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Toward Using Surrogates to Accelerate Solution of Stochastic Electricity Grid Operations Problems

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***Abstract*—Stochastic unit commitment models typically handle uncertainties in forecast demand by considering a finite number of realizations from a stochastic process for the loads. Accurate evaluations of expectations or higher moments for the quantities of interest require a prohibitively large number of model evaluations. In this paper we propose an alternate approach based on using surrogate models valid over the range of the forecast uncertainty. We demonstrate that surrogate models, employing Polynomial Chaos expansions, built using sparse quadrature methods, and used to estimate expected generation cost, can lead to several orders of magnitude reduction in computational cost, relative to using Monte Carlo sampling on the original model, for a given target error threshold.**

Keywords*—*Stochastic Unit Commitment, Monte Carlo, Polynomial Chaos Expansion

I. Introduction

Unit commitment (UC) is the fundamental process of scheduling thermal generating units in advance of operations in the electric power grid [1]. The objective is to minimize overall production costs to satisfy forecasted demand for electricity, while respecting constraints on both transmission (e.g., thermal limits) and generator infrastructure (e.g., ramping limits). Economic dispatch (ED) is a closely related electric grid operations problem, in which cost minimization is performed to identify an optimal set of power output levels of a fixed set of active thermal generating units. UC and ED are respectively formulated as a mixed-integer and linear optimization problems, and solved using commercial solvers. Despite improvements in forecasting technology, next-day demand predictions are imperfect, with errors on average in the 1-3% range and exceeding 10% on specific days. To account for such inaccuracies, reserve margins are universally imposed in UC. These margins implicitly deal with uncertainty in load forecasts, by ensuring there is sufficient generation capacity available to meet unexpectedly high demand during operations.

An alternative approach to dealing with forecast errors in UC is to *explicitly* model the load uncertainty, typically via a finite set of sampled realizations from a stochastic process model of load. This approach results in a stochastic UC model (SUC), in which the objective typically is to minimize the expected cost across the load scenarios [2], [3]. By explicitly representing the inherent uncertainty in load forecasts, a SUC solution ensures sufficient flexibility to meet a range of potential load realizations during next-day operations. Further, by explicitly representing uncertainty, reliance on reserve margins is reduced, yielding less costly solutions than those obtained for the deterministic UC problems. While not considered here, we note that the problems induced by increasing rates of renewables (e.g., wind and solar) generation penetration accentuates the differences between stochastic and deterministic UC problems, due to increased errors in the next-day forecasts relative to load. While conceptually appealing, the computational difficulty of stochastic UC is well-known [4], such that it is not presently used in practice. The difficulty is primarily driven by the number of forecast samples required to achieve high-quality, robust solutions.

Uncertainties such as those found in stochastic UC are ubiquitous in both power systems operations and planning, and the importance of credibly accounting for them is well-recognized. However, lack of advanced methods to handle uncertainty and the limitations of scenario-based approaches have led researchers to seek alternatives. For example, Thiam and DeMarco [5] argue: "*Simply put, when uncertainty is credibly accounted for such methods yield solutions for economic benefit of a transmission expansion in which the "error bars" are often larger than the nominal predicted benefit.*" Instead, Thiam and DeMarco [5] propose an "oblivious approach" to transmission expansion that does not take into account the uncertainties in the inputs. We agree that an oblivious approach is more credible than simply ignoring large error bars in estimations. However, we posit that with proper modeling and sampling algorithms, the errors incurred in such analyses can be drastically reduced. Of course, it is not possible to change the nature of uncertainties, such that if uncertainties are so large that they fail to provide significant information, oblivious approaches may be appropriate. However, as we argue in this paper, it is possible to reduce additional uncertainties and inefficiencies introduced due to poor modeling and sampling.

In this paper, we propose to adopt advanced modeling and sampling techniques from the uncertainty quantification (UQ) community, and leverage them to impact power systems operations problems such as stochastic UC and ED. Such techniques have been successfully applied in many areas of computational science and engineering, with great success [6]. Most studies analyzing uncertain power system operations problems generate forecast scenarios by drawing random samples from a stochastic process using standard Monte Carlo (MC) techniques. In this paper, we consider instead an alternative approach based on using Polynomial Chaos [7] expansions, built using sparse quadrature methods, as surrogate models valid over the range of the forecast uncertainty. We demonstrate that our approach yields a one to two order of magnitude reduction in the number of samples required to estimate expected generation cost, relative to MC, for a given target error threshold. Our approach has the potential to dramatically drop the computational difficulty of stochastic UC and ED, significantly reducing the barriers to its use in practice.

The remainder of this paper is organized as follows. We briefly introduce the stochastic UC formulation in Section II, to provide context for our research. In Section III, we detail our surrogate models of load for stochastic UC, based on Polynomial Chaos expansion. We then empirically analyze the accuracy of our surrogates on standard IEEE test problems in Section IV, and conclude in Section V.

II. Stochastic Commitment and Dispatch

We now introduce a generic formulation of the stochastic UC problem. Our formulation is based on the deterministic mixed-integer linear UC formulation introduced by [1]. Let G and T denote the index sets of thermal generating units and time periods, respectively. We generically define the set of unit commitment constraints (i.e., operational and physical constraints on physical units) as $\mathcal{X}$ and let $\boldsymbol{x}$ denote the vector of unit commitment decisions. The stochastic UC problem is then given as follows:

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & c^u(\boldsymbol{x}) + c^d(\boldsymbol{x}) + \overline{Q}(\boldsymbol{x}) && \text{(1a)} \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{X}, && \text{(1b)} \\ & \boldsymbol{x} \in \{0,1\}^{|G|\times|T|} && \text{(1c)} \end{aligned}$$

The objective terms $c^u(\boldsymbol{x})$ and $c^d(\boldsymbol{x})$ represent generating unit start-up and shut-down costs, respectively, and $\overline{Q}(\boldsymbol{x})$ denotes the expected generation cost. The UC constraints prescribed by $\mathcal{X}$ include: minimum on-off requirements and linearization of startup and shutdown costs.

We treat the loads D^t for all $t \in T$ as random variables (RVs). We begin by setting up a requisite theoretical framework as follows. Define the probability space $(\Omega, \mathfrak{S}, P)$, where Ω is a sample space, $\mathfrak{S}$ is a σ-algebra on Ω, and P is a probability measure on $(\Omega, \mathfrak{S})$. Further, defining the *germ* $\boldsymbol{\xi} = \{\xi_1, \xi_2, \ldots, \xi_{|T|}\}$ as a set of independent identically distributed (*iid*) RVs in $L_2(\Omega, \mathfrak{S}, P)$, to be further specified below, we focus on the probability space $(\Omega, \mathfrak{S}_{\boldsymbol{\xi}}, P)$ employing the sigma algebra generated by $\boldsymbol{\xi}$. We define the uncertain loads as RVs $D^t(\omega) : \Omega \to \mathbb{R}$ in $L_2(\Omega, \mathfrak{S}_{\boldsymbol{\xi}}, P)$, such that we may write, by construction, $D^t := D^t(\boldsymbol{\xi}(\omega))$, $\forall t \in T$.

Given the uncertain loads expressed as RVs, the corresponding generation cost $Q(\boldsymbol{x}, \boldsymbol{\xi}(\omega))$ is similarly uncertain/random. The *expected* generation cost, denoted $\overline{Q}(\boldsymbol{x})$, is defined as

$$\overline{Q}(\boldsymbol{x}) = \mathrm{E}_{\boldsymbol{\xi}} Q(\boldsymbol{x}, \boldsymbol{\xi}(\omega)) \tag{2a}$$

and the uncertain (multi-period) *economic dispatch problem* under a fixed unit commitment $\boldsymbol{x}$ is given by

$$\begin{aligned} & Q(\boldsymbol{x}, \boldsymbol{\xi}(\omega)) = \\ & \min_{\boldsymbol{p},\boldsymbol{q}} \quad \sum_{t\in T}\sum_{g\in G} c_g^P(p_g^t) + \sum_{t\in T} Mq^t && \text{(3a)} \\ & \text{s.t.} \quad \sum_{g\in G} p_g^t - q^t = D^t(\xi_t(\omega)), \quad \forall t \in T && \text{(3b)} \\ & \qquad \underline{P}_g x_g^t \le p_g^t \le \overline{P}_g x_g^t, \quad \forall g \in G, t \in T && \text{(3c)} \\ & \qquad p_g^t - p_g^{t-1} \le RU(x_g^{t-1}, x_g^t), \quad \forall g \in G, t \in T && \text{(3d)} \\ & \qquad p_g^{t-1} - p_g^t \le RD(x_g^{t-1}, x_g^t), \quad \forall g \in G, t \in T. && \text{(3e)} \end{aligned}$$

where

$$\begin{aligned} RU(x_g^{t-1}, x_g^t) &= R_g^u x_g^{t-1} + S_g^u(x_g^t - x_g^{t-1}) + \overline{P}_g(1 - x_g^t) \\ RD(x_g^{t-1}, x_g^t) &= R_g^d x_g^t + S_g^d(x_g^{t-1} - x_g^t) + \overline{P}_g(1 - x_g^{t-1}) \end{aligned}$$

and R_g^u/R_g^d, S_g^u/S_g^d represent nominal ramp-up/ramp-down rates, startup/shutdown ramp rates, respectively.

Note that because the loads are RVs, all solution variables are necessarily RVs. For brevity of notation, we have only included the explicit dependence on $\boldsymbol{\xi}(\omega)$ in D^t and Q, thereby emphasizing the randomness of inputs/output of interest only.

The optimization objective in ED is to minimize the expected total production and loss-of-load costs. The first term in (3a) represents total production cost. The second term represents the loss-of-load penalty, where q^t is the unit (e.g., MW) of load unsatisfied in period t. Typically, the load-shedding penalty is equal to a large number M. Constraints (3b)-(3e) specify operational constraints, and include (in order): power balance at each period (3b); lower and upper bounds for committed generation unit output levels (3c); and generation ramp-up and ramp-down constraints for pairs of consecutive time periods (3d) and (3e).

A quadratic production cost function, given below, is typically employed in scheduling electricity grid operations.

$$c_g^P(p_g^t) = a_g x_g^t + b_g p_g^t + c_g (p_g^t)^2 \tag{4}$$

Equation (4) can be accurately approximated by a set of piecewise linear segments. For conciseness, we omit these standard linearization steps. For a detailed treatment on the linearization of the quadratic cost function please refer to [1].

The stochastic ED problem is embedded as a sub-problem in the stochastic UC problem. The high-level context in stochastic UC is the presence of uncertainty in future loads (and, more generally, renewables and system component failures). In stochastic UC, the first-stage decisions are the unit commitment selections $\boldsymbol{x}$, and the objective is to minimize the expected generation costs. In the second (recourse) decision stage, uncertain loads result in uncertain recourse decisions for the dispatch variables $\boldsymbol{p}$ and $\boldsymbol{q}$, and associated generation and load shedding costs. First-stage unit commitment decisions are determined by taking their future impacts into consideration. These future impacts are quantified by the recourse function $\overline{Q}(\boldsymbol{x})$, which computes the expected value of generation cost for a given unit commitment $\boldsymbol{x}$.

We can *estimate* the expected generation cost by using a finite number of load realizations (i.e., scenarios) $s \in \mathcal{S}$ sampled from the joint density $p(\boldsymbol{D})$, where $\boldsymbol{D} = \{D^1, D^2, \ldots, D^{|T|}\}$. Defining $\rho \equiv 1/|\mathcal{S}|$, formulation (1) can be rewritten as:

$$\min_{\boldsymbol{x}} \quad c^u(\boldsymbol{x}) + c^d(\boldsymbol{x}) + \rho \sum_{s \in \mathcal{S}} Q(\boldsymbol{x}, s) \tag{5a}$$

$$\text{s.t.} \quad \boldsymbol{x} \in \mathcal{X}, \tag{5b}$$

$$\boldsymbol{x} \in \{0,1\}^{|G| \times |T|} \tag{5c}$$

where $Q(\boldsymbol{x}, s) =$

$$\min_{\boldsymbol{p},\boldsymbol{q}} \quad \sum_{t \in T} \sum_{g \in G} c_g^P(p_g^t) + \sum_{t \in T} M q^t \tag{6a}$$

$$\text{s.t.} \quad \sum_{g \in G} p_g^t - q^t = D_s^t, \quad \forall t \in T \tag{6b}$$

$$\underline{P}_g x_g^t \le p_g^t \le \overline{P}_g x_g^t, \quad \forall g \in G, t \in T \tag{6c}$$

$$p_g^t - p_g^{t-1} \le RU(x_g^{t-1}, x_g^t), \quad \forall g \in G, t \in T \tag{6d}$$

$$p_g^{t-1} - p_g^t \le RD(x_g^{t-1}, x_g^t), \quad \forall g \in G, t \in T. \tag{6e}$$

Formulation (5) represents an *extensive form* of the stochastic UC problem, based on $|\mathcal{S}|$ sampled scenarios of load realizations. Formulation (3) can be similarly discretized.

III. Accurate estimation with limited samples

The typical scenario sampling approach described above uses Monte Carlo (MC) sampling to approximate an integration, thereby estimating an expectation. While MC algorithms are commonly used for their convenience and robustness, their poor convergence rate is well-known. The MC estimate of the expectation has the error

$$\mathbb{V}[Q(\boldsymbol{x}, \boldsymbol{\xi})]/\sqrt{|S|}, \tag{7}$$

where $\mathbb{V}$[Q] denotes the variance of the RV Q. Given the significant additional complexity incurred by including stochasticity in the optimization problem, a stochastic formulation, as opposed to a deterministic one, becomes advantageous when the variance is large. Hence, accurate estimation of the expectation is not only an academic exercise but an important practice as well.

According to Eq. (7), this can be achieved by increasing the number of samples, but the a linear decrease in the error requires a quadratic increase in the number of samples, which can quickly render the stochastic optimization problem prohibitively expensive. This illustrates the limitation of MC algorithms in providing accurate estimations; while they are convenient, they do not efficiently yield accurate estimations.

Similar problems arise in uncertainty quantification (UQ) for computational science in general, where each sample point would correspond to a full simulation. At the same time, accuracy of the estimations have paramount importance, since simulation results may lead to scientific discoveries or high-impact policy decisions. The need for accurate estimation of uncertain model outputs, along with the prohibitive cost of MC samples, have lead to the development of efficient alternatives to MC methods in UQ. In this work, we will illustrate how we adopt such methods for stochastic optimization problems in power systems. The details of the proposed method will be explained in the subsequent section. The key utility of the approach is that it enables, in a preprocessing step, *efficient* construction of a compact, accurate, and computationally inexpensive representation of the input-output map of the uncertain system. This representation, is then used as a *surrogate*, for the dependence of select model-output quantities of interest (QoIs) on uncertain model inputs. Using this surrogate, rather than the original system governing equations, enables accurate estimation of uncertain QoIs, and associated expectations, with minimal costs beyond those of surrogate construction.

A. Representation of uncertainty using Polynomial Chaos

Given the formulation in Eq. (3) with uncertain/random loads leading to uncertain/random generation cost, we employ efficient UQ methods, relying on *functional representations* of random variables. Specifically, we use Polynomial Chaos (PC) expansions. A brief description of PC is presented below. For an in-depth description, the reader is referred to a series of publications on this topic [7]–[10].

Considering the above defined *germ* $\boldsymbol{\xi}$ and the associated probability space $(\Omega, \mathfrak{S}_{\boldsymbol{\xi}}, P)$, any RV $X : \Omega \to \mathbb{R}$ where, by construction, $X \in L_2(\Omega, \mathfrak{S}_{\boldsymbol{\xi}}, P)$, can be written as a PC expansion (PCE):

$$X(\omega) = X(\boldsymbol{\xi}(\omega)) = \sum_{k=0}^{\infty} \alpha_k \Psi_k(\boldsymbol{\xi}) \tag{8}$$

where the basis functions Ψ_k are multivariate polynomials* that are orthogonal, by construction, with respect to the density of $\boldsymbol{\xi}$, thus

$$\langle \Psi_i \Psi_j \rangle = \int \Psi_i(\xi) \Psi_j(\xi) dP(\xi) = \delta_{ij} \langle \Psi_i^2 \rangle \tag{9}$$

where δ_{ij} is Kronecker's delta. Further, given this orthogonality, we have

$$\alpha_k = \frac{\langle X \Psi_k \rangle}{\langle \Psi_k^2 \rangle} \tag{10}$$

where the inner product is defined, for any RV $Z(\boldsymbol{\xi})$, by the Galerkin projection

$$\langle Z \rangle = \int Z(\boldsymbol{\xi}) p_{\boldsymbol{\xi}}(\boldsymbol{\xi}) d\boldsymbol{\xi}. \tag{11}$$

Moreover, the Ψ_k are products of univariate polynomials, namely $\Psi_k(\boldsymbol{\xi}) = \psi_{k_1}(\xi_1) \cdots \psi_{k_n}(\xi_n)$, where $n = |T|$. In a practical computational context, one truncates the PCE to order p. The number of terms in the resulting finite PCE

$$X \approx \sum_{k=0}^{P} \alpha_k \Psi_k(\boldsymbol{\xi}) \tag{12}$$

is: $P + 1 = (n+p)!/n!p!$. We dispense with the $\approx$ symbol in the rest of the text, employing for any RV $X(\boldsymbol{\xi})$, its truncated PCE

$$X = \sum_{k=0}^{P} \alpha_k \Psi_k(\boldsymbol{\xi}). \tag{13}$$

Generalized PC (gPC) expansions have been developed by [10] using a broad class of orthogonal polynomials in the Askey scheme [11]. Each family of polynomials corresponds to a

*Generally, other, non-polynomial, basis functions can be used, but we restrict ourselves here, without loss of generality, to the most common polynomial-based usage.

given choice of distribution for the ξ_i and is, by construction, orthogonal with respect to the density of ξ_i. In general, most useful choices for $(\boldsymbol{\xi}, \Psi)$ are uniform RVs with Legendre polynomials and normal RVs with Hermite polynomials.

B. Surrogate Construction

We employ Legendre-Uniform (LU) PC, as it is most useful for purposes of surrogate construction. Further, in this particular context, *i.e.* explicitly for surrogate construction, a key first step is to define the input random variables as *iid* uniform over their ranges of interest. This does *not* restrict the utility of the approach to *iid* uniform load distributions. Rather, the uniform assumption is simply to ensure *uniform* accuracy in the surrogate over the range of loads variability. Once the surrogate is available, providing effectively the input-output map, any $p(\boldsymbol{D})$ can be employed, as is further outlined below.

Since, by construction, for LU PC, $\xi_t \overset{iid}{\sim} U(-1,1)$, $\forall t \in T$, defining $D^t \overset{iid}{\sim} U(D^t_{\min}, D^t_{\max})$, we have the PCE for D^t given simply by

$$D^t = \xi_t \frac{D^t_{\max} - D^t_{\min}}{2} + \frac{D^t_{\max} + D^t_{\min}}{2}, \quad \forall t \in T. \tag{14}$$

In this context we employ Eq. (13) to represent $Q(\boldsymbol{x}, \boldsymbol{\xi})$ with a truncated LU PCE

$$Q_{\mathrm{PC}}(\boldsymbol{x}, \boldsymbol{\xi}) = \sum_{k=0}^{P} c_k(\boldsymbol{x}) \Psi_k(\boldsymbol{\xi}), \tag{15}$$

where, to be specific, $\Psi_k(\boldsymbol{\xi})$ are n-variate Legendre polynomials ($n = |T|$). The coefficients c_k depend on the discrete variable $\boldsymbol{x}$, hence separate PCE approximations for Q will be constructed for each instance of $\boldsymbol{x}$. Given Eq. (11), we have

$$c_k(\boldsymbol{x}) = \frac{\langle Q \Psi_k \rangle}{\langle \Psi_k^2 \rangle} = \frac{1}{\langle \Psi_k^2 \rangle} \int_{[-1,1]^n} Q(\boldsymbol{x}, \boldsymbol{\xi}) \Psi_k(\boldsymbol{\xi}) d\boldsymbol{\xi}. \tag{16}$$

where we have used $p_{\boldsymbol{\xi}}(\boldsymbol{\xi}) = 1$, for $\boldsymbol{\xi} \in [-1,1]^n$.

Given $Q_{\mathrm{PC}}(\boldsymbol{x}, \boldsymbol{\xi})$, then for the given $D^t \overset{iid}{\sim} U(D^t_{\min}, D^t_{\max})$, $\forall t \in T$, we have immediately that

$$\overline{Q}(\boldsymbol{x}) = \mathrm{E}_{\boldsymbol{\xi}}[Q(\boldsymbol{x}, \boldsymbol{\xi})] = \langle Q(\boldsymbol{x}, \boldsymbol{\xi}) \rangle = c_0, \tag{17}$$

being the solution of the stochastic ED problem, as required for the stochastic UC problem Eq. (1).

Beyond this, however, the PCE $Q_{\mathrm{PC}}(\boldsymbol{x}, \boldsymbol{\xi})$ can be used as a *surrogate* for the ED problem solution $Q(\boldsymbol{x}, \boldsymbol{\xi})$. Specifically, $Q_{\mathrm{PC}}(\boldsymbol{x}, \boldsymbol{\xi}(\boldsymbol{D}_s)) \approx Q(\boldsymbol{x}, s)$ (see Eq. 6) for any demand $\boldsymbol{D}_s \in \mathbb{D}$ where $\mathbb{D} = \{\boldsymbol{D} | D^t \in [D^t_{\min}, D^t_{\max}] \forall t \in T\}$, employing

$$\xi_t = \frac{2D^t - (D^t_{\max} + D^t_{\min})}{D^t_{\max} - D^t_{\min}}, \quad \forall t \in T. \tag{18}$$

Thus, for any arbitrary $p(\boldsymbol{D})$, the PCE $Q_{\mathrm{PC}}(\boldsymbol{x}, \boldsymbol{\xi}(\boldsymbol{D}))$ can be used to *efficiently* provide samples $Q(\boldsymbol{x}, s)$ given random samples $\boldsymbol{D}_s \sim p(\boldsymbol{D})$, as long as $\boldsymbol{D}_s \in \mathbb{D}$. Thus, the MC estimation of the expectation $\overline{Q} = \rho^{-1} \sum_{s \in \mathbb{S}} Q(\boldsymbol{x}, s)$ in Eq. (5a) can be done with arbitrarily large $|\mathbb{S}|$ given the low cost of samples. Alternatively, for any $p(\boldsymbol{D})$, one can employ the PCE for $\boldsymbol{D}(\boldsymbol{\eta})$, where $\boldsymbol{\eta}$ is suitable PC germ, and use sparse-quadrature methods outlined below to evaluate projection integrals, but now using as forward model the surrogate $Q_{\mathrm{PC}}(\boldsymbol{x}, \boldsymbol{\xi}(\boldsymbol{D}(\boldsymbol{\eta})))$, to arrive at the PCE for $Q(\boldsymbol{x}, \boldsymbol{\eta})$, from which one easily has $\overline{Q} \equiv c_0$, as long as the sparse-quadrature samples $\boldsymbol{D}(\boldsymbol{\eta}_q) \in \mathbb{D}$. The only remaining issue is the evaluation of the projection integrals in Eq. (16). This is discussed below.

C. Evaluation of the Projection Integrals

Several methods can be employed to evaluate the projection integrals in Eq. (16). MC methods can be used in principle, but are impractical given their slow convergence rate. Alternatively, for smooth integrands, and particularly in low-moderate dimensional problems, sparse quadrature methods [12]–[14] can provide highly accurate results with smaller numbers of deterministic samples. Figure 1 shows the locations of deterministic samples we use, with a sparse grid employing Clenshaw-Curtis quadraure. Several nested levels are shown in the figure, with "+", "x", and "o" markers to illustrate the fact that model evaluations can be re-used if higher-order approximations are necessary. The number of requisite samples employing sparse-quadrature evaluation of the projection integrals, for a given requisite surrogate accuracy, is much smaller than the corresponding number of MC samples, as we illustrate in the following.

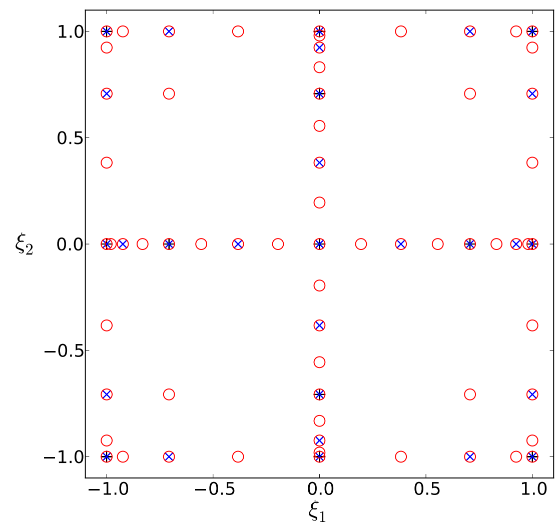

Fig. 1. Placement of deterministic samples via a sparse grid approach.

IV. Numerical Results

We now present numerical results comparing the calculation of the expected cost of ED via our proposed surrogate model with the classical approach via Monte Carlo scenario sampling. We consider two cases: a 9-bus example [15] and the IEEE 118-bus test system [16]. We vary the number of time periods $|T|$ from 6 to 24. Following the model and analysis in [1], we relax transmission constraints and focus on generating unit characteristics. In Figure 2, we show a typical load series for the IEEE 118 bus test system. The red bars depict a 20% uncertainty range around the nominal values.

We first proceed to test the accuracy of the PCE surrogate for $Q(\boldsymbol{x}, D)$ with respect to full model evaluation. For this

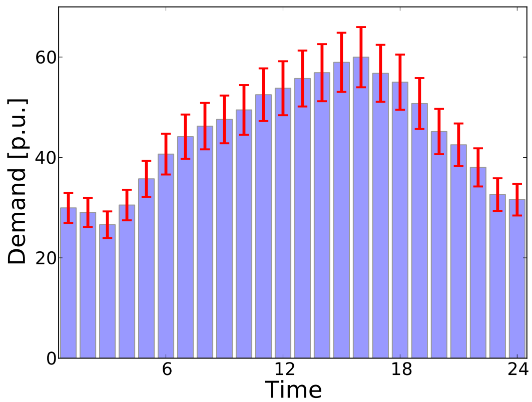

Fig. 2. Uncertain demand values for the IEEE 118-bus example. The error range is 20% around the nomical values.

exercise we select the 9-bus model since the full model evaluation is cheap, hence we can explore well the high-dimensional demand space. Table I shows the relative L_2 error computed with the discrepancies observed at quadrature points between truncated PCE's, in Eq. (15), and the cost values via direct evaluations. Several sparse quadrature levels are employed. The table shows results for PCE orders from 1 through 4. A three-level sparse quadrature, L3 in the table, is sufficient to construct a second-order PCE model with a negligible error compared to the full model. Moreover, depending on the purpose of the PCE model, a first-order approximation can also be sufficient. Cross-validation tests using ensembles with 10^6 random samples, results not shown, confirm the high accuracy of the PCE model. Similarl tests indicate that point-

Order	Sparse Quadrature			
	L2, 85p	L3, 389p	L4, 1457p	L5, 4865p
1	1.62e-05	2.90e-05	2.15e-05	2.18e-05
2	-	7.48e-07	2.17e-07	7.83e-08
3	-	-	1.92e-07	5.36e-08
4	-	-	-	2.10e-08

TABLE I. RELATIVE L_2 ERROR AT TRAINING POINTS FOR SEVERAL PCE SURROGATES AND SPARSE QUADRATURE LEVELS. POWER GENERATION COST DISCRETIZED USING 10 SEGMENTS.

wise discrepancies between the PCE surrogate and the full model are less than 0.5% throughout the computational domain for a 2-nd order PCE.

Figure 3 illustrates the dependence of cost on the load in specific time periods. This figure shows 2-dimensional slices though the load spaces for the 9-bus (top frame) and 118-bus (bottom frame) model. The cost Q in these figure is normalized by the expected value for each case.

The goal of this exercise is to demonstrate the efficiency of PCE to compute the expected cost over all possible load realizations. Fig. 4 shows the convergence of the standard deviation of the expected cost for the 9-bus model with 6 load periods, evaluated using an number of scenarios between 10^2 and 10^5. These results indicate that about 10^4 samples are necessary to reduce the relative error of the expected cost to below 10^{-3}.

Figure 5 shows sample convergence rates for MC compared

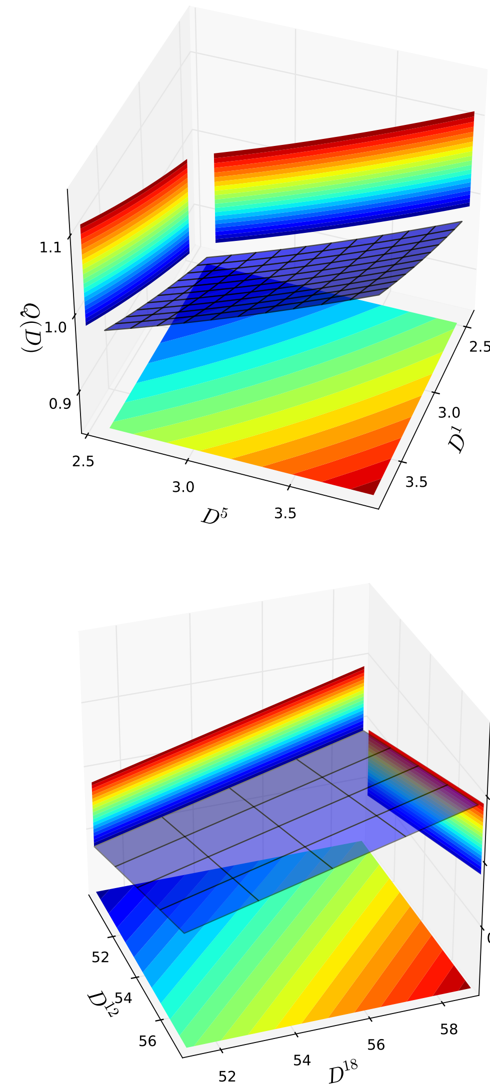

Fig. 3. Top frame: Slice through a 6-dimensional 2-nd order PCE model for the 9-bus system. Bottom frame: Slice through a 24-dimensional PCE model for the 118-bus system. The transparent surface shows the 3D dependence of Q on the corresponding loads. Filled contours are projected on the side planes to provide a qualitative view for the dependence on each load.

to the PCE approach. For MC, a relative error of 10^{-3} requires about 10^4 model evaluations. The number of model evaluations is independent of the number of loads periods, and is nearly the same for the 9-bus and 118-bus systems. For the surrogate model approach, the number of model evaluations represents the cost of building the PCE. This cost is higly dependent on the dimensionality of the surrogate, for example the number of model evaluations for a 24-dimensional PCE is one-two orders of magnitude larger compared to a 6-dimensional PCE. Once the PCE is constructed, subsequent evaluations of $\overline{Q}(\boldsymbol{x})$ incur negligible cost. Hence the efficiency of the surrogate model approach over the routine MC approach is proportional to the number of times $\overline{Q}$ needs to be evaluated during a simulation. For the 118-bus model, 10^3 evaluations of Eq. (2) results in a computational time about 10^4 times smaller for the PCE approach compared to the traditional MC approach

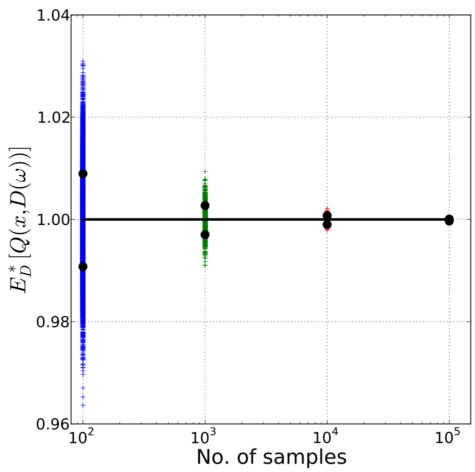

Fig. 4. Convergence of $\mathbb{E}_{\boldsymbol{D}} Q(\boldsymbol{x}, D(\omega))$ computed via MC vs PCE results. Scatter plots show normalized $\mathbb{E}_{\boldsymbol{D}} Q(\boldsymbol{x}, D(\omega))$ computed an increasing number of MC samples. The filled circles show $\pm\sigma$ for each ensenmble, and the horizontal line shows the values computed via PCE. The expectation values are normalized by the "true" value obtained by MC using 10^6 samples.

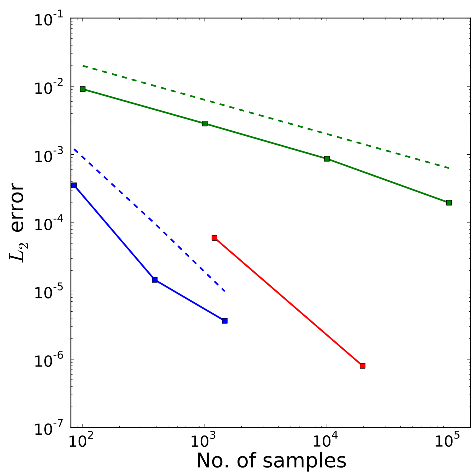

Fig. 5. Convergence of $\mathbb{E}_{\boldsymbol{D}} Q(\boldsymbol{x}, D(\omega))$ computed via MC (solid green line) vs PCE results. The results for a 6-dimensional PCE corresponding to 9-bus with 6 load periods is shown with solid blue line, while the results for a 118-bus 24 load perionds and equivalent 24-dimensional PCE is shown with solid red line. The dashed lines show theoretical convergence rates of $1/2$ (green) and 2 (blue).

V. Conclusion

In this paper we present an efficient approach to reduce the computational cost of the forecast samples necessary in stochastic Unit Commitment simulations. This approach is based on surrogate models for the generation cost that cover the uncertainty of future load. The surrogate models are constructed using Polynomial Chaos expansions. The construction of the terms in the surrogate models is based on the projection of the model on increasingly higher basis modes. Thereby the global error in an L_2 sense between the surrogate model and the actual simulations is easily controlled.

We present computational results using 9-bus and 118-bus test cases. For both these cases quadratic surrogate models for the generation cost showed global L_2 errors less that 10^{-4} while pointwise errors were less than 1% throughout the uncertain demand space. The construction of Polynomial Chaos surrogate models typically requires a much smaller number of samples, typically one to two orders of magnitude for the examples considered in this paper, compared to Monte Carlo evaluation of the expected generation cost for a given requisite accuracy. Subsequent evaluation of the generation cost statistics via surrogate model incurrs negligible additional cost, thereby potentially reducing the computational expense of the forecast ensembles in a stocastic unit commitment by several orders of magnitude.

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