

Presentation Start

SAND2008-2569C

Hydrogen Behavior – Myth Busting



Jay Keller,
Sandia National Laboratories

II International Forum: Hydrogen Technologies
for the Developing World

Moscow, Russia

April 21-33, 2008



Acknowledgements



The author wishes to recognize the following people for their contribution to the science discussed in this presentation.

Bénard, Pierre; Université du Québec à Trois-Rivières

Groethe, Mark; SRI International

Houf, Bill; Sandia National Laboratories

Moen, Chris; Sandia National Laboratories

Schefer, Bob; Sandia National Laboratories

Andrei V. Tchouvelev; Tchouvelev & Associates





Hydrogen Myths

- ⇒ Hindenburg
 - Hydrogen Caused the Disaster
- ⇒ Hydrogen Molecular Diffusivity is 3.8 times that of CH_4
 - Therefore it diffuses rapidly and mitigates any hazard
- ⇒ Hydrogen is 14.4 times lighter than air
 - Therefore it rapidly moves upward and out of the way
- ⇒ We do not know the flammability limits for H_2
- ⇒ Hydrogen heats upon expansion
 - This is the cause of auto-ignition (Joule-Thomson Effect)





Hydrogen Myths

- ⇒ We just do not understand hydrogen combustion behavior
 - Hydrogen release is different than other fuels
 - Radiation is different than other fuels
- ⇒ Hydrogen hazards can be compared favorably to experiences with other hydrocarbon fuels
 - Less dangerous than gasoline, methane ...
- ⇒ Hydrogen is toxic and will cause environmental harm
 - "... We need to be indemnified against a hazardous toxic hydrogen spill ..." – Generic Insurance Company





Hydrogen Myths

- ⇒ Hindenburg
 - Hydrogen Caused the Disaster
- ⇒ Hydrogen Molecular Diffusivity is 3.8 times that of CH_4
 - Therefore it diffuses rapidly and mitigates any hazard
- ⇒ Hydrogen is 14.4 times lighter than air
 - Therefore it rapidly moves upward and out of the way
- ⇒ We do not know the flammability limits for H_2
- ⇒ Hydrogen heats upon expansion
 - This is the cause of auto-ignition (Joule-Thomson Effect)

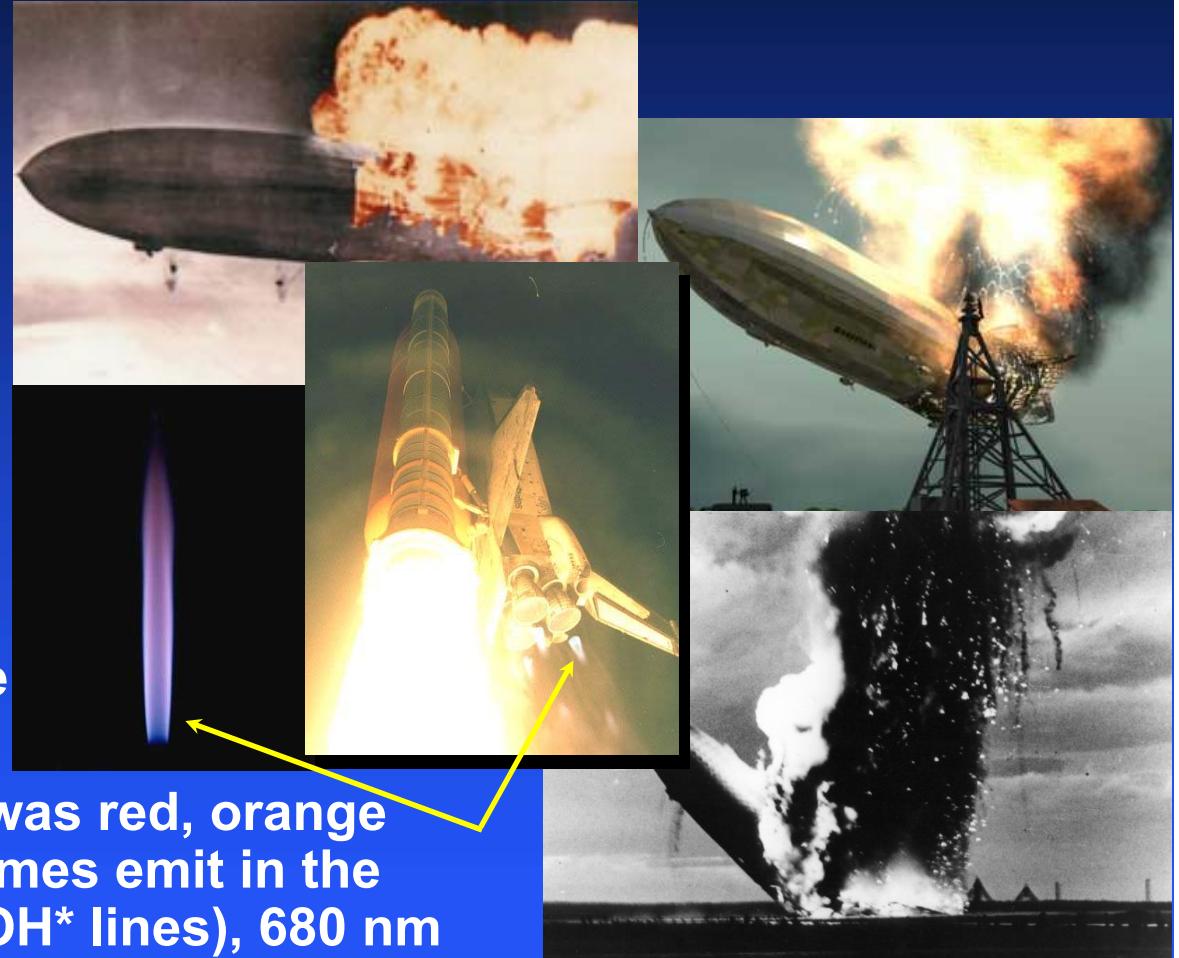


Lets get this out of the way!

Hindenburg Disaster



- ⇒ 36 out of 97 died mostly trapped by the fire of fabric, diesel fuel, chairs, tables ... (not hydrogen)
- ⇒ The craft did not explode but burned – and while burning stayed aloft (Hydrogen was still in the nose)
- ⇒ The craft fell to the ground tail first – the nose was still full of hydrogen
- ⇒ Radiation from the flame was red, orange and yellow – hydrogen flames emit in the near UV ~304 to 350 nm (OH* lines), 680 nm to 850 nm (vibrationally excited H₂O), and ~0.5 to 23 mm (water bands)



Lets get this out of the way! *Hindenburg Disaster (Cont'd)*



- ⇒ The covering was coated with cellulose nitrate or cellulose acetate -- both flammable materials. Furthermore, the cellulose material was impregnated with aluminum flakes to reflect sunlight. -- Dr. Addison Bain
- ⇒ A similar fire took place when an airship with an acetate-aluminum skin burned in Georgia -- it was full of helium!
- ⇒ “I guess the moral of the story is, don’t paint your airship with rocket fuel.”
-- Dr. Addison Bain



Courtesy of Dr. Addison Bain and the National Hydrogen Association





Hydrogen Myths

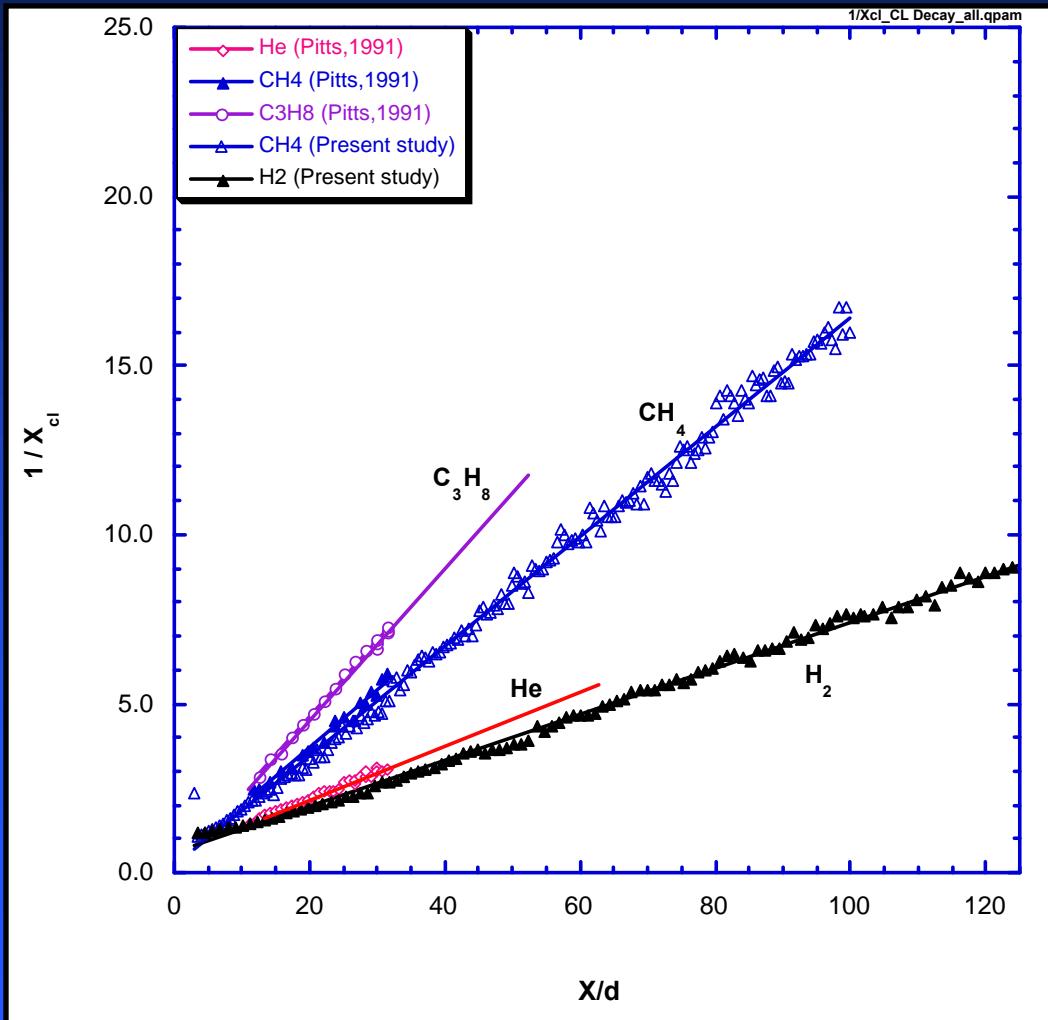
- ⇒ Hindenburg
 - Hydrogen Caused the Disaster
- ⇒ Hydrogen Molecular Diffusivity is 3.8 times that of CH_4
 - Therefore it diffuses rapidly and mitigates any hazard
- ⇒ Hydrogen is 14.4 times lighter than air
 - Therefore it rapidly moves upward and out of the way
- ⇒ We do not know the flammability limits for H_2
- ⇒ Hydrogen heats upon expansion
 - This is the cause of auto-ignition (Joule-Thomson Effect)



Small Unignited Releases: Momentum-Dominated Regime



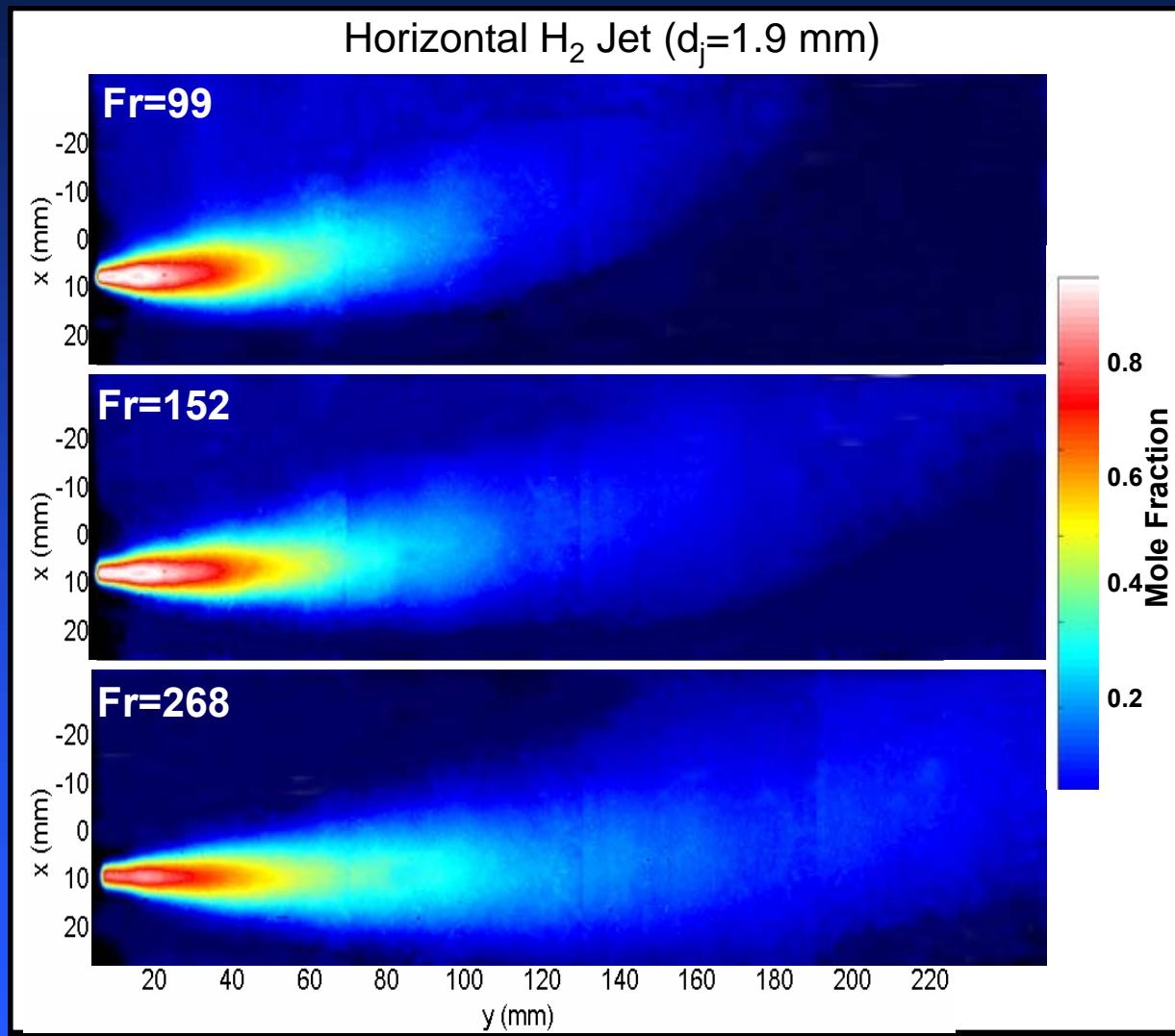
Data for round turbulent jets



- ⇒ In momentum-dominated regime, the centerline decay rate follows a $1/x$ dependence for all gases.
- ⇒ The centerline decay rate for mole fraction increases with increasing gas density.
- ⇒ The decay rate for H₂ is significantly slower than methane and propane.



Buoyancy effects are characterized by Froude number



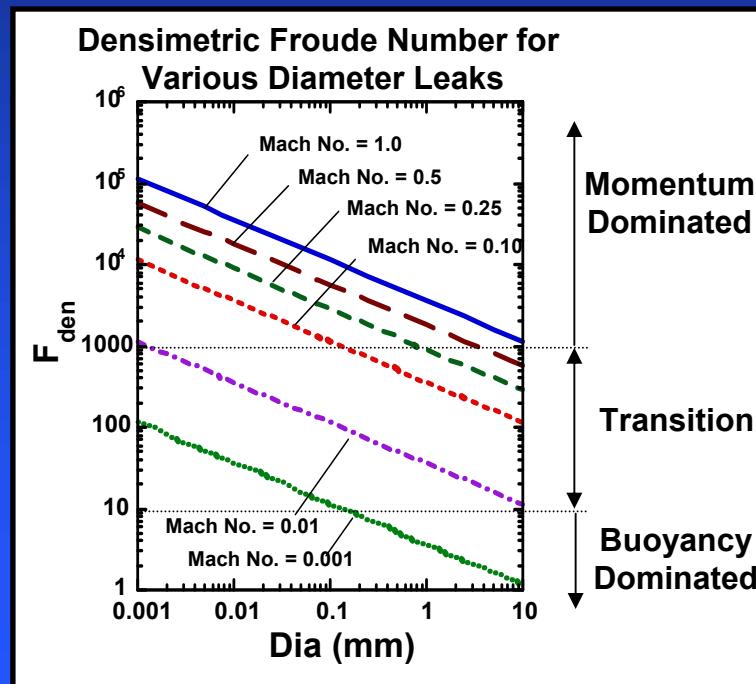
- ⇒ Time-averaged H₂ mole fraction distributions.
- ⇒ Froude number is a measure of strength of momentum force relative to the buoyant force
- ⇒ Increased upward jet curvature is due to increased importance of buoyancy at lower Froude numbers.



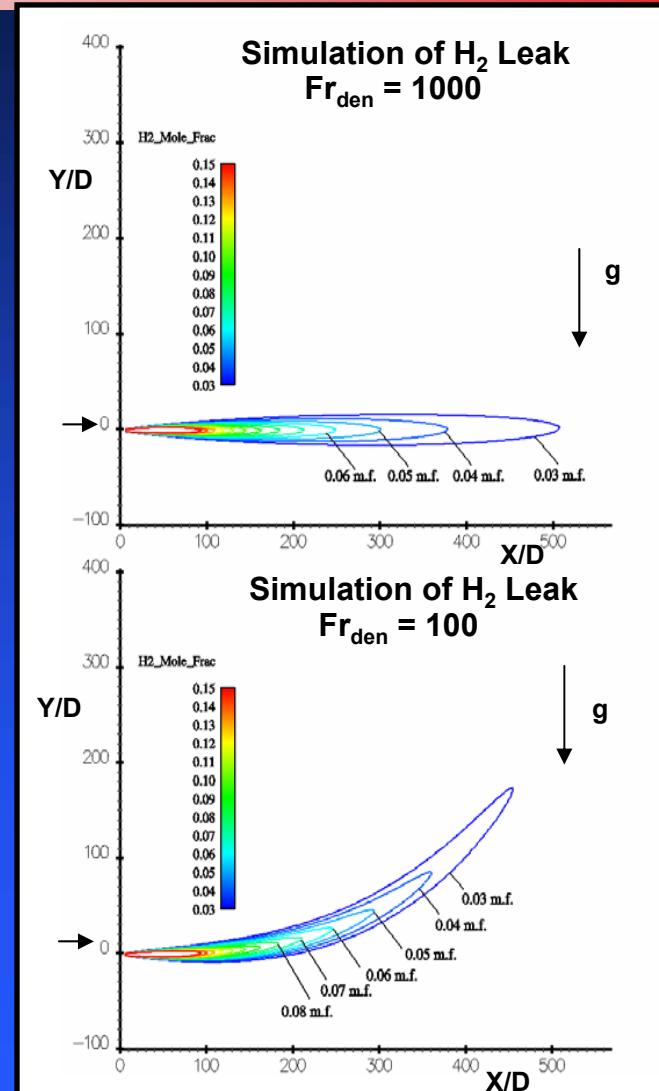
Influence of buoyant force is quantified by the dimensionless Froude number



- ⇒ Jets from choked flows (Mach 1.0) are typically momentum-dominated.
- ⇒ Lower source pressures or very large pressure losses through cracks lead to subsonic, buoyancy-dominated plumes.



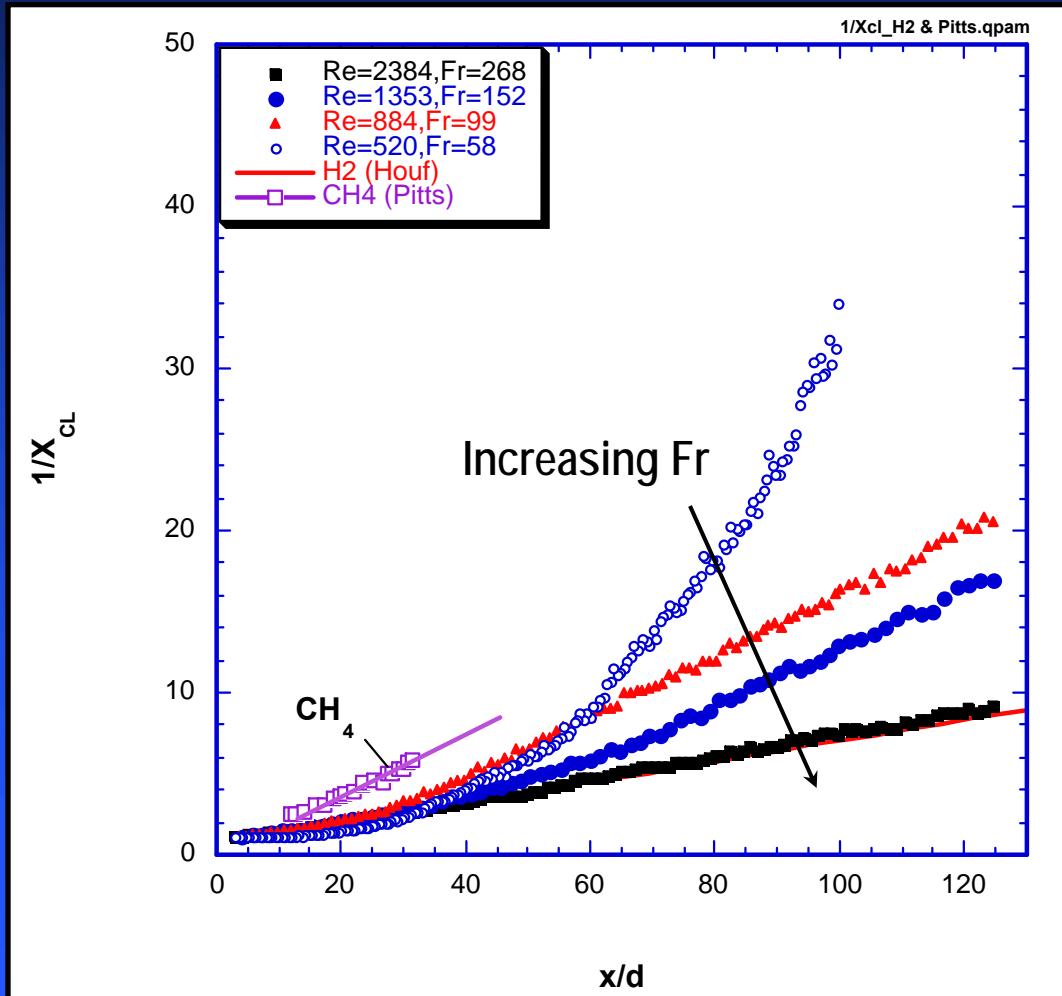
$$Fr_{den} = U_{exit} / (gD(\rho_{amb} - \rho_{exit})/\rho_{exit})^{1/2}$$





Small Unignited Releases: Buoyancy Effects

⇒ Data for round H_2 Jets ($d_j=1.91$ mm)



- ⇒ At the highest Fr , $1/X_{CL}$ increases linearly with axial distance, indicating momentum dominates.
- ⇒ As Fr is reduced buoyancy forces become increasingly important and the centerline decay rate increases.
- ⇒ The transition to buoyancy-dominated regime moves upstream with decreasing Fr .





Hydrogen Myths

- ⇒ Hindenburg
 - Hydrogen Caused the Disaster
- ⇒ Hydrogen Molecular Diffusivity is 3.8 times that of CH_4
 - Therefore it diffuses rapidly and mitigates any hazard
- ⇒ Hydrogen is 14.4 times lighter than air
 - Therefore it rapidly moves upward and out of the way
- ⇒ We do not know the flammability limits for H_2 Hydrogen heats upon expansion
 - This is the cause of auto-ignition (Joule-Thomson Effect)



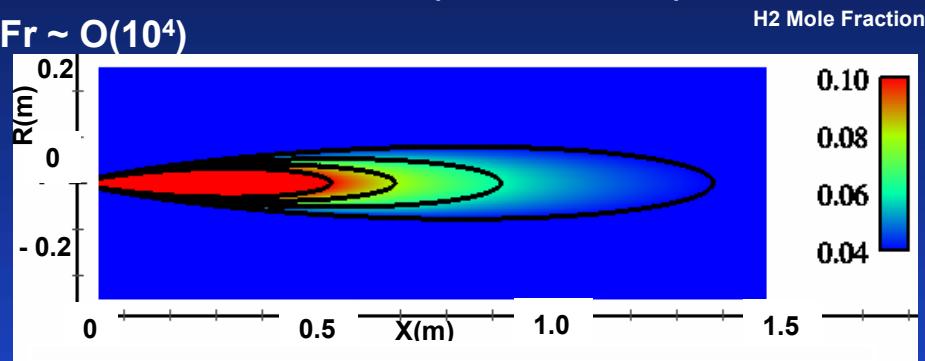
Choked & Unchoked Flows at 20 SCFM



Tank Pressure = 3000 psig, Hole Dia. = 0.297 mm

Exit Mach Number = 1.0 (Choked Flow)

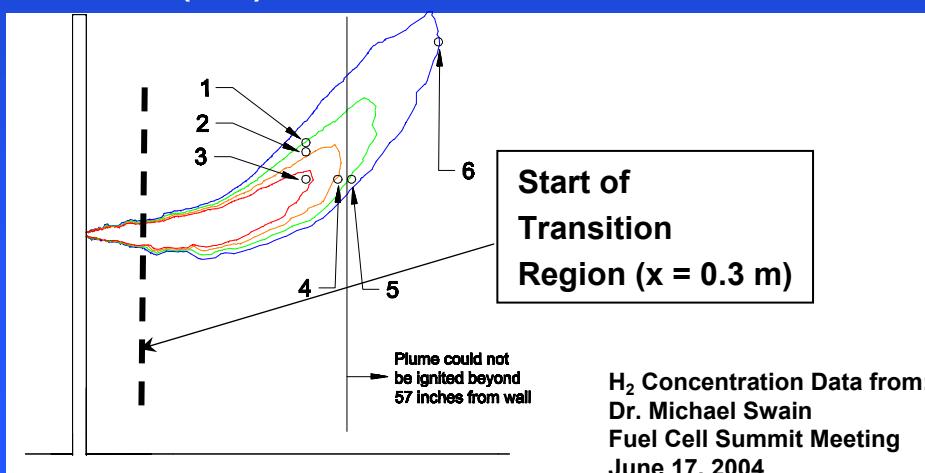
$Fr \sim O(10^4)$



Flowrate = 20 scfm, Hole Dia. = 9.44 mm

Exit Mach Number = 0.1 (Unchoked Flow)

$Fr \sim O(100)$



- ⇒ Correlations based on experimental data
- ⇒ Start Intermediate Region
 - $x/D = 0.5 F^{1/2}(\rho_{exit}/\rho_{amb})^{1/4}$
- ⇒ End Intermediate Region
 - $x/D = 5.0 F^{1/2}(\rho_{exit}/\rho_{amb})^{1/4}$
- ⇒ F = Exit Froude No.

$$= U_{exit}^2 \rho_{exit} / (g D (\rho_{amb} - \rho_{exit}))$$

Start Transition Region -> $x = 6.3$ m

- ⇒ Assuming gases at 1 Atm, 294K (NTP)
 - Red – 10.4%
 - Orange – 8.5%
 - Green – 5.1%
 - Blue – 2.6%

*(Chen and Rodi, 1980)





Hydrogen Myths

- ⇒ Hindenburg
 - Hydrogen Caused the Disaster
- ⇒ Hydrogen Molecular Diffusivity is 3.8 times that of CH_4
 - Therefore it diffuses rapidly and mitigates any hazard
- ⇒ Hydrogen is 14.4 times lighter than air
 - Therefore it rapidly moves upward and out of the way
- ⇒ We do not know the flammability limits for H_2
- ⇒ Hydrogen heats upon expansion
 - This is the cause of auto-ignition (Joule-Thomson Effect)



Flammability Limits for H_2



Upward Flame Propagation

Tube Dimensions, cm		Firing end	Limits, percent		Water Vapor Content	Reference
Diameter	Length		Lower	Higher		
7.5	150	Closed	4.15	75.0	Half-saturated	356
5.3	150					
5.3	150					
5.3	150					
5.0	150					
5.0	150					
4.8	150					
4.5	80					
4.5	80					

Horizontal Flame Propagation

Tube Dimensions, cm		Firing end	Limits, percent		Water Vapor Content	Reference
Diameter	Length		Lower	Higher		
7.5	150	Closed	6.5	----	Half-saturated	356
5.0	150		N			
2.5	150		N			

Propagation in a Spherical Ves

Capacity, cc	Firing end	Limits, percent		Water Vap Content
		Lower	Higher	
Not stated	Closed	9.2	----	Saturated
Not stated	N	8.5	67.5	N
1,000	N	8.7	75.5	N
810	N	5.0	73.5	N
350	N	4.6	70.3	N
35	N	9.4	64.8	N

Downward Flame Propagation

Tube Dimensions, cm		Firing end	Limits, percent	
Diameter	Length		Lower	Higher
21.0	31	Open	9.3	----
8.0	37	Closed	8.9	68.8
7.5	150	N	8.8	74.5
7.0	150	N	----	74.5
6.2	33	Open	8.5	----
6.0	120	N	9.45	----

N

325

National Laboratories



Flammability Limits for H_2



Upward Flame Propagation

Tube Dimensions, cm		Firing end	Limits, percent		Water Vapor Content	Reference
Diameter	Length		Lower	Higher		
7.5	150	Closed	4.15	75.0	Half-saturated	356

- ⇒ 78 investigations of hydrogen flammability limits were identified between 1920 and 1950.
- ⇒ Hydrogen flammability limits are well established.

Tube Dimensions, cm		Firing end	Limits, percent	
Diameter	Length		Lower	Higher
21.0	31	Open	9.3	----
8.0	37	Closed	8.9	68.8
7.5	150	N	8.8	74.5
7.0	150	N	----	74.5
6.2	33	Open	8.5	----
6.0	120	N	9.45	----

Tube Dimensions, cm	Firing end	Lower	Higher	Content
				Saturated
Not stated	Closed	9.2	----	
Not stated	N	8.5	67.5	N
1,000	N	8.7	75.5	N
810	N	5.0	73.5	N
350	N	4.6	70.3	N
35	N	9.4	64.8	N

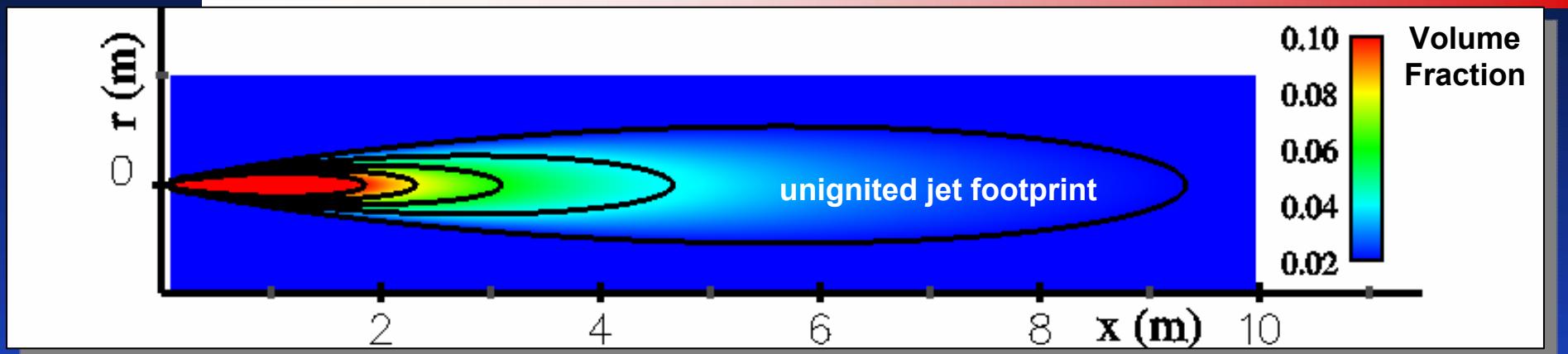
N

325

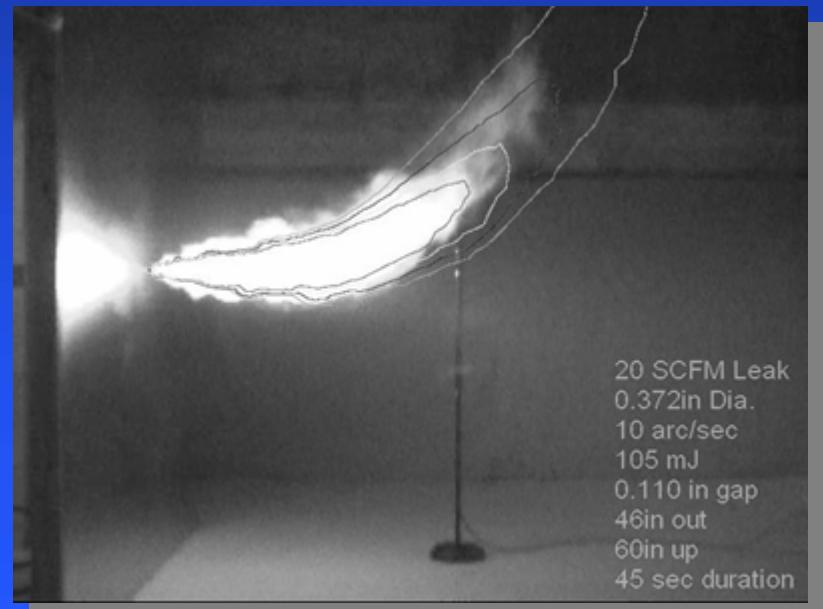
National Laboratories



What is a Reasonable Flame Stabilization Limit?



- ⇒ Which volume fraction contour is relevant:
 - lean flammability limit? ... 4% or 8%
 - detonation limit? ... 18%
 - a fraction of the lowest lean flammability limit?
... 1%
- ⇒ ***Ignition of hydrogen in turbulent jets occurs around 8% as measured by Swain.***
 - This is consistent with the downward propagating limit of 8%





Hydrogen Myths

- ⇒ Hindenburg
 - Hydrogen Caused the Disaster
- ⇒ Hydrogen Molecular Diffusivity is 3.8 times that of CH_4
 - Therefore it diffuses rapidly and mitigates any hazard
- ⇒ Hydrogen is 14.4 times lighter than air
 - Therefore it rapidly moves upward and out of the way
- ⇒ We do not know the flammability limits for H_2
- ⇒ Hydrogen heats upon expansion
 - This is the cause of auto-ignition (Joule-Thomson Effect)



Joule-Thomson Effect



High-pressure H₂ Jet



- ⇒ A rapidly expanding gas can increase or decrease in temperature.
- ⇒ The direction and magnitude of temperature change is determined by the Joule-Thomson coefficient.
- ⇒ Definition:
$$\mu_{JT} = (\delta T / \delta P)_H = (\Delta T / \Delta P)_H$$
- ⇒ Above the inversion temperature, the expanding gas temperature rises.
- ⇒ The inversion temperature of H₂ is between 28 and 200 K (depending on pressure); at ambient temperature the expanding H₂ increases in temperature.

- ⇒ For initial compressed gas pressure of 14 MPa, the estimated temperature rise is approximately 6 C.
- ⇒ At pressures up to 250 MPa, the maximum estimated coefficient is 0.53 K/MPa. Thus, at H₂ storage pressures of 100 MPa, the maximum temperature rise would be 53 C, (gas temperature is only ~75C).

Given the H₂ auto-ignition temperature of 585 C, Joule-Thomson heating is insufficient to cause ignition



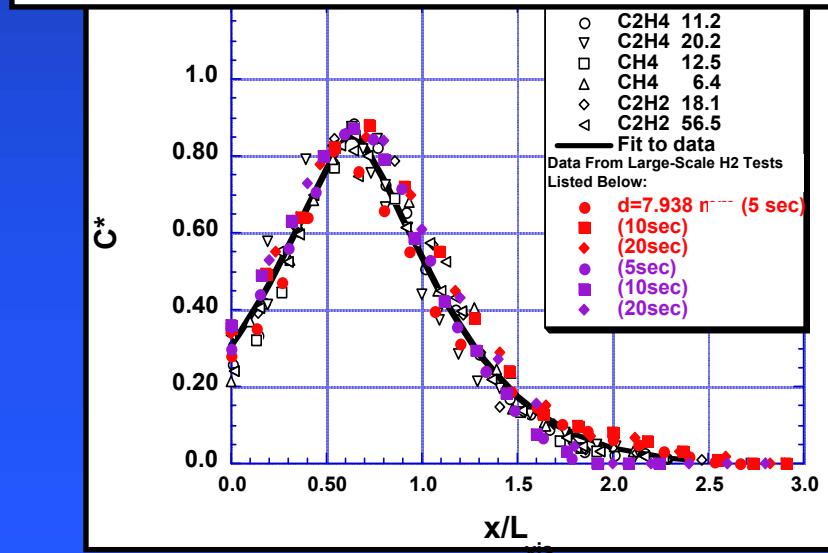
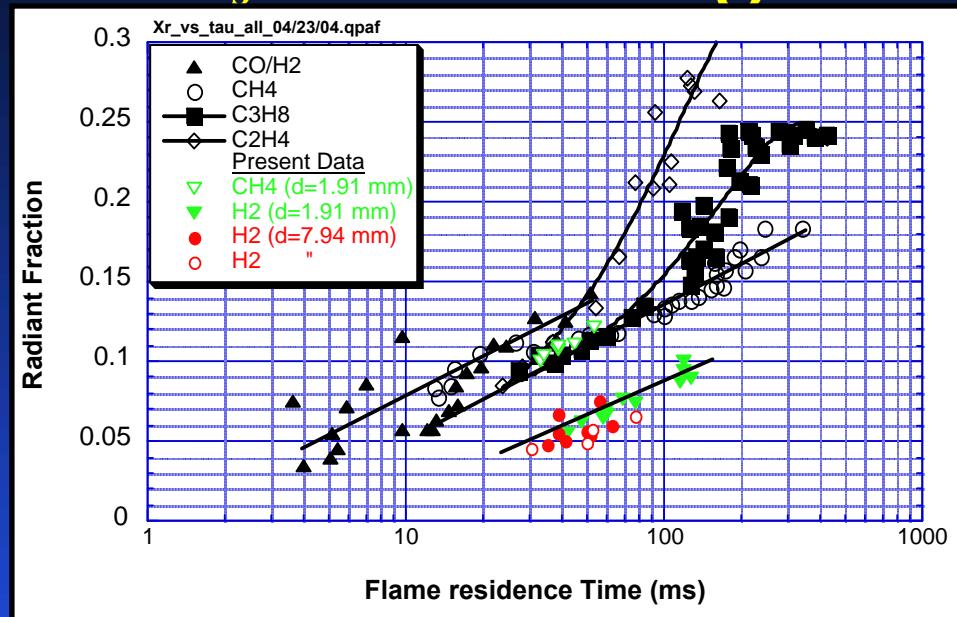


Hydrogen Myths

- ⇒ We just do not understand hydrogen combustion behavior
 - Hydrogen release is different than other fuels
 - Radiation is different than other fuels
- ⇒ “Hydrogen hazards can be compared favorably to experiences with other hydrocarbon fuels
 - Less dangerous than gasoline, methane ...
- ⇒ Hydrogen is toxic and will cause environmental harm
 - “... We need to be indemnified against a hazardous toxic hydrogen spill ...” – Generic Insurance Company

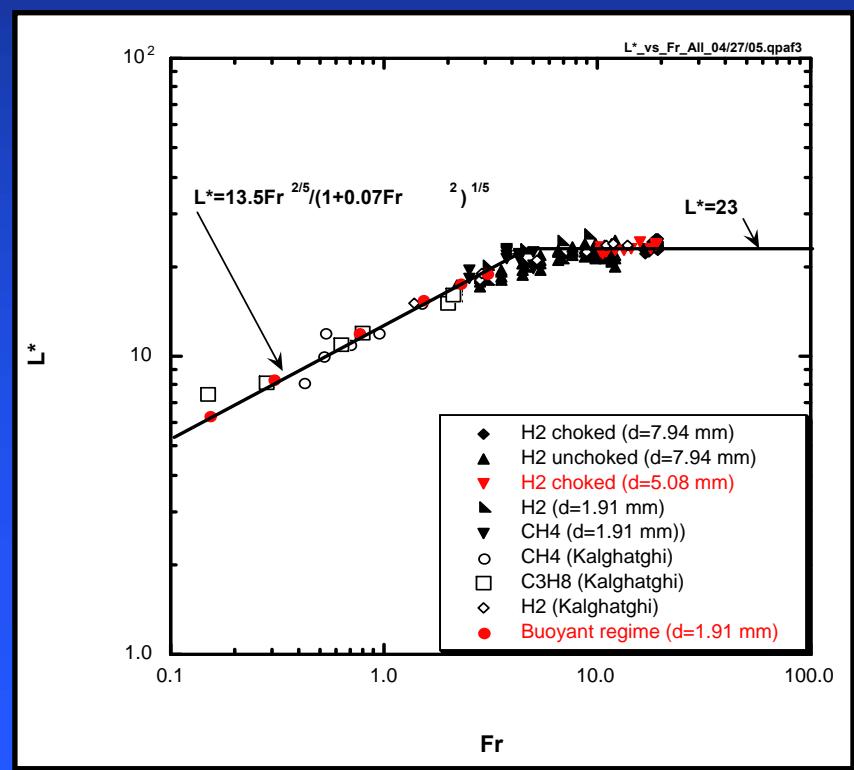


Hydrogen jets and flames are similar to other flammable gases



9/11/07

- ⇒ Fraction of chemical energy
- ⇒ Converted to thermal radiation
- ⇒ Radiation heat flux distribution
- ⇒ Jet length

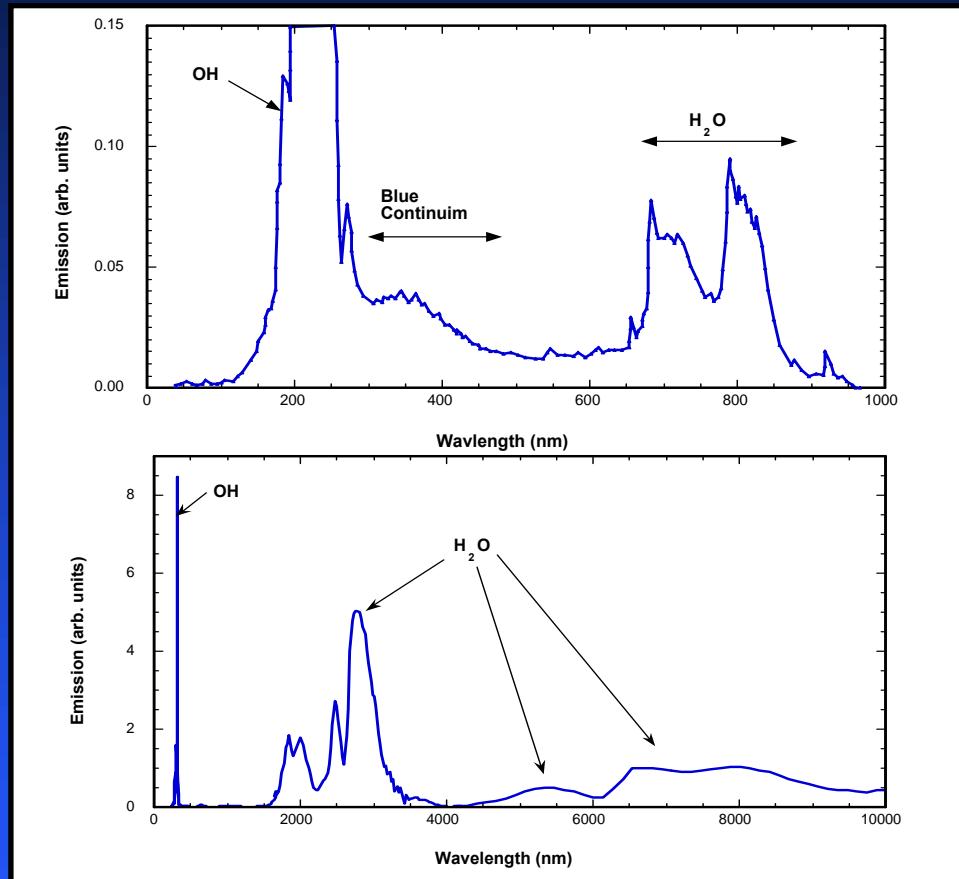


23

Sandia National Laboratories



H₂ Flame Radiation

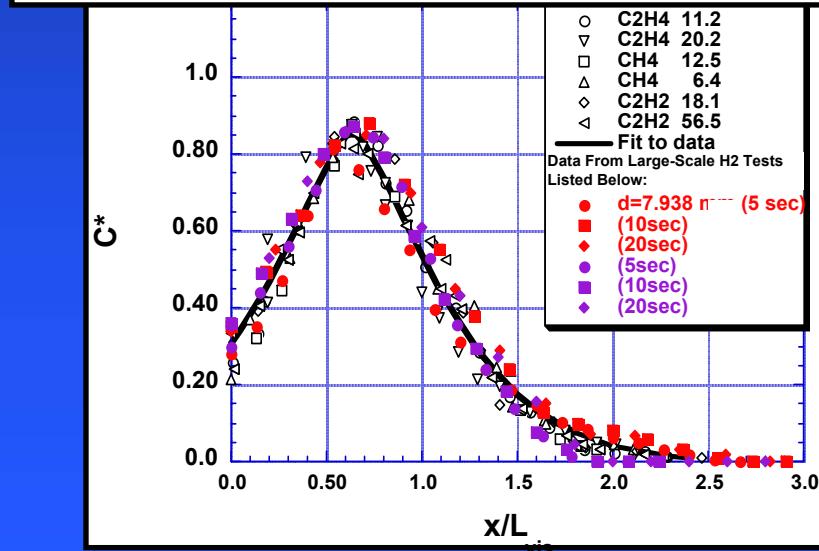
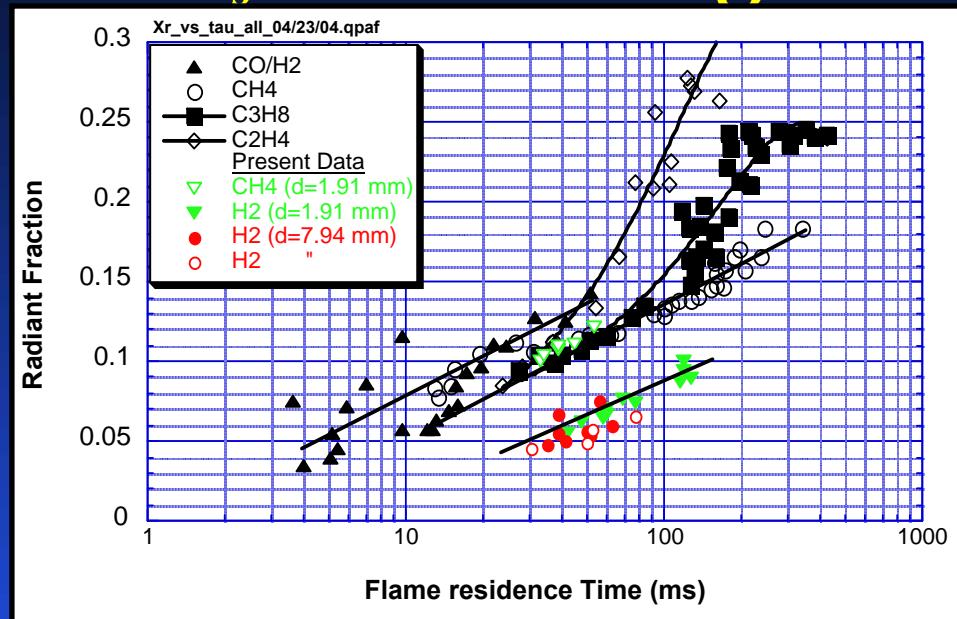


- ⇒ Orange emission due to excited H₂O vapor
- ⇒ Blue continuum due to emission from OH + H => H₂O + h_v
- ⇒ UV emission due to OH*
- ⇒ IR emission due to H₂O vibration-rotation bands

H₂O emission in IR accounts for 99.6% of flame radiation

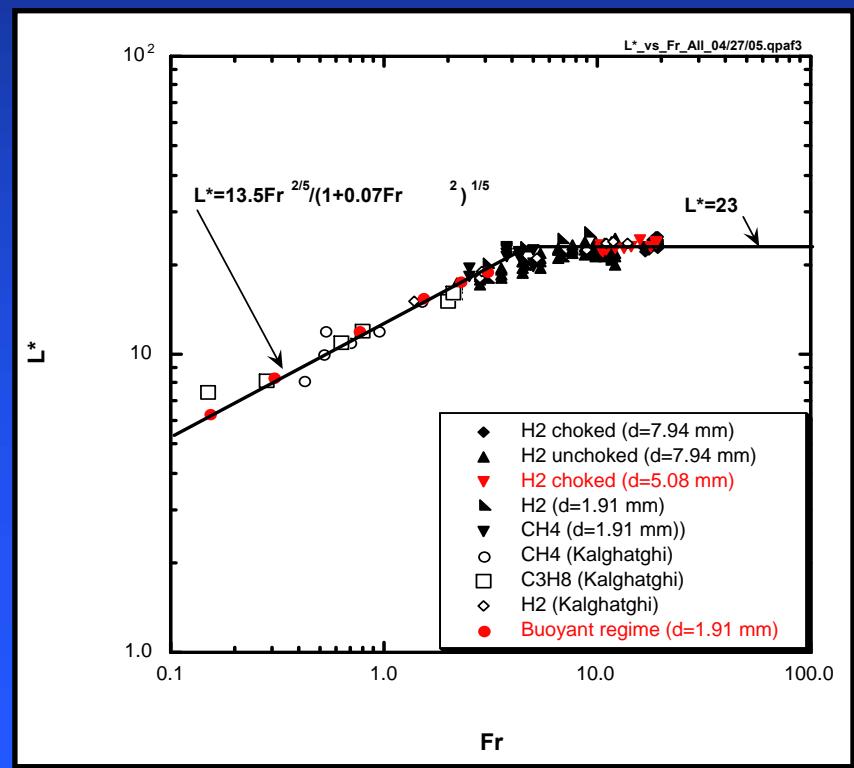


Hydrogen jets and flames are similar to other flammable gases



9/11/07

- ⇒ Fraction of chemical energy
- ⇒ Converted to thermal radiation
- ⇒ Radiation heat flux distribution
- ⇒ Jet length

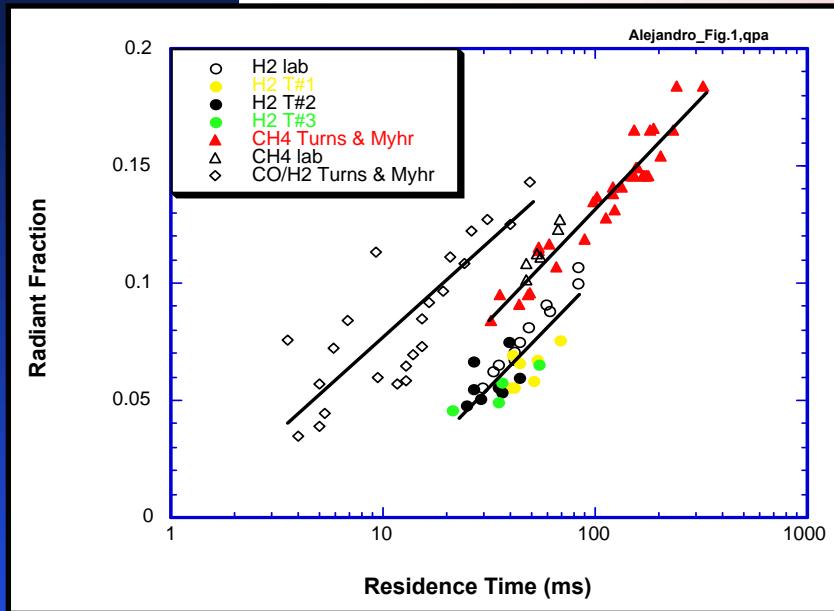


25

Sandia National Laboratories

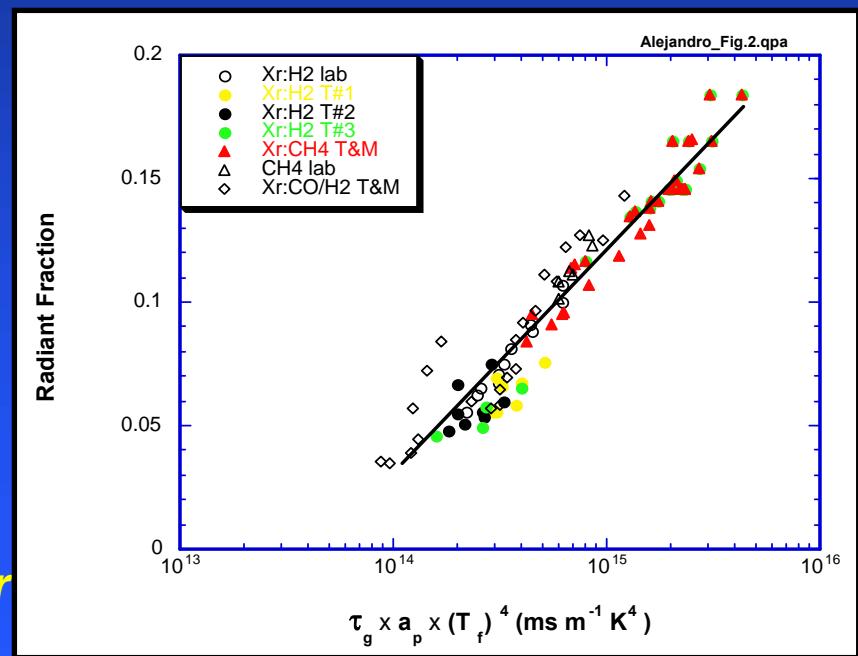


Thermal Radiation from Hydrogen Flames



- ⇒ Radiation heat flux data collapses on single line when plotted against product $\tau_g \times a_p \times T_f^4$.
- ⇒ a_p (absorption coefficient) is factor with most significant impact on data normalization
- ⇒ *Plank mean absorption coefficient for different gases must be considered*

- ⇒ Previous radiation data for *nonsooting* CO/H₂ and CH₄ flames correlate well with flame residence time.
- ⇒ Sandia's H₂ flame data is a factor of two lower than the hydrocarbon flame data.



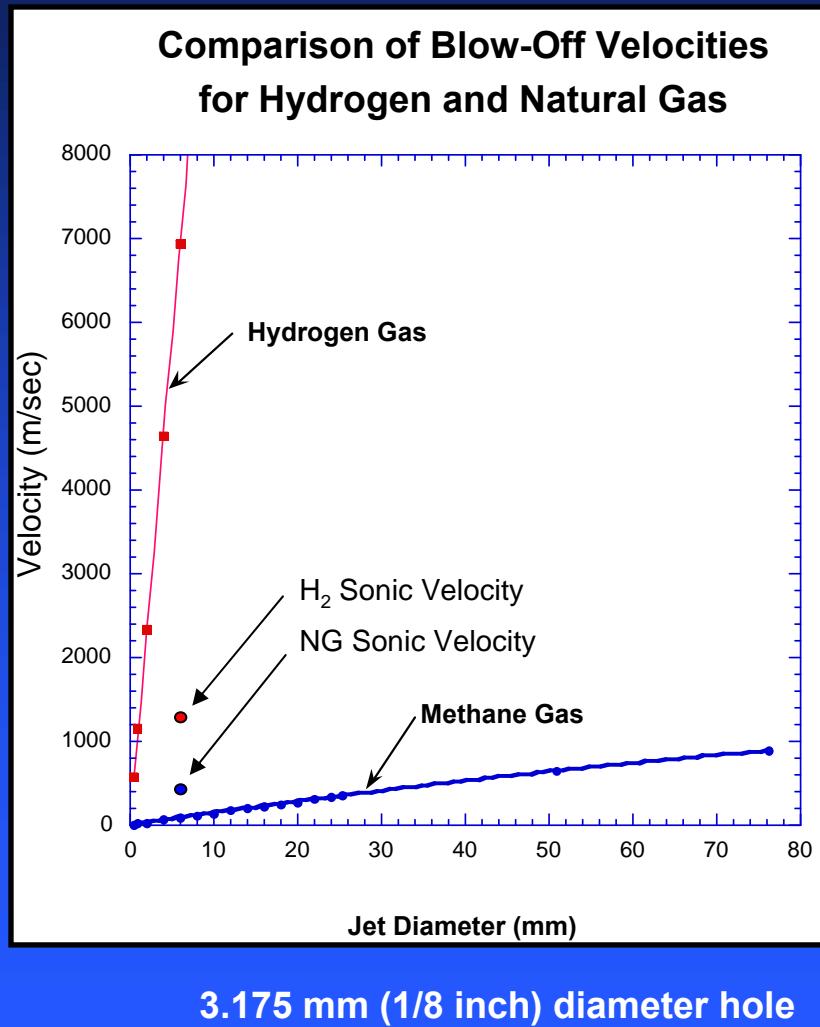


Hydrogen Myths

- ⇒ We just do not understand hydrogen combustion behavior
 - Hydrogen release is different than other fuels
 - Radiation is different than other fuels
- ⇒ “Hydrogen hazards can be compared favorably to experiences with other hydrocarbon fuels”
 - Less dangerous than gasoline, methane ...
- ⇒ Hydrogen is toxic and will cause environmental harm
 - “... We need to be indemnified against a hazardous toxic hydrogen spill ...” – Generic Insurance Company



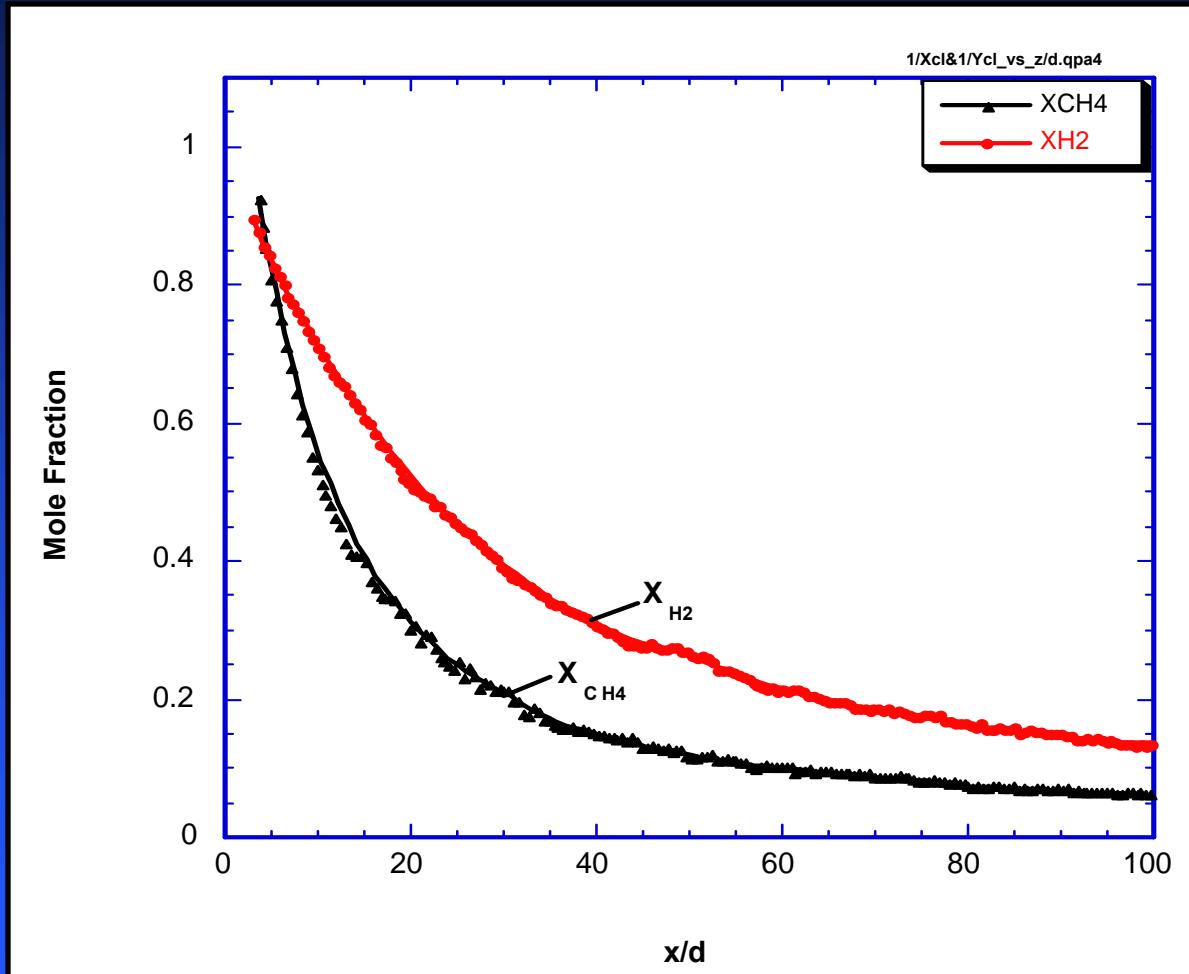
Comparisons of NG and H₂ Behaviors



- ⇒ Assume 3.175 mm (1/8 inch) dia. hole
- ⇒ Unignited jet lower flammability limits
 - LFL H₂ - 4% mole fraction
 - LFL NG - 5% mole fraction
- ⇒ Flame blow-off velocities for H₂ are much greater than NG
- ⇒ Flow through 1/8" diameter hole is choked
 - $V_{sonic} = 450$ m/sec for NG (300K)
 - $V_{sonic} = 1320$ m/sec for H₂ (300K)
- ⇒ Hole exit (sonic) velocity for NG is greater than NG blow-off velocity
 - No NG jet flame for 1/8" hole
- ⇒ Hole exit (sonic) velocity for H₂ is much less than blow-off velocity for H₂
 - H₂ jet flame present for 1/8" hole



Small Unignited Releases: Momentum-Dominated Regime



⇒ Decay rate for H_2 mole fraction is slower than CH_4 .



Unignited jet concentration decay distances for natural gas and hydrogen.



Distance on Jet Centerline to Lower Flammability Limit for Natural Gas and Hydrogen

Tank Pressure	Hole Diameter	Distance to 5% Mole Fraction Natural Gas	Distance to 4% Mole Fraction Hydrogen
18.25 bar (250 psig)	3.175 mm (1/8 inch)	1.19 m (3.90 ft)	4.24 m (13.91 ft)
	1.587 mm (1/16 inch)	0.59 m (1.93 ft)	2.12 m (6.95 ft)
207.8 bar (3000 psig)	3.175 mm (1/8 inch)	3.92 m (12.86 ft)	13.54 m (44.42 ft)
	1.587 mm (1/16 inch)	1.96 m (6.43 ft)	6.77 m (22.21 ft)

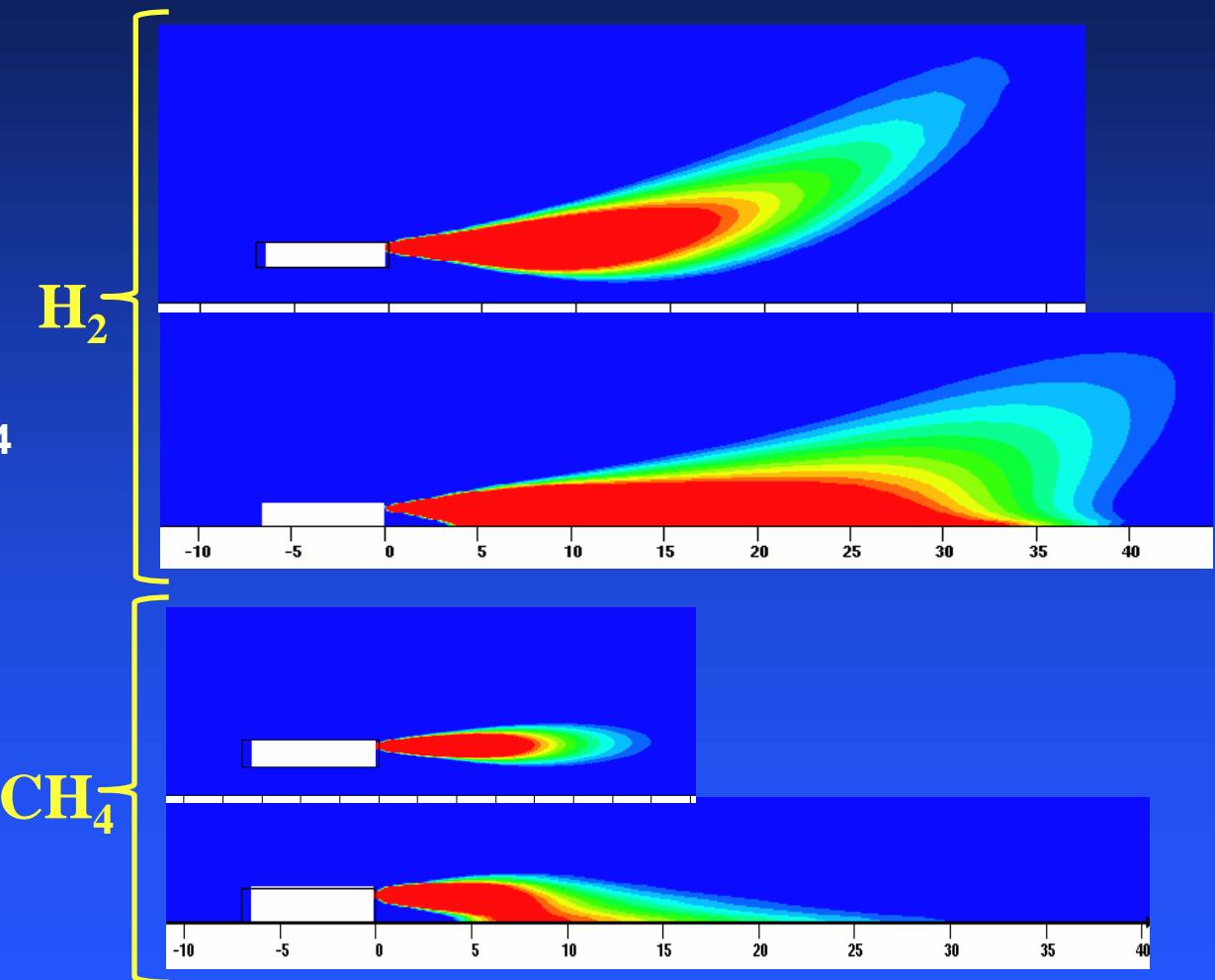
Distance to the lower flammability limit for hydrogen is about 3 times longer than for natural gas





Effects of surfaces ?

- ⇒ While both flammable envelopes lengths are increased, the increase is more pronounced for CH_4 jets than H_2 jets
- ⇒ “Transient puffs” seems to lead to a larger temporary increase of extent of horizontal H_2 surface jets

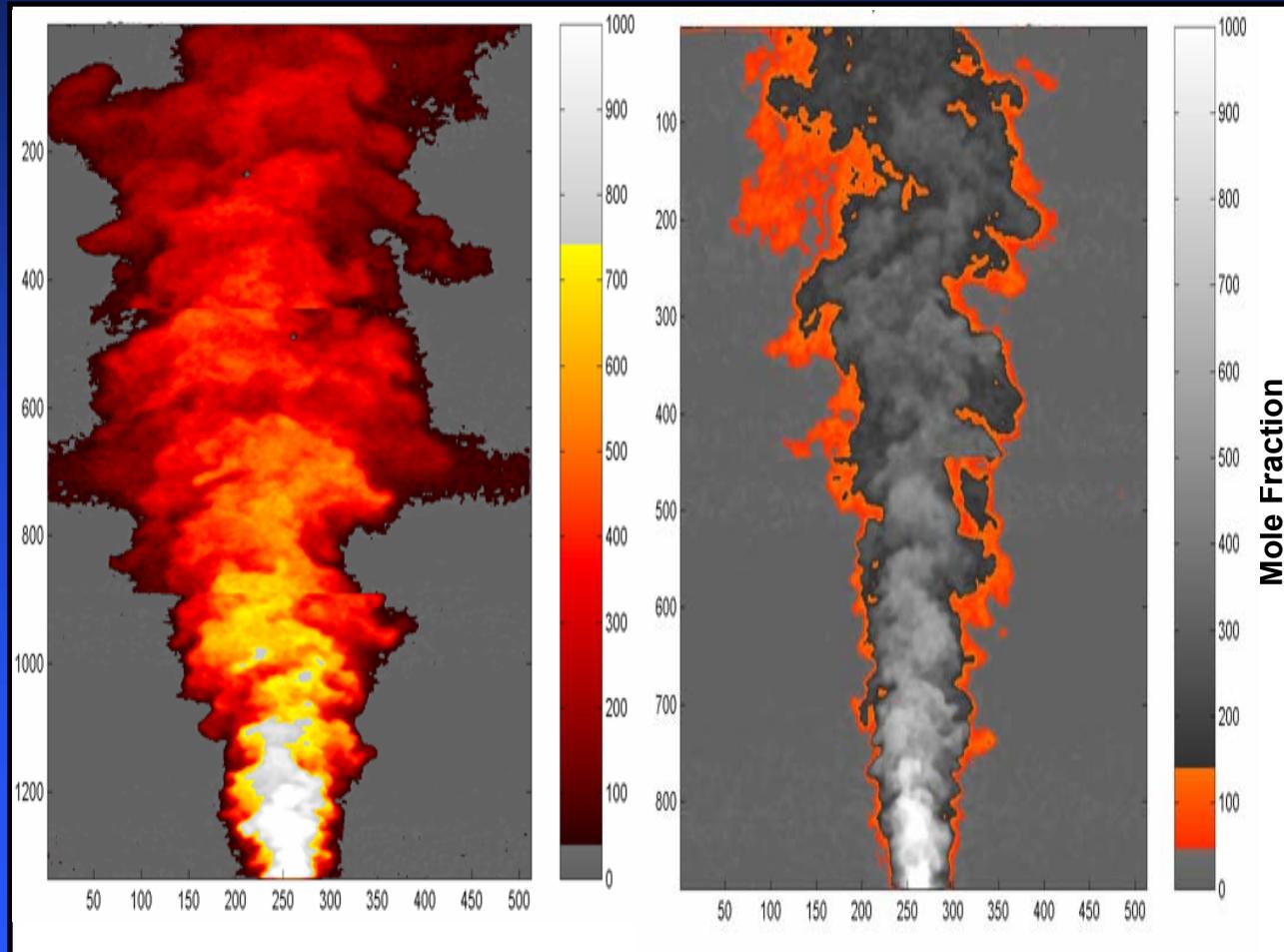


Small Unignited Releases: Ignitable Gas Envelope



H_2 Jet at $\text{Re}=2,384$; $\text{Fr} = 268$

CH_4 Jet at $\text{Re}=6,813$; $\text{Fr} = 478$



⇒ H_2 flammability limits: LFL 4.0%; RFR 75%

⇒ CH_4 flammability limits: LFL 5.2%; RFR 15%

Radial profiles in H_2 jet, $d = 1.91$ mm, $\text{Re} = 2384$

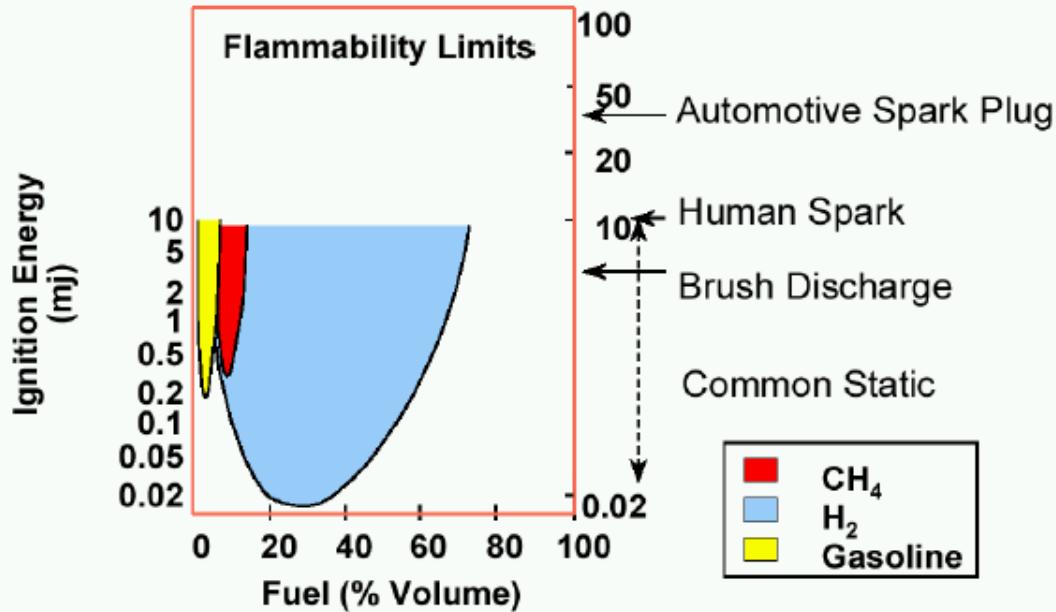


Is there a myth about the minimum ignition energy?



- Lower ignition energy of H_2 is the lowest of the flammable gases at stoichiometry
 - Over the flammable range of CH_4 (~below 10%), however, H_2 has a comparable ignition energy.

Ignition Energy of H_2 , CH_4 and gasoline with Air



© Air Products & Chemicals, Inc., 2001

AIR
PRODUCTS

Figure 1: Flammability Limits vs. Ignition Energy of H_2 , CH_4 , and Gasoline in Air





Hydrogen Myths

- ⇒ We just do not understand hydrogen combustion behavior
 - Hydrogen release is different than other fuels
 - Radiation is different than other fuels
- ⇒ “Hydrogen hazards can be compared favorably to experiences with other hydrocarbon fuels
 - Less dangerous than gasoline, methane ...
- ⇒ Hydrogen is toxic and will cause environmental harm
 - “... We need to be indemnified against a hazardous toxic hydrogen spill ...” – Generic Insurance Company



Some people just do not get it!

⇒ H_2

- is not toxic,
- it is environmentally benign
- We just borrow it -- ($2H_2O + E \rightarrow 2H_2 + O_2$; then $2H_2 + O_2 \rightarrow 2H_2O + E$)

⇒ H_2 is a fuel and as such has stored chemical energy

- It has hazards associated with it
 - It is no more dangerous than the other fuels that store chemical energy
 - IT IS JUST different; -- ***WE UNDERSTAND THE SCIENCE***

We will learn how to safely handle H_2 in the commercial setting just as we have for our hydrocarbon fuels.



Publication list



Nighttime photograph of ~40 MPa large-scale H₂ jet-flame test (d_j = 5.08mm, L_{vis} = 10.6 m) from Sandia/SRI tests.

- (1) Houf and Schefer, "Predicting Radiative Heat Fluxes and Flammability Envelopes from Unintended Releases of Hydrogen," accepted for publication Int. Jour. of Hydrogen Energy, Feb. 2006.
- (2) Schefer, Houf, San Marchi, Chernicoff, and Englom, "Characterization of Leaks from Compressed Hydrogen Dispensing Systems and Related Components," Int. Jour. of Hydrogen Energy, Vol. 31, Aug. 2006.
- (3) Molina, Schefer, and Houf, "Radiative Fraction and Optical Thickness in Large-Scale Hydrogen Jet Flames," Proceedings of the Combustion Institute, April, 2006.
- (4) Houf and Schefer, "Rad. Heat Flux & Flam. Env. Pred. from Unintended Rel. of H₂," Proc. 13th Int. Heat Tran. Conf., Aug., 2006.
- (5) Schefer, Houf, Williams, Bourne, and Colton, "Characterization of High-Pressure, Under-Expanded Hydrogen-Jet Flames," submitted to Int. Jour. of Hydrogen Energy, 2006.
- (6) Houf and Schefer, "Predicting Radiative Heat Fluxes and Flammability Envelopes from Unintended Releases of Hydrogen," 16th NHA Meeting, Washington, DC, March 2005.
- (6) Schefer, R. W., Houf, W. G., Bourne, B. and Colton, J., "Turbulent Hydrogen-Jet Flame Characterization", Int. Jour. of Hydrogen Energy, 2005.
- (7) Schefer, R. W., Houf, W. G., Bourne, B. and Colton, J., "Experimental Measurements to Characterize the Thermal and Radiation Properties of an Open-flame Hydrogen Plume", 15th NHA Meeting, April 26-30, 2004, Los Angeles, CA.
- (8) Schefer R. W., "Combustion Basics," in National Fire Protection Association (NFPA) Guide to Gas Safety, 2004.
- (9) P. Bénard (2007), Chapter 3 – Hydrogen Release and Dispersion - Release of hydrogen - section a.1, , Biennial Report on Hydrogen Safety, HySafe.
- (10) B. Angers, A. Hourri, P. Bénard, P. Tessier and J. Perrin (2005), "Simulations of Hydrogen Releases from a Storage Tank: Dispersion and Consequences of Ignition". International Conference on Safety 2005, Sept 8-10, 2005, Pisa, Italy.
- (11) A.V. Tchouvelev, P. Bénard, V. Agranat and Z. Cheng (2005), "Determination of Clearance Distances for Venting of Hydrogen Storage". International Conference on Safety 2005, Sept 8-10, 2005, Pisa, Italy (NRCAN, AUTO 21).
- (12) Tchouvelev A., P. Bénard, D. R. Hay, V. Mustafa, A. Hourri, Z. Cheng, Matthew P. Large, Quantitative Risk Comparison of Hydrogen and CNG Refuelling Options, Final Technical Report to Natural Resources Canada for the Codes and Standards Workshop of the CTFCA, August 2006 (194 pages).
- (13) Bénard, P., Tchouvelev, A., Hourri, A., Chen, Z., Angers, B. High Pressure Hydrogen Jets in a Presence of a Surface. Proceedings of International Conference on Hydrogen Safety, San Sebastian, Spain, September 2007.
- (14) Tchouvelev, A.V., Howard, G.W. and Agranat, V.M. Comparison of Standards Requirements with CFD Simulations for Determining of Sizes of Hazardous Locations in Hydrogen Energy Station. Proceedings of the 15th World Hydrogen Energy Conference, Yokohama, June 2004.



Presentation End



Toss-up

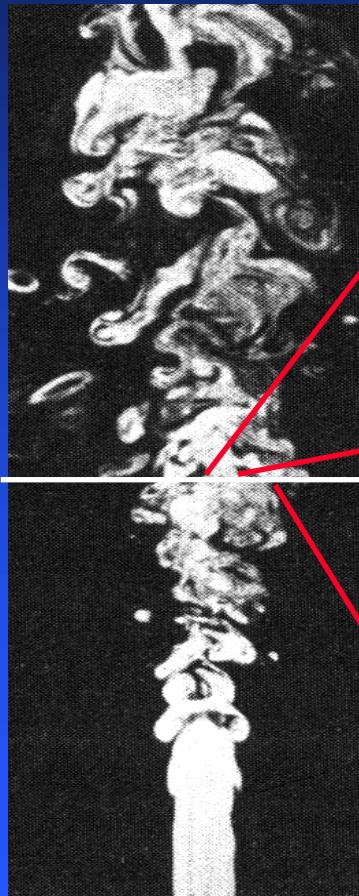
- ⇒ In practice, most of a hydrogen jet close to the release point is ignitable, and significant regions of the jet have ignitable concentrations higher than 10%
 - A hydrogen jets thus remains more likely to ignite than natural gas.
- ⇒ For a slow and uniform build-up of hydrogen, however, the risks remain comparable provided detectors are used, depending on the location of the ignition source with respect to the leak
 - The low minimum ignition energy issue remains, overall, a concern.



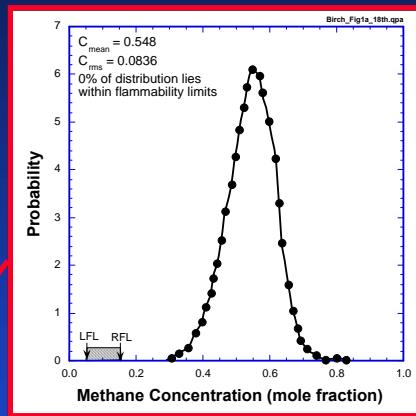
Jet Ignition Probability



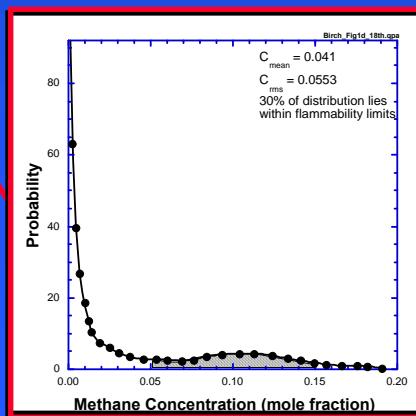
- Methane jet into ambient air (Birch et. al., 1981)



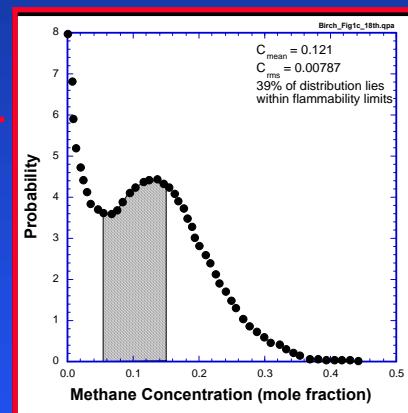
-5 0 5
r/D



$r/D = 0.0$



$z/D = 10$



$r/D = 1.5$

$r/D = 1.8$

- Probability distributions quantify intermittent nature of turbulent flows.

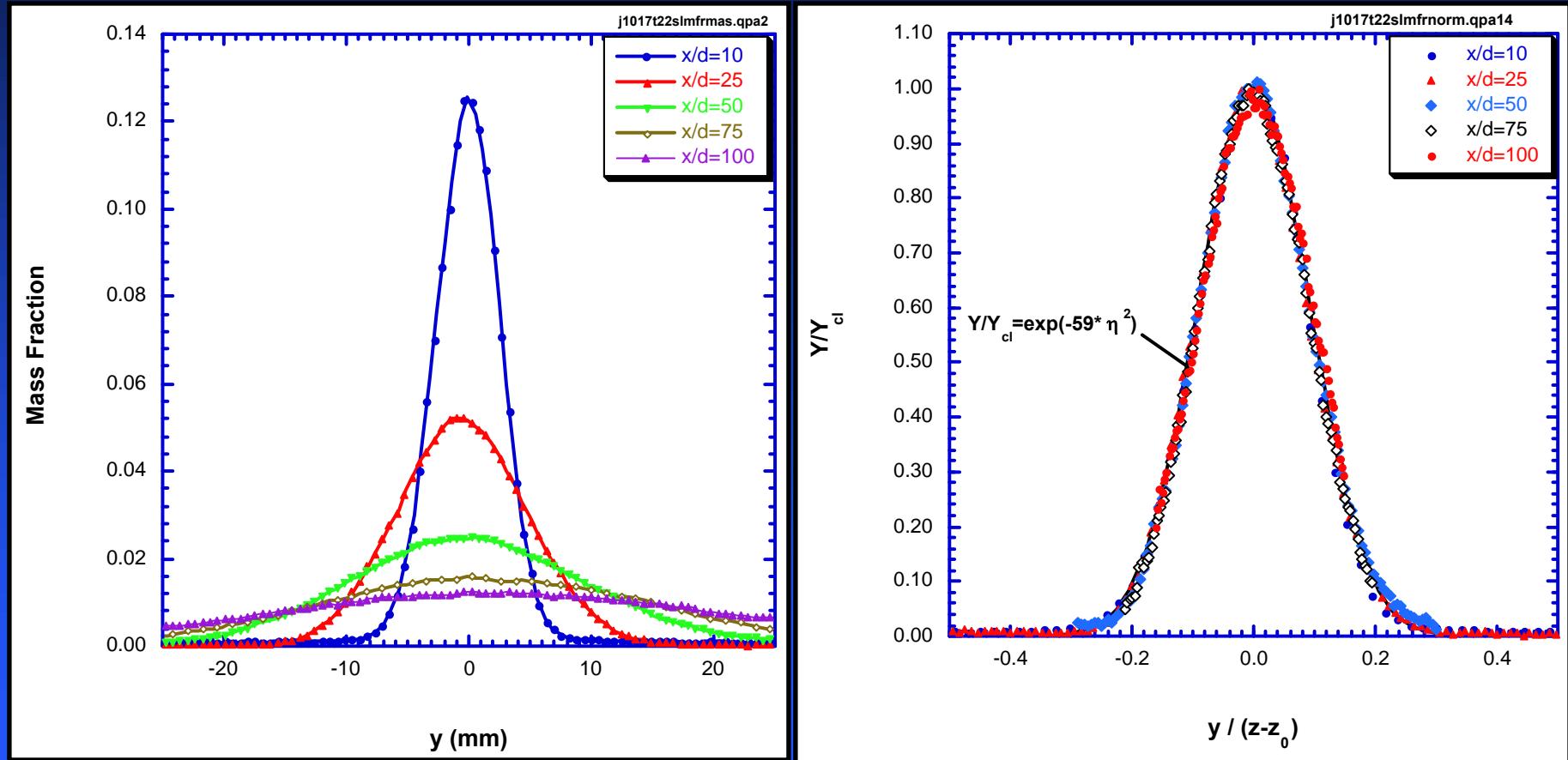
Flammability Factor is defined as the cumulative probability of a potentially flammable mixture occurring at a given point.



Small Unignited Releases: Momentum-Dominated Regime



Radial profiles in H₂ jet, d=1.91 mm, Re = 2384



Radial profiles of H₂ mass fraction collapse onto a single curve in agreement with CH₄ jet data.



Momentum-Dominated Jets are within the Ignition Region



Flow between exit and 4% mole fraction (LFL) remains in jet momentum dominated regions
Choked flow conditions

Unignited Jet Separation Distance Length Scales

Pressure = ~20 MPa (~3000 psig)

Hole Diameter	Flowrate	Xmax - Distance to 4% mole fraction	Start of Intermediate Region
3.175 mm (1/8 inch)	9.718×10^{-2} Kg/sec (2,463 ft ³ /min)*	14.80 m (48.55 ft)	20.7 m (67.9 ft)
1.5875 mm (1/16 inch)	2.430×10^{-2} Kg/sec (615.9 ft ³ /min)*	7.40 m (24.28 ft)	14.6 m (48.0 ft)
0.794 mm (1/32 inch)	6.075×10^{-3} Kg/sec (154.1 ft ³ /min)*	3.70 m (12.14 ft)	10.3 m (33.9 ft)

*@NTP = 21° C (70° F), 101 kPa (14.7 psia)

- Start Intermediate Region

$$x/D = 0.5 F^{1/2} (\rho_{exit}/\rho_{amb})^{1/4}$$

$$F = \text{Exit Froude No.} = U_{exit}^2 \rho_{exit} / (gD(\rho_{amb} - \rho_{exit}))$$

