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Quantum Graph Analysis with Trapped Yb Ion Qubits

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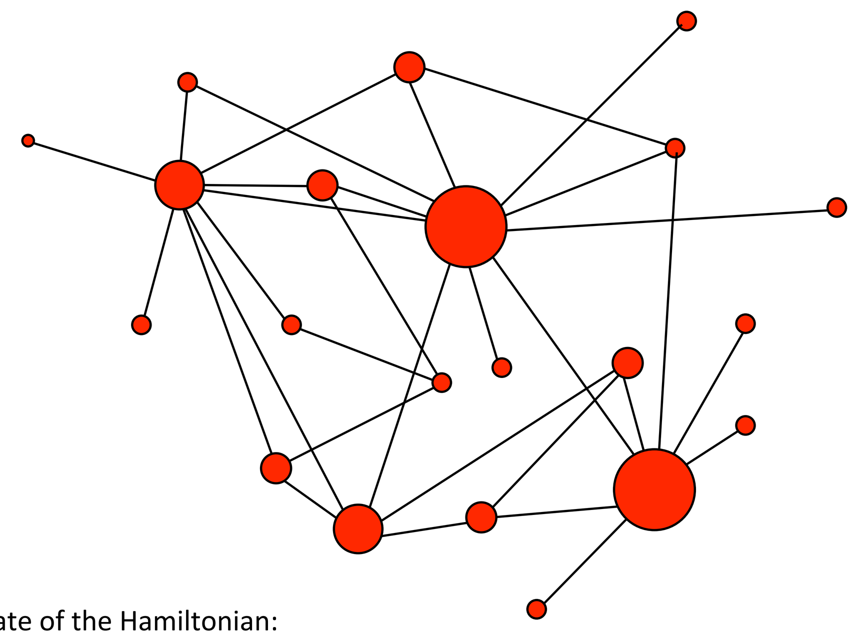
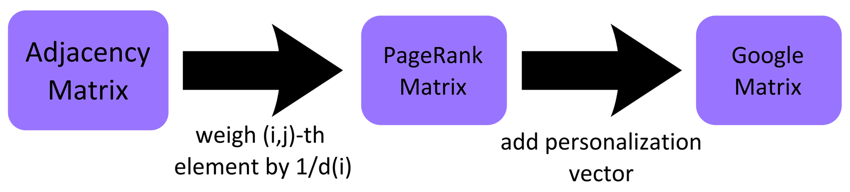
Adiabatic Quantum Algorithm for PageRank

Recently, Garnerone *et al.* proposed an adiabatic quantum algorithm for search engine ranking based on the PageRank graph analysis algorithm utilized by Google [1]. Here we discuss the algorithm as a starting point for our work.

PageRank Algorithm

- The PageRank Algorithm [2] finds the stationary distribution of a random walk on a graph
- The stationary state is an eigenvector of the Google matrix, G
- The Google matrix is a matrix formed with the addition of a stochastic personalization vector





Adiabatic Quantum Algorithm for PageRank

- In mapping to a quantum version, we are looking for the ground state of the Hamiltonian:

$$h^{(p)} = (1-G)^\dagger(1-G)$$

- This problem illustrates several features:
  - Needs only log(N) qubits for an N vertex graph
  - Requires full control of Hamiltonian. i.e. encoding an arbitrary graph requires arbitrary matrix elements of the Hamiltonian.
- Can be easily extended to a single qubit per node, realizing an XY model:

$$H = \frac{1}{2}\sum_i (\mathbb{1}_i - Z_i) + \frac{1}{2}\sum_{i<j} h^{(p)}_{ij}\,(X_iX_j + Y_iY_j)$$

References

[1] Garnerone, *et al. Phys. Rev. Lett.* **108**, 230506 (2012)
[2] S. Brin and L. Page, *Computer Networks and ISDN Systems*, **30**, 107 (1998)

Experimental Status

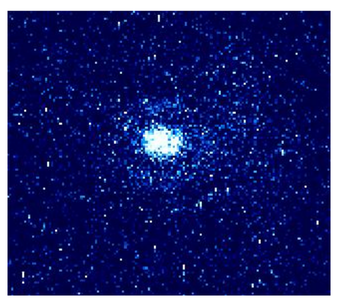
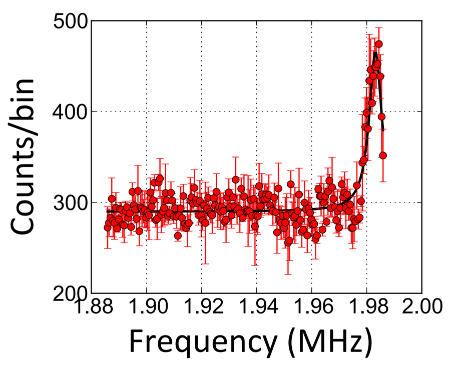


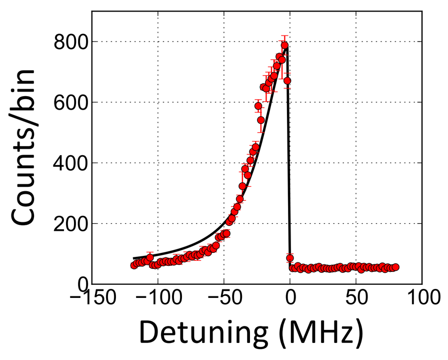
Image of a single ^{174}Yb ion

- Have trapped both ytterbium 174 and 171
- Applied RF: ~200V at 20MHz
- Trap characterization with ytterbium 174
- Secular frequencies: 0.500 MHz (axial), 1.98 MHz (radial)

Experimental Control

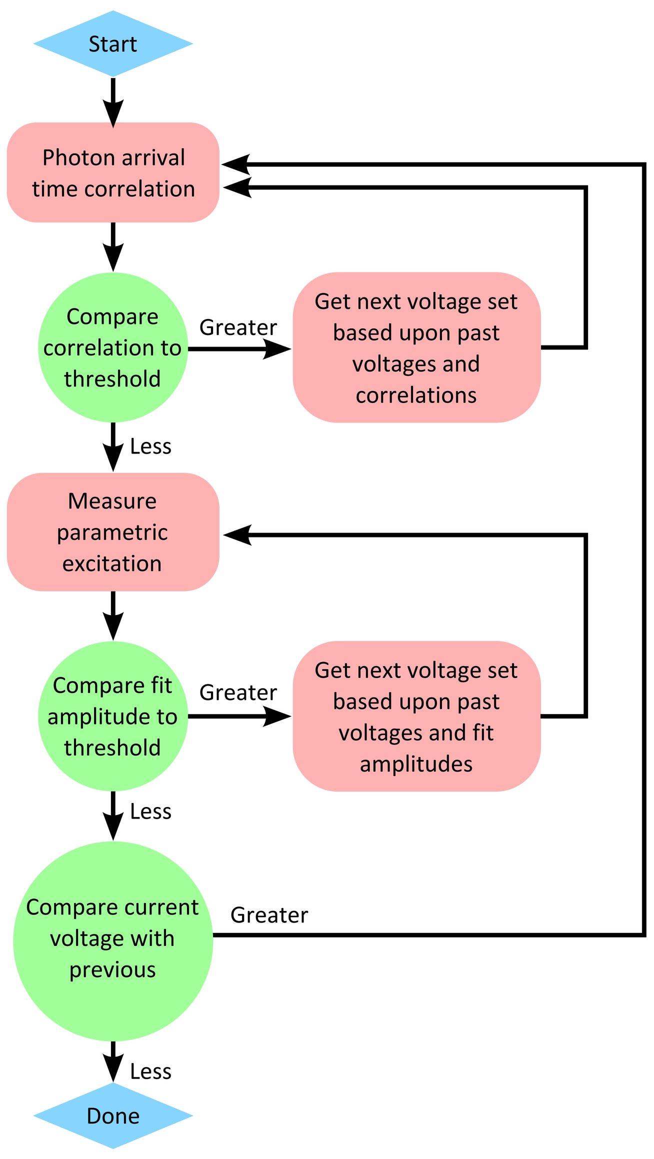
- Python control software for voltage control and instrumentation
- Can control 96 independent voltages on the fly
- Provides feedback onto applied voltages and instruments based upon photon counts
- FPGA based pulsing system with photon counting, time-to-digital converter, and 6 direct digital synthesizers



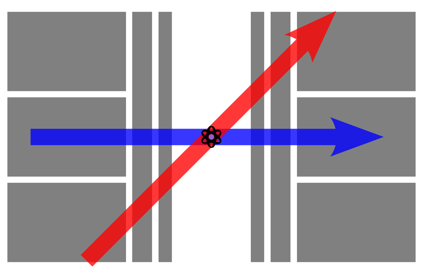


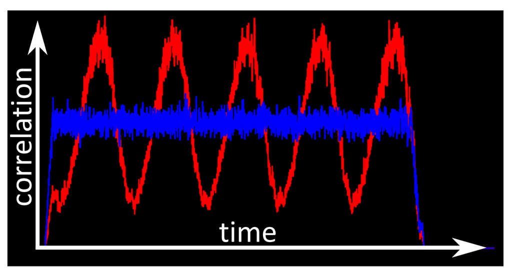
Micromotion Compensation

- Automated control sequence compensates micromotion based on both photon arrival time [1], and parametric excitation [2]



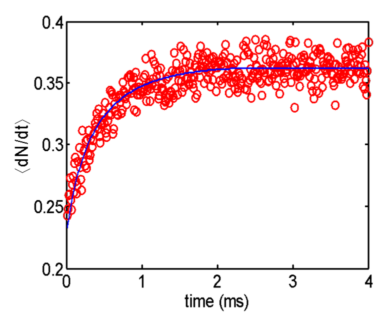
- In our experiments, we have two beams, on normal to trap axis (blue arrow below), and one at 45 degrees (red).
- Minimization of photon correlations with blue results in significant correlations with the red beam.
- This implies a large amount of axial micromotion. To compensate in all directions, we will need to use multiple beams in our minimization routine.

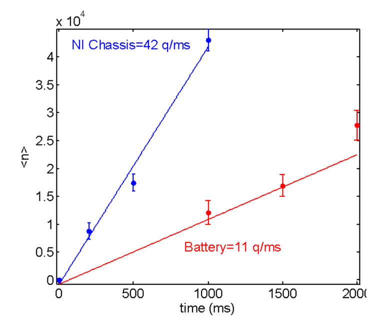




Heating Rates

- Measuring heating rates via doppler recooling [2].
- Measurements performed with calcium in another chamber with identical trap.
- Heating rate: ~42 quanta/ms with NI Chassis, 27quanta/ms with batteries

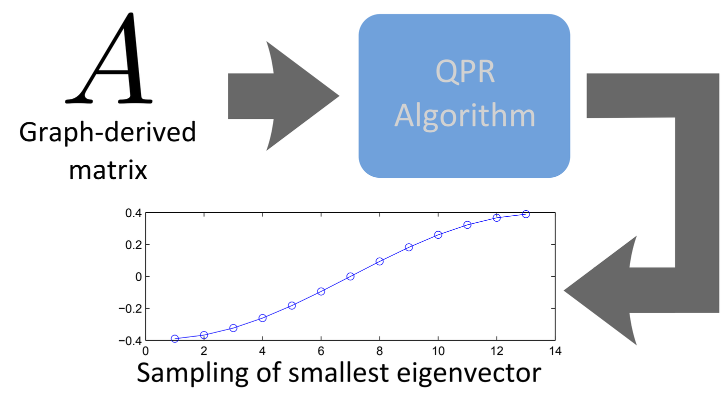




References

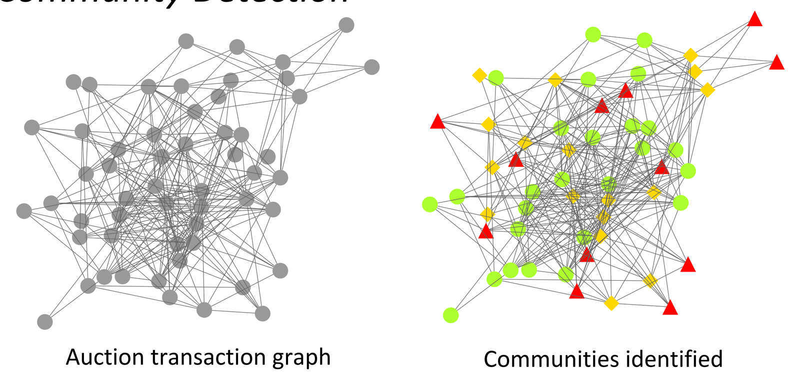
[1] D. Berkeland, *et al. Journal of Applied Physics*, **83**, 5025 (1998)
[2] S. Narayanan, *et al. Journal of Applied Physics*, **110**, 114909 (2011)
[3] J. H. Wesenberg, et al.Physical Review A **76**, 053416 (2007)

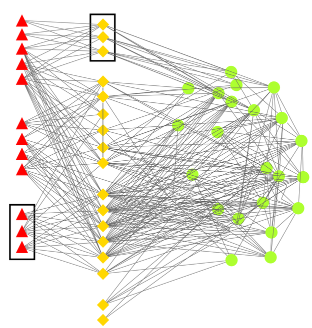
Applications of Graph Theory



- QPR algorithm provides sampling of smallest eigenvector
  - Sampling probability proportional to magnitude of entry
  - Potential sublinear time algorithm for top entries
- Apply to Graph-derived matrices (spectral algorithms)
  - Examples: Adjacency, Laplacian, Normalized Laplacian
- Instances need large spectral gap
- Dense instances imply non-local Hamiltonians

Community Detection

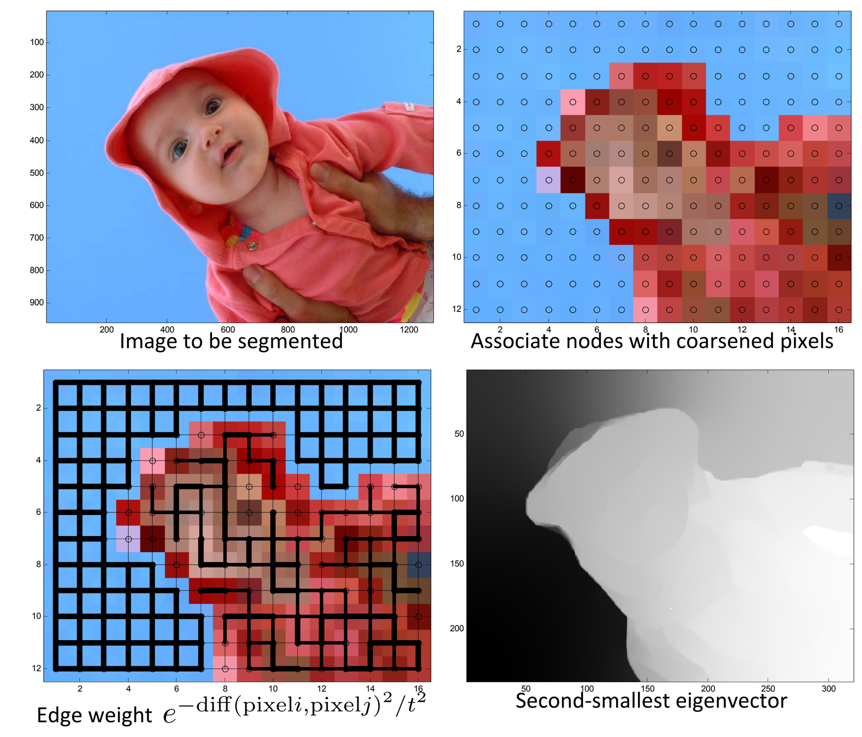




Near-bipartite structure:
Fraud perpetrators
Accomplices
Honest users

- Communities typically identified by (weighted) edge structure
  - Internally dense with sparse extrenal connections
- Issue: *elusive communities* may be internally sparse?
  - Some communities elude direct detection
  - Collusion or fraud may prevent detection of edges
  - How to find such communities?
- Use "targets" to identify elusive communities
  - (Near-)Bipartite core: two densely connected very sparse sets
  - One set elusive community, other its "targets"
  - Spectral approaches are promising
  - Applications: web communities [2], auction site fraud [1] (figure derived from [1])

Spectral Graph partitioning



Edge weight $e^{-\mathrm{diff}(\mathrm{pixel}i,\mathrm{pixel}j)^2/t^2}$

- Use eigenvectors to partition graph
  - Each entry is associated with a node
  - E.g. sign of entry determine partitions
- Second-smallest eigenvector effective for partitioning
- Sampling gives "important" nodes in partitions
- Employ recursive hierarchical partitioning for > 2 partitions
- Application: image segmentation [3] (figure derived from [4])

References

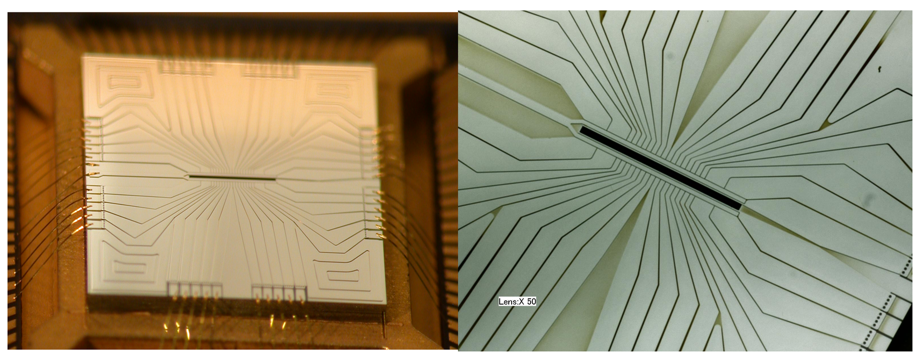
[1] Chau et al. *Proc. ECML-PKDD conf.*, (2006)
[2] Kumar at al. *Proc. 8th International WWW conf.*, (1999)
[3] Shi and Malik. *IEEE Trans. on Pattern Anal. and Mach. Intel.*, 22, 8 (2000)
[4] Spielman. *Spectral Graph Theory and its Applications*, http://cs.yale.edu/homes/spielman/sgta/SpectTut.pdf

Fabrication Technology

Utilizing the ion trap fabrication technology available at Sandia, we are currently using a workhorse linear trap to develop our ytterbium qubit capabilities, as well as developing advanced ion trap structures such as rings and junctions.

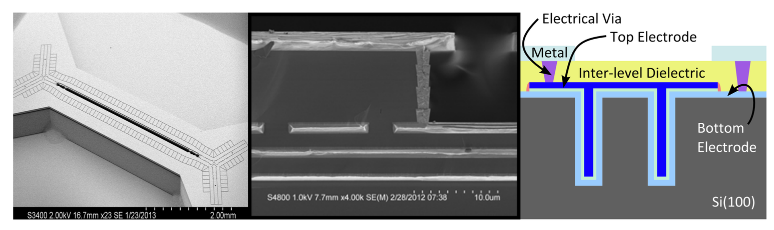
Thunderbird Linear Trap

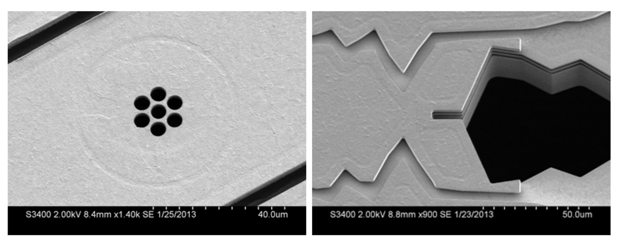
- Microfabricated linear trap using 2-level metallization
- Has been used in labs around the world to trap calcium, ytterbium, and magnesium
- 48 control electrodes



Fabrication Techniques

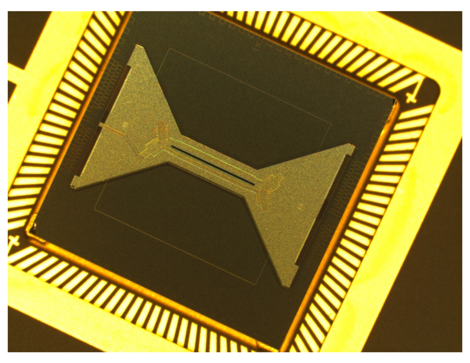
- Integrated trench capacitors for RF pickup filtering
- Four level metallization for routing/grounding
- Small through holes for loading
- Custom die shapes for a tight beam focus

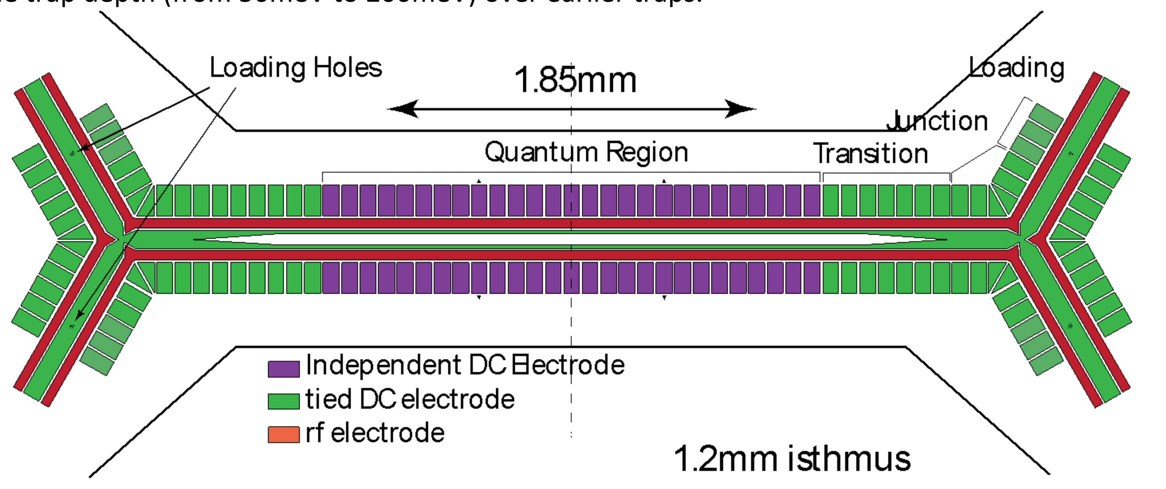




High Optical Access Trap

- Surface skimming beams with waists as small as 4μm
- Slot to accommodate tightly focused beams vertical to surface
- Loading regions and junctions to control ordering of a multi-species ion chain
- Designed to have a 4x improvement to the trap depth (from 50meV to 200meV) over earlier traps.

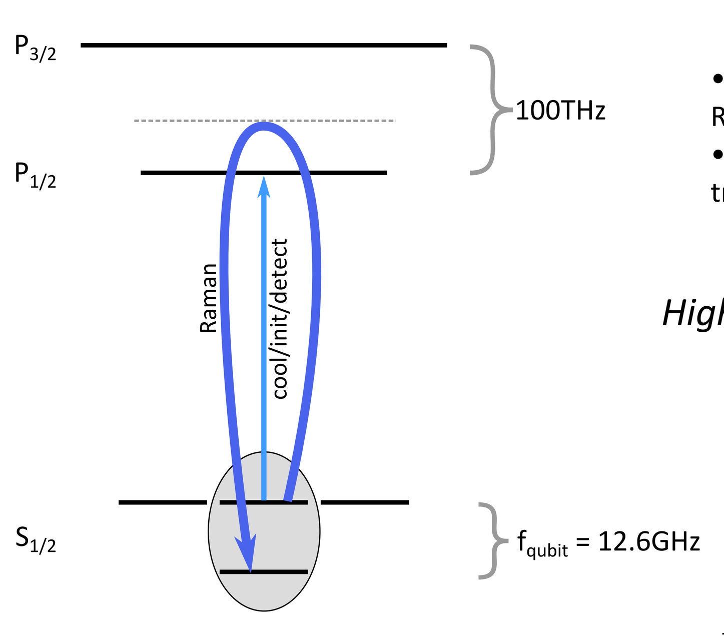




Implementing Graph Analysis with Trapped Ions

Ytterbium ions are good candidates for quantum bits due to long storage and coherence times [1], and high fidelity gates can be performed with high power pulsed lasers [2]. Additionally, they currently are used for quantum simulation of the transverse Ising model [3]. By leveraging this technology we hope to realize the spin-spin couplings needed for graph algorithms (such as the XY model).

Relevant Atomic Structure of $^{171}Yb^+$



Qubit levels

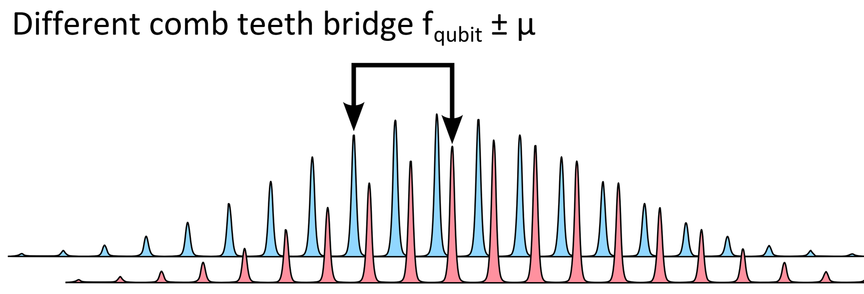
- $|S_{1/2}$ F=0 m_F=0> = |0>
- $|S_{1/2}$ F=1 m_F=0> = |1>
- Can have long coherence times (>1s) due to 1st order insensitivity to magnetic field fluctuations

Implementing the circuit model

- Single Qubit rotations:
  - Global Rotations: microwaves at 12.6GHz
  - Individual rotations: Raman beams at carrier
- Multi Qubit gates: Møllmer-Sørensen gate with Raman beams
- Initialization and detection: 369nm light on $S_{1/2}$-> $P_{1/2}$ transition

High power Raman Interactions

Different comb teeth bridge $f_{qubit} \pm \mu$



- Pulsed laser with large bandwidth can provide frequencies needed for gates/spin-spin interactions
- Tuning the relative phase of the two envelopes determines the type of interaction (carrier-envelope phase stabilization not necessary)

Adiabatic Realization of XX interactions [3]

- Adiabatic quantum simulation of the transverse Ising model with trapped ions
- Carrier Raman --> Transverse B field (y direction)
- Møllmer-Sørensen --> XX coupling (tuning phase)
- Spin Spin interaction tuned by power and detuning
- Residual spin-motion terms can be elimiated by large detunings

$$H = \sum_i B_y\sigma_y^{(i)} + \sum_{i<j} J_{i,j}\sigma_x^{(i)}\sigma_x^{(j)}$$

In general: spin-spin interaction is $\sigma^i_\phi\sigma^j_\phi$. Phase angle ϕ is tuned by relative phase between RF sources in modulators

Rabi Frequencies

$$J_{i,j} = -\sum_m \frac{\eta_{i,m}\Omega_i\eta_{j,m}\Omega_j\nu_m}{\mu^2-\nu_m^2}$$

Detuning from motional modes
μ = detuning from carrier

Experimental Issues

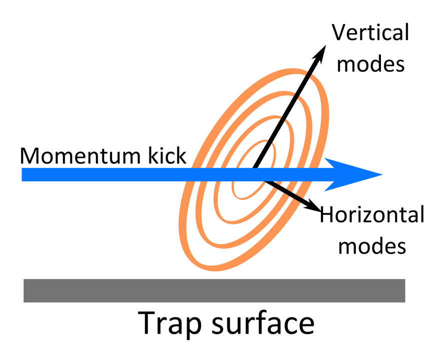
High power 355nm laser
- Far detuned from any transition
- Extremely low spontaneous emission rate
- Differential Stark shifts small
- High power may damage trap (effects unknown)

Ti: Sapphire
- Lower power, tune closer to $P_{1/2}$
- Possibly non-negligible spontaneous emission
- May produce differential Stark shifts
- Lower power will not damage trap

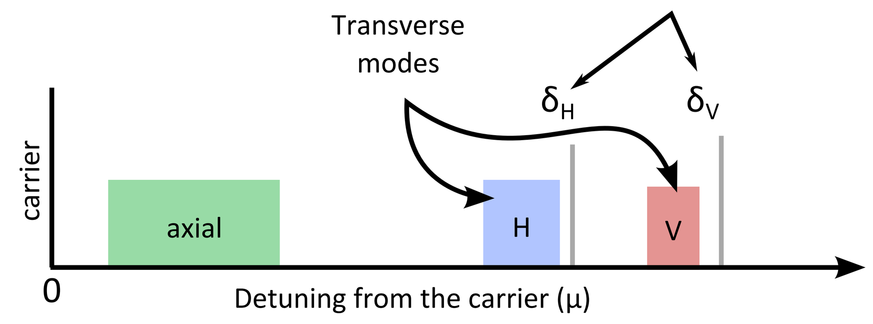
- What are the full set of experimental control knobs available?
- Given the set of controllable paramters, what is the set of realizable graphs?
- Can we realize algorithms with just an XX coupling? Or adiabatically tuning the phase in the ϕϕ interaction between XX and YY?

Extension to the XY model

- Applying two sets of Møllmer-Sørensen gates with a π/2 phase shift results in cross talk between modes.
- Possible solution: address different motional modes,
  - Multiple beams with different orientations to provide orthogonal kicks
  - Rotating the principal axes to change direction of kick
  - Splitting the horizontal and vertical in frequency space to help eliminate cross talk



δ_k has components in both principal directions, allowing us to address both sets of modes with detunings δ_H and δ_V



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[2] D. Hayes, *et al. Physical Review Letters*, **104**, 140501 (2010)
[3] K. Kim, *et al. New Journal of Physics*, **13**, 105003 (2011)

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