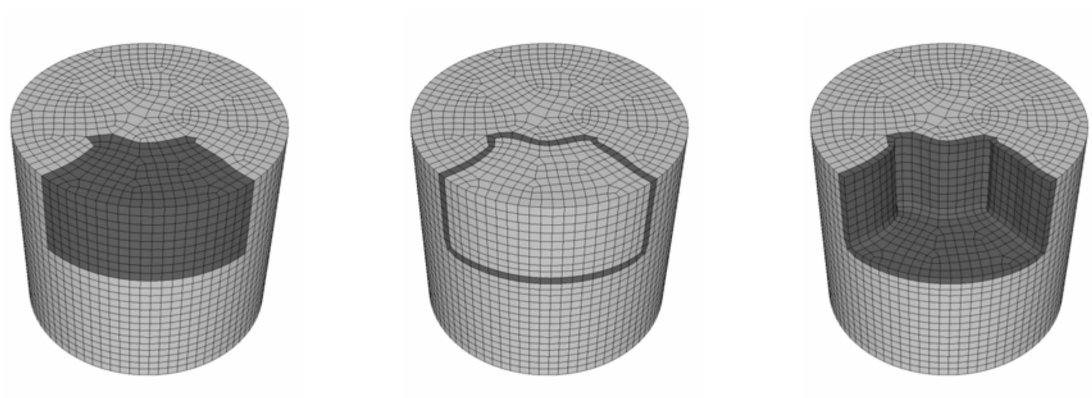

Figure 8. Pillowing a specified coarsening region.

This additional sheet provides columns that are confined to the coarsening region. By collapsing some of these columns, local sheets can be formed and then extracted as shown in Figure 9. Some elements from the pillow are left behind, but the overall number of elements in the region is reduced.

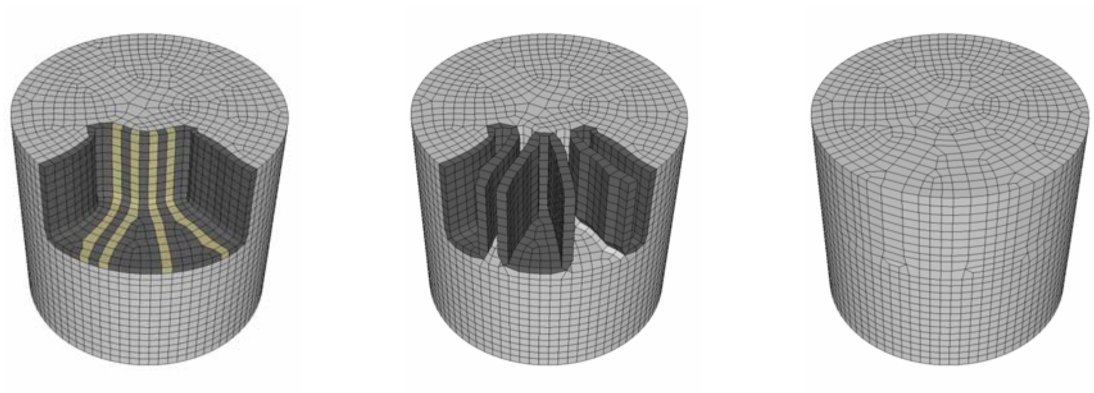

Figure 9. Local operations to produce local coarsening.

This process can be automated and multiple iterations can be performed to achieve a desired level of coarsening. Figure 10 shows a region that has undergone automated coarsening. Approximately sixty percent of the elements in the region were removed.

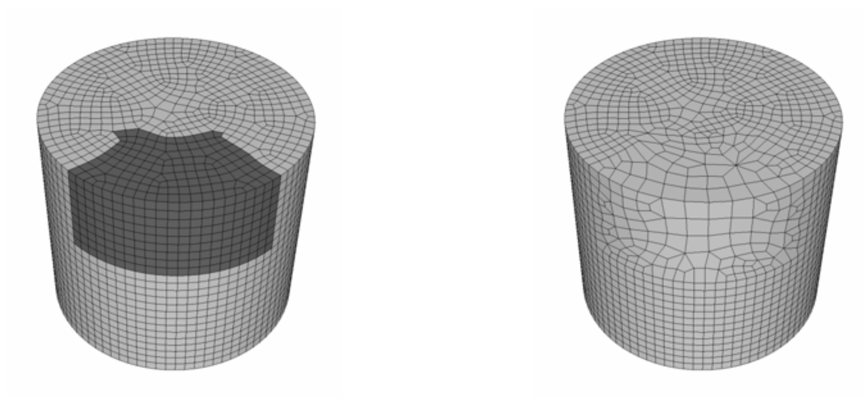

Figure 10. Automated coarsening of a selected region.

Conclusions

This paper has proposed an effective way to increase the functionality of conformal all-quadrilateral and all-hexahedral finite element adaptation schemes by introducing conformal coarsening. We note that inherent to coarsening is an accompanying loss of quality of the mesh. However, such loss of quality is generally acceptable because coarsened regions are likely portions of the model in which computational accuracy is less important.

Acknowledgements

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