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Advection Scheme in CAM-SE, Focus on Stabilization

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Outline

Spectral Element Method (SEM) and stabilization

CAM-SE model for atmosphere

Variable resolutions

Hyper-viscosity for stabilization

- New tensor hyper-viscosity
- CFL condition

Shallow water tests

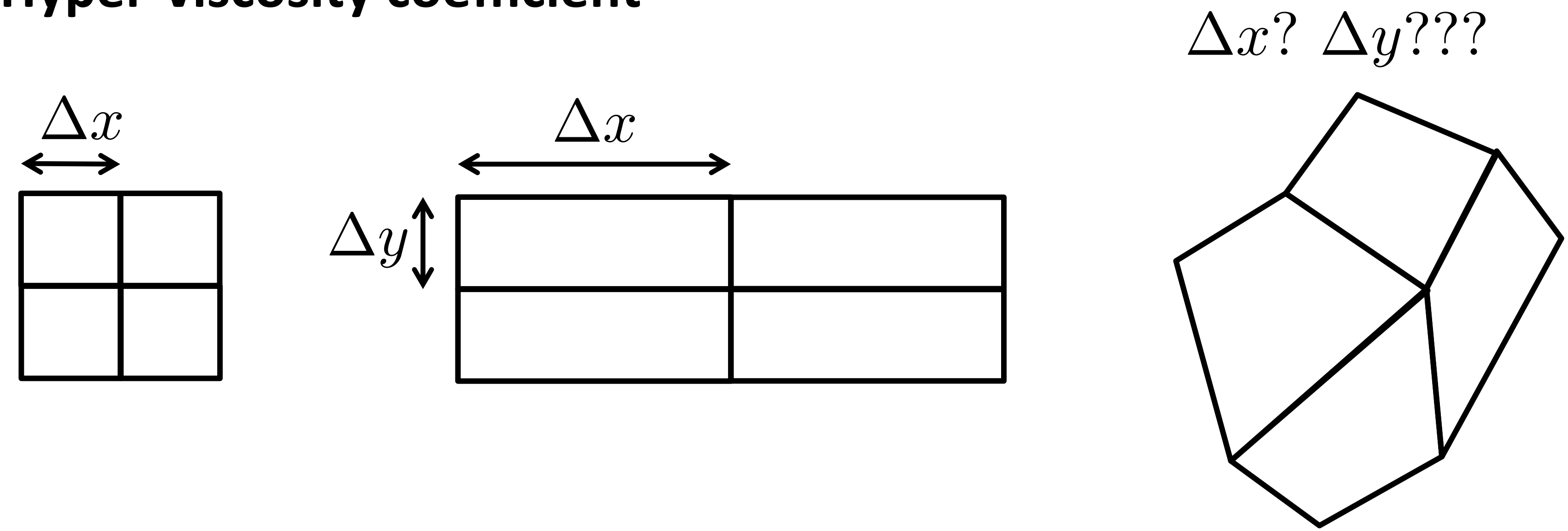
- Refined highly distorted meshes
- Convergence and performance

Spectral Element Method and Stabilization

SEM

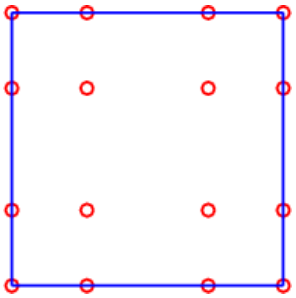
- Continuous Galerkin finite element method with diagonal mass matrix and Gauss quadrature => highly scalable
- Mimetic properties
- Requires stabilization => hyperviscosity with a coefficient, $C(\Delta x)^4\Delta^2$

Hyper-viscosity coefficient



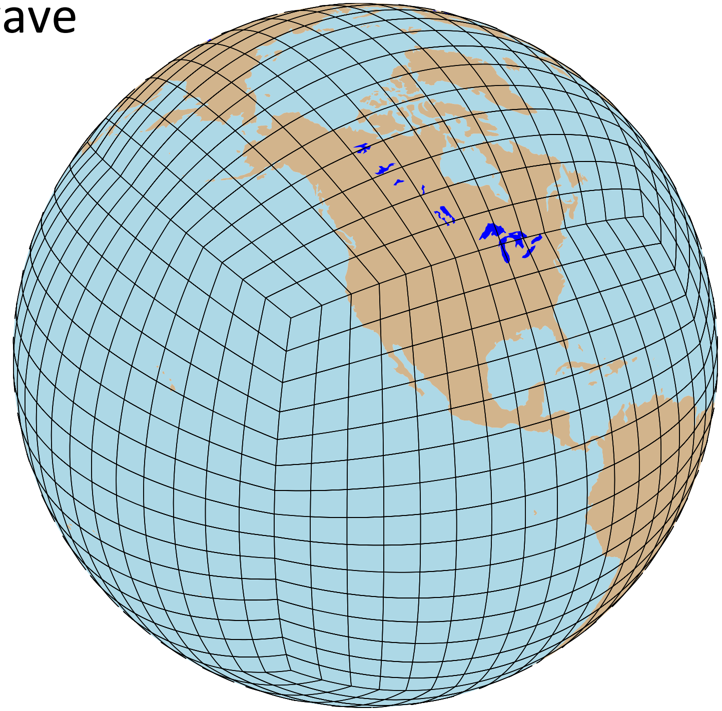
CAM-SE

Both dynamics and tracer advection use vertical Lagrangian remap => 2D only



Scalability, mimetic properties

Stabilization needed for both damping of 2dx wave and modeling enstrophy cascade



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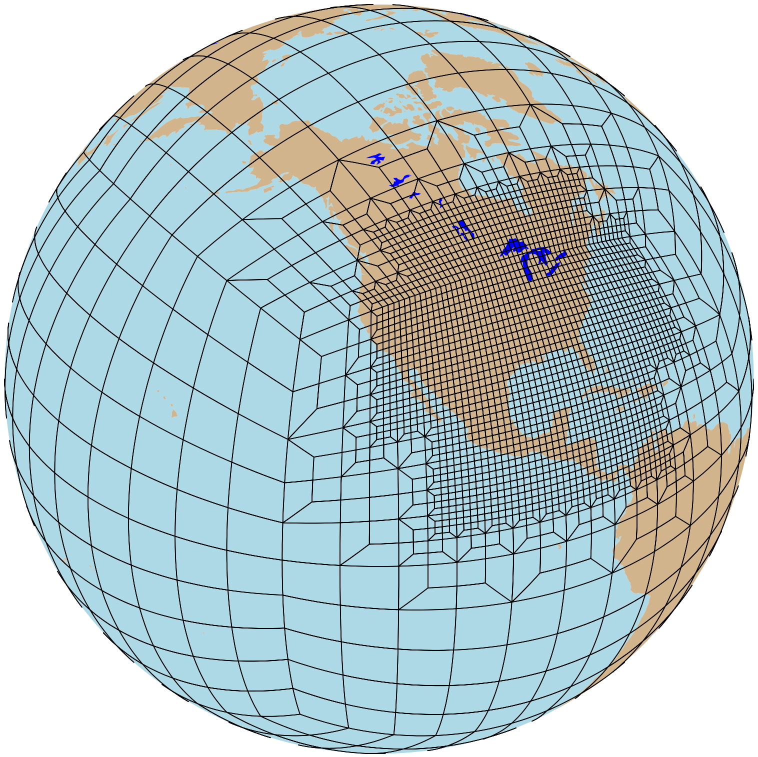
Why variable meshes for climate community

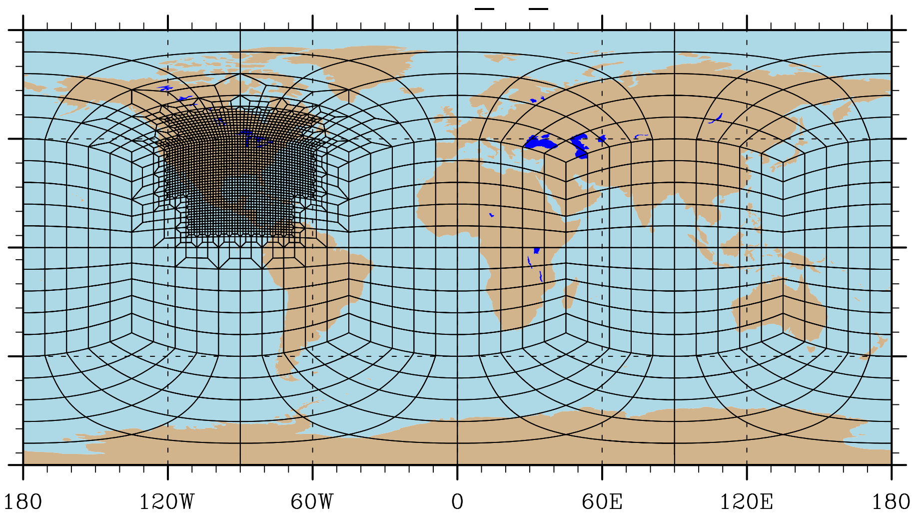
Goals: resolve fine scales with not-so-high cost to calibrate parametrizations, developing parametrizations, forecast, etc.

Example: Variable resolution runs are 10-100 times faster, hundreds of runs are needed. For the mesh below, approximately 46 times less DOFs.

Mesh refined 8 times, from 333 km to 42 km



(a) orthographic



(b) stereographic

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Hyper-viscosity (HV)

Stabilization technique for tracers is $\nu\Delta^2 q$ and for vector fields $\nu\Delta^2\vec{u}$
Coefficient ν scales like $(\Delta x)^{-p}$ with $p = 4$ or $p = 3.2$

Works well. Problem is highly-distorted elements with uneven scales.

In CAM-SE, HV is implemented by

$$\int_{sphere} \phi_i q_t = \int_{sphere} \phi_i \Delta q = -\int_{sphere} \nabla\phi_i \cdot \nabla q$$

We focus on a local part:

$$\int_{element} \nabla\phi_i \cdot \nabla q$$

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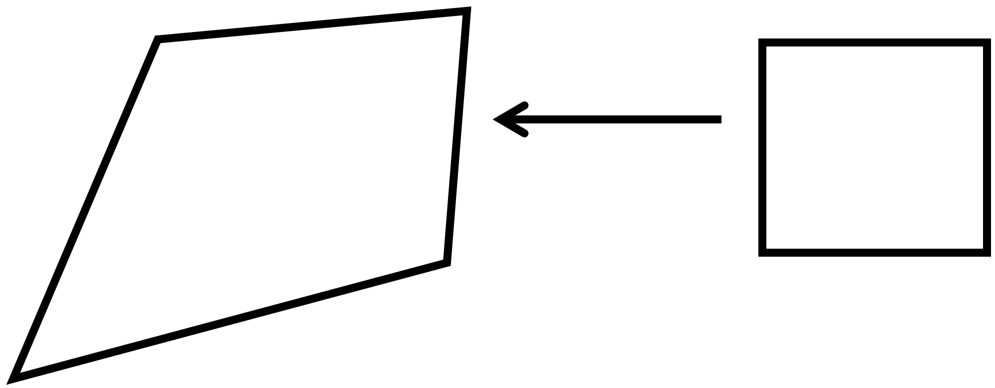
Elements in Physical and Reference Spaces

Transform:

$$x(\xi,\eta), y(\xi,\eta) \qquad \xi,\eta \in [-1,1]\times[-1,1]$$



$$\int_{element} \nabla_{xy}\phi_i \cdot \nabla_{xy} q$$

$$= \int_{[-1,1]\times[-1,1]} D^{-T}\nabla_{\xi\eta}\phi_i \cdot D^{-T}\nabla_{\xi\eta} q$$

$$= \int_{[-1,1]\times[-1,1]} \nabla_{\xi\eta}\phi_i \cdot D^{-1}D^{-T}\nabla_{\xi\eta} q$$

$$D = \begin{pmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{pmatrix}$$

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Dimensions from Metric Tensors

Focus on an inverse metric tensor

$$D^{-1}D^{-T} = (D^T D)^{-1} = E\Lambda E^T$$

$$E\Lambda E^T = E \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} E^T = E \begin{pmatrix} \left(\frac{2}{\Delta x}\right)^2 & 0 \\ 0 & \left(\frac{2}{\Delta y}\right)^2 \end{pmatrix} E^T$$

$\Delta x, \ \Delta y$ are interpreted as dimensions of an element

Tensor hyper-viscosity:

Instead of $\nabla_{\xi\eta} \ \phi_i \cdot D^{-1}D^{-T}\nabla_{\xi\eta} \ q$ take

$$\nabla_{\xi\eta} \ \phi_i \cdot D^{-1}\mathbf{V}D^{-T}\nabla_{\xi\eta} \ q$$

$$\mathbf{V} = DE \begin{pmatrix} \left(\frac{2}{\Delta x}\right)^{2-p} & 0 \\ 0 & \left(\frac{2}{\Delta y}\right)^{2-p} \end{pmatrix} E^T D^T$$

Tensor hyper-viscosity, motivation

In case of uniform elements, $(\Delta x)^p \Delta q$ leads to

$$\nabla_{\xi\eta}\ \phi_i \cdot E \begin{pmatrix} \left(\frac{1}{\Delta x}\right)^{2-p} & 0 \\ 0 & \left(\frac{1}{\Delta x}\right)^{2-p} \end{pmatrix} E^T \nabla_{\xi\eta}\ q$$

For distorted elements,

$$\nabla_{\xi\eta}\ \phi_i \cdot E \begin{pmatrix} \left(\frac{1}{\Delta x}\right)^{2-p} & 0 \\ 0 & \left(\frac{1}{\Delta y}\right)^{2-p} \end{pmatrix} E^T \nabla_{\xi\eta}\ q$$

Technicalities: we project all 4 elements of $\mathbf{V}$.
It is well-defined across elements' edges.

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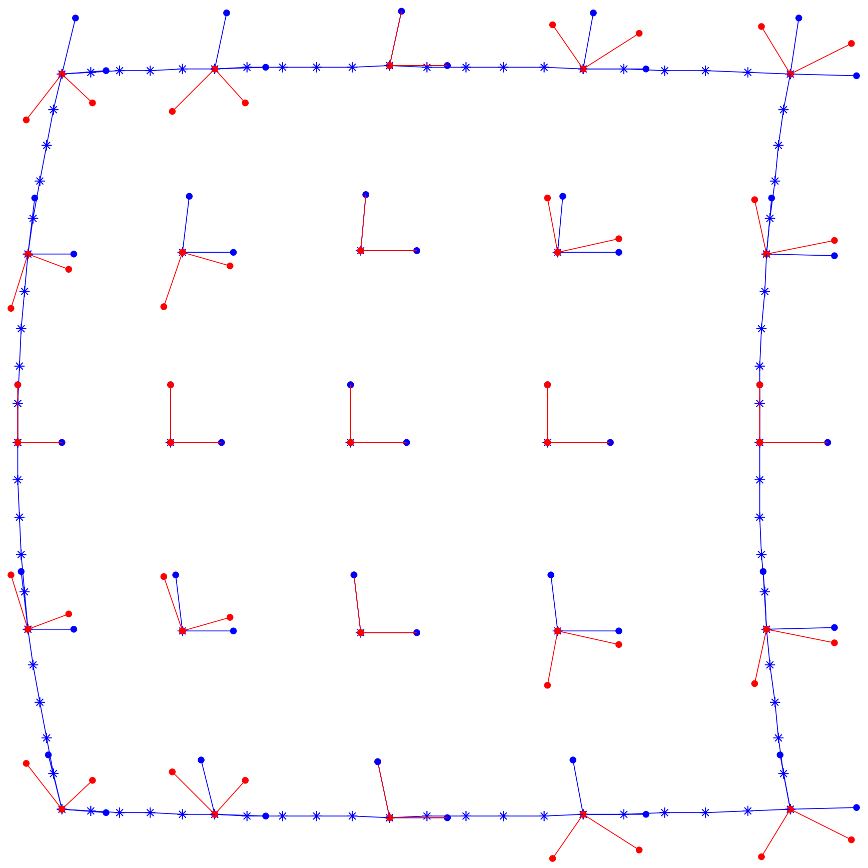
CFL, matrix E

CFL estimates follow from 1D analysis.

Columns of matrices E as vectors: 'uniform' quad on a sphere, blue segments represent covariant bases, red segments represent columns of matrix E.



Shallow Water Tests

Tests as in Williamson et al. (JCP, 1992)

Test Case #2: Global steady state nonlinear zonal geostrophic flow.

Convergence rates for a numerical scheme are expected to be same as in theory.

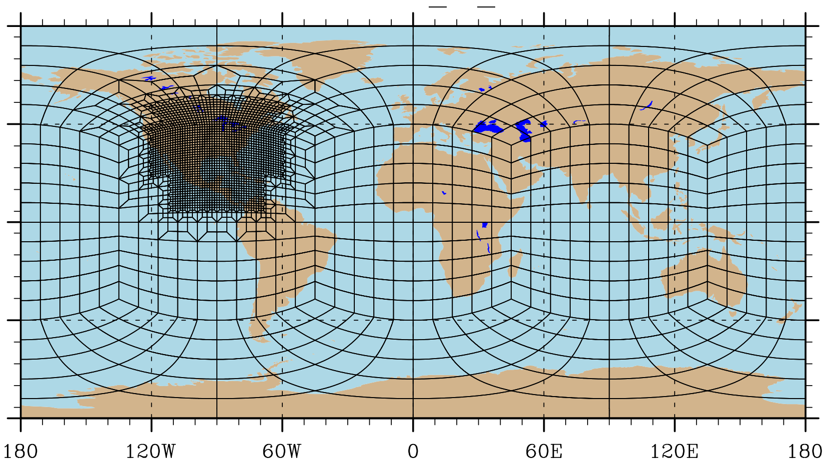
plots

Test Case #5: Zonal flow over a mountain

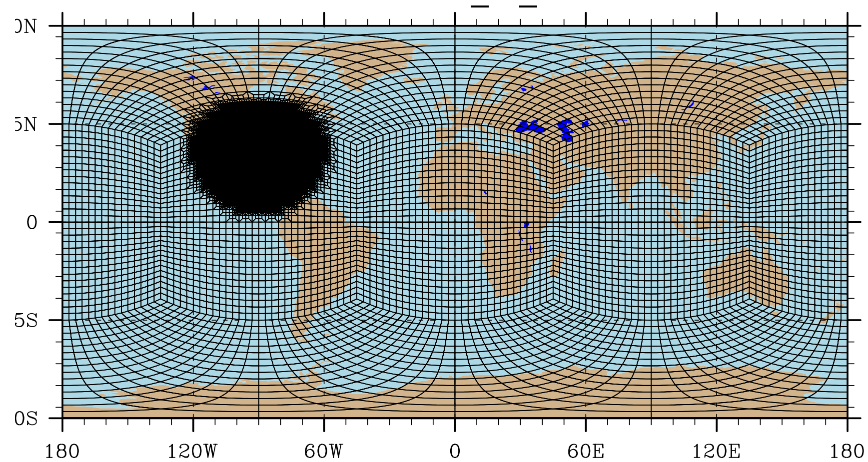
Analytic solution does not exist. Errors are obtained with a hi-res solution.

The mountain is given by C0 function. Theoretical convergence rates are not expected; vorticity field is examined for oscillations.

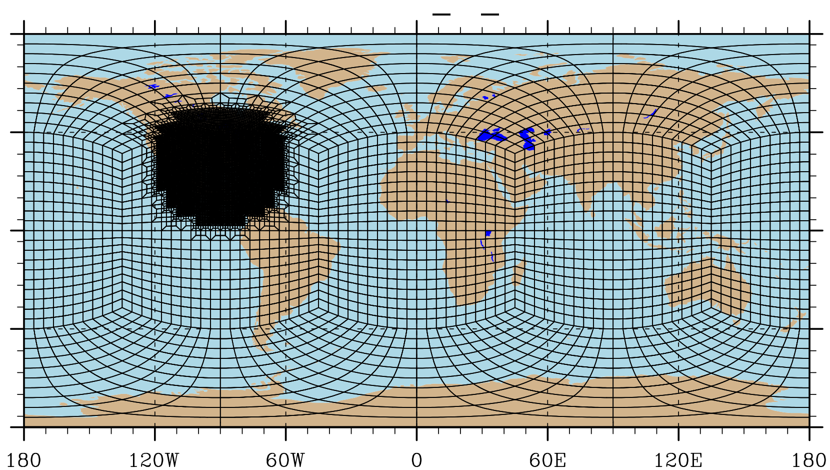
Meshes



Coarse resolution 333 km



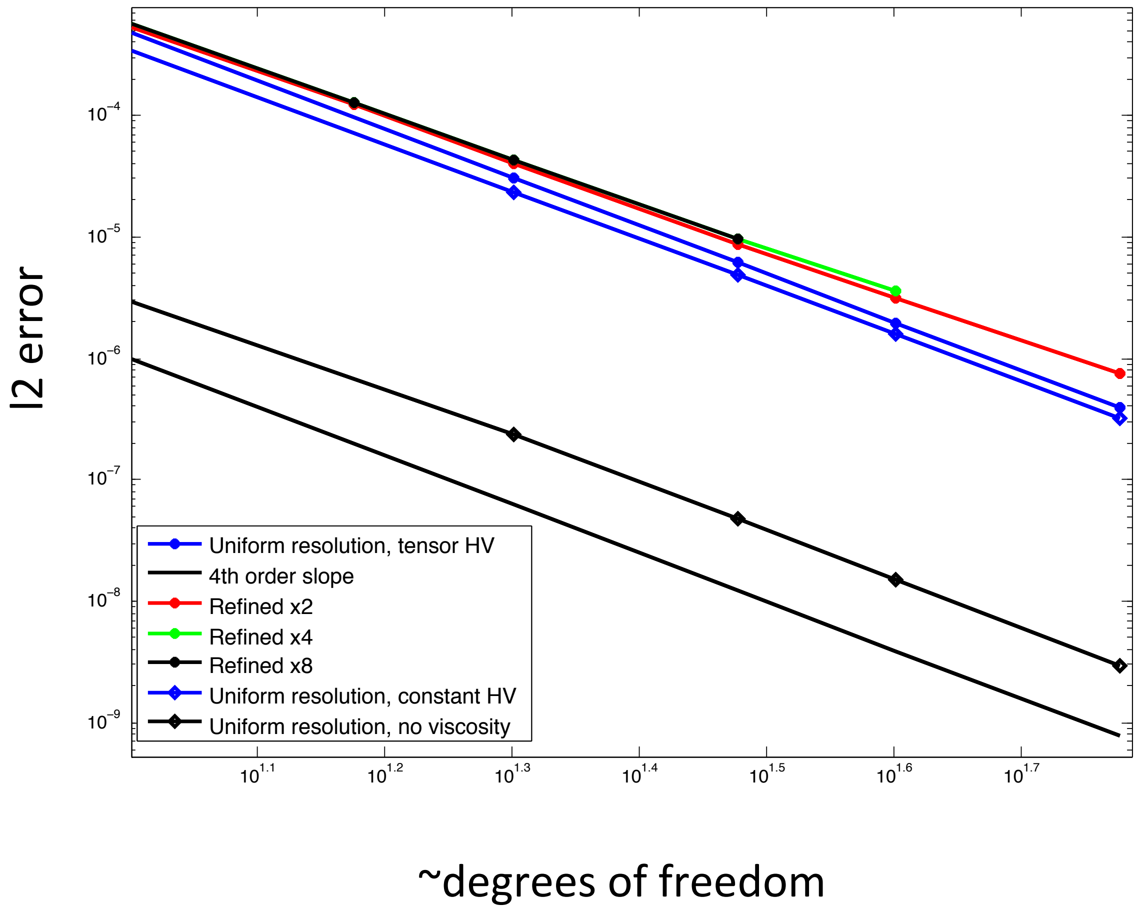
Coarse resolution 111 km



Coarse resolution 167 km

Goal is to show that refined meshes do 'no harm' compared to uniform meshes of the same coarse resolution

Convergence for TC2



Tensor HV, uniform resolution:
4th order

Refinement with x2:
3.7th order

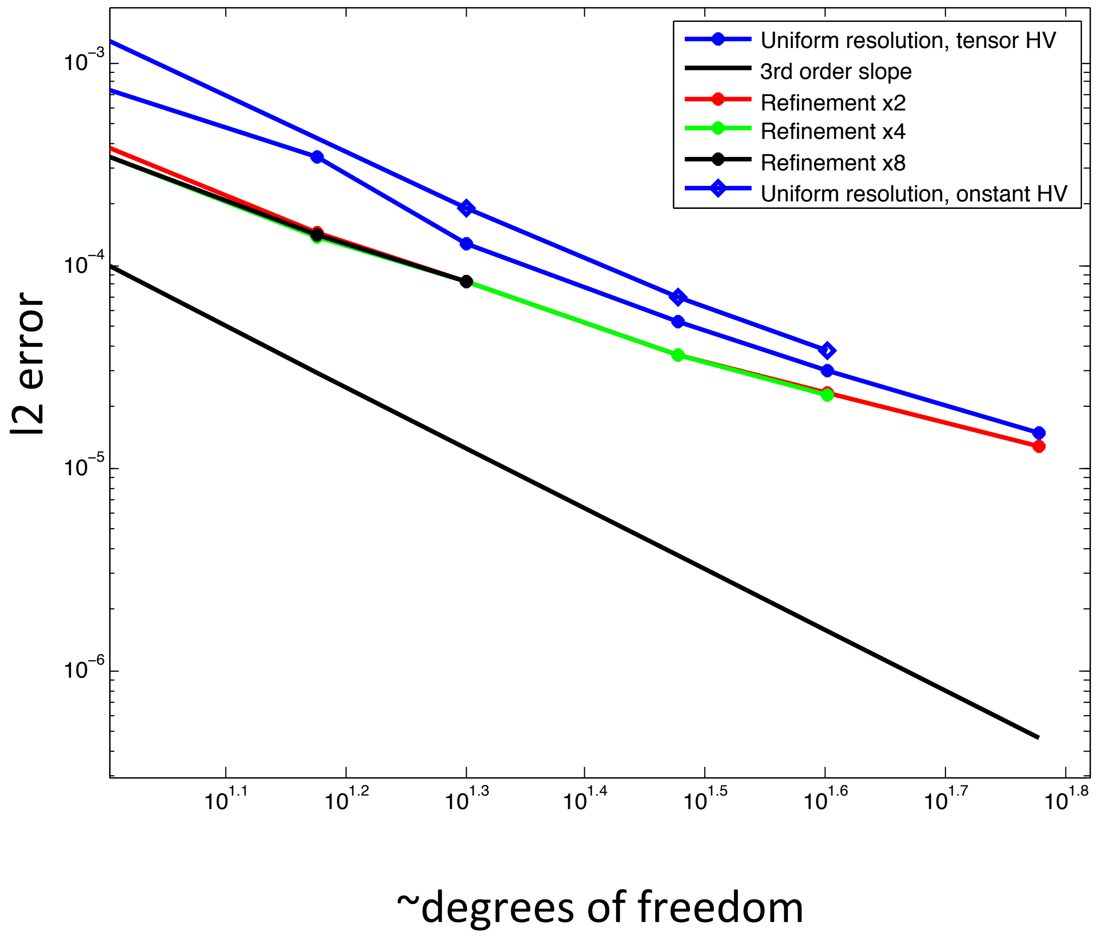
Refinement with x4:
3.7th order

Refinement with x8:
3.7th order

Constant (traditional) HV:
3.9th order

No hyperviscosity:
3.9th order

Performance of TC5



Note that refined regions are over the mountain which improves errors

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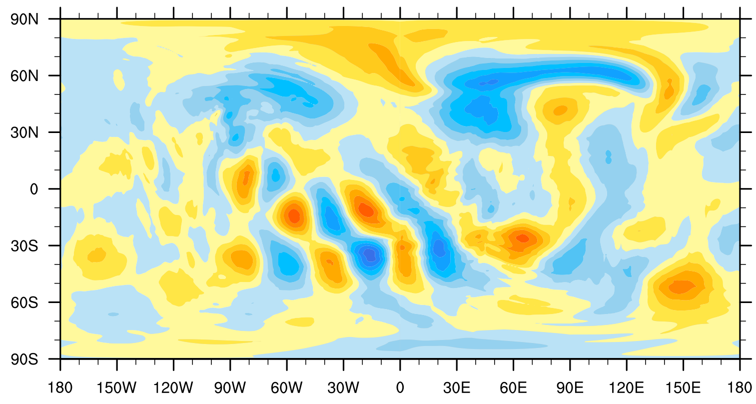
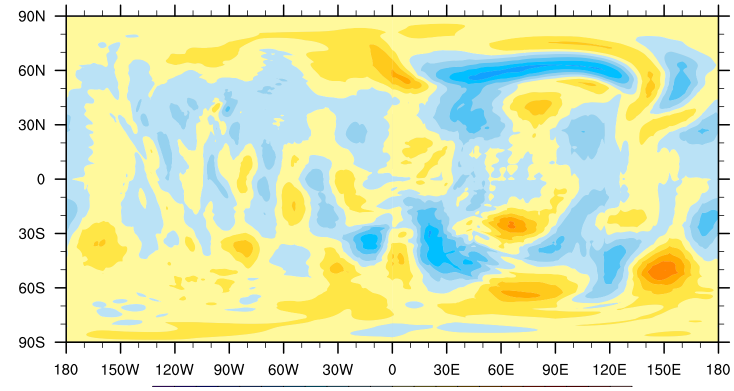
Performance of TC5 (cont.)

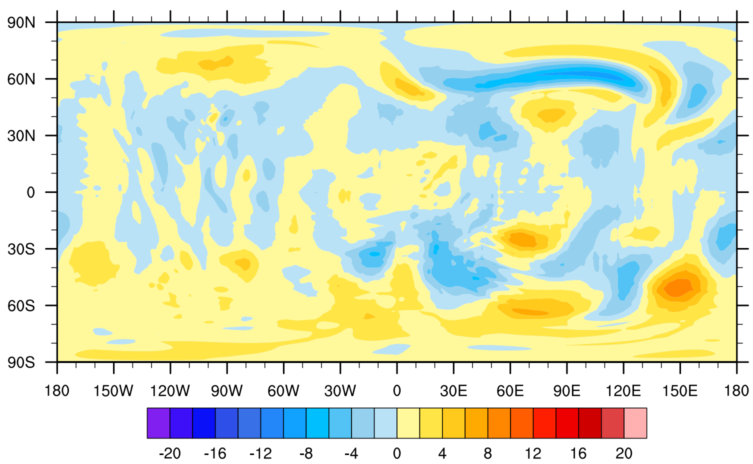
Uniform Mesh



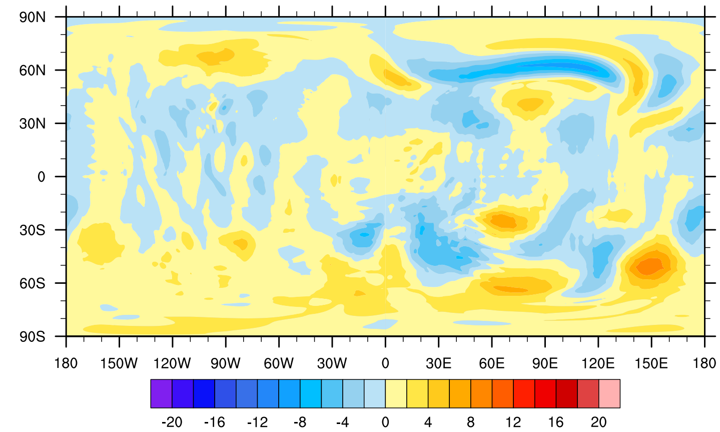
k=2 2x local refinement



k=4 4x local refinement



K=8 8x local refinement



- Refinement over mountain has almost zero impact on error
- Local refinement simulations have slightly smaller error – probably due to differences in hyperviscosity operator not mesh refinement (tensor vs. const. coeff)

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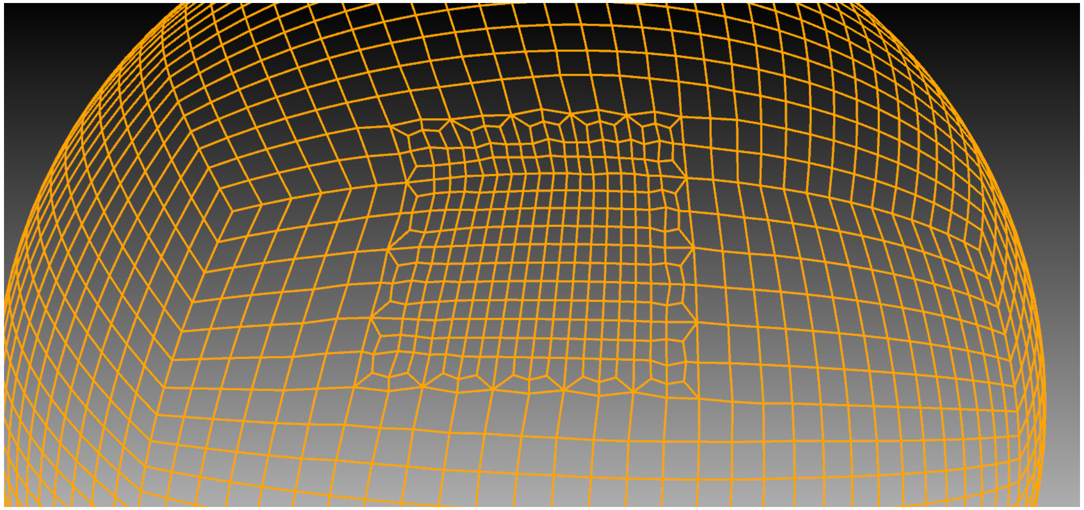
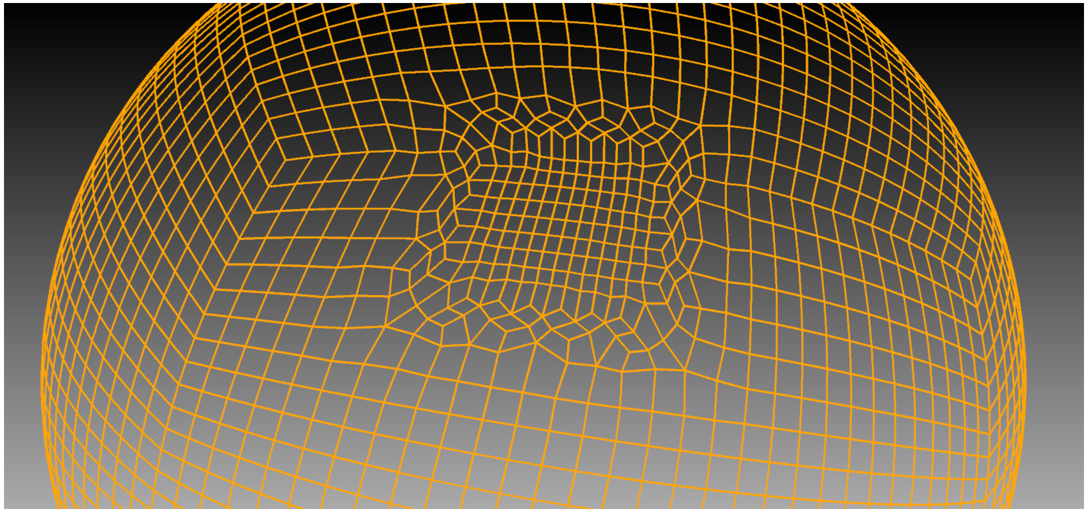
Things To Do



CUBIT refined mesh



Refinement 'by hand'

Tensor $\mathbf{V}$ smoothness
Is important

Quality of meshes
(less nodal valence,
smoothness) will improve
results further

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