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Fundamental analysis and prediction of turbulent premixed combustion: Status and prospects

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Outline of presentation

- Status of burning-rate analysis
- Analysis for weak turbulence
- Studies of the strong-turbulence regime
- Conclusions

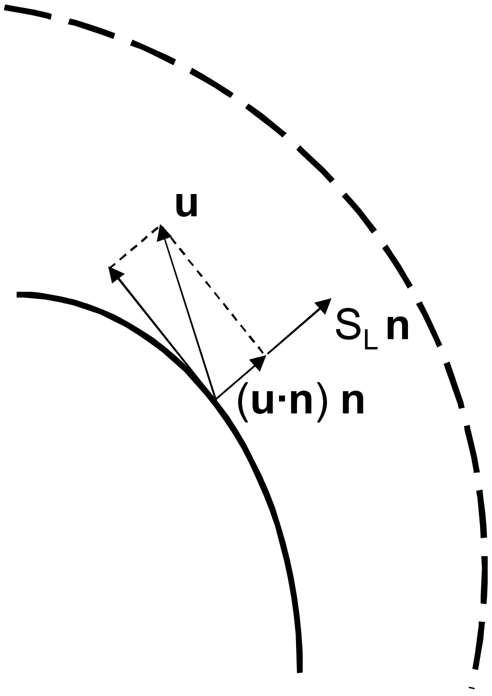
Propagation of a flame through a turbulent mixture is a theoretical as well as a technological challenge

- Burning-rate measurements show wide scatter, indicating sensitivity to apparatus details
- Usual modeling strategy: Start from a generic idealization (e.g., homogeneous isotropic flow, constant density), then add complicating details empirically
- <u>Key obstacle</u>: Even for idealized cases, neither a consensus on the governing physics nor a sound mathematical framework for burning-rate analysis has been established

Turbulent premixed combustion is idealized as an advected propagating (Huygens) front

<u>Idealization</u> ('Huygens propagation'):

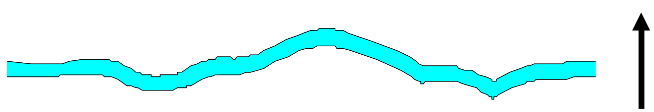
At a given location on the front, if the flow velocity is **u**, the front normal vector is **n**, and the front propagation speed (laminar flame speed) is S_L, then the front advancement velocity is $(\mathbf{u} \cdot \mathbf{n} + S_L)\mathbf{n}$



This idealization omits flow effects on the laminar flame speed and combustion effects on the flow, such as thermal expansion. Also, homogeneous turbulence (constant rms velocity fluctuation u′) is assumed.

Front wrinkling by turbulence increases the burning velocity

A wrinkled front has greater surface area A and thus sweeps through more volume per unit time AS_L, resulting in faster propagation



Turbulent burning velocity: $u_T \propto$ surface area

Studies to date yielded diverse predictions, but no reliable way to evaluate them

Weak turbulence ($u' << S_L$):

- $u_T/S_L - 1 \sim (u'/S_L)^2$
  - Clavin and Williams (1979)
- $u_T/S_L - 1 \sim (u'/S_L)^{4/3}$ predicted for random flow; quadratic dependence attributed to periodic flow
  - Kerstein and Ashurst (1992)
- Quadratic dependence demonstrated for a random flow
  - Akkerman and Bychkov (2003)

Strong turbulence ($u' >> S_L$):

- $u_T \sim u'/[\log (u'/S_L)]^{1/2} << u'$
  - Yakhot (1988); others propose $u_T/S_L \sim (u'/S_L)^p$ for $0<p<1$
- $u_T \sim u'$
  - Pocheau (1994) and others
- $u_T \sim u'Re^{1/4} >> u'$
  - Upper bound implied by Fedotov (1997)

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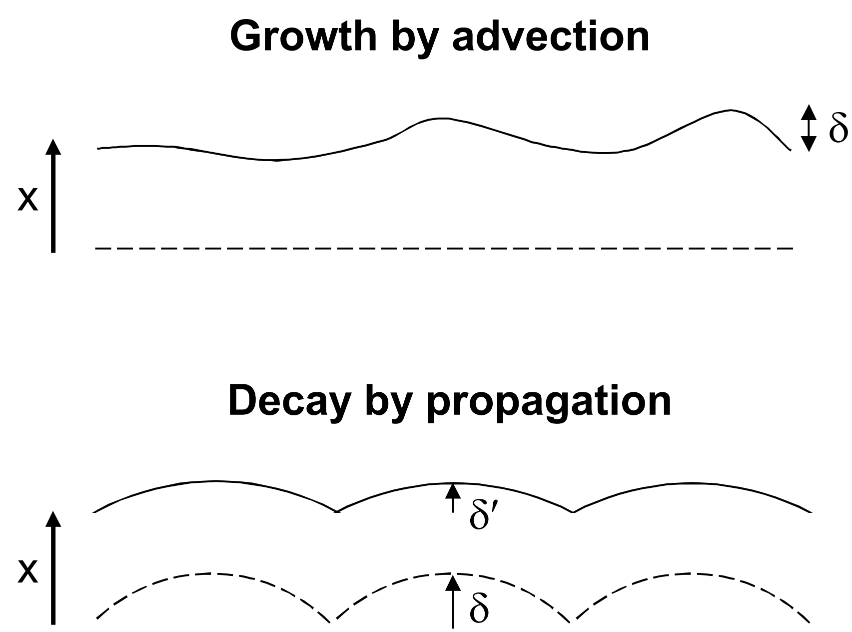
Weak-turbulence 4/3 scaling: Derived heuristically, supported by simulations of idealized flows

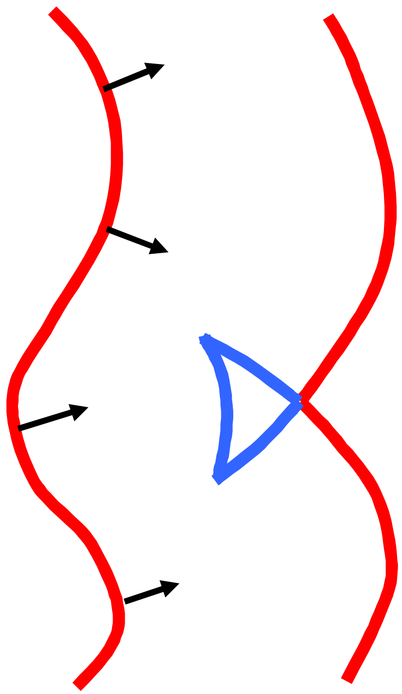
- For $u' << S_L$, $u_T/S_L - 1 \sim (\delta/\xi)^2$ (δ is fluctuation amplitude, ξ is correlation length)
- Balance of growth (by advection) and decay (by propagation) processes determines δ
- Growth: $\delta \sim \delta_\xi (x/\xi)^{1/2}$ (random walk scaling, where $\delta_\xi \sim \xi u' / S_L$)
- Decay: $d\delta/dx \sim -(\delta/\xi)^2$
- Balance occurs at downstream distance $x \sim (u'/S_L)^{-2/3}\xi$, giving $u_T/S_L - 1 \sim (u'/S_L)^{4/3}$



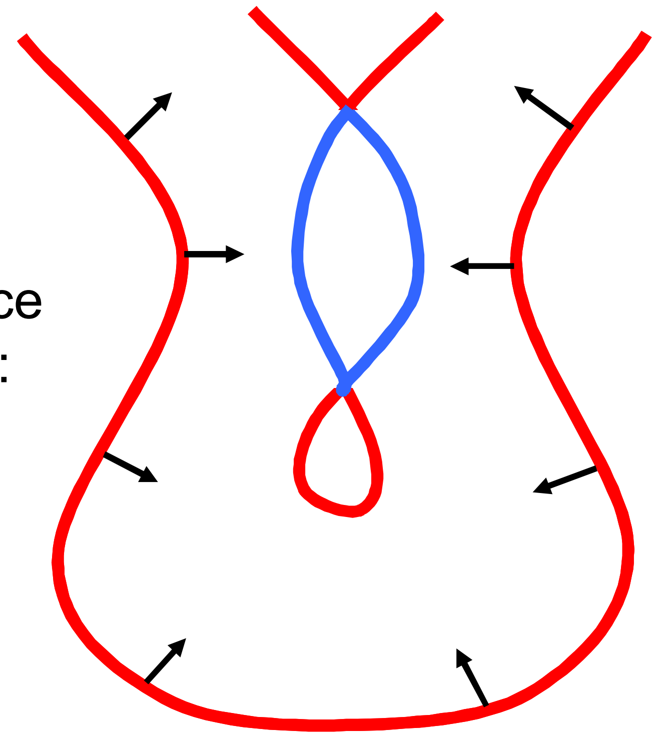
(Kerstein and Ashurst 1992)

Front surface reduction isn't mathematical disappearance, but a transition from first to later passage through the fluid

Weak turbulence scenario:



Strong turbulence scenario:



For weak turbulence, the problem reduces to a formulation amenable to quantum field theory

Steps in the analysis

- For u′ << S_L, propagation + advection reduces to propagation for **x**-dependent S_L (which idealizes heterogeneous propellant combustion; Kerstein, 1987)
- A Lagrangian formulation allows exact problem reduction, giving
  - Immediate extraction of 4/3 scaling
  - Prefactor = energy density of noise-driven Burgers flow
- Previous field-theory analysis of Burgers 1D flow provides a framework that allows the prefactor to be bounded as a function of the spatial autocorrelation of S_L(**x**)

Results

- Formally established this intuitive result
- Generalized a cusp analysis by White (1984)
- Applied a theorem of Iturriaga and Khanin (2003)
- Obtained a new explicit solution within Blum's (1994) formal framework
- Found new relationships among advection/propagation, polymer conformation statistics, and a quantum multi-particle system

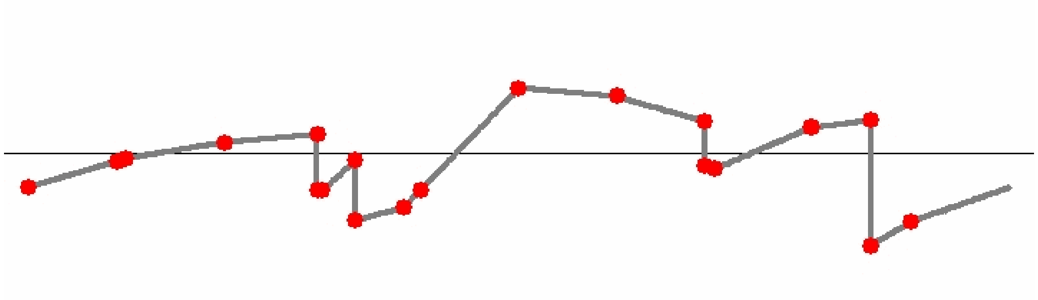
(Mayo and Kerstein 2008)

Links among diverse realms of physics contribute to the analysis

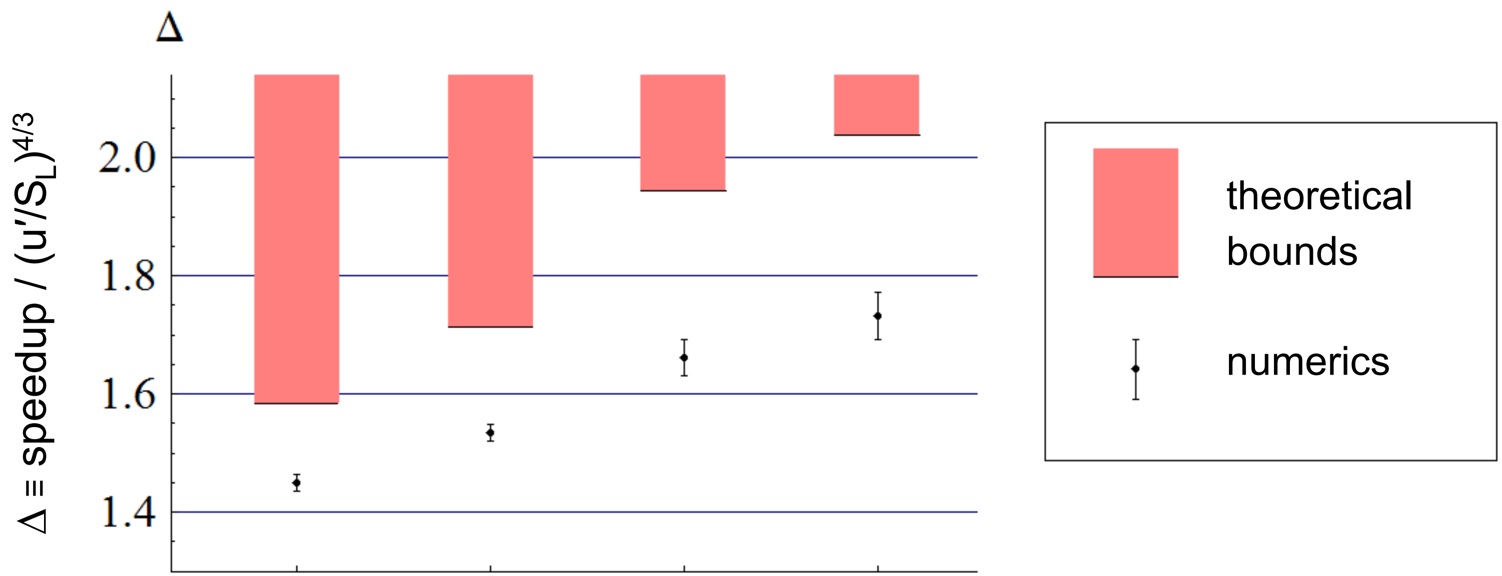
Physical system	*Branch of physics*	*Small-scale smoothing parameter*
Propagation of flames	Combustion	Markstein length
↓ Weak perturbations		
First passage of rays	Geometrical optics	Wavelength
↓ White noise		
The Burgers equation	Fluid dynamics	Viscosity
↓ Reinterpretation		
Directed polymers	Statistical mechanics	Temperature
↓ Replicas		
The Schrödinger equation	Quantum mechanics	Planck constant

A Lagrangian numerical method allows systematic testing of the predicted bounds for 2D propagation

- Represent an inviscid Burgers velocity field by piecewise linear sections, with discontinuities at shocks
  - Burgers velocity represents tilt of the front
  - Burgers shocks represent cusps
- Evolve freely (retaining exact piecewise linearity) for a timestep, then ‘kick’ the fluid with an impulsive force
  - synthesized from a given S_L autocorrelation
  - taken as piecewise linear
- The number of marker points
  - increases as kicks occur, but
  - stabilizes as shocks form and merge
- Convergence of the steady-state energy density is observed with
  - timestep
  - forcing grid
  - length of the periodic domain

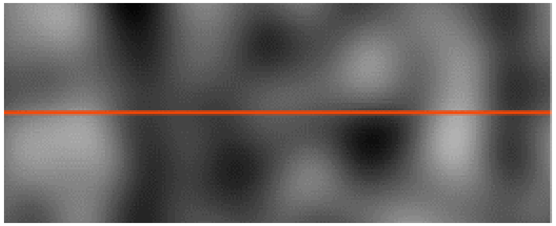


2D simulations demonstrate bound accuracy, raising confidence in untested 3D predictions

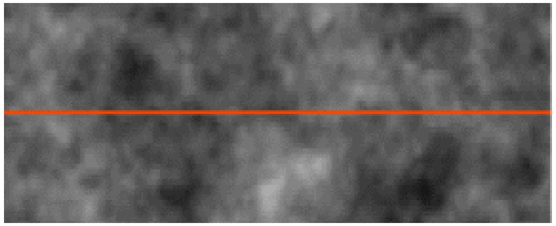


S_L autocorrelation: Modified Gaussian | Gaussian | Modified exponential | Exponential

Gaussian



Exponential



The results resolve an apparent discrepancy and have important implications for $u' >> S_L$

- Analysis assumes isotropic random flow
  - Certain unphysical forms of anisotropy can change the scaling
  - This anomaly explains the quadratic scaling found by Akkerman and Bychkov
- Significant dependence of u_T on flow structure is found
  - For $u' << S_L$, both the power spectrum of fluctuations and u'/S_L affect u_T
  - Analysis suggest even greater sensitivity to details for $u' >> S_L$
  - Implication: no expression for u_T/S_L that depends only on u'/S_L can capture all physics
- Because u_T must decrease as S_L decreases,
  - The derived bound on Δ constrains u_T scaling for $u' >> S_L$
  - In particular, it rules out $u_T >> u'$ in the absence of 'pathological intermittency'

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It is useful to analyze a field *G* whose level sets are instantiations of the front

$$\frac{\partial G}{\partial t} + \vec{u} \bullet \nabla G = S_L \left| \nabla G \right|$$

Because the propagation term smoothes wrinkles, it is dissipative in the same sense as the Laplacian term in the scalar equation

$$\frac{\partial \theta}{\partial t} + \vec{u} \bullet \nabla \theta = \kappa \nabla^2 \theta$$

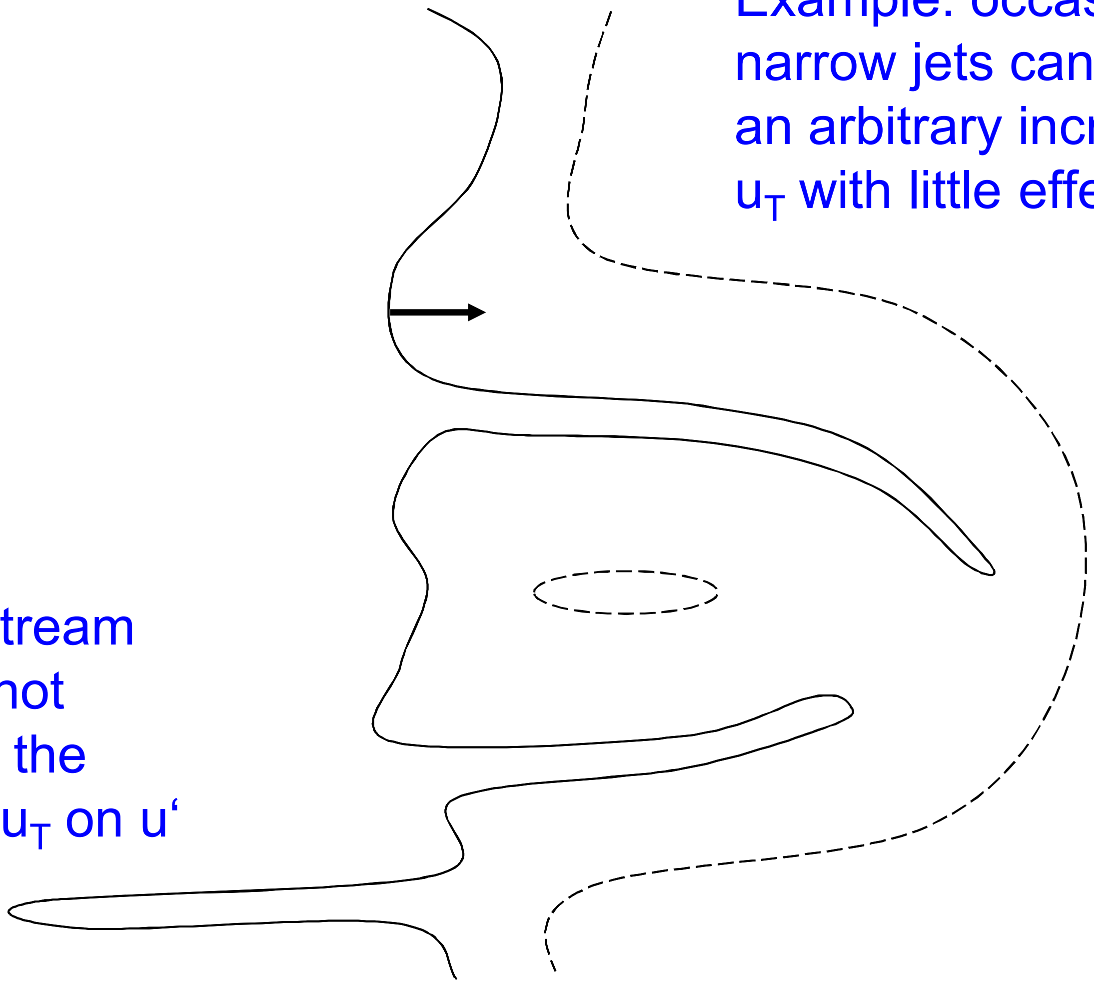
From *G*-equation analysis, Chertkov and Yakhot (1998) reached significant conclusions, but …

- Their claims:
  - u_T is determined by asymptotically high-order intermittency statistics (extreme rare events)
  - A regime is predicted in which $u_T << u'$
  - Unconventional behavior of the Gibson scale (lower cutoff of front fluctuations) is predicted
- The concerns:
  - Does the analysis capture the dominant wrinkling phenomenology in all instances, or does the result $u_T << u'$ miss a leading-order effect?
  - **Quantitative inferences are based on the statistics of the θ field, not the *G* field**

$u_T << u'$ implies that 'bending' of the (otherwise linear) u' dependence can occur in the flamelet regime, with important implications for data interpretation

Front propagation is a first-passage process, so $u_T >> u'$ is hard to rule out, but $u_T << u'$ seems counter-intuitive

Example: occasional narrow jets can cause an arbitrary increase of u_T with little effect on u'



Extreme downstream fluctuations do not obviously affect the dependence of u_T on u'

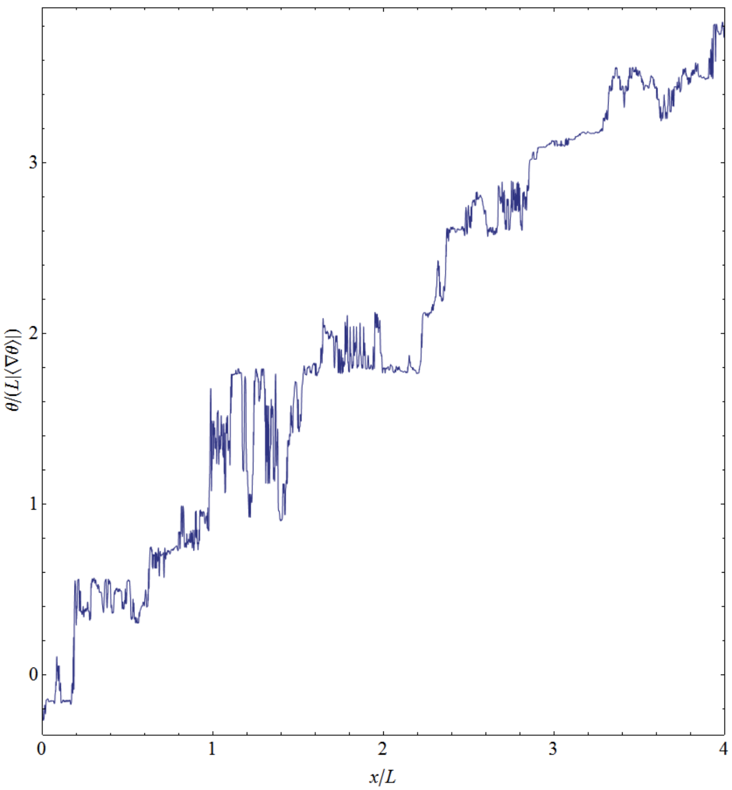
Scalar intermittency has been studied only for Laplacian dissipation, motivating a study of *G*-field intermittency

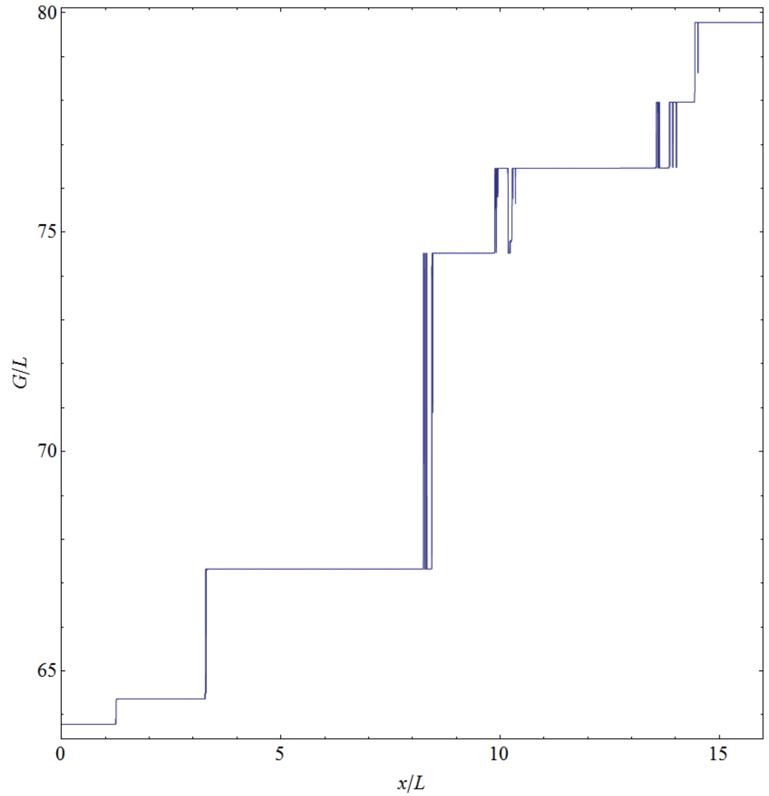
- High-order structure functions are key signatures of intermittency:

$$S_n(r) = \left\langle \left| \theta(x+r) - \theta(x) \right|^n \right\rangle \propto r^{\zeta(n)}$$

- Large-*n* saturation of $\zeta(n)$ signals that the largest small-scale gradients are caused by extreme compression of large-scale property differences
- For Laplacian dissipation, theory and empirical evidence do not rule out:
  - no saturation
  - large-*n* asymptotic saturation
  - saturation at finite *n*
- We are studying *G*-field intermittency numerically using
  - 1D stochastic simulation involving map-based advection (Linear-Eddy Model)
  - 2D synthetic white-in-time velocity field (Rapid-Change Model)

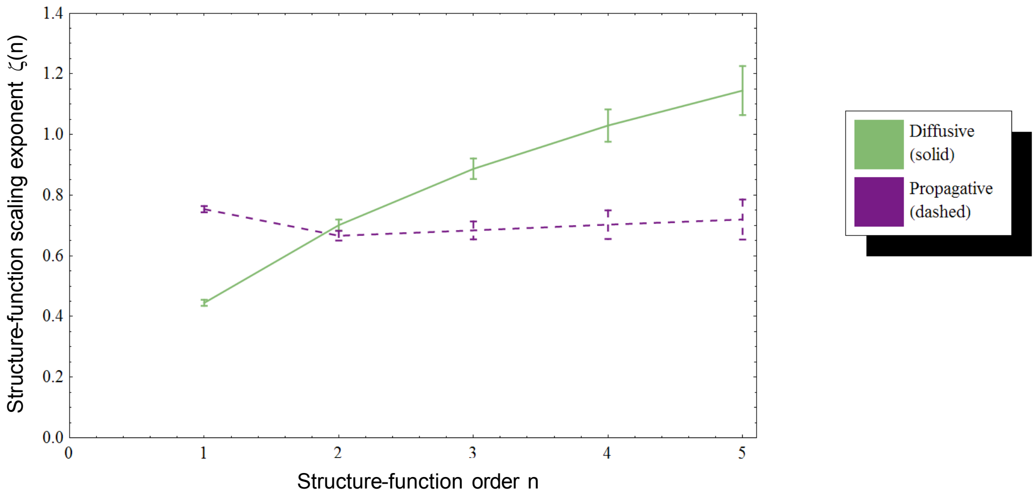
Linear-Eddy simulations show major differences between the θ field (left) and the G field (right)





In the simulations, 1D maps create local extrema leading to stair-step structure, but the multi-dimensional G field has no extrema

Linear-Eddy *G*-field results indicate nearly complete saturation, a predicted but previously unseen ideal limit



- For Laplacian dissipation, results agree well with measurements
- For the *G* field, saturation and the *n*=1 exponent value imply
  - A theoretical perspective that yields no $u_T << u'$ regime
  - Intuitive (rather than counter-intuitive) intermittency effect on the Gibson scale

This first hint of distinctive intermittency behavior of a non-Laplacian scalar suggests a fruitful direction for further research, starting with use of the Rapid-Change Model

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Studies of the weak-turbulence and strong-turbulence regimes of front propagation are complementary

- Weak-turbulence results are relevant to strong turbulence
  - There cannot be a unique dependence of u_T/S_L solely on u'/S_L
  - The strong-turbulence result $u_T >> u'$ is possible only for pathological intermittency scenarios
- For $u' >> S_L$, u_T dependence on high-order fluctuations has been proposed
  - This proposal should be further evaluated
  - If correct, it calls for study of the fluctuations of the relevant property: $G(\boldsymbol{x},t)$
  - Preliminary investigation suggests interesting statistical features