

## ASSESSMENT OF VALIDATION METRICS FOR UNDEX SIMULATIONS

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### INTRODUCTION

A necessary phase in the development of a ship model for simulation of response to underwater explosion (UNDEX) events is the quantification of the model's adequacy for the intended purpose. Validation of a model is based on comparisons of calculated predictions and experimental results for selected measures of response. Metrics based on these response measures are validation metrics that indicate the predictive accuracy of the modeling. Various response measures have been used for this application, historically based in the time domain since UNDEX is a transient event.

This paper explores the use of alternate response measures and novel implementation of some standard measures for validating UNDEX simulations. These response measures include windowed acceleration spectra, windowed pseudo-velocity spectra, windowed RMS of the time signal, band-limited temporal moments, windowed relative input energy, and windowed peak strain energy. The continuous response measures are discretized through windowing into vectors for subsequent use in quantifying margin and uncertainty. The validation metrics obtained from the response measures are applied to a single barge shock test where acceleration time histories are compared between 10 predictions and the experiment for eight locations on the barge. The performances of the validation metrics are examined and the parametric sensitivities of the response measure definitions such as window width, overlap, and number of windows are explored.

### DESCRIPTION OF VALIDATION METRICS

Various response quantities of interest (QoIs) are used to compare analytical and experimental results for UNDEX events. These response measures include windowed acceleration shock response spectra, windowed pseudo-velocity shock response spectra, windowed RMS of the time signal, band-limited central moments, windowed relative input energy, and windowed peak strain energy. The windowing process, which applies to both spectral and temporal quantities, is described below.

Acceleration and pseudo-velocity shock spectra are common QoIs used for assessing shock response. A SRS is a frequency based compilation of the maximum response of a single degree of freedom (SDOF) system to a particular base excitation. An acceleration SRS is the maximum absolute value of acceleration response for the SDOF system across the frequency range of interest. The pseudo-velocity shock spectrum (PVSS) is the maximum absolute value of the relative displacement of the SDOF system for each frequency multiplied by that frequency which results in units of velocity. The maxi-max values of the SRSs are used for the validation assessments where the peak response is identified both during and after the base motion has been applied to the SDOF systems.

The RMS of a time history signal is calculated as a running, temporal RMS. An averaging interval is specified and a window with the length of the averaging interval is defined. The window is centered at each point in the time history and the product between each point on the window and the square of each point on the time history is computed. The sum of these products is evaluated where the sum is the estimated RMS at the center point of the window. The computation is performed for every point in the time history. In the limit, the interval could be defined as the length of the transient to produce a single RMS value for the complete time history. The windowing operation as described here is the time-domain equivalent of the frequency-domain computation described, graphically, in Figure 2.

Spectral quantities lack any temporal information (e.g., phasing) which may be important for validating analytical models. Temporal moments have been used to supplement spectral information in the characterization of time history signals [8]. Temporal moments [1,2] are defined as

(1)

the weighted summations of the time signal squared. Typically, temporal moments calculated according to Equation 1 above are normalized to generate central moments defined as

$$\begin{aligned}
 E &= M_0, & T &= \frac{M_1}{M_0}, & D^2 &= \frac{M_2}{M_0} - \left( \frac{M_1}{M_0} \right)^2 \\
 S_t^3 &= \frac{M_3}{M_0} - 3 \left( \frac{M_1 M_2}{M_0^2} \right) + 2 \left( \frac{M_1}{M_0} \right)^3, & S &= \frac{S_t}{D} \\
 K_t^4 &= \frac{M_4}{M_0} - 4 \left( \frac{M_1 M_3}{M_0^2} \right) + 6 \left( \frac{M_1^2 M_2}{M_0^3} \right) - 3 \left( \frac{M_1}{M_0} \right)^4, & K &= \frac{K_t}{D}
 \end{aligned} \tag{2}$$

These central moments have unique physical interpretations as described in Table 1 which can prove useful when comparing the characteristics of similar signals. All five of the central moments listed in Table 1 are included in the validation metric set. The use of these central moments as validation metrics is further refined by applying them on band-pass filtered components of the signal of interest. The original signal is separated using a four-pole Butterworth filter into meaningful bandwidth components.

**Table 1: Physical Interpretation of Central Moments**

Order	Name	Symbol	Interpretation
0	Energy	E	Energy integral of the signal envelope
1	Centroid	T	Locates the center of the energy signal with respect to time
2	RMS Duration	D	Measure of the time duration of the signal's energy
3	Skewness	S	A positive skewness indicates the signal decays with time
4	Kurtosis	K	Related to the peakedness of the envelope

The application of relative energies as intensity measures and indicators of damage potential was pioneered in earthquake engineering [3,4,5] to examine the energy input and distribution throughout a structure. The energy balance is evaluated where the energy input to the structure by the earthquake is balanced by providing adequate energy dissipation capacity [6]. The amount of base motion energy that is input to the individual modes of a structure is characterized by the input energy. Peak strain energy quantifies the potential energy that a structure exercises in each mode. The descriptor “relative” is used because the expressions are formulated in a relative coordinate frame.

The relative input energy to a single degree of freedom (SDOF) base excited system is defined as

$$\frac{E_I^R}{m} = - \int \ddot{x} \dot{z} dt = \Psi \quad (3)$$

where  $m$  – mass,  
 $\ddot{x}$  – base acceleration, and  
 $\dot{z}$  – relative velocity of mass.

This is described as relative input energy because the work is integrated over the relative displacement of the mass. Absolute energies include rigid body motion while relative energies only consider deformation which is a desirable characteristic. Relative input energy is typically expressed in the mass normalized form as shown in Equation 3 so that the quantity will scale with the mass of the SDOF system. The strain energy is simply the stored potential energy in the SDOF spring.

It has been recognized that all energy measures do not scale linearly with the input which is why the quantity of energy equivalent velocity [7] was developed which does scale linearly. An energy equivalent velocity is developed by equating the internal strain energy and input energy for a SDOF system which assumes that all input energy is directed into deforming the structure. The resulting expression for energy equivalent velocity is

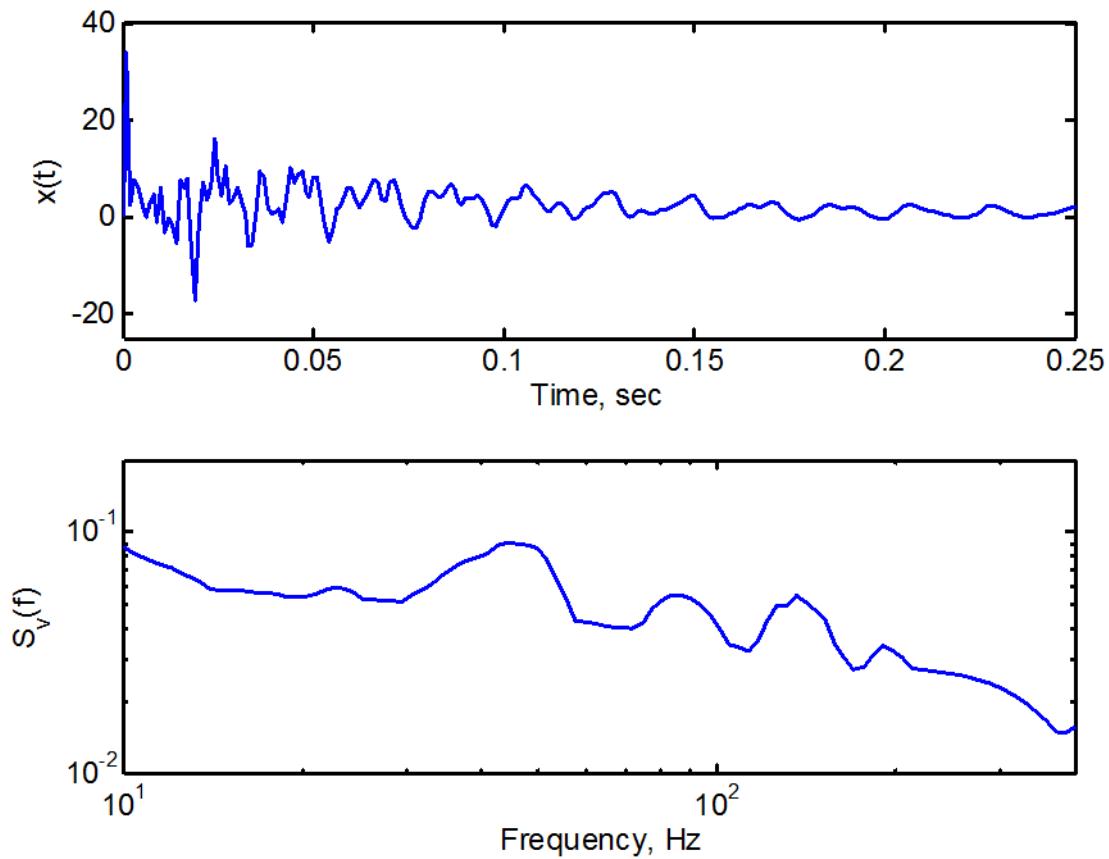
$$EEV = \sqrt{2\Psi} \quad (4)$$

Note that the energy equivalent velocity is calculated on the basis of input energy instead of the relative displacement that is used for the pseudo-velocity even though both metrics have units of velocity.

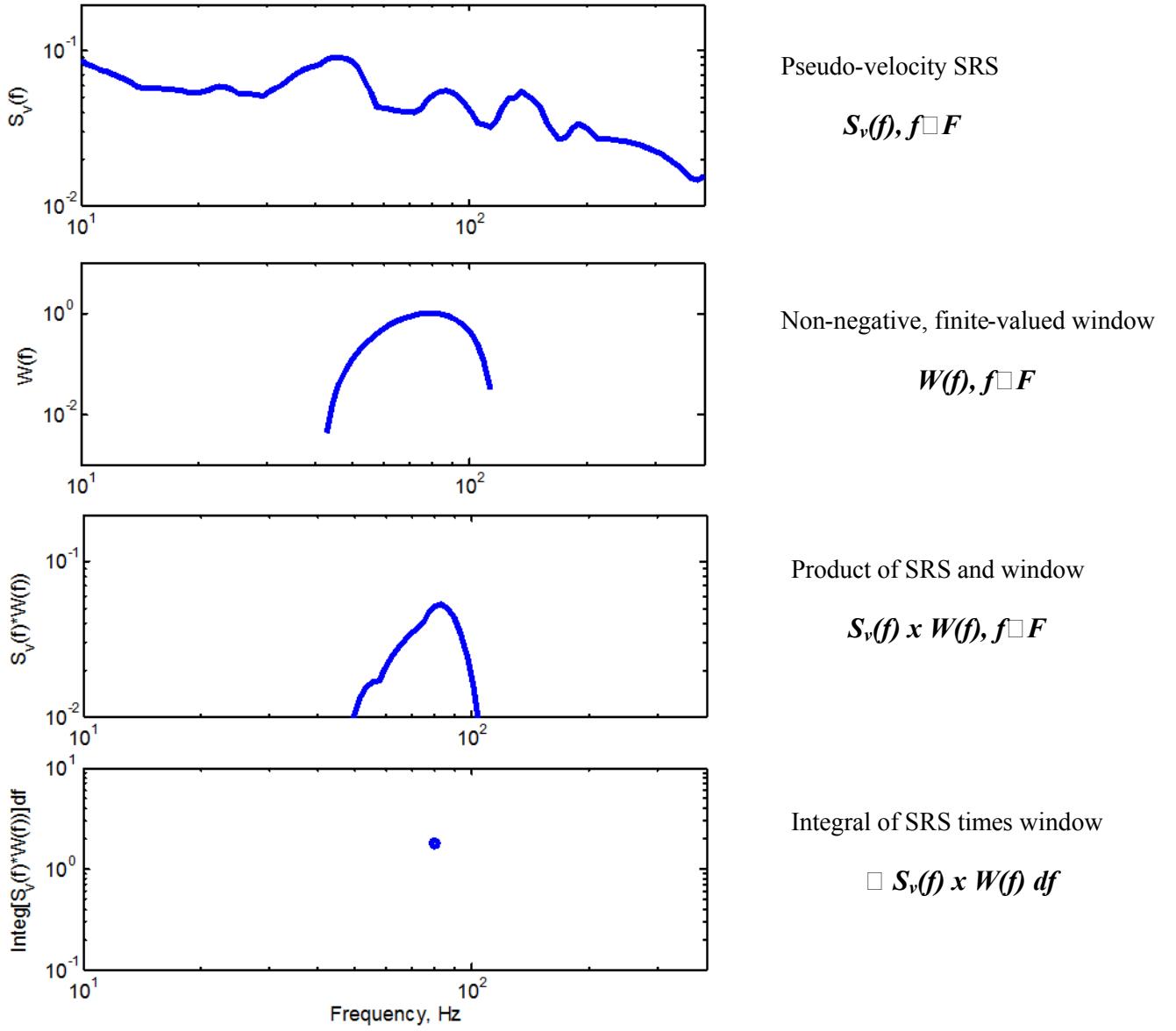
## WINDOWING FUNCTION

An example of windowing a shock response spectrum is provided below although the same methodology applies to windowing a time history (e.g., windowed RMS). An acceleration time history signal and its corresponding pseudo-velocity response spectrum are shown in Figure 1. A non-negative, finite-valued window (typically with unit area) is defined in the frequency domain and multiplied by the shock response spectrum (SRS) as indicated in Figure 2. This product of the SRS and window function is then integrated to produce a discrete measure of the SRS in the frequency domain.

Figure 1: Acceleration Signal and Corresponding Pseudo-Velocity Shock Response Spectrum

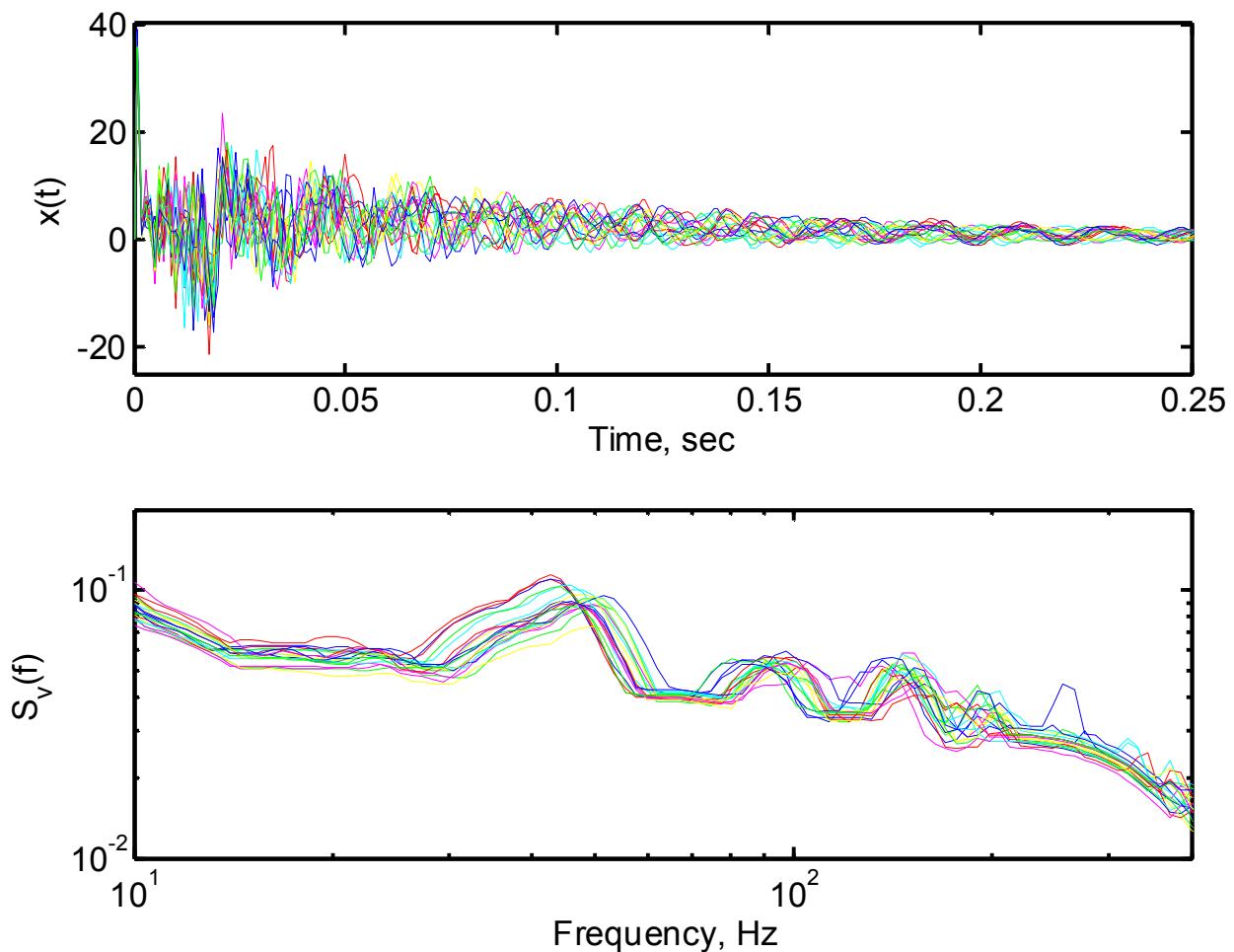


**Figure 2: Application of Windowing to the Shock Response Spectrum of Figure 1**

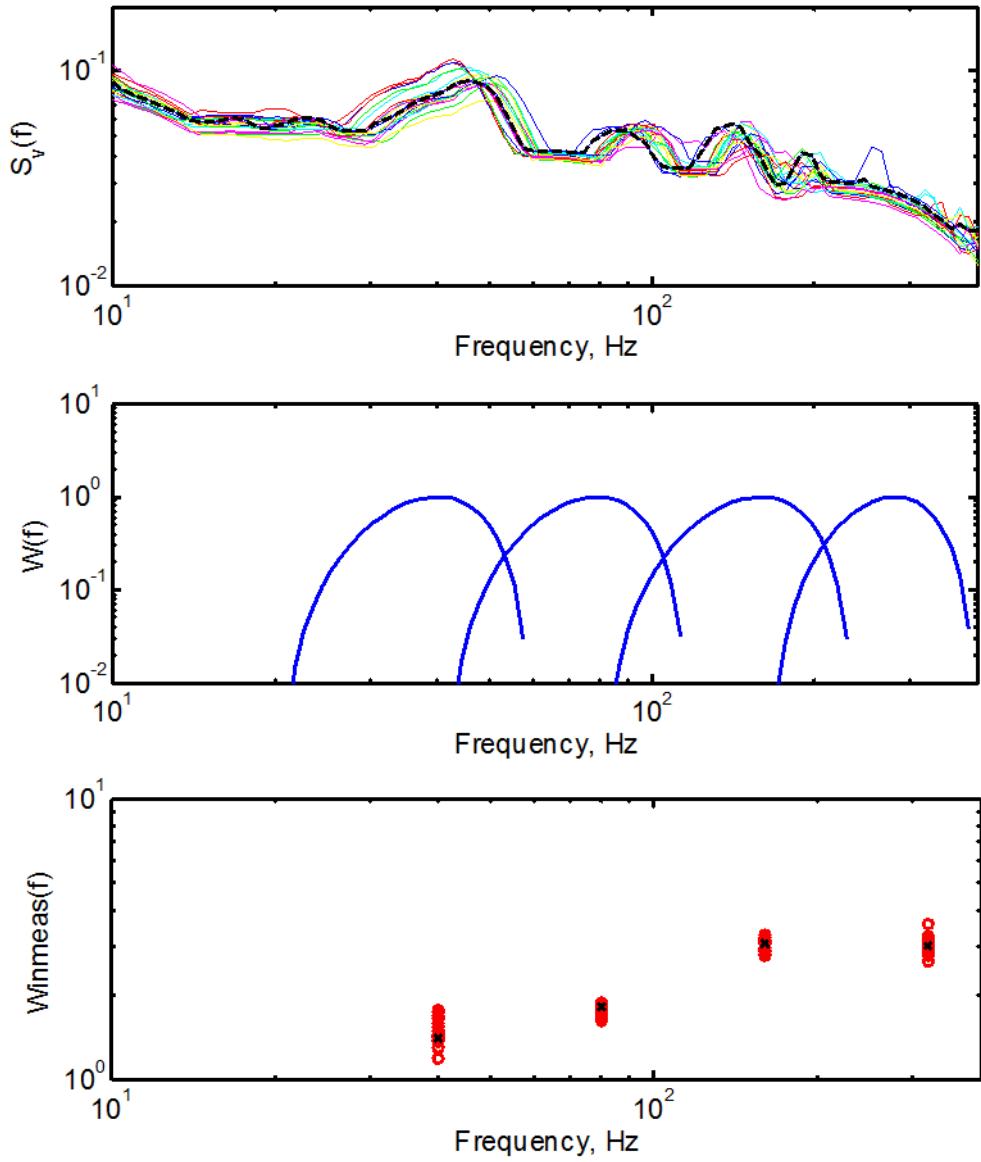


Parameter uncertainties in a system result in multiple computed shock responses where each shock response has its own SRS as shown in Figure 3. The goal is to compute a distribution of windowed measures from analytical predictions for comparison to windowed experimental response(s) in a quantifiable manner. Multiple windows are used to ensure coverage of the frequency range of interest and the windowed measures of the model generated SRS (red) and experimental SRS (black) are shown in Figure 4. This vector form of the metrics where there is a windowed SRS value at each window frequency facilitates algorithmic comparison (e.g., hypothesis testing) of analytical and experimental responses. Implementation of the procedure described here permits objective comparison of model-predicted to experimental responses.

**Figure 3: Multiple Computed Shock Responses**



**Figure 4: Distribution of Windowed Metrics from Figure 3**

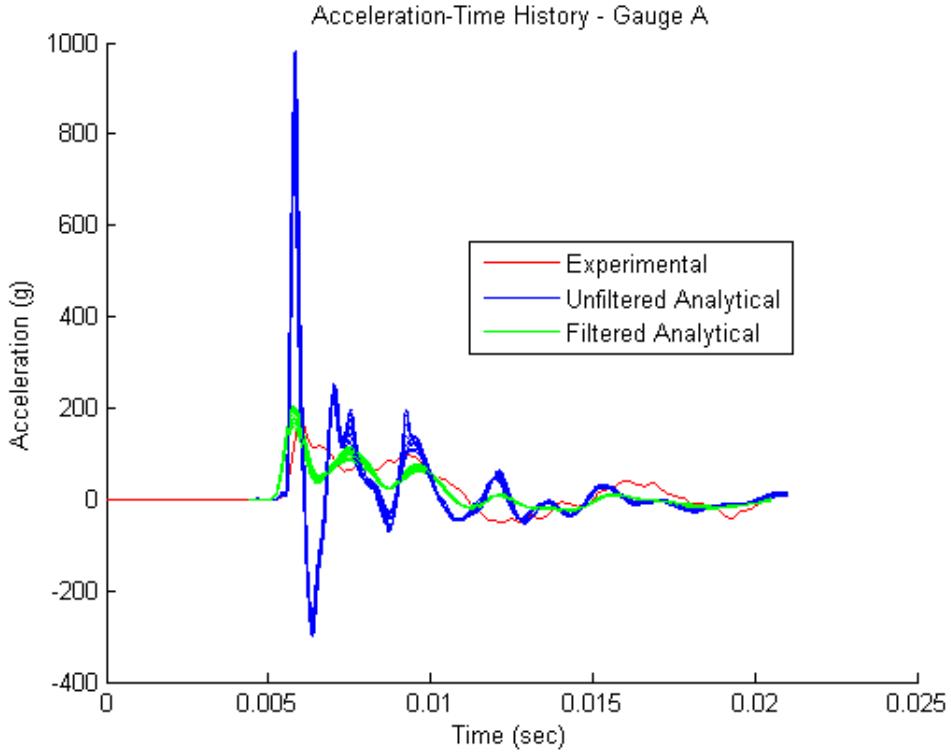


(1) Windowed measures of model generated and experimental SRS are in red and black, respectively.

### DEMONSTRATION PROBLEM

A standard data set has been identified consisting of a single barge shock test with measurements at eight gauge locations, A through F. The experimental data was obtained using a bandpass 2-pole Bessel filter set at 0.25 and 250 Hz, respectively. A velocity-time history record for Gauge A is shown in Figure 5 where the recorded event duration is 0.14 sec. A corresponding set of 10 UNDEX simulations were conducted where the only model parameter that was changed was the charge density.

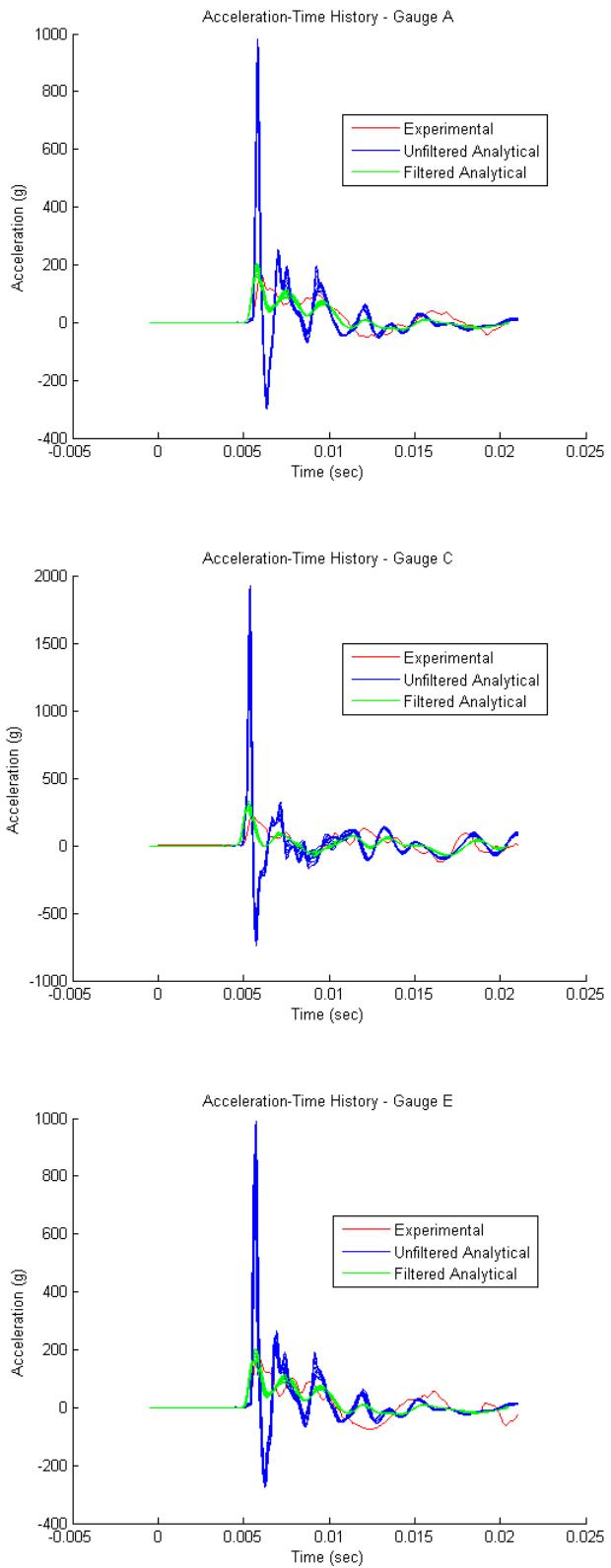
**Figure 5: Velocity-Time History for Gauge A**



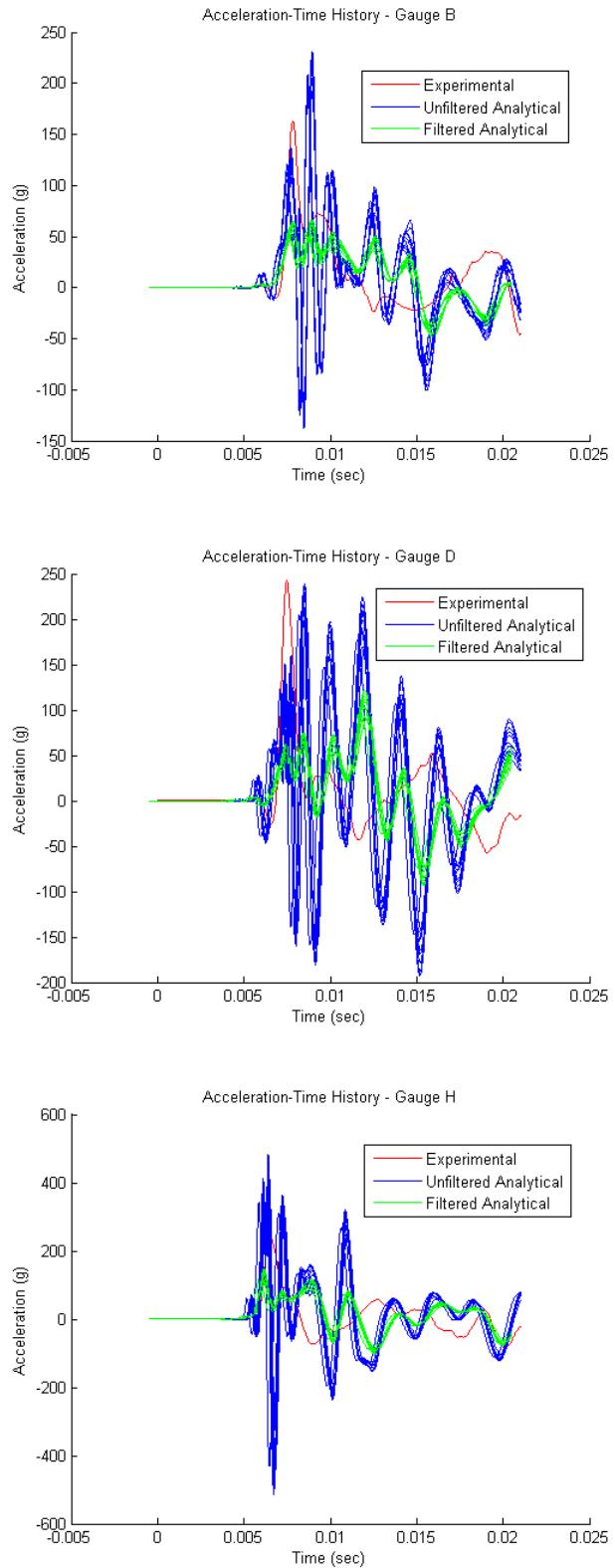
The simulations were conducted with minimal damping which produced significant high frequency content compared to the filtered experimental results. This high frequency content confounds the visual comparisons of results so the analytical results were filtered using a bandpass 2-pole Bessel filter set at 0.25 and 250 Hz, respectively, for consistency with the experimental data. The procedure described in [8] was used for digital implementation of the analog Bessel filter and the resulting temporal lag (0.0005 sec) was negated during plotting of the results.

The filtered analytical data were used for visual comparison purposes only and were not used for any subsequent QoI calculations so that metric performance would be unbiased. The criterion of comparison is based on two factors: the initial peak response and the ringdown phase that follows. A “good” comparison correlates well for both the peak and ringdown phases of response while a “bad” comparison does not correlate well with either response phase. A “questionable” comparison agrees with either the peak response or the ringdown phase, but not both response characteristics. Comparisons of the filtered results at the eight gauge locations were visually ranked as three good (Gauges A, C and E), three bad (Gauges B, D and H), and two questionable (Gauges F and G) cases which are shown in Figures 6, 7, and 8, respectively. These subjective rankings were used to assess metric performance as discussed below.

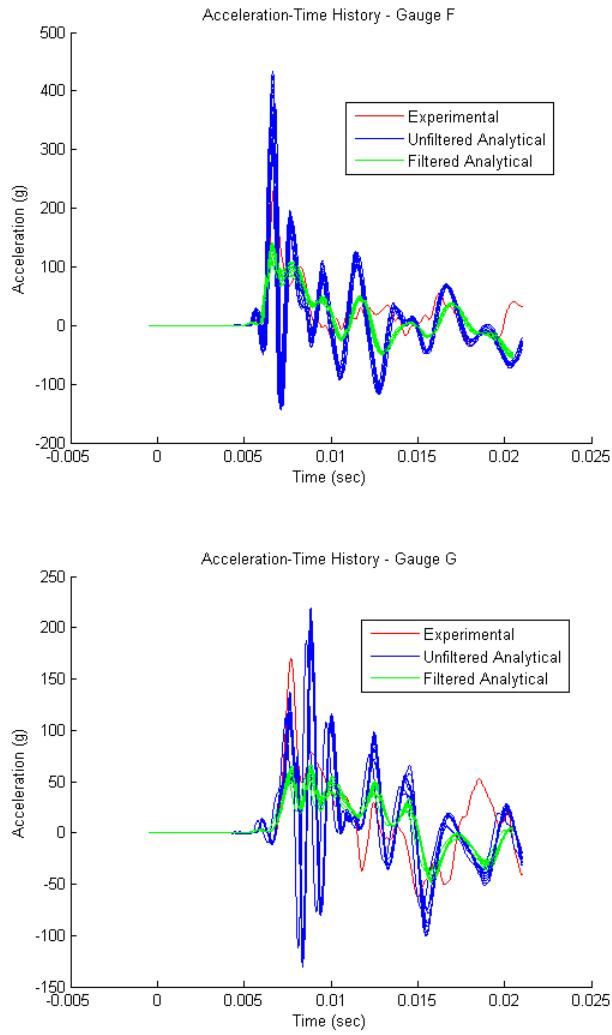
**Figure 6: Good Comparisons of Results**



**Figure 7: Bad Comparisons of Results**



**Figure 8: Questionable Comparisons of Results**



#### SPECTRAL QOI PARAMETER DEFINITIONS

All spectral quantities (i.e., acceleration SRS, pseudo-velocity SRS, input energy, strain energy and energy equivalent velocity) were calculated from 1-250 Hz in 40 logarithmically spaced increments using 5% modal damping. Gaussian windows were used for all spectral applications. The center frequencies for windowing the spectral quantities were specified as 10, 20, 40, 80 and 160 Hz with corresponding window widths of 10, 20, 40, 80 and 160 Hz, respectively. These center frequencies and widths were chosen to maximize coverage of the 1-250 Hz frequency range with increased emphasis on the lower frequencies which are most significant for this class of structure.

The spectral calculation routines are currently formulated to process acceleration records which are not directly compatible with the velocity records produced during UNDEX testing. The measured velocity-time histories were numerically differentiated to produce the acceleration histories used for the spectral calculations. The differentiation routine fits a cubic equation to nine points of the velocity time history centered about a timestep and the slope of the

fitted cubic is the acceleration value for that timestep. This smoothing approach was taken to eliminate any high frequency numerical noise that could potentially corrupt the derivative calculations.

### **TEMPORAL QOI PARAMETER DEFINITIONS**

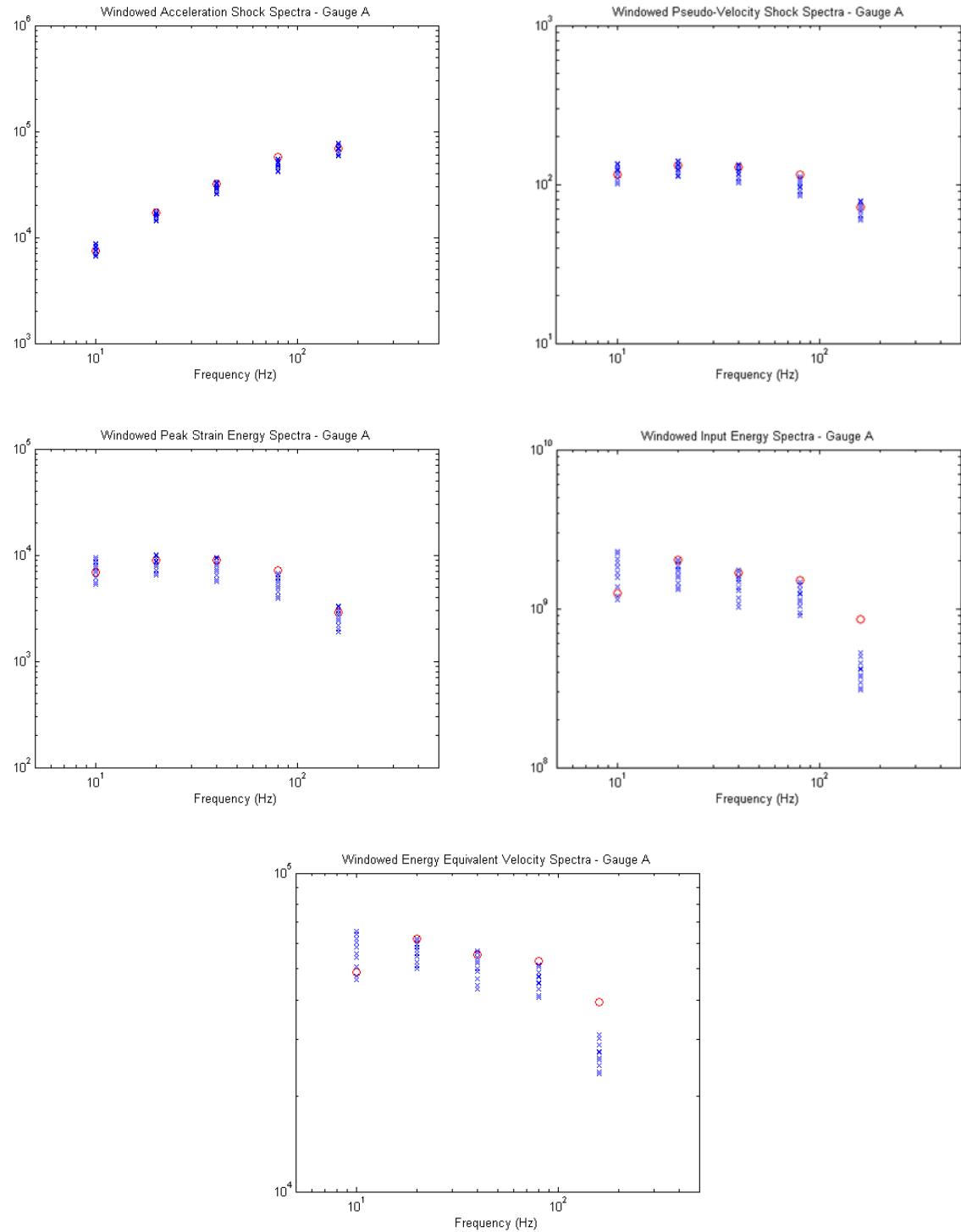
The barge shock event is characterized by an impulsive load applied to the system followed by a damped, ringdown response. The initial period of response is the most significant for equipment survivability and, as a result, the first 0.021 sec of response was evaluated for the temporal QoIs. Gaussian windows were used for all temporal applications. A 1.5 msec interval width with no overlap was used for the RMS calculations in the time domain where an RMS value is calculated for each point in the time history. Once the smoothed RMS values had been determined, 10 equally spaced windows without overlap for the 0 – 0.21 sec time history were used as the windowing function for the metric evaluation.

The 1-250 Hz frequency range was divided into three equal, non-overlapping component bandwidths for the band-limited, central moment calculations. The resulting bandwidth regions were 1-84 Hz, 84-167 Hz and 167-250 Hz with center frequencies of 42.5, 125.5 and 208.5 Hz, respectively. The temporal calculations (i.e., windowed RMS and band-limited central moments) operate on the velocity records directly so numerical differentiation was not required.

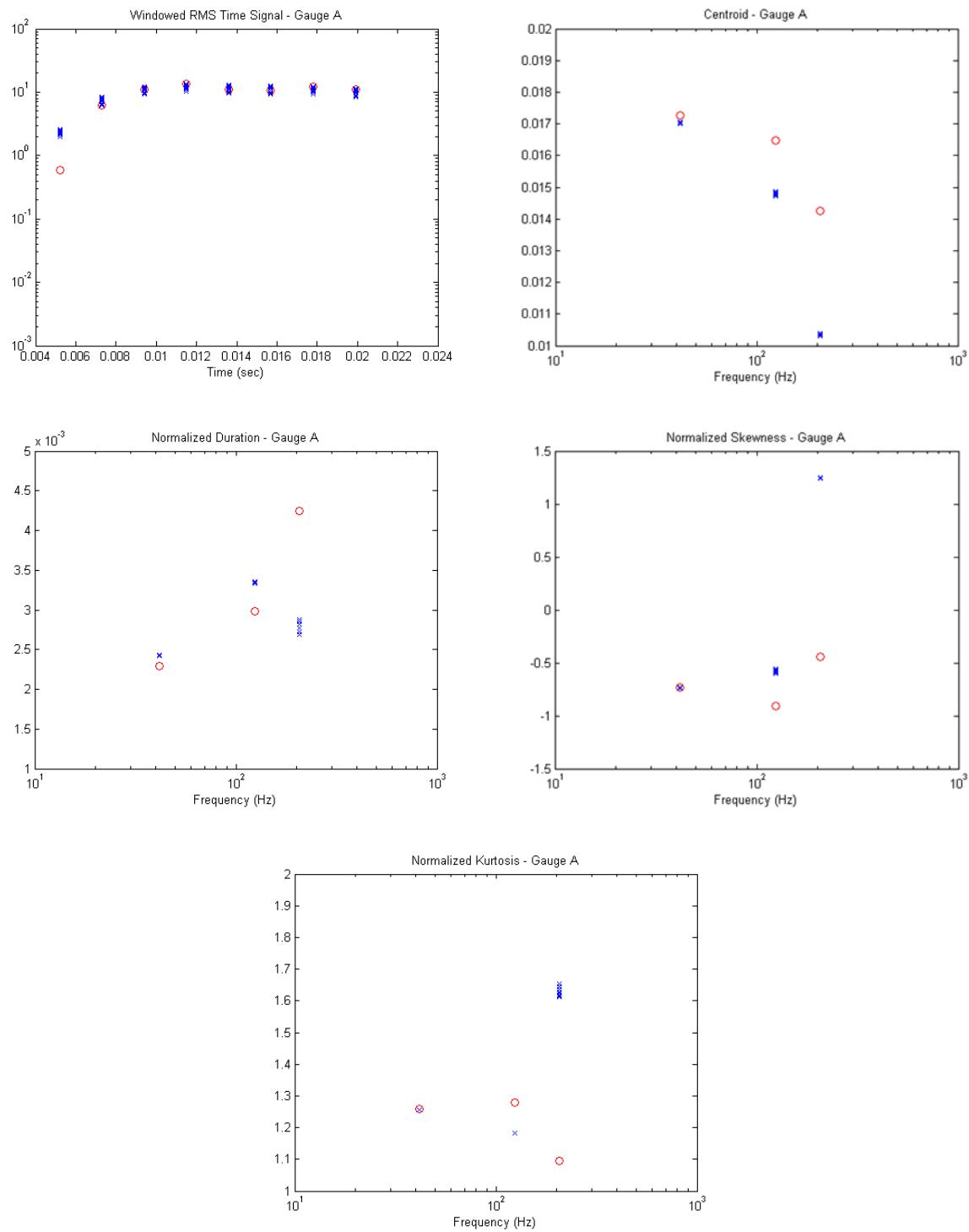
### **DISCUSSION OF VALIDATION METRIC PERFORMANCE**

The windowing operation produces a discretized version of the original function but the quantities are changed by the temporal or spectral integrations, thus losing their physical significance. The windowed quantities are intended to be used on a relative basis for comparing similar response quantities and the discretization facilitates subsequent hypothesis testing. The validation metrics were applied to the demonstration problem described above to evaluate their performance. Spectral and temporal QoIs calculated for a single case of the good results (i.e., Case A) are provided in Figures 9 and 10, respectively. Similarly, spectral and temporal QoIs calculated for a single case of the bad results (i.e., Case D) are provided in Figures 11 and 12, respectively. Finally, spectral and temporal QoIs calculated for a single case of the questionable results (i.e., Case F) are provided in Figures 13 and 14, respectively.

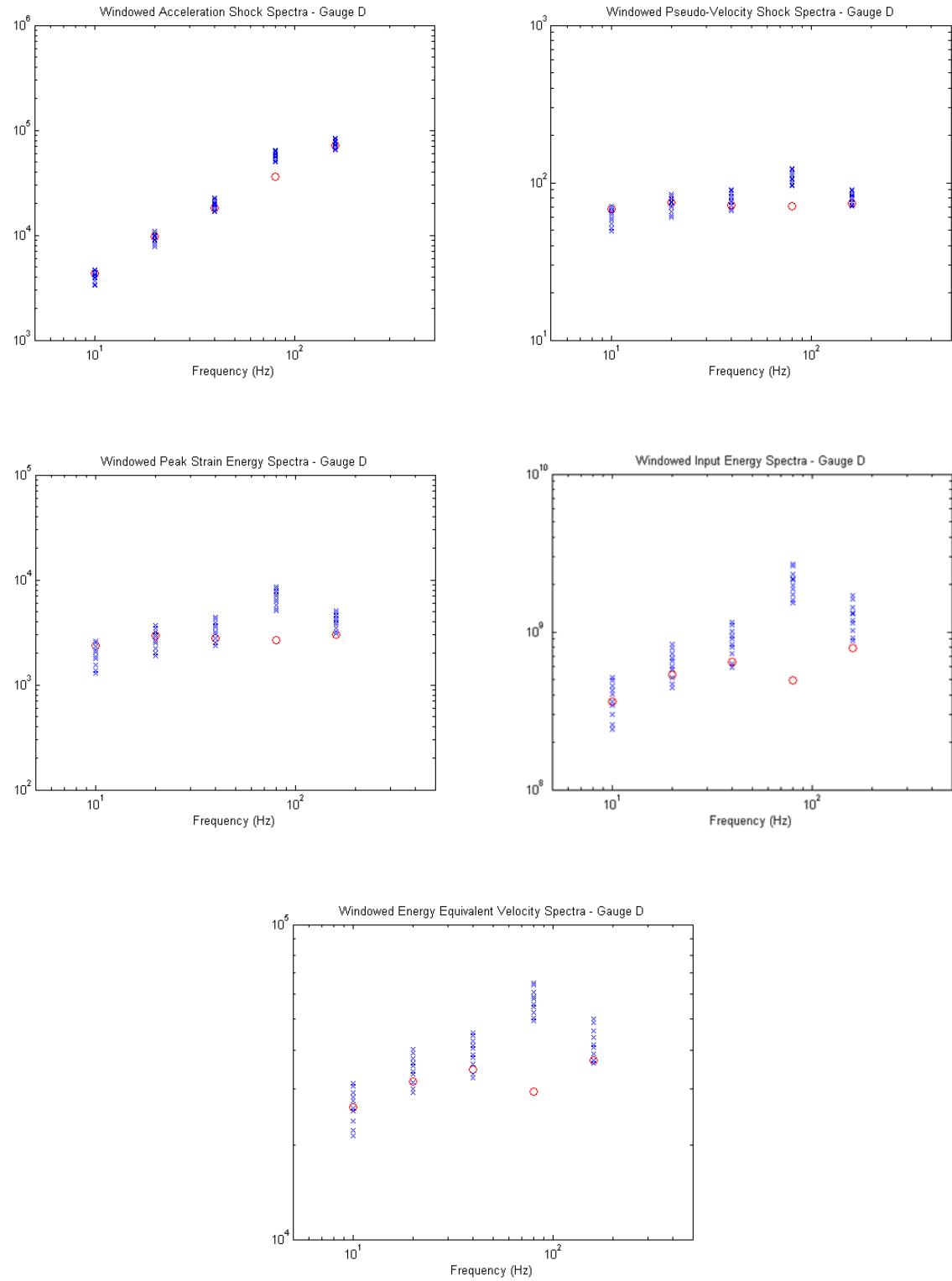
**Figure 9: Spectral QoIs Calculated for Good Results**



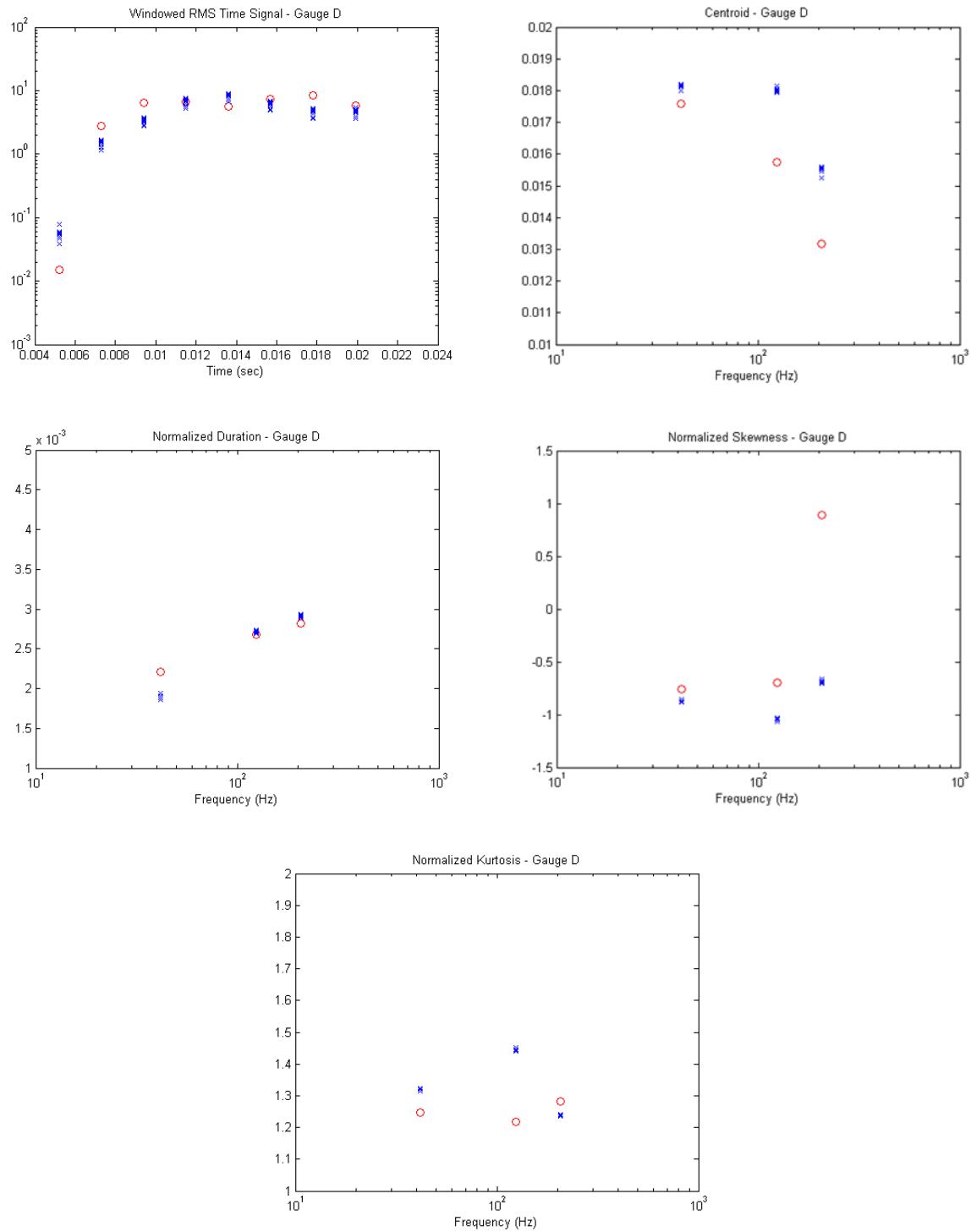
**Figure 10: Temporal QoIs Calculated for Good Results**



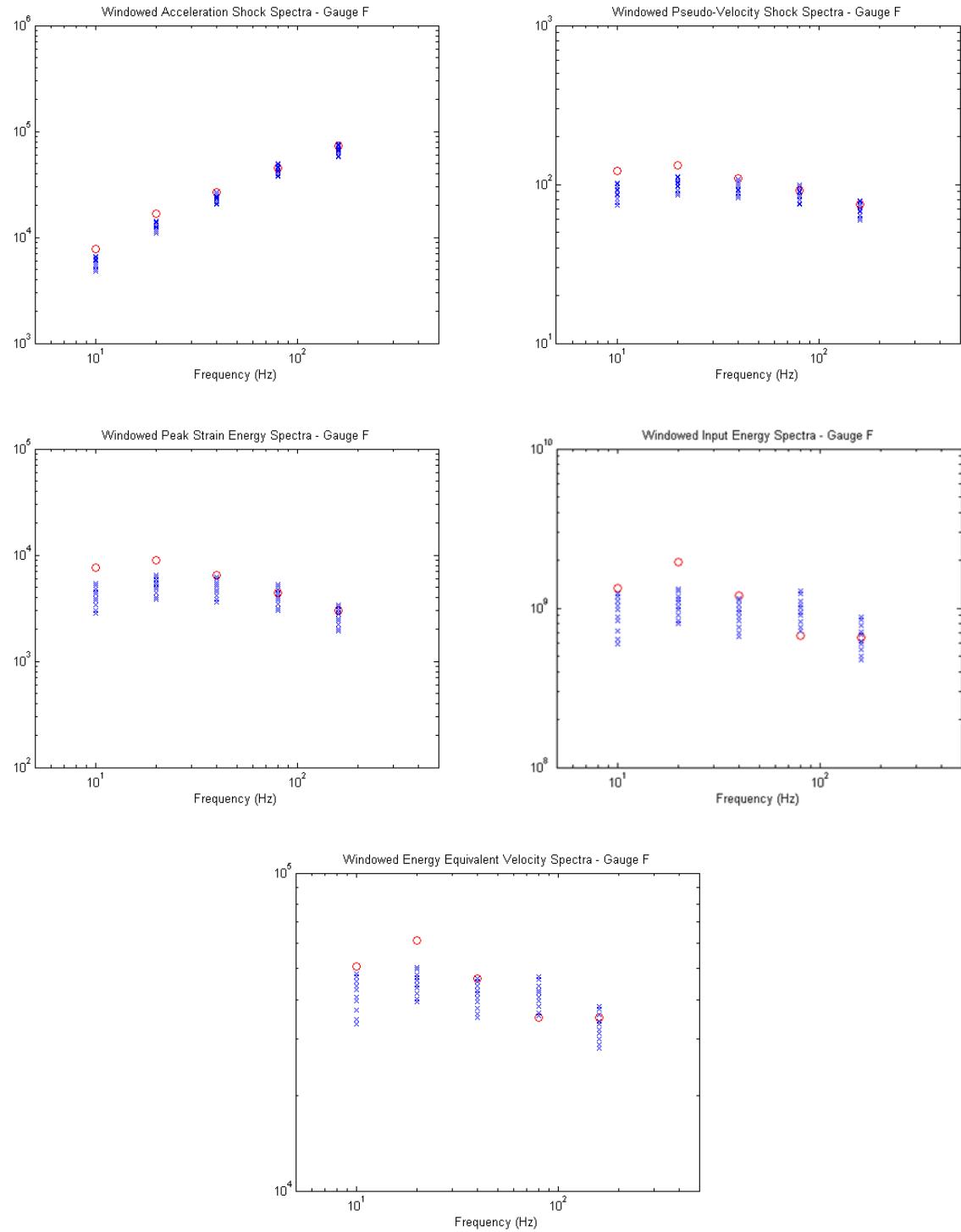
**Figure 11: Spectral QoIs Calculated for Bad Results**



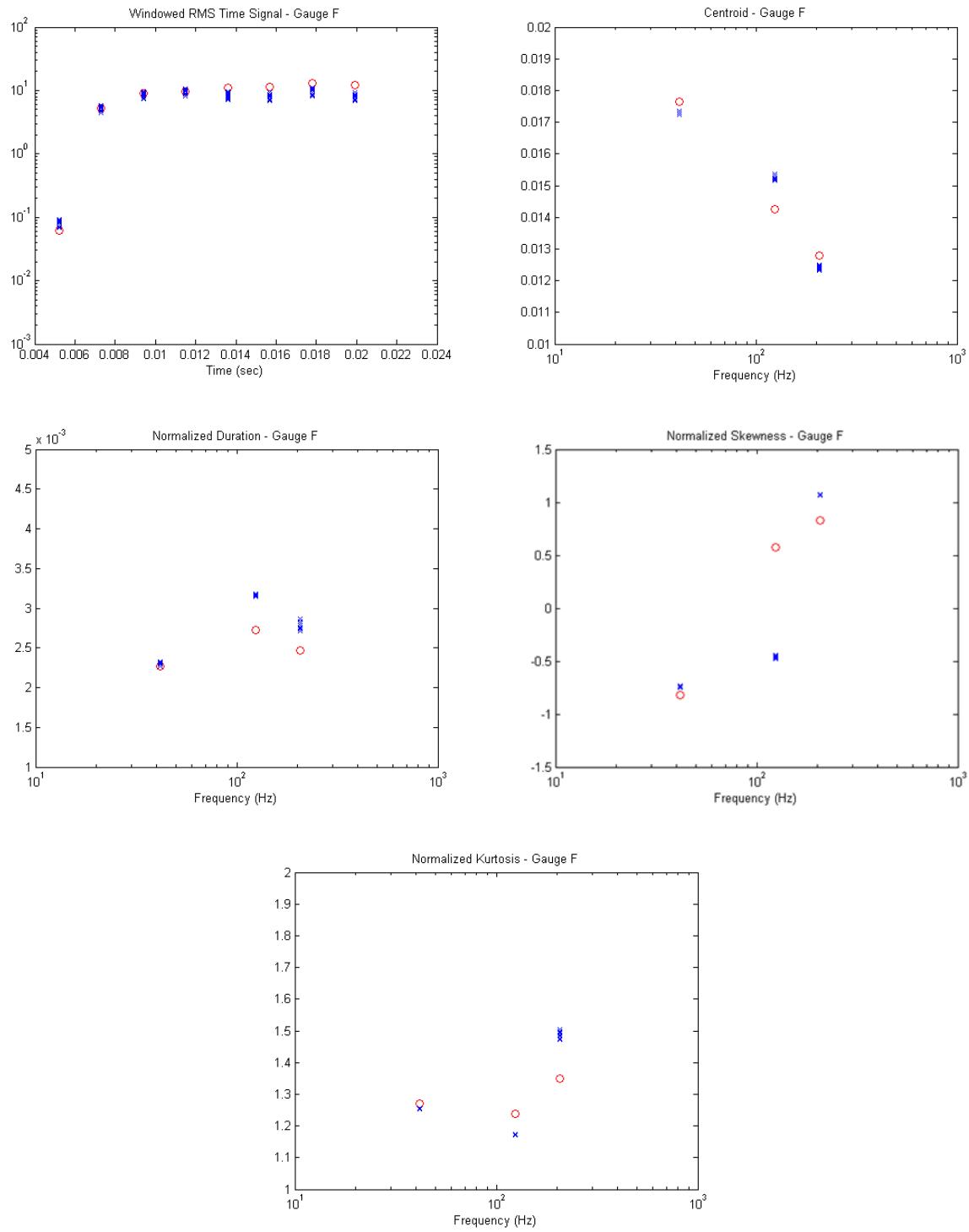
**Figure 12: Temporal QoIs Calculated for Bad Results**



**Figure 13: Spectral QoIs Calculated for Questionable Results**

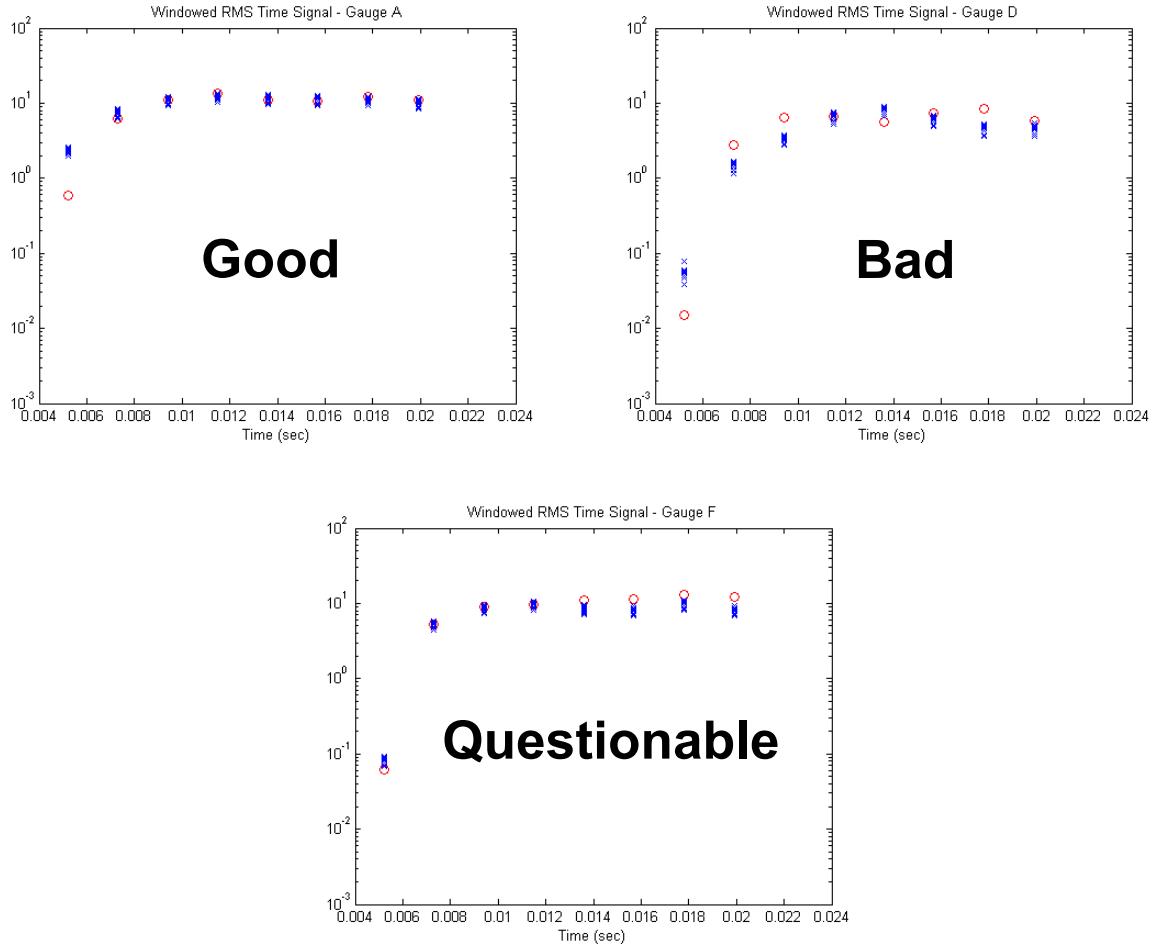


**Figure 14: Temporal QoIs Calculated for Questionable Results**



The QoI parameter definitions produced a set of measures that exhibit significant sensitivity to differences in the response quantities. This is necessary to discern critical differences between test and analysis results. Low QoI sensitivity would lead to the conclusion that all models are validated. Sensitivities of the windowed QoIs discussed in this report require further evaluation to establish acceptance thresholds for hypothesis testing. The windowed RMS time signal provides the best correlation with the visual assessment as shown for Gauge A in Figure 15. The trend of the windowed RMS correlation holds for the remaining gauge locations. The degree of correlation at early times dominates one's judgment of validation for the windowed RMS measure.

**Figure 15: Windowed RMS Correlation**



Spectral quantities lack any temporal information (e.g., phasing) which may be important for validating analytical models. To this end, temporal moments have been used to supplement spectral information [9]. A time-evolving spectral calculation is a natural extension to capture the temporal dependency while condensing the volume of data with a spectral representation. Evolutionary spectral density (ESD) [10] is the culmination of this approach but many weightings of ESD are possible, each with its own interpretation. Wavelets are one form of ESD which are currently being applied as a validation metric for UNDEX simulations.

## CONCLUSIONS

The QoIs examined in this paper provide discrete measures of system response that can be used to quantitatively compare test and analysis results. The validation process can be formalized by applying test-of-hypothesis to compare model-predicted to experimental response measures and make an assessment of model validity. The sensitivities of the QoIs discussed in this report must be further examined to establish acceptable tolerance limits for hypothesis testing. This set of QoIs does not consider spatial variation or ranking which is important to validate an analytical model for a specific purpose. Assuming all gauges have equal importance for the above demonstration problem, it would be concluded that the model is not validated due to bad and questionable correlation constituting five out of eight gauge response measures.

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