SAND2013-2232C

Comparison of statistical properties of advected diffusive scalars and advected propagating fronts, and implications for turbulent premixed flame propagation

Jackson Mayo[1] and Alan Kerstein[2]

1. Sandia National Labs, Livermore, CA
2. Consultant, Danville, CA

April 10, 2013

SIAM Conference on Numerical Combustion

The U.S. Department of Energy, Office of Basic Energy Sciences, Division of Chemical Sciences, Geosciences, and Biosciences supported this work. Sandia is a multiprogram laboratory operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy under contract DE-AC04-94AL85000.

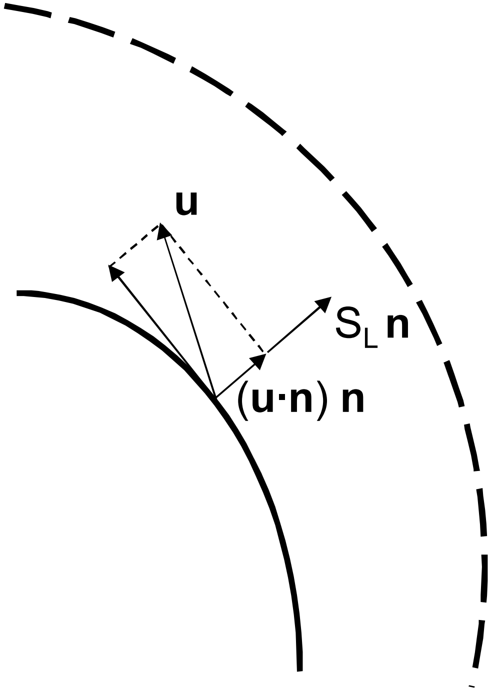
Propagation of a flame through a turbulent mixture is a theoretical as well as a technological challenge

- Burning-rate measurements show wide scatter, indicating sensitivity to apparatus details
- Usual modeling strategy: Start from a generic idealization (e.g., homogeneous isotropic flow, constant density), then add complicating details empirically
- Key obstacle: Even for idealized cases, neither a consensus on the governing physics nor a sound mathematical framework for burning-rate analysis has been established

Turbulent premixed combustion is idealized as an advected propagating (Huygens) front

Idealization ('Huygens propagation'):

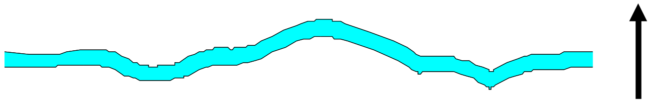
At a given location on the front, if the flow velocity is **u**, the front normal vector is **n**, and the front propagation speed (laminar flame speed) is S_L, then the front advancement velocity is $(\mathbf{u}\cdot\mathbf{n} + S_L)\mathbf{n}$



This idealization omits flow effects on the laminar flame speed and combustion effects on the flow, such as thermal expansion. Also, homogeneous turbulence (constant rms velocity fluctuation u′) is assumed.

Front wrinkling by turbulence increases the burning velocity

A wrinkled front has greater surface area A and thus sweeps through more volume per unit time AS_L, resulting in faster propagation



Turbulent burning velocity: $u_T \propto$ surface area

There are diverse predictions of the burning velocity of a passively propagating front advected by turbulence

The turbulent burning velocity u_T depends minimally on:

- u′: root-mean-square velocity fluctuation (turbulence intensity)
- S_L: laminar flame speed (subsumes all chemistry and its coupling to transport)

- Idealization: both of these parameters are assumed constant in space and time

Predictions for strong turbulence ($u' >> S_L$):

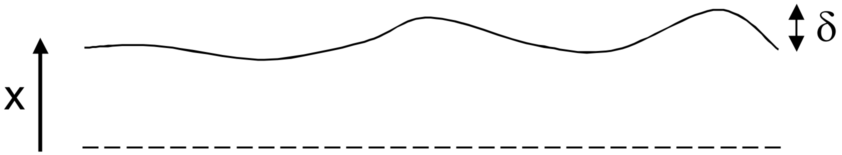
- $u_T \sim u'/[\log(u'/S_L)]^{1/2} << u'$
  - Yakhot (1988); others propose $u_T/S_L \sim (u'/S_L)^p$ for $0<p<1$
- $u_T \sim u'$
  - Pocheau (1994) and others
- $u_T \sim u'Re^{1/4} >> u'$
  - Upper bound implied by Fedotov (1997)

Front wrinkling is governed by a balance of surface area growth and decay mechanisms

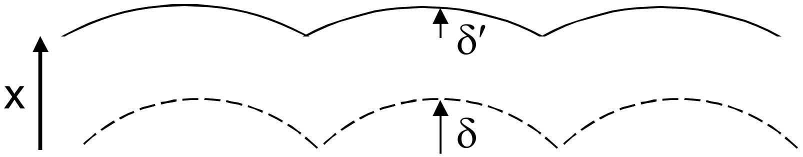
The front is

- wrinkled by turbulent advection
- smoothed by front propagation

Growth by advection



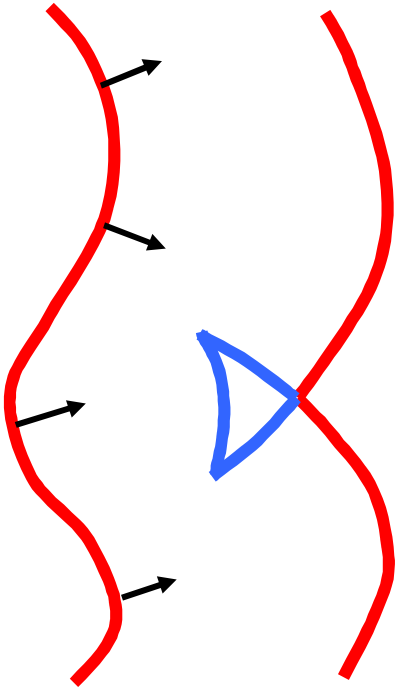
Decay by propagation



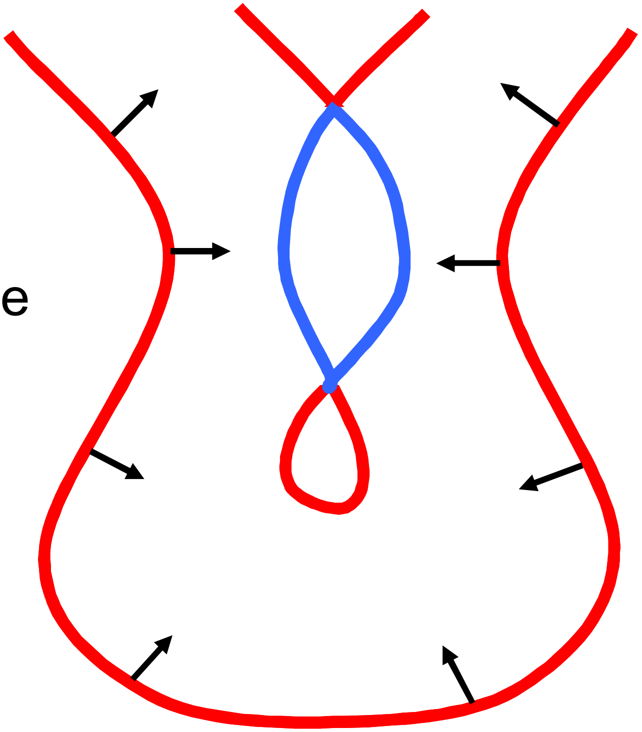
The balance of these effects determines the surface area, and thus u_T

Front surface reduction isn’t mathematical disappearance, but a transition from first to later passage through the fluid

Weak turbulence scenario:

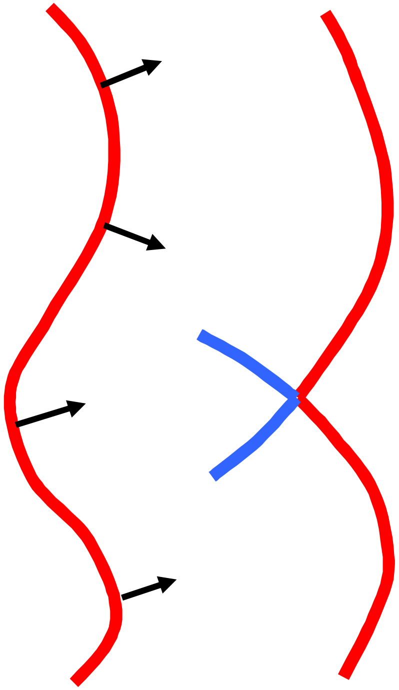


Strong turbulence scenario:

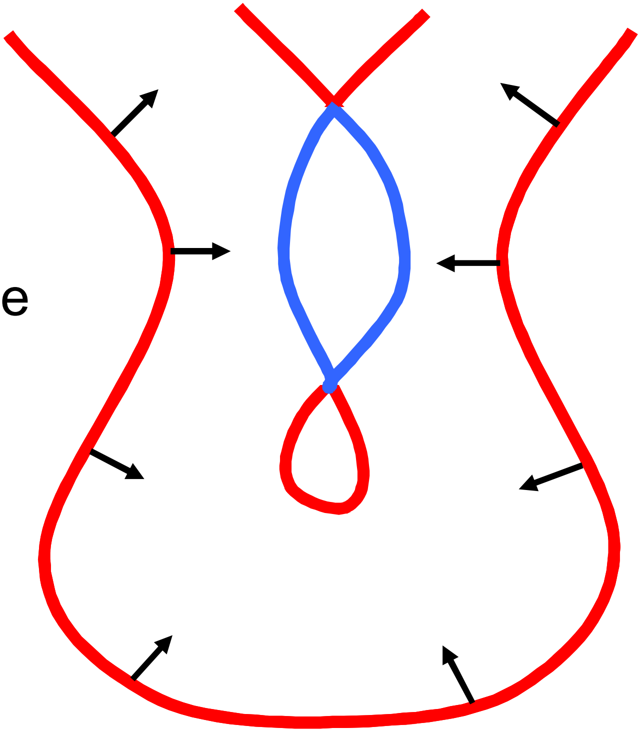


Front surface reduction isn't mathematical disappearance, but a transition from first to later passage through the fluid

Weak turbulence scenario:



Strong turbulence scenario:

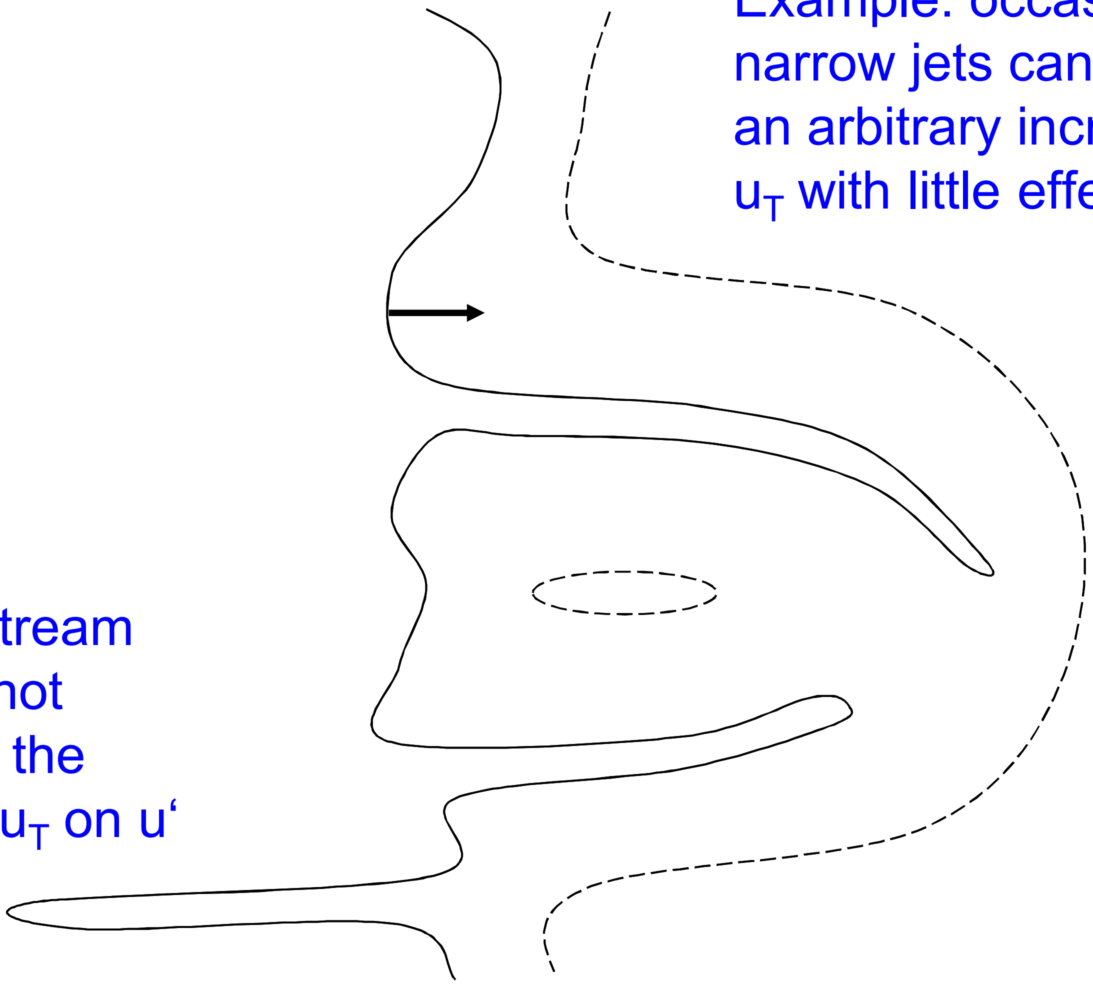


From *G*-equation analysis, Chertkov and Yakhot (1998) reached significant conclusions, but …

- Their claims:
  - u_T is determined by asymptotically high-order intermittency statistics (extreme rare events)
  - A regime in which $u_T << u'$ is predicted
  - Unconventional behavior of the Gibson scale (lower cutoff of front fluctuations) is predicted
- The concerns:
  - Does the analysis capture the dominant wrinkling phenomenology in all instances, or does the result $u_T << u'$ miss a leading-order effect?
  - **Quantitative inferences are based on the statistics of the θ field, not the *G* field**

$u_T << u'$ implies that 'bending' of the (otherwise linear) u' dependence can occur in the flamelet regime, with important implications for data interpretation

Front propagation is a first-passage process, so $u_T >> u'$ is hard to rule out, but $u_T << u'$ seems counter-intuitive



Example: occasional narrow jets can cause an arbitrary increase of u_T with little effect on u'

Extreme downstream fluctuations do not obviously affect the dependence of u_T on u'

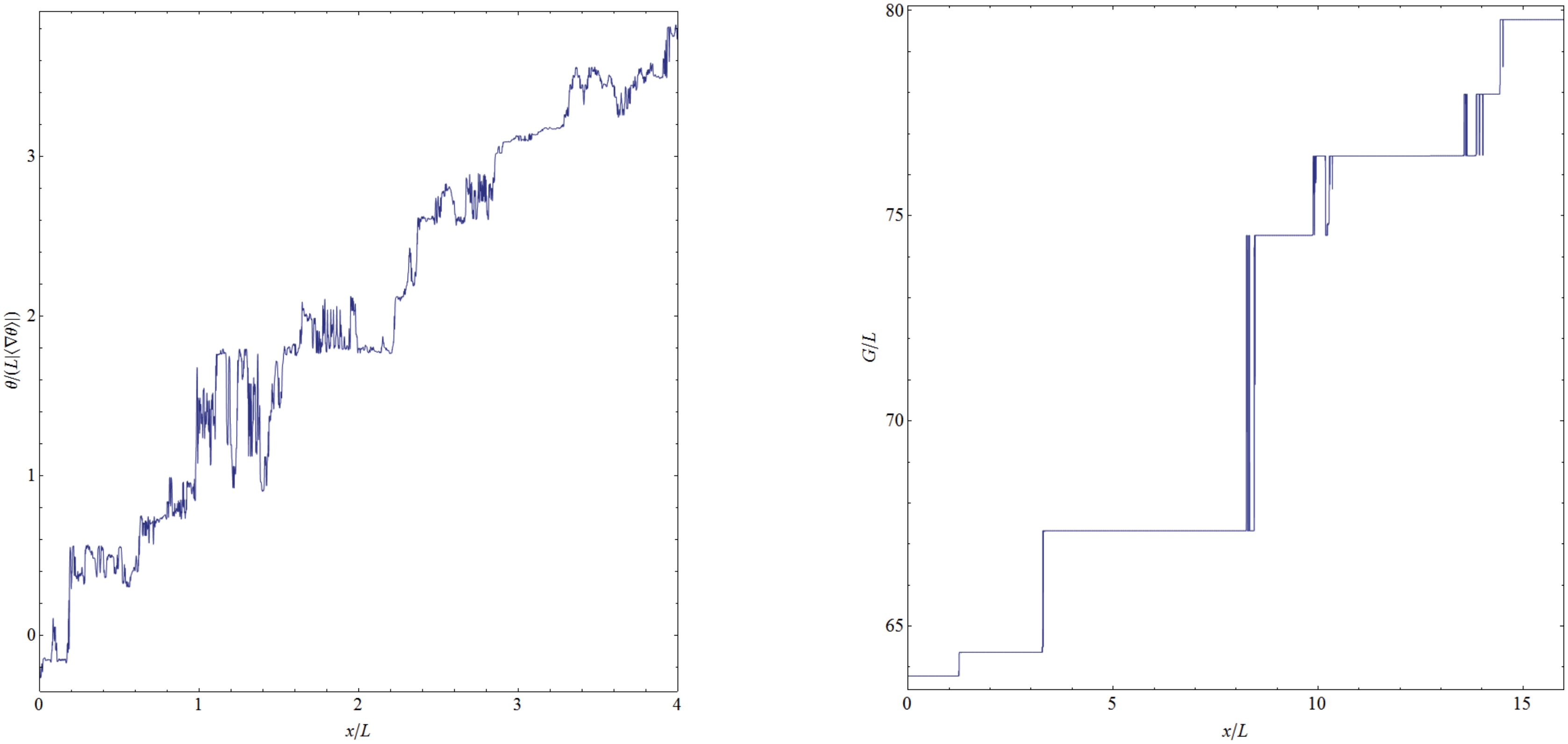
Scalar intermittency has been studied only for Laplacian dissipation, motivating a study of *G*-field intermittency

- High-order structure functions are key signatures of intermittency:

$$S_n(r) = \left\langle \left| \theta(x+r) - \theta(x) \right|^n \right\rangle \propto r^{\zeta(n)}$$

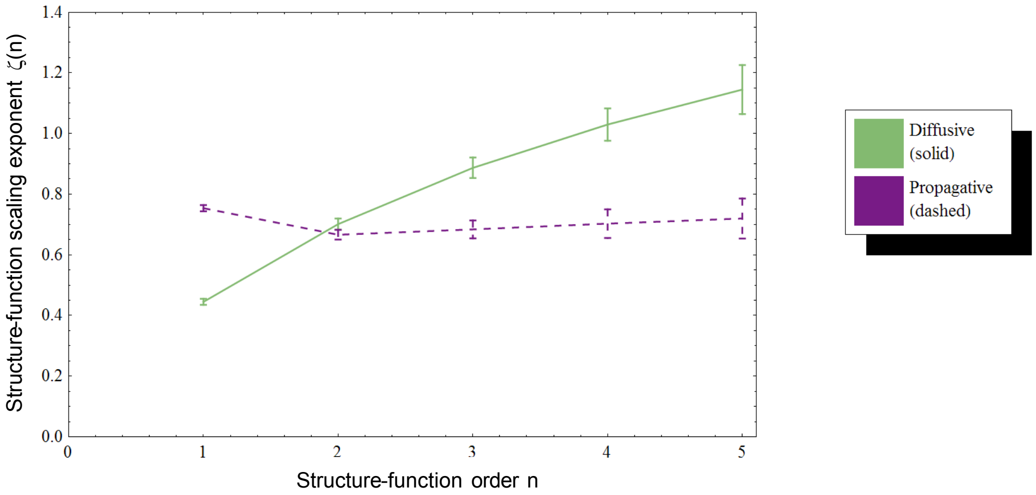
- Large-*n* saturation of *ζ(n)* signals that the largest small-scale gradients are caused by extreme compression of large-scale property differences
- For Laplacian dissipation, theory and empirical evidence do not rule out:
  - no saturation
  - large-*n* asymptotic saturation
  - saturation at finite *n*
- We are studying *G*-field intermittency numerically using
  - 1D stochastic simulation involving map-based advection (Linear-Eddy Model)
  - 2D synthetic white-in-time velocity field (Rapid-Change Model)

Linear-Eddy simulations show major differences between the θ field (left) and the *G* field (right)



In the simulations, 1D maps create local extrema leading to stair-step structure, but the multi-dimensional *G* field has no extrema

Linear-Eddy *G*-field results indicate nearly complete saturation, a predicted but previously unseen ideal limit



- For Laplacian dissipation, results agree well with measurements
- For the *G* field, saturation and the *n*=1 exponent value imply
  - A theoretical perspective that yields no $u_T << u'$ regime
  - Intuitive (rather than counter-intuitive) intermittency effect on the Gibson scale

This first hint of distinctive intermittency behavior of a non-Laplacian scalar suggests a fruitful direction for further research, starting with use of the Rapid-Change Model

For $u' \gg S_L$, possible u_T dependence on high-order fluctuations suggests new research directions

- The proposed dependence should be further evaluated
- If correct, it calls for study of the fluctuations of the relevant property: $G(\mathbf{x},t)$
- Preliminary investigation suggests interesting statistical features