

# Limits to Predictability in Nonlinear Dynamical Systems and the Challenge of Modeling Pervasive Fracture

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In the field of nonlinear dynamics it is well known that many deterministic nonlinear systems are inherently unpredictable beyond a certain critical time, the so called predictability horizon. This limit on system predictability is not due to a lack of physics or choice of numerical approximation but rather extreme sensitivity to initial conditions. Examples include event-driven dynamical systems, vibro-impact systems, and pervasive fracture. Beyond the predictability horizon, the system can no longer be described deterministically even though the governing equations are deterministic. Instead, statistical descriptions are needed along with new definitions of convergence with mesh refinement.

A pervasive fracturing process is one in which a multitude of cracks are dynamically active, propagating in arbitrary directions, coalescing, and branching. Examples include fragmentation and progressive structural collapse due to blast effects or seismic events. Pervasive fracturing is a highly nonlinear process involving complex material constitutive behavior, post-peak material softening, localization, new surface generation, and ubiquitous contact. A pure Lagrangian computational method is proposed for simulating the pervasive fracturing of structures by allowing new cohesive fracture surfaces to nucleate at the inter-element faces of a random polyhedral mesh. The random polyhedral mesh is obtained from a randomly close-packed (RCP) Voronoi tessellation. The *a priori* crack paths of the RCP Voronoi mesh are viewed as instances of realizable random crack paths within a random field representation of the continuum material properties. Mesh convergence in a pervasive fracture simulation is viewed in a distributional sense rather than at the level of a single realization.

## Bio

Joe Bishop received his Ph.D. in Aerospace Engineering from Texas A&M University in 1996. His graduate research was in the general area of composite materials and fracture mechanics. From 1997 to 2004 he worked in the Synthesis & Analysis Department of the Powertrain Division of General Motors Corporation, performing thermal-structural analysis of internal combustion engines. Joe joined Sandia National Laboratories in 2004 in the Computational Solid Mechanics group. He has worked on various analysis projects involving earth penetration, blast effects on structures, and the geomechanics related to CO<sub>2</sub> sequestration. His research interests include computational mechanics, pervasive fracture modeling, multi-scale analysis, and uncertainty quantification in highly-nonlinear systems.

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<sup>1</sup> Sandia is a multi-program laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under Contract DE-AC04-94AL85000.