SAND2013-1712C

Exceptional service in the national interest

Sandia
National
Laboratories

Error correction in adiabatic quantum computation

Kevin Young, Mohan Sarovar, Robin Blume-Kohout

Sandia National Laboratories

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Outline

- Introduction to adiabatic quantum computation
  - Why is this an interesting model for quantum computation?
- Introduction to quantum stabilizer codes
  - Error correction in circuit model QC
- Suppression schemes in AQC
  - Zeno effect
  - Dynamical decoupling
  - Energy gap protection
- Master equation describing evolution
- Limitations of error suppression
  - Final state decoding difficult “almost certain to be out of the code space”
- A closer look at stabilizer codes
- Limitations of error correction
  - Self-correcting quantum memories

Analog simulation

- We want to implement some (time-dependent?) Hamiltonian
  - **Adiabatic**

$$H(t) = f(t)H_0 + g(t)H_1$$

$$H_0 = \sum_i \epsilon_i X_i$$

$$H_{\mathrm{QUBO}} = \sum_i \alpha_i Z_i + \sum_{ij} \beta_{ij} Z_i Z_j$$

  - **Heisenberg**

$$H = J \sum_{\langle i,j \rangle} X_i X_j + Y_i Y_j$$

  - **Hubbard**

$$H = \sum_i U_i n_{i,\uparrow} n_{i,\downarrow} + \sum_{i,j} \sum_{\sigma=\uparrow,\downarrow} t_{ij} c^\dagger_{i,\sigma} c_{i,\sigma}$$

- Calculate some properties
  - Correlation functions
  - Energy gaps
  - Expectation values of operators

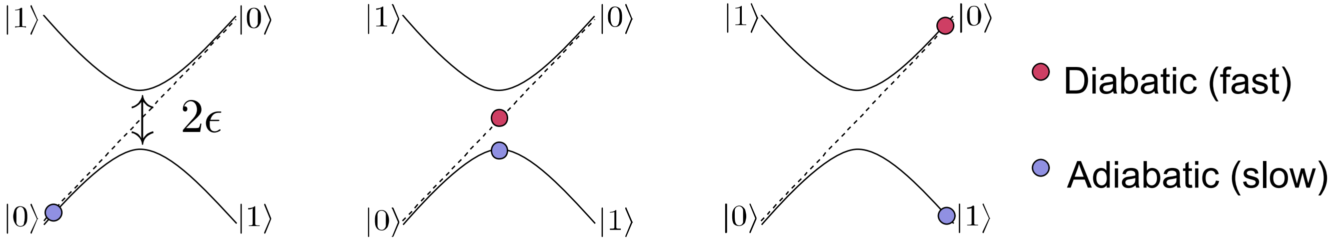
What can go wrong?

- Well, pretty much everything.
  - You implement the wrong Hamiltonian (**solve the wrong problem**)

$$H = 0.7532 \sum_i X_i + 0.7011 \sum_i X_i X_{i+1}$$

$$H = 0.6923 \sum_i X_i + 0.739 \sum_i Z_i Z_{i+1}$$

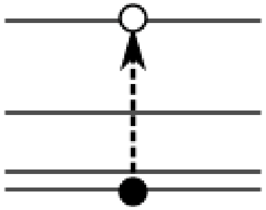
  - Blow through an adiabatic transition (**control errors**)



  - Coupling to environmental degrees of freedom (**noise**)

$$H(t) = H_{\mathrm{AQC}}(t) + \sum_k \gamma_k E_k \otimes B_k + H_{\mathrm{Bath}}(t)$$

$$H(t) = H_{\mathrm{AQC}}(t) + \sum_k \gamma_k \eta_k(t) E_k$$



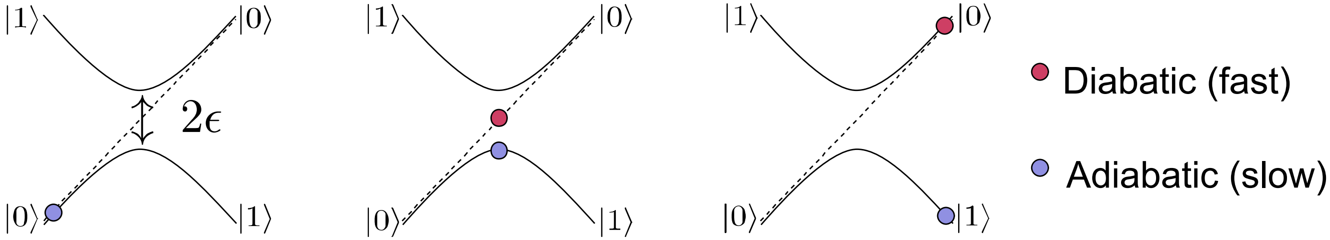
What can go wrong?

- Well, pretty much everything.
  - You implement the wrong Hamiltonian (**solve the wrong problem**)

$$\xcancel{\begin{gathered} H = 0.7532 \sum_i X_i + 0.7011 \sum_i X_i X_{i+1} \\ H = 0.6925 \sum_i X_i + 0.739 \sum_i Z_i Z_{i+1} \end{gathered}}$$

Tomography?

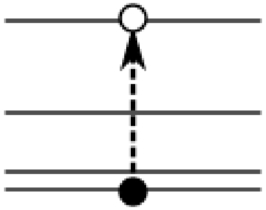
  - Blow through an adiabatic transition (**control errors**)



  - Coupling to environmental degrees of freedom (**noise**)

$$H(t) = H_{\mathrm{AQC}}(t) + \sum_k \gamma_k E_k \otimes B_k + H_{\mathrm{Bath}}(t)$$

$$H(t) = H_{\mathrm{AQC}}(t) + \sum_k \gamma_k \eta_k(t) E_k$$



What can go wrong?

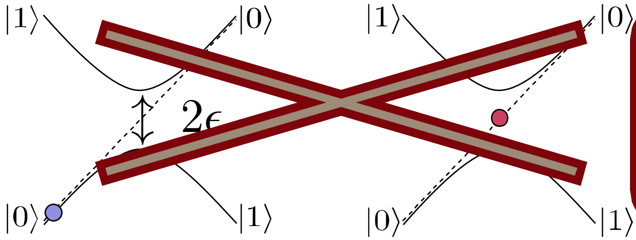
- Well, pretty much everything.
  - You implement the wrong Hamiltonian (**solve the wrong problem**)

$$\xcancel{H = 0.7532 \sum_i X_i + 0.7011 \sum_i X_i X_{i+1}}$$

$$\xcancel{H = 0.6625 \sum_i X_i + 0.739 \sum_i Z_i Z_{i+1}}$$

Tomography?

  - Blow through an adiabatic transition (**control errors**)

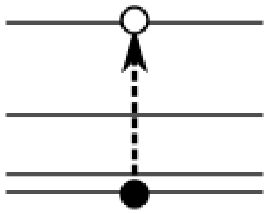


- Move slower (at certain places)
- Increase Hamiltonian magnitude

  - Coupling to environmental degrees of freedom (**noise**)

$$H(t) = H_{\mathrm{AQC}}(t) + \sum_k \gamma_k E_k \otimes B_k + H_{\mathrm{Bath}}(t)$$

$$H(t) = H_{\mathrm{AQC}}(t) + \sum_k \gamma_k \eta_k(t) E_k$$



What can go wrong?

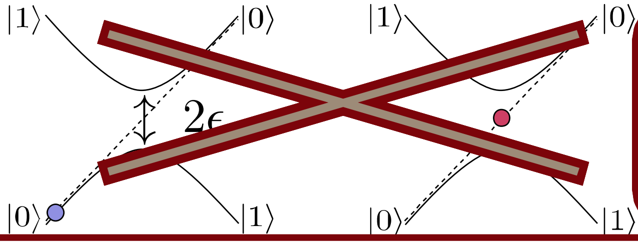
- Well, pretty much everything.
  - You implement the wrong Hamiltonian (**solve the wrong problem**)

$$\xcancel{H = 0.7532 \sum_i X_i + 0.7011 \sum_i X_i X_{i+1}}$$

$$\xcancel{H = 0.6625 \sum_i X_i + 0.739 \sum_i Z_i Z_{i+1}}$$

Tomography?

  - Blow through an adiabatic transition (**control errors**)

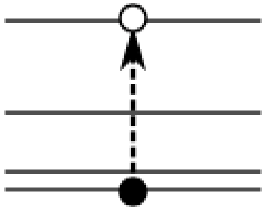


- Move slower (at certain places)
- Increase Hamiltonian magnitude

  - Coupling to environmental degrees of freedom (**noise**)

$$H(t) = H_{\mathrm{AQC}}(t) + \sum_k \gamma_k E_k \otimes B_k + H_{\mathrm{Bath}}(t)$$

$$H(t) = H_{\mathrm{AQC}}(t) + \sum_k \gamma_k \eta_k(t) E_k$$



Error detecting stabilizer codes

- Encode few logical qubits into many physical qubits
- Example: Bit flip code (*not a quantum code*)
  - Two stabilizer generators

$$\mathcal{S} = \langle ZZI, IZZ \rangle$$

Encoded states

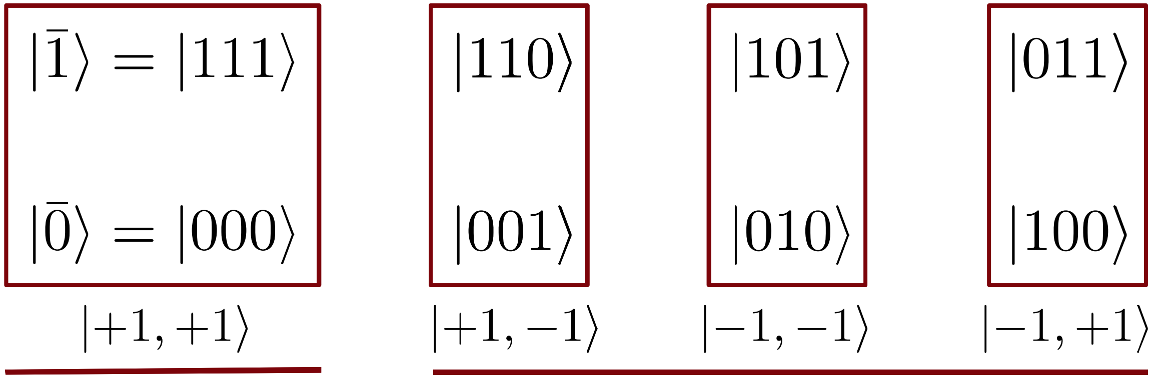
$$|\bar{0}\rangle = |000\rangle\,, \quad |\bar{1}\rangle = |111\rangle$$

  - Can detect (and correct)

$$E \in \{XII, IXI, IIX\}$$

Logical operator

$$\bar{X} = XXX$$



$$|\bar{1}\rangle = |111\rangle \qquad |110\rangle \qquad |101\rangle \qquad |011\rangle$$

$$|\bar{0}\rangle = |000\rangle \qquad |001\rangle \qquad |010\rangle \qquad |100\rangle$$

$$|+1,+1\rangle \qquad |+1,-1\rangle \qquad |-1,-1\rangle \qquad |-1,+1\rangle$$

code-space

error-spaces
labeled by ***syndromes***

Error detecting stabilizer codes

- A code comes with:
  - A set of mutually-commuting stabilizer generators
    - An error operator is ***detectable*** if it anti-commutes with at least one stabilizer generator.

$$E_{\mathbf{k}} S_i = (-1)^{\mathbf{k_i}} S_i E_{\mathbf{k}}$$

  - A set of logical operators
    - Commute with the stabilizer group, so there *exists a basis of simultaneous eigenstates*

$$\bar{L} S_i = S_i \bar{L}$$

  - +1 Eigenspace of the stabilizer generators defines the ***code space***
  - States outside the code space belong to an ***error space***
- *Stabilizer codes permit the detection of an error without destroying the encoded quantum information*

Suppressing errors

- Leads to several of strategies for suppressing errors:
  - **Energy Gap Protection**
    - Add stabilizer generators to Hamiltonian, increasing energy gap to excited states

$$H \rightarrow H + \sum_i \kappa_i S_i$$

S. Jordan, Error-correcting codes for adiabatic quantum computation, PRA 74, 052322 (2006)

    - If noise power spectrum drops exponentially, then so does the transition rate out of the code-space
  - **Dynamical Decoupling**
    - Apply stabilizer generators periodically as unitary operators

$$H \rightarrow S_i H S_i$$

D. Lidar, Towards fault tolerant adiabatic quantum computation, PRL, 100, 160506 (2008)

    - Error terms are suppressed in the average Hamiltonian
  - **Zeno Effect Suppression**
    - Frequently measure stabilizer generators
    - Zeno effect tends to keep the state in the code-space

Suppressing errors

- Leads to several of strategies for suppressing errors:
  - **Energy Gap Protection**
    - Add stabilizer generators to Hamiltonian, increasing energy gap to excited states

$$H \rightarrow H + \sum_i \kappa_i S_i$$

S. Jordan, Error-correcting codes for adiabatic quantum computation, PRA 74, 052322 (2006)

    - If noise power spectrum drops exponentially, then so does the transition rate out of the code-space
  - **Dynamical Decoupling**
    - Apply stabilizer generators periodically as unitary operators

$$H \rightarrow S_i H S_i$$

D. Lidar, Towards fault tolerant adiabatic quantum computation, PRL, 100, 160506 (2008)

    - Error terms are suppressed in the average Hamiltonian
  - ~~**Zeno Effect Suppression**~~
    - ~~Frequently measure stabilizer~~
    - ~~Zeno effect tends to keep the~~

- Shown by Lidar et. al to be mathematically equivalent to dynamical decoupling

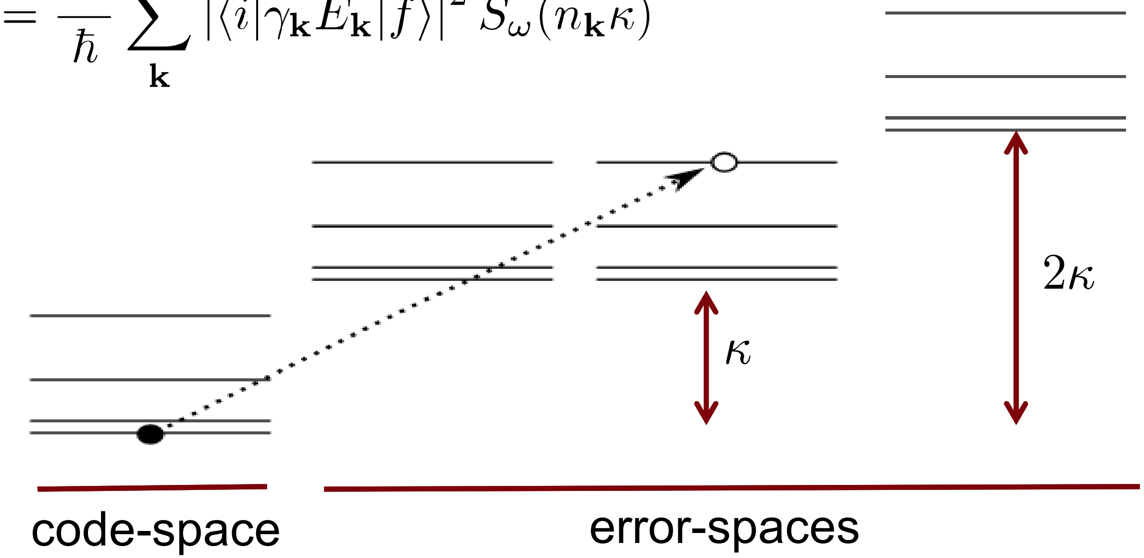
Energy Gap Protection

- Addition of stabilizer generators increases gap to error-subspaces

$$H \to H + \sum_i \kappa S_i$$

  - Fermi's golden rule says this will reduce the rate if the power spectrum is decreasing

$$\begin{aligned}\Gamma_{if} &= \frac{2\pi}{\hbar} \left|\langle i|H|f\rangle\right|^2 S_\omega(E_f - E_i) \\ &= \frac{2\pi}{\hbar} \sum_{\mathbf{k}} \left|\langle i|\gamma_{\mathbf{k}} E_{\mathbf{k}}|f\rangle\right|^2 S_\omega(n_{\mathbf{k}}\kappa)\end{aligned}$$



S. Jordan, Error-correcting codes for adiabatic quantum computation, PRA 74, 052322 (2006)

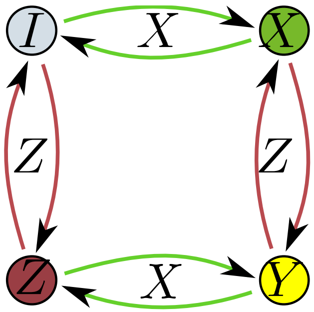
Dynamical Decoupling

- Detectable errors anti-commute with stabilizer generators
  - Therefore anti-commute with *exactly half* of the full stabilizer group, so

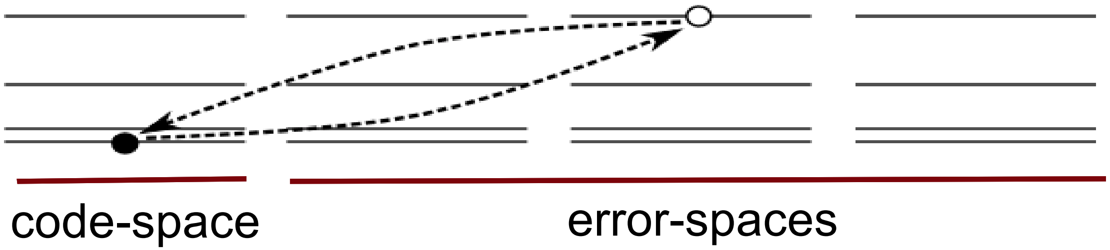
$$\sum_{s \in \langle \mathcal{S} \rangle} s E_{\mathbf{k}} s = 0$$

  - Apply the stabilizer generators as unitary operators such that they complete a Hamilton cycle on the Cayley graph of the stabilizer group.

$$\begin{aligned} H_{\text{int}} &\simeq \frac{1}{T}\int_0^T H_{\text{int}}(t)dt \\ &= \sum_{\mathbf{k}} \sum_{s} \gamma_{\mathbf{k}} s E_{\mathbf{k}} s \otimes B_{\mathbf{k}} \\ &= 0 \end{aligned}$$



$$\begin{aligned} &E_{\mathbf{k}} \\ \rightarrow &X E_{\mathbf{k}} X \\ \rightarrow &Z X E_{\mathbf{k}} X Z = Y E_{\mathbf{k}} Y \\ \rightarrow &X Z X E_{\mathbf{k}} X Z X = Z E_{\mathbf{k}} Z \end{aligned}$$



D. Lidar, Towards fault tolerant adiabatic quantum computation, PRL, 100, 160506 (2008)

DD and QES relationship

- The encoded system with control is,

$$H(t) = H_{\mathrm{AQC}}(t) + \sum_{\mathbf{k}} \gamma_{\mathbf{k}} E_{\mathbf{k}} \otimes B_{\mathbf{k}}(t) + \text{control}$$

$$\text{control} = \begin{cases} H_{\mathrm{QES}} = \frac{\Omega}{2} \sum_i S_i & \\ U_{\mathrm{DD}} \in \{S_i\} & \text{at frequency } \Omega \end{cases}$$

- Move to **interaction picture** with respect to control

$$\tilde{H}(t) = U_{\mathrm{ctrl}}(t)\ \left(H(t) - H_{\mathrm{ctrl}}(t)\right)\ U_{\mathrm{ctrl}}^{\dagger}(t)$$

$$\tilde{H}_{\mathrm{int}}(t) = \begin{cases} \sum_{\mathbf{k}} \gamma_{\mathbf{k}} (-1)^{g_{\mathbf{k}}(t)} E_{\mathbf{k}} \otimes B(t) & \mathrm{DD} \\ \sum_{\mathbf{k}} \gamma_{\mathbf{k}} \exp\left(2i\Omega t\, \mathbf{k} \cdot \vec{S}\right) E_{\mathbf{k}} \otimes B(t) & \mathrm{QES} \end{cases}$$

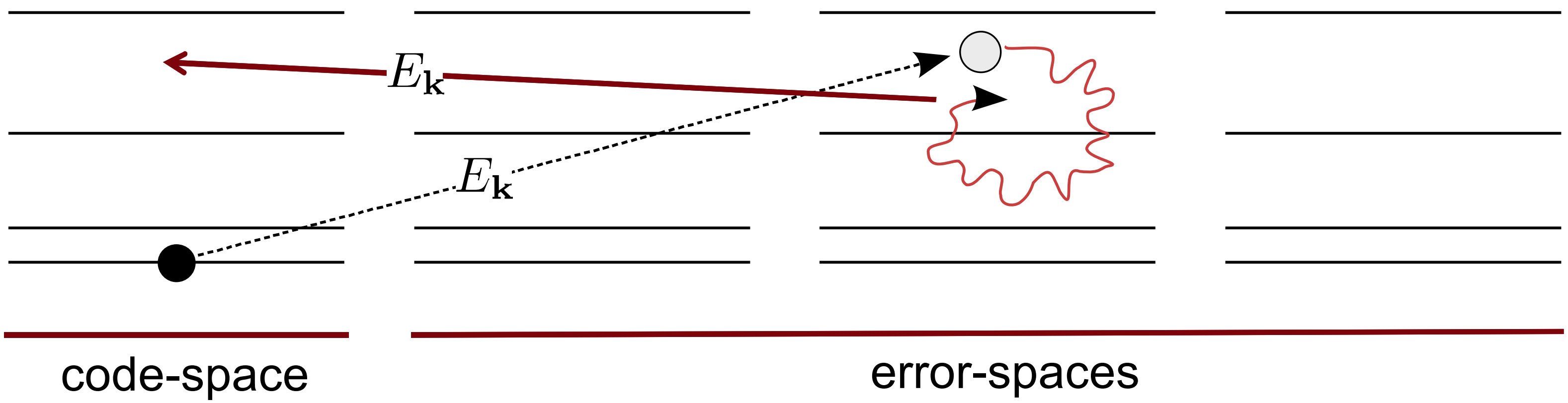
- In rotating frame, both QES and DD result in periodic modulation of noise.
- Effective noise rate is similar in both cases.
  - Fermi Golden Rule yields roughly equal leakage from the ground state.

What if an error **does** happen?

- Coherent evolution (by logical Hamiltonian) leads to mixing of states.
  - Hamiltonian and error operator don't exactly commute or anti-commute, so $E_{\mathbf{k}} |\psi\rangle$ is not necessarily an eigenstate. This state then evolves under the action of the logical Hamiltonian before it is corrected.

$$\boxed{\begin{gathered} E_{\mathbf{k}} e^{-i(H^+_{\mathbf{k}} + H^-_{\mathbf{k}})t} E_{\mathbf{k}} |\psi\rangle = e^{-i(H^+_{\mathbf{k}} - H^-_{\mathbf{k}})t} |\psi\rangle \\ H = H^+_{\mathbf{k}} + H^-_{\mathbf{k}} \qquad [E_{\mathbf{k}}, H^+_{\mathbf{k}}] = \{E_{\mathbf{k}}, H^-_{\mathbf{k}}\} = 0 \end{gathered}}$$

  - A low weight physical error gets "dressed" by the (always on) logical Hamiltonian and gets converted into a high-weight uncorrectable error
  - Diffusion rate obeys Lieb-Robinson bound, but connectivity of encoded system is high



Circuit model vs. Hamiltonians

- Error correcting codes behave very well in the circuit model
  - Stabilizer measurements destroy coherences between spaces

$$\boxed{|\psi\rangle = \sum_{\mathbf{k}} \alpha_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{k}\rangle = \sum_{\mathbf{k}} \alpha_{\mathbf{k}} E_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle}$$

Perform gate ⟶ $\bar{L}|\psi\rangle = \sum_{\mathbf{k}} \alpha_{\mathbf{k}} \bar{L} E_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle = \sum_{\mathbf{k}} (-1)^{g(\bar{L},\mathbf{k})} \alpha_{\mathbf{k}} E_{\mathbf{k}} \bar{L} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle$

Measure stabilizers ⟶ $|\psi\rangle \to \bar{L} |\psi_{\mathbf{k}}\rangle |\mathbf{k}\rangle \quad \text{with probability } \alpha_{\mathbf{k}}^2$

  - Logical operators are equivalent up to stabilizers

Perform gate ⟶ $\bar{L} S_i |\psi\rangle = \sum_{\mathbf{k}} \alpha_{\mathbf{k}} \bar{L} S_i E_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle = \sum_{\mathbf{k}} (-1)^{g(\bar{L},\mathbf{k}) + \mathbf{k}_i} \alpha_{\mathbf{k}} E_{\mathbf{k}} \bar{L} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle$

Measure stabilizers ⟶ $|\psi\rangle \to \bar{L} |\psi_{\mathbf{k}}\rangle |\mathbf{k}\rangle \quad \text{with probability } \alpha_{\mathbf{k}}^2$

Circuit model vs. Hamiltonians

- Stabilizer encodings don't play so well with the adiabatic model
  - Logical operators act as ***Hamiltonians*****,** so they can be ***added***: $H = \bar{L} + \bar{M}$

$$\boxed{|\psi\rangle = \sum_{\mathbf{k}} \alpha_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{k}\rangle = \sum_{\mathbf{k}} \alpha_{\mathbf{k}} E_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle}$$

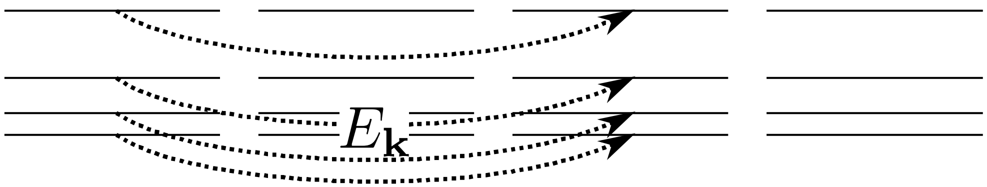
- Hamiltonian acts differently on each syndrome space

$$\begin{aligned} H|\psi\rangle &= \sum_{\mathbf{k}} \alpha_{\mathbf{k}} (\bar{L} + \bar{M}) E_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle \\ &= \sum_{\mathbf{k}} \alpha_{\mathbf{k}} E_{\mathbf{k}} ((-1)^{g(\bar{L},\mathbf{k})} \bar{L} + (-1)^{g(\bar{M},\mathbf{k})} \bar{M}) |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle \\ &= \sum_{\mathbf{k}} \alpha_{\mathbf{k}} E_{\mathbf{k}} H_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle \end{aligned}$$

- $|\psi_{\mathbf{k}}\rangle$ may not even be an eigenstate of $H_{\mathbf{k}}$

Protected Hamiltonians

- We want errors to map **eigenstates to eigenstates**



  - Want error to be pseudo-eigenoperator of Hamiltonian

$$[H, E_{\mathbf{k}}]\Pi_0 = \epsilon_{\mathbf{k}} E_{\mathbf{k}} \Pi_0$$

- Exploit freedom in choosing logical operators
  - **Multiplication by stabilizers** does not affect action on code space:

$$S_{\mathbf{j}} L \simeq L$$

    - Repetition code example:

$$ZZI \cdot XXX = YYX \simeq XXX$$

  - Construct **protected Hamiltonian** by conjugation with correctable errors

$$\bar{H} = \sum_{\mathbf{k}} E_{\mathbf{k}} \, H \, \Pi_0 \, E_{\mathbf{k}}$$

    - This is *terribly high weight*

Logical/Syndrome operators

- Think about syndrome degrees of freedom as *another subsystem*

$$U\,|q_1, q_2, \cdots, q_N\rangle = \sum_{\mathbf{i}} \alpha_{\mathbf{i}}\,|\psi_{\mathbf{i}}\rangle \otimes |\mathbf{i}\rangle$$

  - Naïve logical operators act *highly nontrivially* on syndrome subspace

$$\begin{aligned} L \rightarrow L \otimes (&IIII - IIIZ - IIZI + IIZZ - IZII + IZIZ + IZZI - IZZZ + \\ &ZIII + ZIIZ + ZIZI - ZIZZ + ZZII - ZZIZ - ZZZI + ZZZZ)/16 \end{aligned}$$

  - The protected Hamiltonian acts as the identity on the syndrome subsystem

$$\bar{H} \rightarrow H \otimes I \qquad\qquad \bar{H} = \sum_{\mathbf{k}} E_{\mathbf{k}}\, H\, \Pi_0\, E_{\mathbf{k}}$$

  - Summing equivalent logical operators reduces nontrivial behavior
    - Sums of all min-weight logical operators give effective operators of the form

$$\sum_{\text{min weight}} L \rightarrow L \otimes (\Pi_0 + I)$$

Protected operators: [[5,1,3]] code

- The quantum [[5,1,3]] code is usually defined by the operators

$$\mathcal{S} = \langle IXZZX, XIXZZ, ZXIXZ, ZZXIX \rangle$$

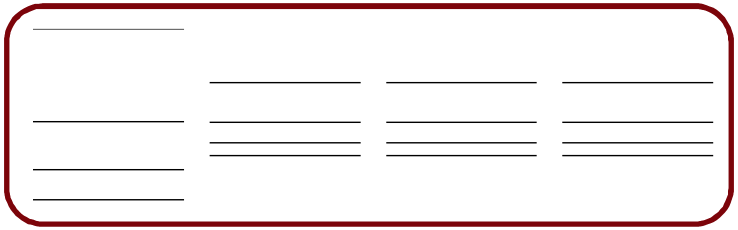
$$\bar{Z} = ZZZZZ \quad \bar{X} = XXXXX$$

- But standard logical operators *do not act consistently across subspaces*

$$\begin{aligned} 4\,U[\bar{X}]U^\dagger = X \otimes (&-IIII + IIIZ + IIZI + IIZZ + IZII - IZIZ + IZZI + IZZZ + \\ &ZIII - ZIIZ - ZIZI - ZIZZ + ZZII - ZZIZ + ZZZI + ZZZZ) \\ 4\,U[\bar{Z}]U^\dagger = Z \otimes (&-IIII + IIIZ + IIZI - IIZZ + IZII + IZIZ - IZZI - IZZZ + \\ &ZIII + ZIIZ + ZIZI + ZIZZ - ZZII + ZZIZ - ZZZI + ZZZZ) \end{aligned}$$

- Improved operators act consistently

$$\begin{aligned} \bar{X}_3 =& IIZXZ + IXIYY + IYYIX + IZXZI + XIYYI + \\ & XZIIZ + YIXIY + YYIXI + ZIIZX + ZXZII \\ \bar{Z}_3 =& IIYZY + IXXIZ + IYZYI + IZIXX + XIZIX + \\ & XXIZI + YIIYZ + YZYII + ZIXXI + ZYIIY \end{aligned}$$



$$\boxed{\begin{aligned} U\bar{X}_3U^\dagger &= -\frac{1}{2}X \otimes (4\Pi_0 + \mathbb{I}) \\ U\bar{Z}_3U^\dagger &= -\frac{1}{2}Z \otimes (4\Pi_0 + \mathbb{I}) \end{aligned}}$$

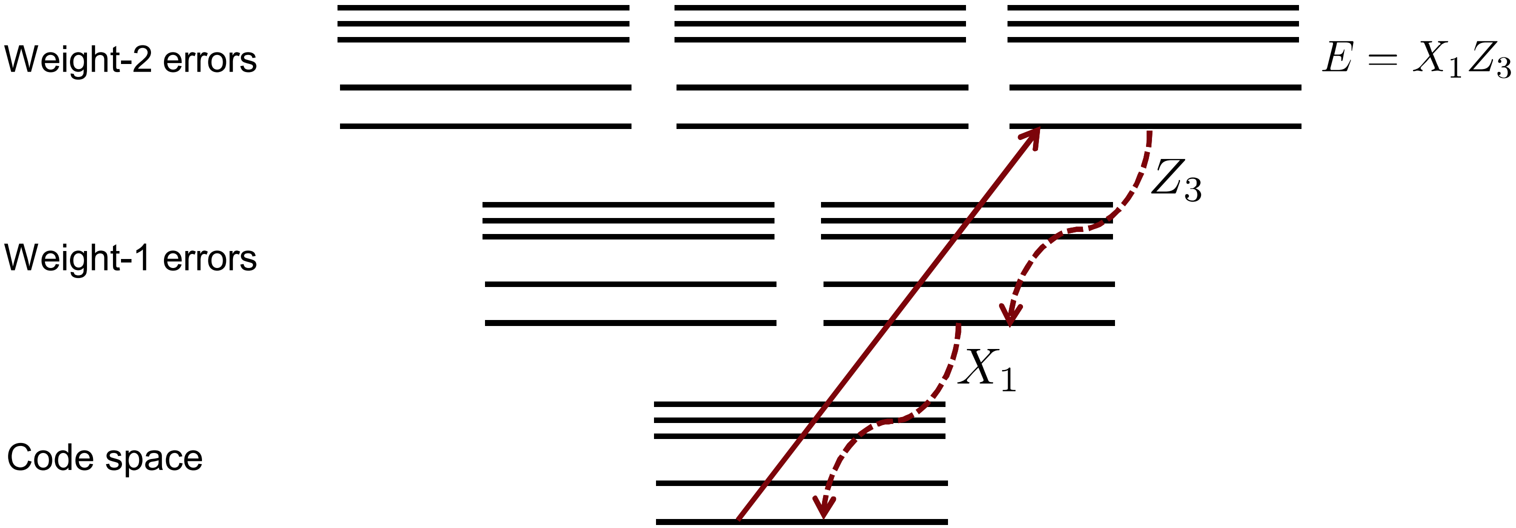
What about adding error correction?

- A continuous form of error correction is appealing for AQC – meshes better with the analog simulation philosophy.

- Stabilizer Hamiltonians, Hamiltonians composed entirely of stabilizer elements, naturally fit into a continuous error correction framework because:
    - Local error operators promote the ground state to excited energy eigenstates (logical Hamiltonian = 0 in this case).
    - Hence these errors can be reversed (corrected) by another application of a local operator.
    - We can *cool* the system by embedding it in a cold reservoir that couples locally and linearly to the system.
    - As long as the temperature of this reservoir can be maintained below the noise temperature, the system will be protected (error correction will overcome noisy fluctuations).
    - NOTE: The localized cooling implementation of error correction is effective because of the local structure of stabilizer Hamiltonian excitations.

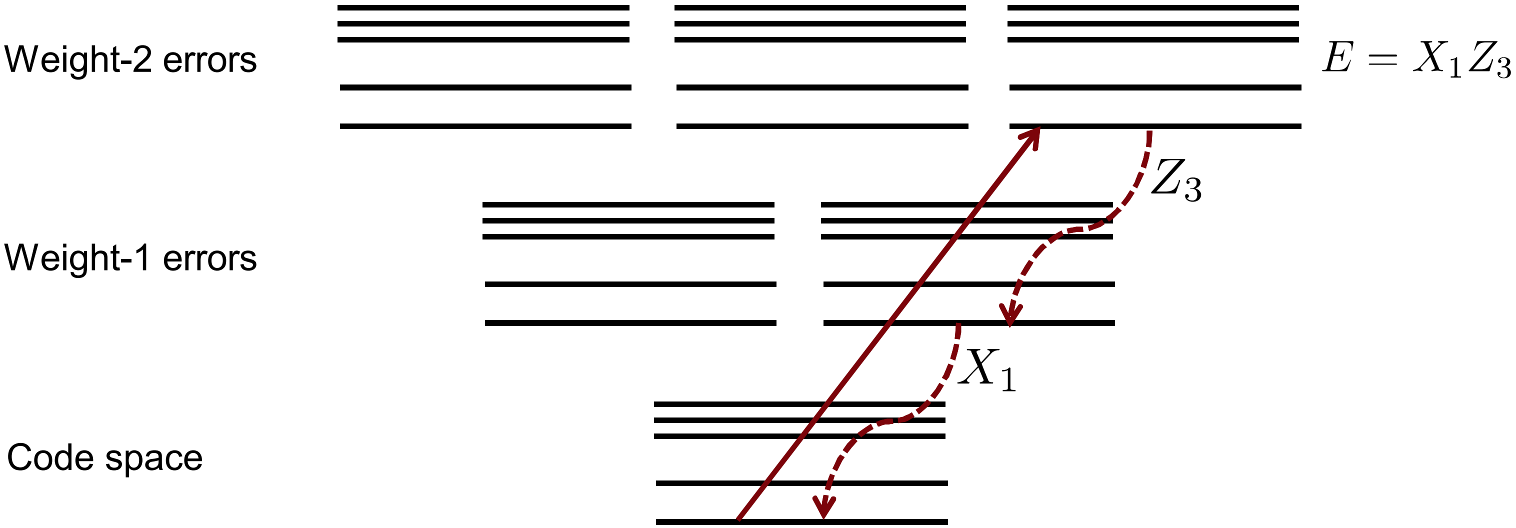
Ideal error correction

- Errors are penalized according to their **weight**



Ideal error correction

- Errors are penalized according to their **weight**



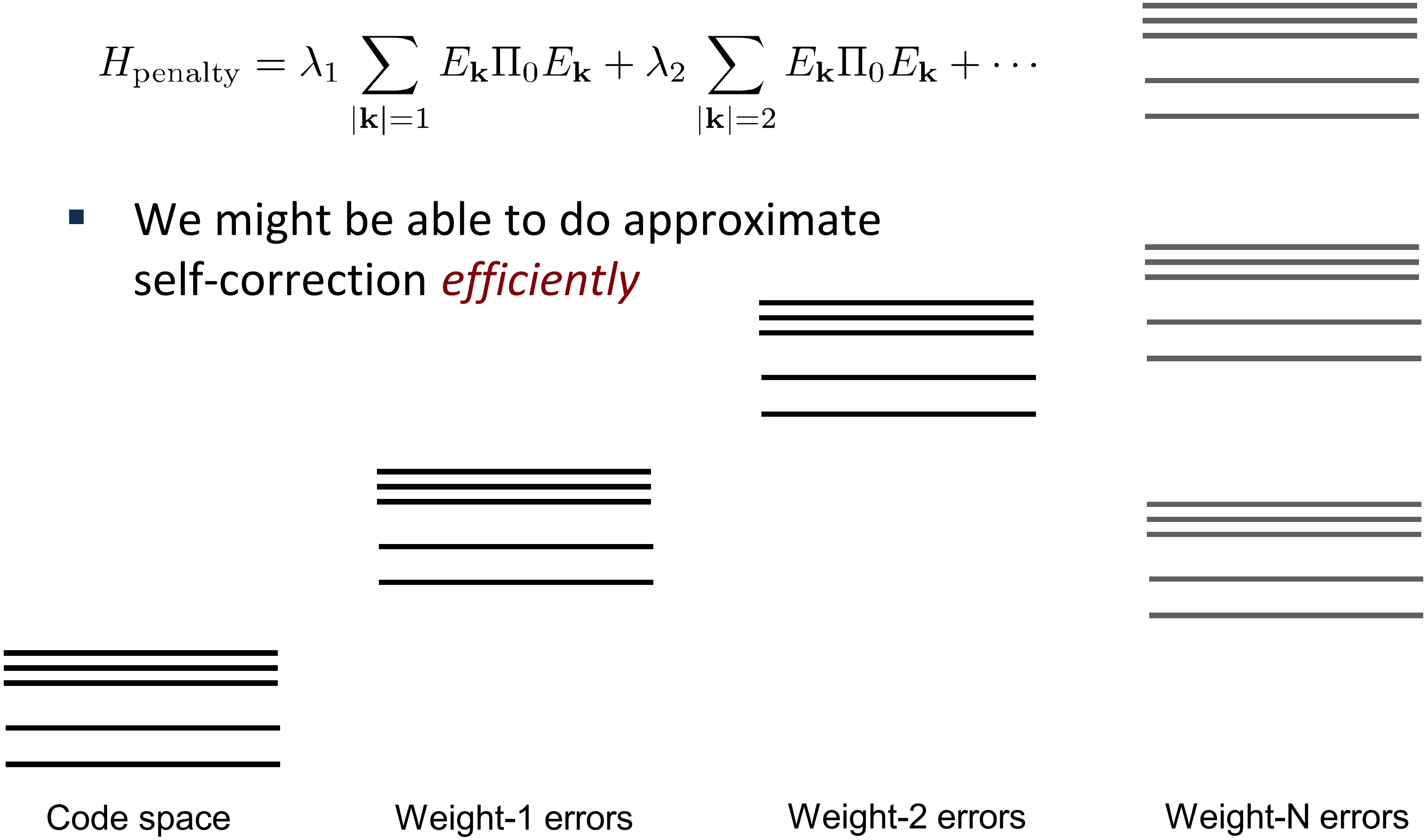
- But this is a self **correcting quantum memory**
  - No-go theorem in 2D: Bravyi and Terhal
  - Imbed toric code in ferromagnet to get string tension: Loss

Ideal error correction

- But we can produce **incomplete** self-correcting codes

$$H_{\text{penalty}} = \lambda_1 \sum_{|\mathbf{k}|=1} E_{\mathbf{k}} \Pi_0 E_{\mathbf{k}} + \lambda_2 \sum_{|\mathbf{k}|=2} E_{\mathbf{k}} \Pi_0 E_{\mathbf{k}} + \cdots$$

- We might be able to do approximate self-correction *efficiently*



Summary

- Error suppression techniques in AQC are fundamentally equivalent to one another
  - Performance characteristics of one can be mimicked by another
- Stabilizer codes act differently in the adiabatic setting
  - Uncorrected operators can scramble erred states
  - Operators can be ***protected*** so they act consistently across subspaces
    - May enable active error correction
- For more information, see:
  - ***Equivalence and limitations of error suppression techniques for adiabatic quantum computing,*** Kevin Young and Mohan Sarovar, quant-ph/1208.6371

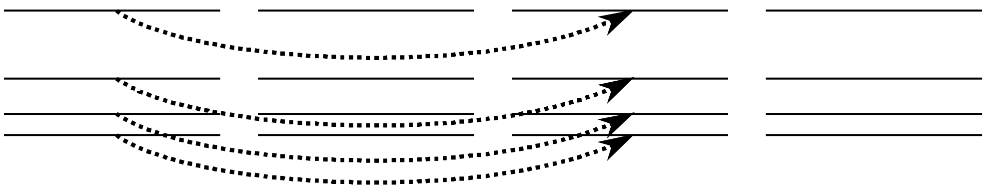
Logical/Syndrome operators

- Define operators which convert physical state to logical/syndrome state

$$U\left|q_1, q_2, \cdots, q_N\right\rangle = \sum_{\mathbf{i}} \alpha_{\mathbf{i}} \left|\psi_{\mathbf{i}}\right\rangle \otimes \left|\mathbf{i}\right\rangle$$

  - States in the code space are defined *in terms of the error operators*

$$\left|\psi\right\rangle \otimes \left|\mathbf{k}\right\rangle \equiv U\left[E_{\mathbf{k}}\right]U^{\dagger}\left|\psi\right\rangle \otimes \left|\mathbf{0}\right\rangle$$



  - We want to find logical operators that respect this definition:
    - Recall that previous logical operators did not, and this caused all kinds of problems
  - The operators must obey

$$U\left[\bar{L}\right]U^{\dagger}\left|\psi\right\rangle\left|\mathbf{k}\right\rangle = U\left[\bar{L}\right]U^{\dagger}\left|\psi\right\rangle\left|\mathbf{0}\right\rangle$$

$$U\left[\bar{L}\right]U^{\dagger} = L \otimes \mathbb{I}$$

Logical/Syndrome operators

- We can construct these protected operators explicitly

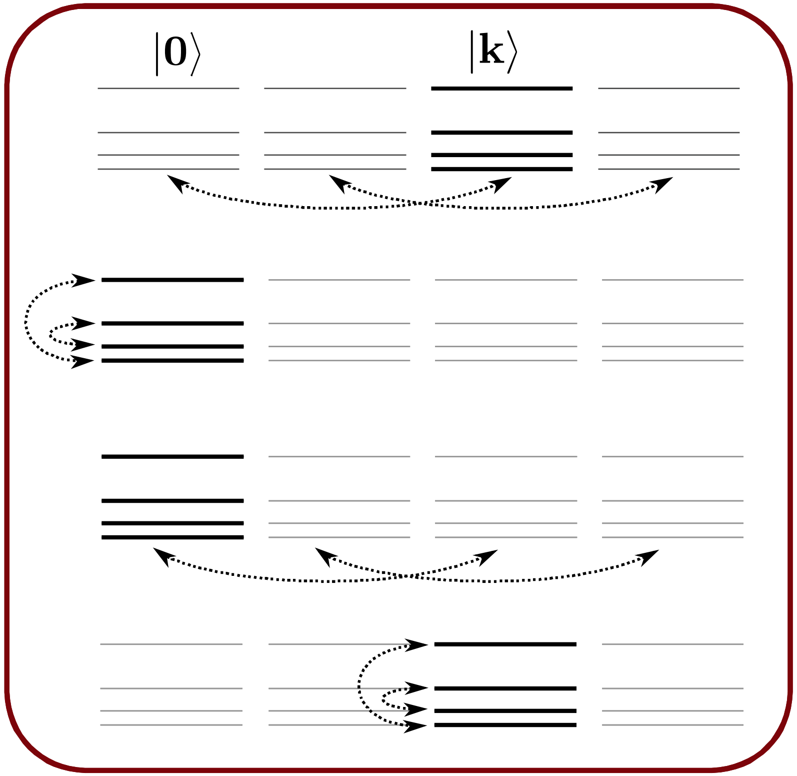
$$L \otimes \mathbb{I} = U \left[\sum_{\mathbf{k}=\mathbf{0}}^{\mathbf{1}} E_{\mathbf{k}} \bar{L}\, \Pi_{\mathbf{0}} E_{\mathbf{k}} \right] U^{\dagger}$$

  - Where the code-space projector is:

$$\Pi_{\mathbf{0}} = \prod_j \frac{1}{2}(1 + S_j)$$

  - The syndrome operator for a given logical operator is then:

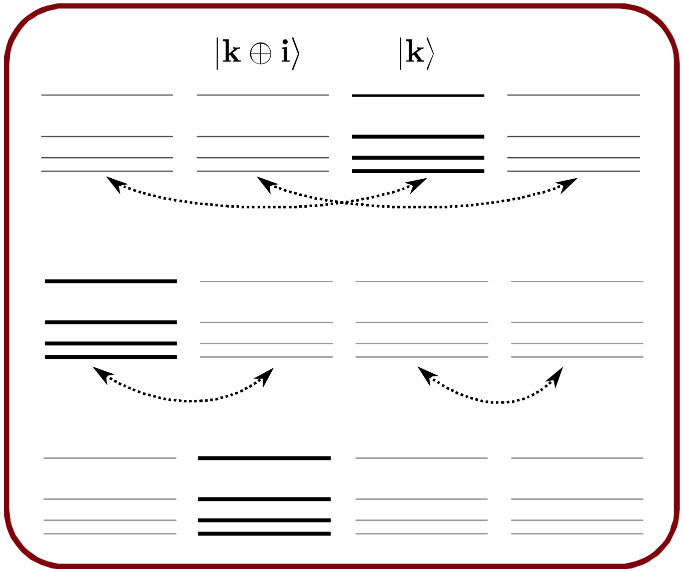
$$\mathbb{I} \otimes S_{\bar{L}} = U \left[\bar{L} \sum_{\mathbf{k}} E_{\mathbf{k}} \bar{L} \Pi_{\mathbf{0}} E_{\mathbf{k}} \right] U^{\dagger}$$



Logical/Syndrome operators

- Construct the syndrome operators explicitly:
  - Syndrome Z-operators are the stabilizer generators

$$\mathbb{I} \otimes Z_{\mathbf{k}} = U \left[\prod_j S_j^{\mathbf{k}_j} \right] U^\dagger$$



  - Syndrome X operators are

$$\mathbb{I} \otimes X_{\mathbf{k}} = U \left[\sum_{\mathbf{i}=\mathbf{0}}^{\mathbf{1}} E_{\mathbf{i}\oplus\mathbf{k}} \Pi_0 E_{\mathbf{i}} \right] U^\dagger$$

  - This gives a full set of operators for a ***subsystem encoding***

$$\mathcal{H} = \mathbb{C}_2^{\otimes N} = \mathcal{H}_{\mathbf{0}} \oplus \mathcal{H}_{\text{err}} = \mathcal{H}_{\text{logic}} \otimes \mathcal{H}_{\text{synd}}$$

“Solution” 1: Timescales

- We want the error correction procedure to be effective
    - Between error correction steps, the state must *not* evolve out of the correctable space
    - Equivalent to demanding that

$$\tau \ll \frac{\hbar}{||H_{\mathrm{logical}}||}$$

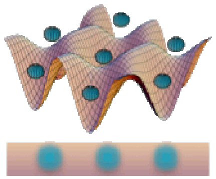
    - But the TOTAL time that the system runs is proportional to the norm of the logical Hamiltonian (increase norm, decrease run time), implying that no matter the norm, you will always require the same number of error correction steps.
    - If the error correction steps are driven by coupling to a bath, then the coupling should be such that

$$\frac{J\tau}{\hbar} \simeq \frac{\pi}{2}$$

        - This ensures a complete step is completed in time τ
- If this is fast enough, the resonance issue may be taken care of by lifetime broadening of the resonance line.

AQUARIUS

- **A**diabatic **QU**antum **AR**chitectures **I**n **U**ltra-cold **S**ystems
  - Sandia National Laboratories
    Grand Challenge Laboratory Directed Research and Development Program
  - Three teams
    - *Neutral atoms*
      - Demonstrate an adiabatic quantum algorithm in a neutral atom array
    - *Semiconducting quantum dots*
      - Demonstrate an adiabatic quantum algorithm in a quantum dot system
    - ***Architectures***
      - Consider advantages and disadvantages of adiabatic quantum computation
      - Support experimental groups
      - Extend notions of circuit-model fault tolerance to adiabatic systems

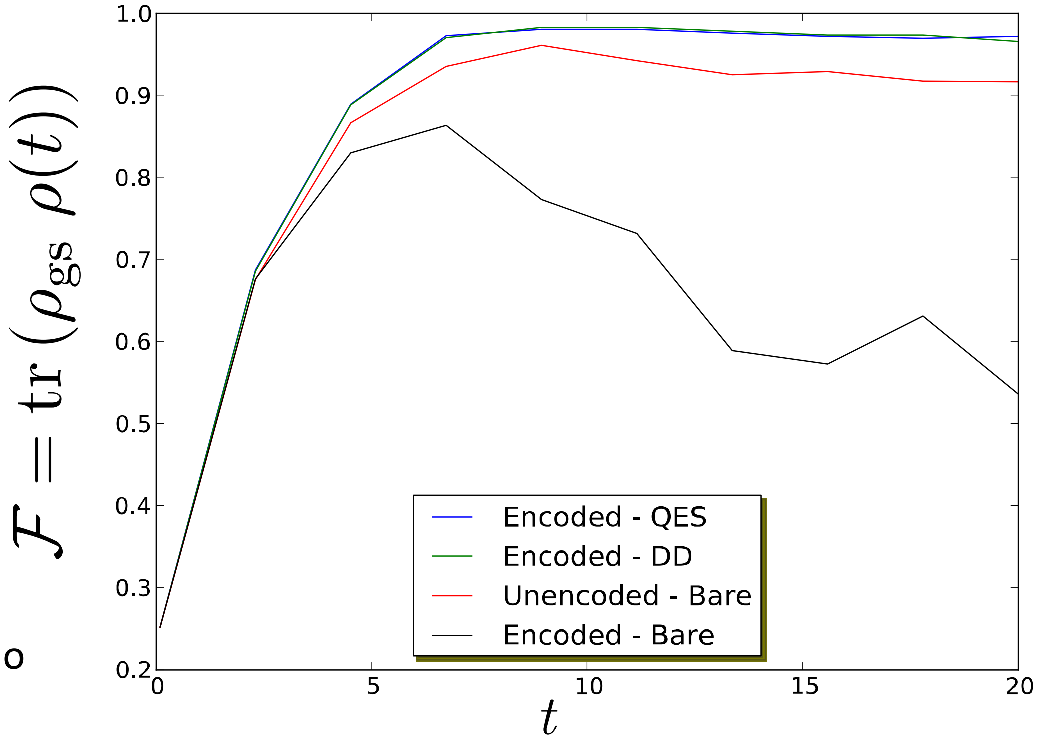


AQUARIUS™

Adiabatic quantum architectures in ultracold systems

Simulations

- Simulations of [[4,2,2]] QUBO problem showing equivalent performance of DD and QEC - Calculated by Monte Carlo with 1/f noise
  - Increase in performance with time because more adiabatic
  - Long-time decrease due to accumulation of error
  - Encoding without correction doubles likelihood of error
  - QES and DD perform nearly equally well
  - Squiggly lines because of poorly converged Monte Carlo



Master equation

- DD/EGP equivalence lets us use *whatever formalism* is most convenient to study error suppression
    - We can derive a master equation describing the population remaining in the code space assuming EGP and Orenstein-Uhlenbeck type noise

$$\begin{aligned}\frac{\mathrm{d}P_c(t)}{\mathrm{d}t} \approx 2\sum_k \int_0^t d\tau \Re\Big\{C_k(\tau)e^{-2i\int_{t-\tau}^t ds\ \sum_m^k f_m(s)}\mathrm{Tr}\left[\mathbf{P}\Xi_k(t,\tau)\mathbf{Q}\breve{\rho}(t)\mathbf{Q}E_k\mathbf{P}\right]\Big\} \\ -\Re\Big\{C_k(\tau)e^{2i\int_{t-\tau}^t ds\ \sum_m^k f_m(s)}\mathrm{Tr}\left[\mathbf{P}E_k\Xi_k(t,\tau)\mathbf{P}\breve{\rho}(t)\mathbf{P}\right]\Big\}\end{aligned}$$

$$C_k(t) \propto \exp(-\gamma t)$$

- Population left in code space (derived assuming no logical Hamiltonian)

$$\left.\frac{\mathrm{d}P_c(t)}{\mathrm{d}t}\right|_{H_{\mathrm{AQC}}=0} = -\sum_k r_k^+(t)P_c(t) + \sum_k r_k^-(t)P_{e1}(t)$$

- Rates are modified by error suppression

$$r_k^\pm(t) \propto \frac{2\left(\gamma - \gamma e^{-\gamma t}\cos[2\kappa w_k t] + 2\kappa w_k e^{-\gamma t}\sin[2\kappa w_k t]\right)}{(2\kappa w_k)^2 + \gamma^2}$$

Limitations of error suppression in AQC

- “Decode at the end” is unlikely to be useful
    - State might be in correctable subspace, but ***logical errors*** have happened
    - Error suppression must be so strong that state is in code-space with high probability.
        - For large computations, this is very unlikely
        - Circuit model results imply the state will almost certainly not be in the code-space
- For EGP implementation of error suppression, require high-weight (many-body) terms to enforce energy penalties.
    - DD does not have this problem:
        - For example:

$$XXXXX = e^{-i\pi\sigma_x^{(1)}/2}e^{-i\pi\sigma_x^{(2)}/2}e^{-i\pi\sigma_x^{(3)}/2}e^{-i\pi\sigma_x^{(4)}/2}e^{-i\pi\sigma_x^{(5)}/2}$$

- Our options:
    - Fixing the logical operators
    - Active error correction
    - ***BOTH***?

Circuit model vs. AQC encodings

- Error correcting codes behave very well in the circuit model
  - Stabilizer measurements destroy coherences between spaces

$$\boxed{|\psi\rangle = \sum_{\mathbf{k}} \alpha_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{k}\rangle = \sum_{\mathbf{k}} \alpha_{\mathbf{k}} E_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle}$$

Perform gate ⟶ $\bar{L} |\psi\rangle = \sum_{\mathbf{k}} \alpha_{\mathbf{k}} \bar{L} E_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle = \sum_{\mathbf{k}} (-1)^{g(\bar{L},\mathbf{k})} \alpha_{\mathbf{k}} E_{\mathbf{k}} \bar{L} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle$

Measure stabilizers ⟶ $|\psi\rangle \to \bar{L} |\psi_{\mathbf{k}}\rangle |\mathbf{k}\rangle \quad \text{with probability } \alpha_{\mathbf{k}}^2$

  - Logical operators are equivalent up to stabilizers

Perform gate ⟶ $\bar{L} S_i |\psi\rangle = \sum_{\mathbf{k}} \alpha_{\mathbf{k}} \bar{L} S_i E_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle = \sum_{\mathbf{k}} (-1)^{g(\bar{L},\mathbf{k})+\mathbf{k}_i} \alpha_{\mathbf{k}} E_{\mathbf{k}} \bar{L} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle$

Measure stabilizers ⟶ $|\psi\rangle \to \bar{L} |\psi_{\mathbf{k}}\rangle |\mathbf{k}\rangle \quad \text{with probability } \alpha_{\mathbf{k}}^2$

Circuit model vs. AQC encodings

- Stabilizer encodings don't play so well with the adiabatic model
  - Logical operators act as ***Hamiltonians***, so they can be ***added***: $H = \bar{L} + \bar{M}$

$$\boxed{|\psi\rangle = \sum_{\mathbf{k}} \alpha_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{k}\rangle = \sum_{\mathbf{k}} \alpha_{\mathbf{k}} E_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle}$$

    - Hamiltonian acts differently on each syndrome space

$$\begin{aligned} H|\psi\rangle &= \sum_{\mathbf{k}} \alpha_{\mathbf{k}} (\bar{L} + \bar{M}) E_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle \\ &= \sum_{\mathbf{k}} \alpha_{\mathbf{k}} E_{\mathbf{k}} ((-1)^{g(\bar{L},\mathbf{k})} \bar{L} + (-1)^{g(\bar{M},\mathbf{k})} \bar{M}) |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle \\ &= \sum_{\mathbf{k}} \alpha_{\mathbf{k}} E_{\mathbf{k}} H_{\mathbf{k}} |\psi_{\mathbf{k}}\rangle |\mathbf{0}\rangle \end{aligned}$$

    - $|\psi_{\mathbf{k}}\rangle$ may not even be an eigenstate of $H_{\mathbf{k}}$