

*A physically-based model for  
low-temperature plasticity in BCC metals*

**Thomas E. Buchheit, Corbett C. Battaile,  
Christopher R. Weinberger, Elizabeth A. Holm**

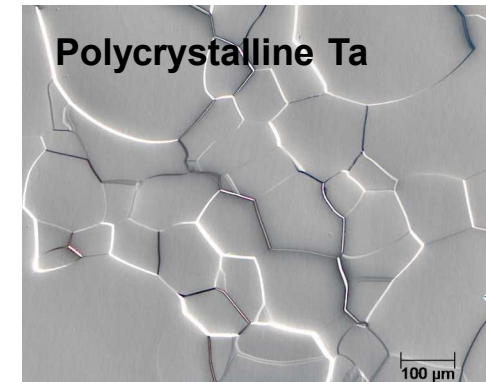
Computational Materials Science and Engineering Department,  
Sandia National Laboratories, Albuquerque, NM 87185-1411 U.S.A.

# Motivation for developing low-temperature BCC deformation models

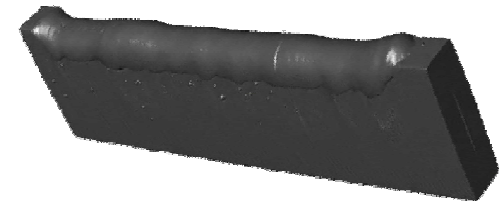
- BCC metals are scientifically interesting.
  - Technologically important.
    - *Refractories: W, Mo, Ta*
    - *Steel*
  - Underrepresented in computational materials science studies.
    - *Complex response, compared to FCC*
    - *Most models are phenomenological*
  - Favorable properties for experimental studies.
    - *Can prepare microstructures ranging from single crystal to nanocrystalline.*
    - *Favorable properties for microscopy and EBSD analysis.*



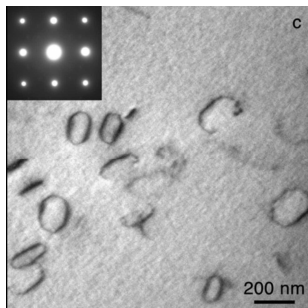
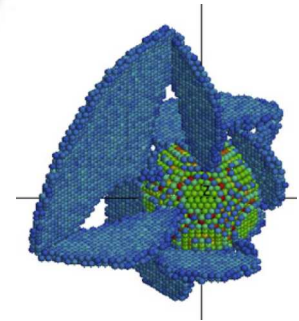
Single-crystal Ta



304L stainless weldment

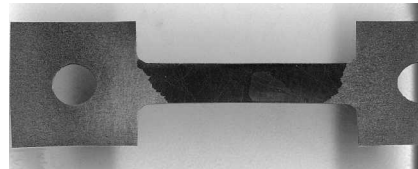
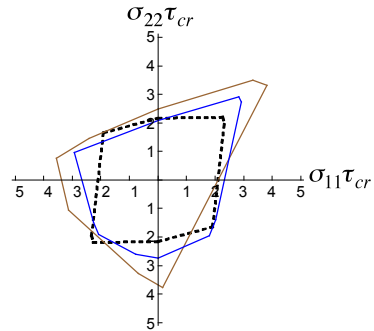


# Including microstructure in design and analysis



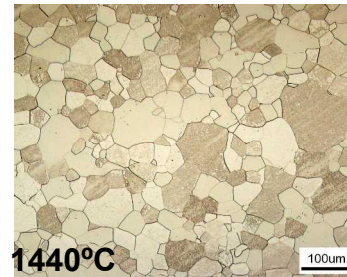
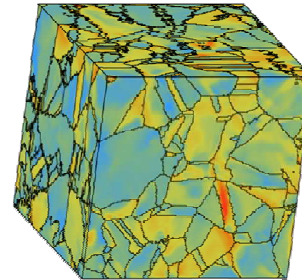
**Atomic scale  
phenomena**

$10^{-9}$  m  
 $10^{-9}$  s



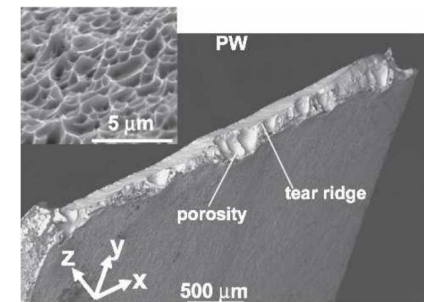
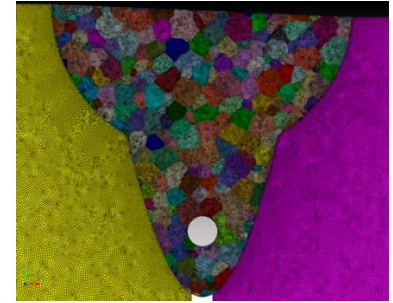
**Single crystal  
behavior**

$10^{-6}$  m  
 $10^0$  s



**Microstructural  
effects**

$10^{-3}$  m  
 $10^3$  s



**Material  
performance**

$10^0$  m  
 $10^6$  s

# How BCC and FCC crystal plasticity differ

- In BCC metals at low temperatures, slip occurs via the motion of screw dislocations along  $\langle 111 \rangle$  directions on  $\{110\}$  planes.

The plastic strain rate is given by:

$$\mathbf{D} = \sum_s \dot{\gamma}^{(s)} \mathbf{m}^{(s)} \rightarrow \begin{array}{l} \text{Schmid factor} \\ \dot{\gamma}^{(s)} = G \left( \frac{\mathbf{m}^{(s)} : \boldsymbol{\sigma}}{\tau^{(s)}} \right) \end{array}$$

**Note:**

FCC slip system:  $\langle 110 \rangle \{111\}$

BCC slip system:  $\langle 111 \rangle \{110\}$

$\therefore \mathbf{m}$  is the same for BCC and FCC

The lattice resistance on slip system  $s$  is:

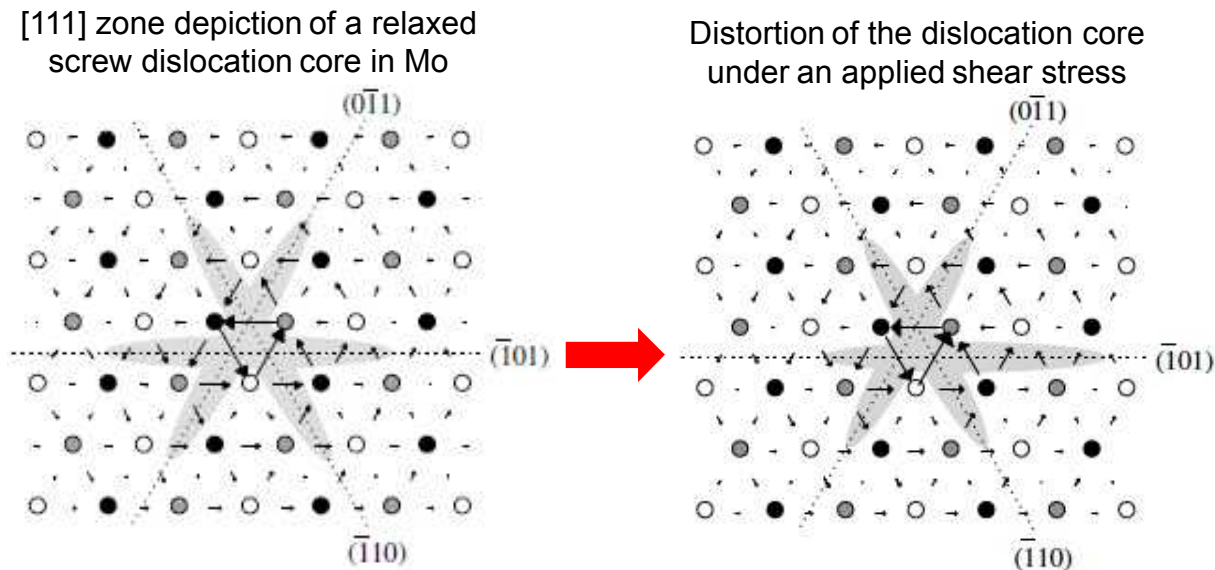
$$\tau^{(s)} = \tau(T, \sigma) = \tau_{\text{obs}} + \tau_{\text{fric}}(T, \sigma) \rightarrow \begin{array}{l} \text{Peierls stress} \\ \text{Obstacle stress} \end{array}$$

In FCC metals,  $\tau_{\text{obs}} \gg \tau_{\text{fric}} \quad \tau_{\text{fric}} \approx 0$

In BCC metals,  $\tau_{\text{fric}} \gg \tau_{\text{obs}} \quad \tau_{\text{obs}} \approx 0$

# Atomic Scale: Physical model for dislocation motion in BCC metals

- Atomic scale simulations show dislocation core spreading onto adjacent (110) planes in BCC metals.
  - Core spreading creates a **significant Peierls barrier** to dislocation motion.
  - Because the dislocation spreads onto three planes, motion can be affected by stress components outside the preferred slip plane, i.e. **non-Schmid stresses**.



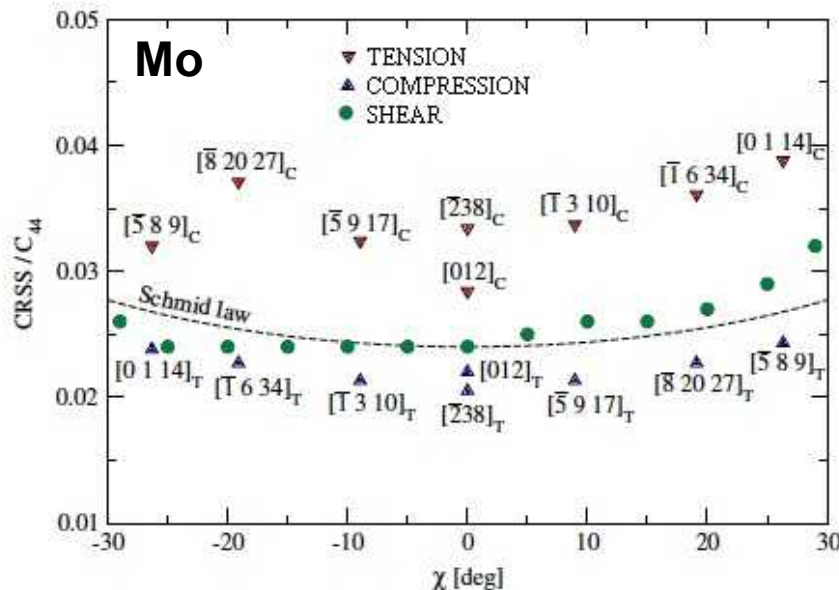
Groger, Vitek et al. *Acta Mat.* **56** (2008) 5412

# Implications of non-Schmid deformation

- The non-Schmid stress components arise from two causes:
  - Asymmetry within the slip plane (twinning/anti-twinning) is a minor effect.
  - Contributions by stress components outside the slip plane are significant.

*“...glide of the  $1/2[111]$  screw dislocation [on the  $(-101)$  plane] depends on shear stresses both parallel and perpendicular to the Burgers vector that act not only in the slip plane but also in other  $\{110\}$  planes of the  $[111]$  zone.”*

-Groger, Vitek et al. *Acta Mat.* **56** (2008) 5412.



**The non-Schmid stress components cause the widely observed tension-compression asymmetry in BCC metals**



## Single crystal behavior: BCC crystal plasticity model

---

The atomic results can be fit to a yield criterion given by:

$$\underbrace{\sigma_{cr}^{app}}_{\substack{\downarrow \\ \text{applied stress}}} \left[ \underbrace{a_0 \mathbf{m}^{(s)} \mathbf{n}^{(s)} + a_1 \mathbf{m}^{(s)} \mathbf{n}^{(s')} + a_2 \left( \mathbf{n}^{(s)} \times \mathbf{m}^{(s)} \right) \mathbf{n}^{(s)} + a_3 \left( \mathbf{n}^{(s)} \times \mathbf{m}^{(s)} \right) \mathbf{n}^{(s')}}_{\substack{\text{stress projection tensor, } \mathbf{P}_{\sigma}^{(s)}}} \right] = \underbrace{\tau_{cr}}_{\substack{\downarrow \\ \text{yield stress}}}$$

We use this form to derive the generalized stress state on a slip system:

$$\tau^{(s)} = \mathbf{P}_{\sigma}^{(s)} : \boldsymbol{\sigma}^{app}$$

Which leads to a single-crystal constitutive law:

$$\dot{\gamma}^{(s)} = \frac{\tau^{(s)}}{\tau_{cr}} \left| \frac{\tau^{(s)}}{\tau_{cr}} \right|^{\frac{1}{m}-1}$$

Which gives the plastic strain rate:

$$\mathbf{D} = \sum_s \dot{\gamma}^{(s)} \mathbf{m}^{(s)}$$





## Material-specific constitutive parameters

---

- The parameters  $a_0$ ,  $a_1$ ,  $a_2$  and  $a_3$  are determined from bond order potential atomistic simulations.

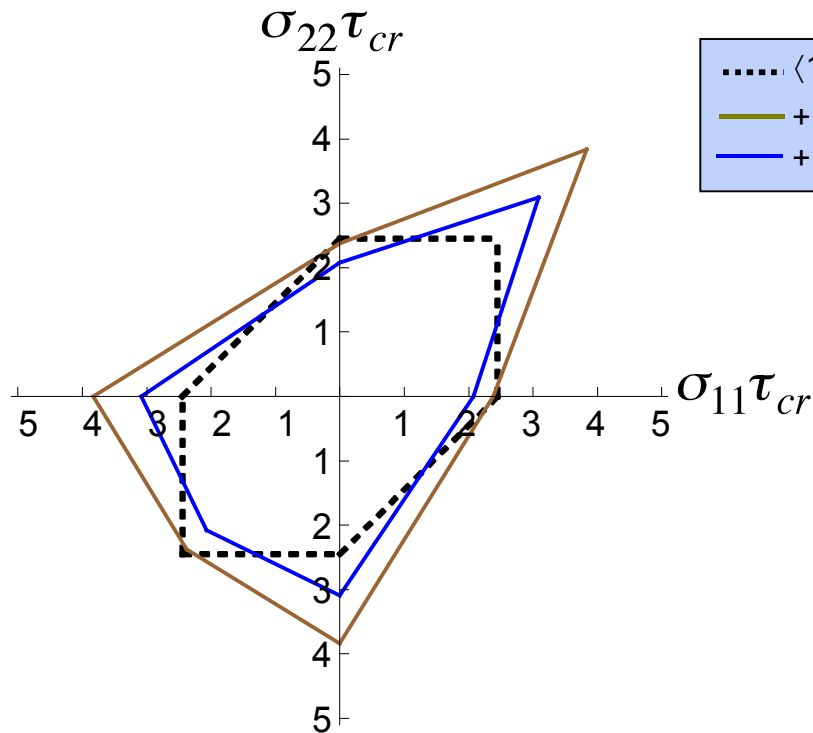
Parameter	FCC	W	Mo	
$a_0$	1	1	1	Schmid stress
$a_1$	0	0	0.24	twinning/anti-twinning
$a_2$	0	0.56	0	out-of-plane effects
$a_3$	0	0.75	0.35	out-of-plane effects
$\tau_{cr}$	1	1.36	1.26	

- Parameters are normalized such that  $\tau_{CRSS} = 1$ .

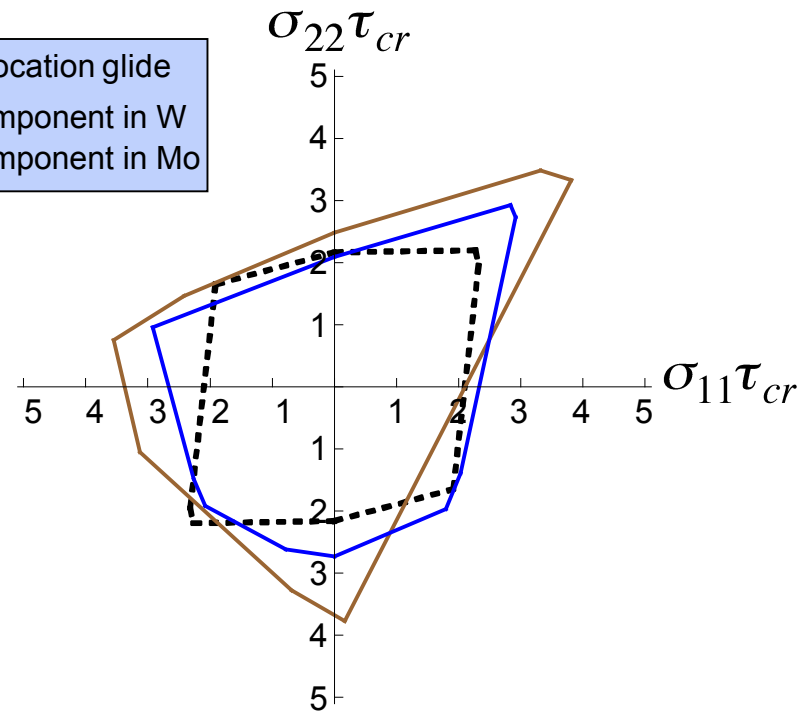
**Gap: To develop similar models for other BCC metals, such as Ta and Fe, we need valid interatomic potential functions.**



# Single Crystal Results: BCC single crystal yield surfaces



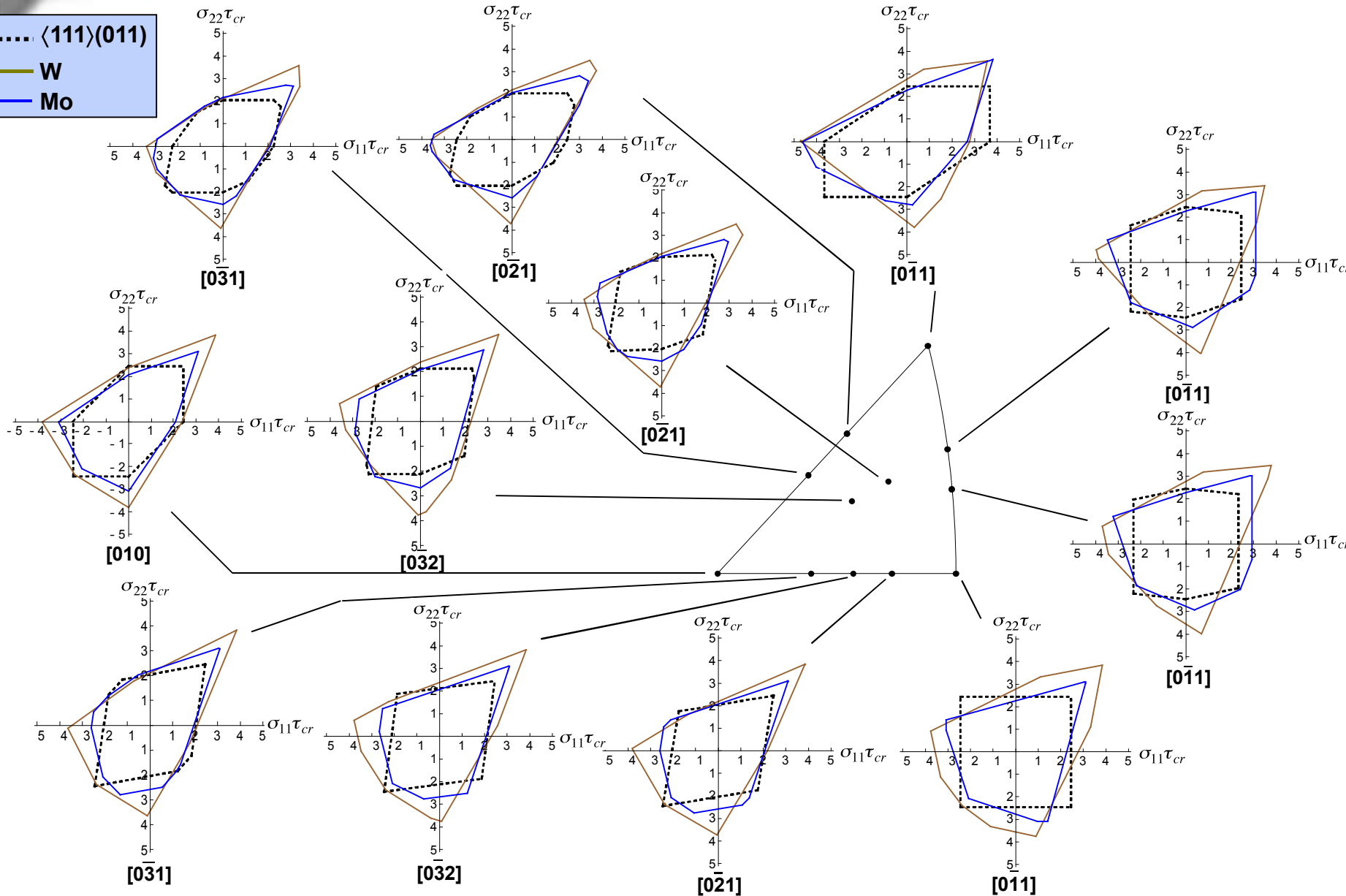
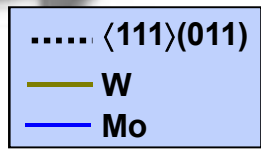
$\langle 100 \rangle \langle 010 \rangle$  orientation  
*highly symmetric*



$\langle -0.180, 0.575, 0.798 \rangle, (0, -0.811, 0.585)$  orientation  
*asymmetric*

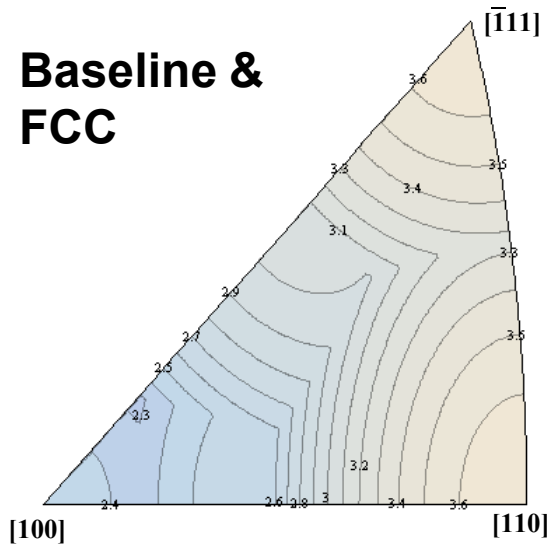
- BCC yield surfaces are considerably different from FCC yield surfaces.
- The yield surfaces of W and Mo are quite distinct.
- Tension/compression asymmetry is apparent.

# There is significant biaxial tension-compression asymmetry in BCC yield surfaces

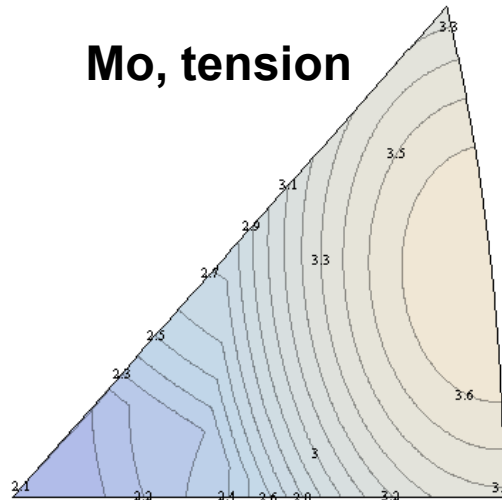


# *Non-Schmid stresses significantly alter the Taylor factor landscape in BCC metals*

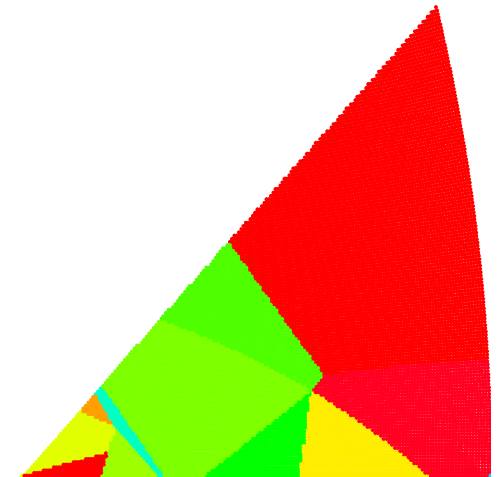
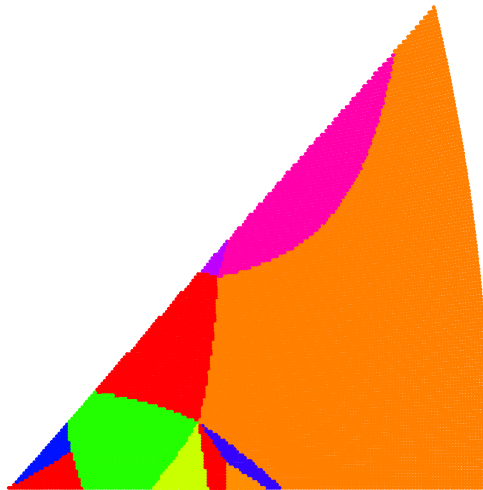
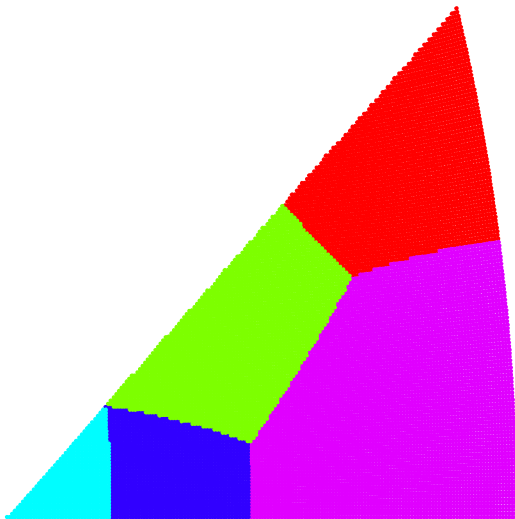
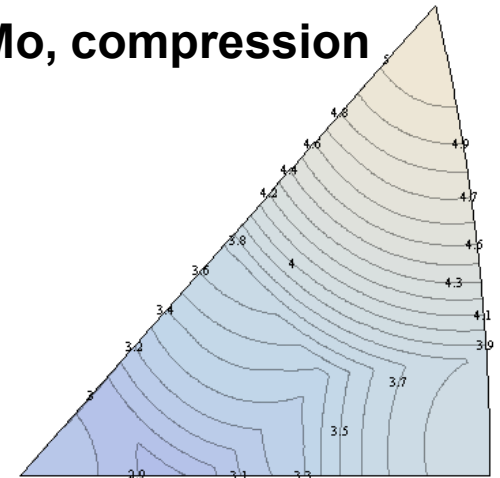
**Baseline & FCC**



**Mo, tension**

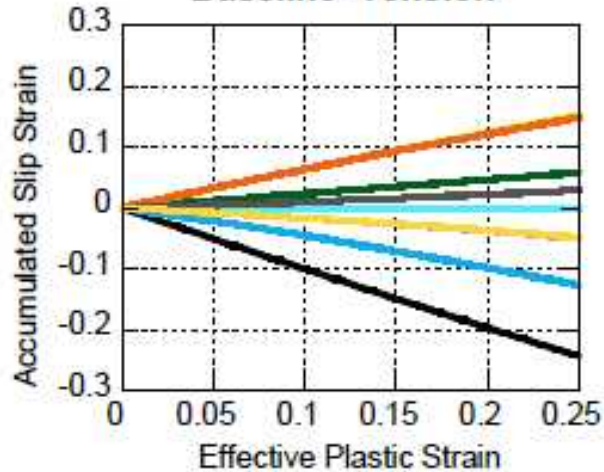


**Mo, compression**

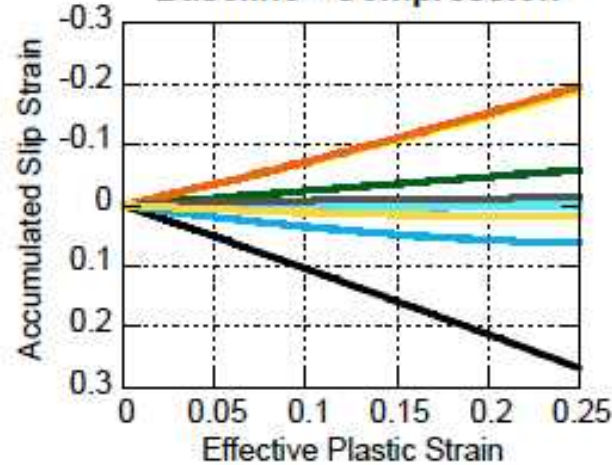


# Non-Schmid stresses significantly alter slip system activity in BCC metals

Baseline- Tension



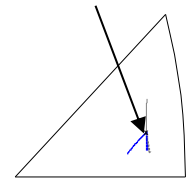
Baseline - Compression



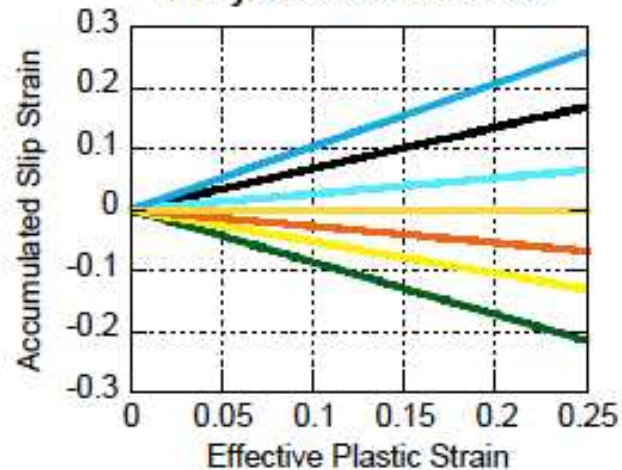
slip system



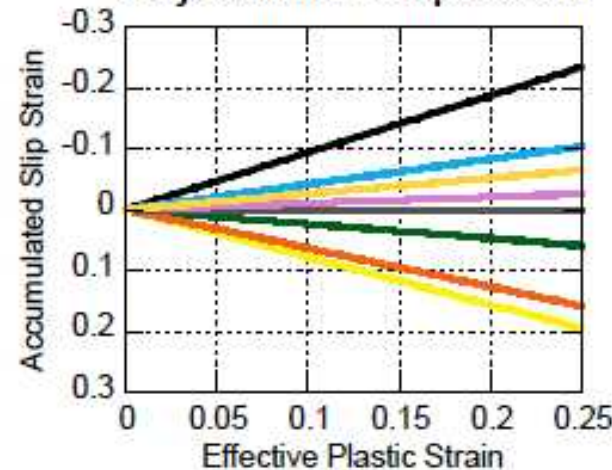
crystal orientation:  
[-.180, 0.575, 0.798]



Molybdenum- Tension



Molybdenum- Compression



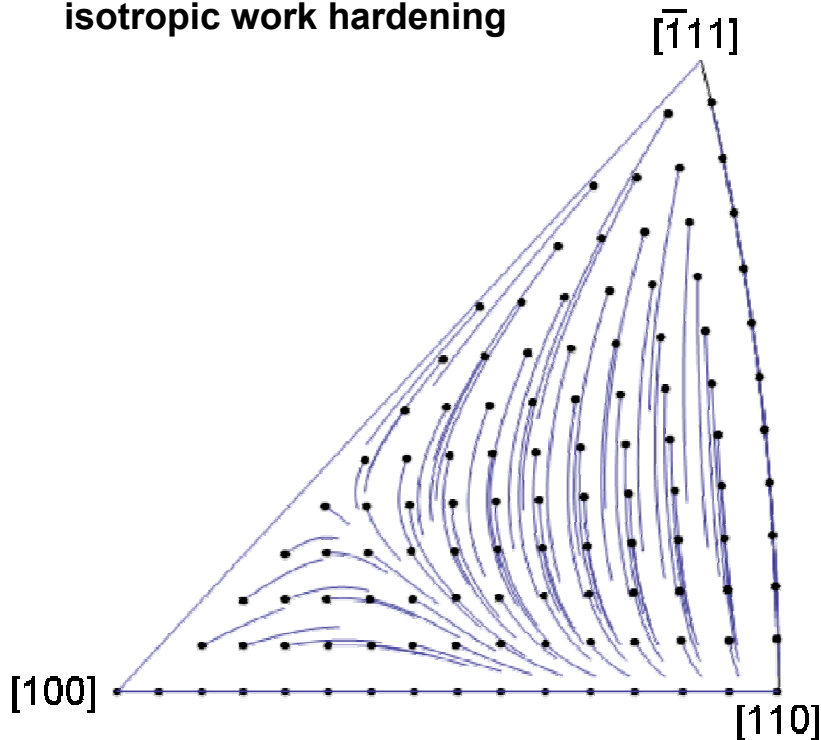
- Baseline is the same in tension and compression.
- Mo differs in tension and compression.
- Mo differs from baseline in tension and compression.

# *Non-Schmid stresses significantly alter crystal rotations in BCC metals*

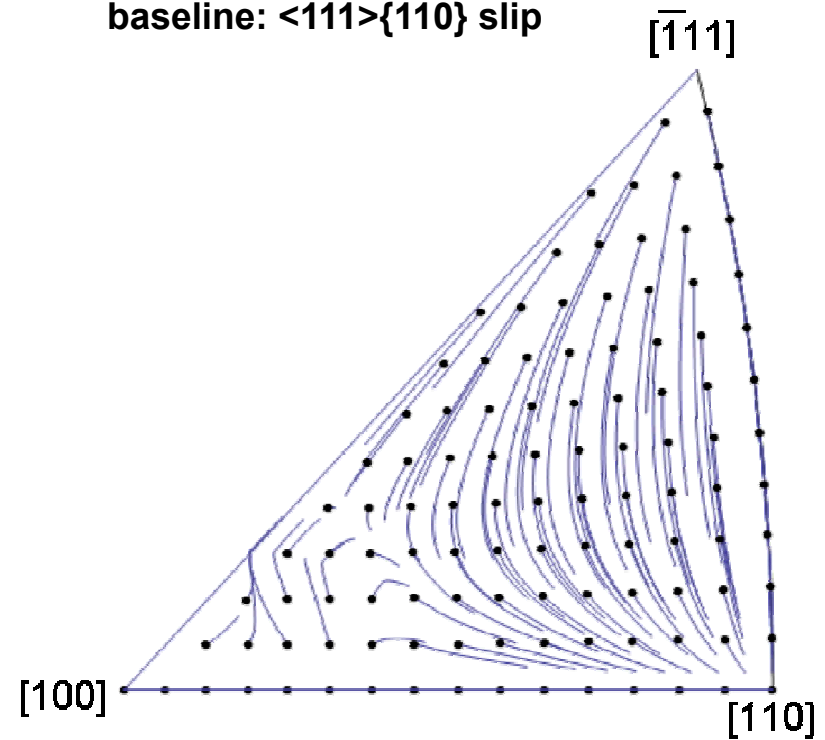
Isochoric Deformation to 50% strain:

$$\dot{\varepsilon} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & -1/2 \end{pmatrix} dt$$

FCC crystal plasticity model with  
isotropic work hardening



BCC crystal plasticity  
baseline:  $\langle 111 \rangle \{110\}$  slip

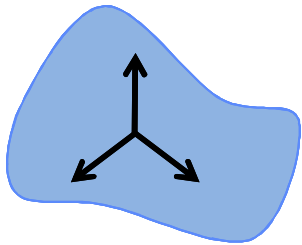




# Extending single crystal behavior to capture microstructural effects

- Polycrystal plasticity models reveal how individual grains take part in polycrystalline deformation

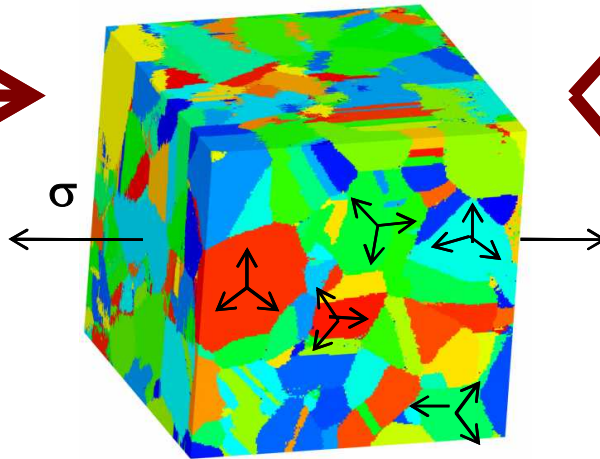
## Single crystal plasticity



Constitutive law

$$\dot{\gamma}^{(s)} = \frac{\tau^{(s)}}{\tau_{cr}} \left| \frac{\tau^{(s)}}{\tau_{cr}} \right|^{\frac{1}{m}-1}$$

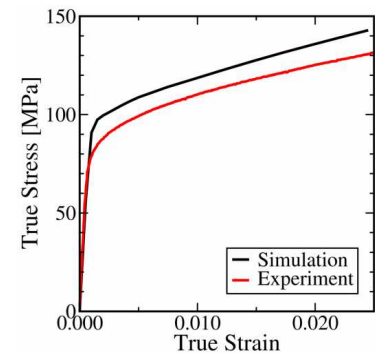
## Polycrystal plasticity



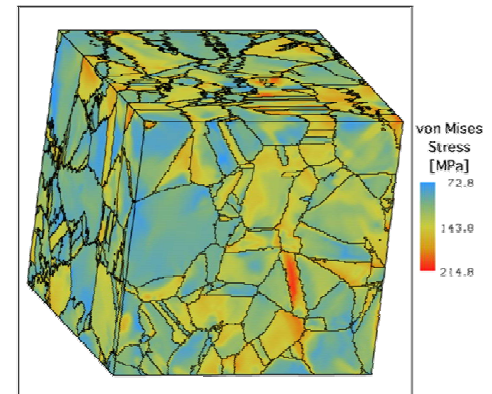
Each grain responds via the orientation-dependent constitutive law

## Results

Overall mechanical response



Individual grain response (rotation, stress, etc.)

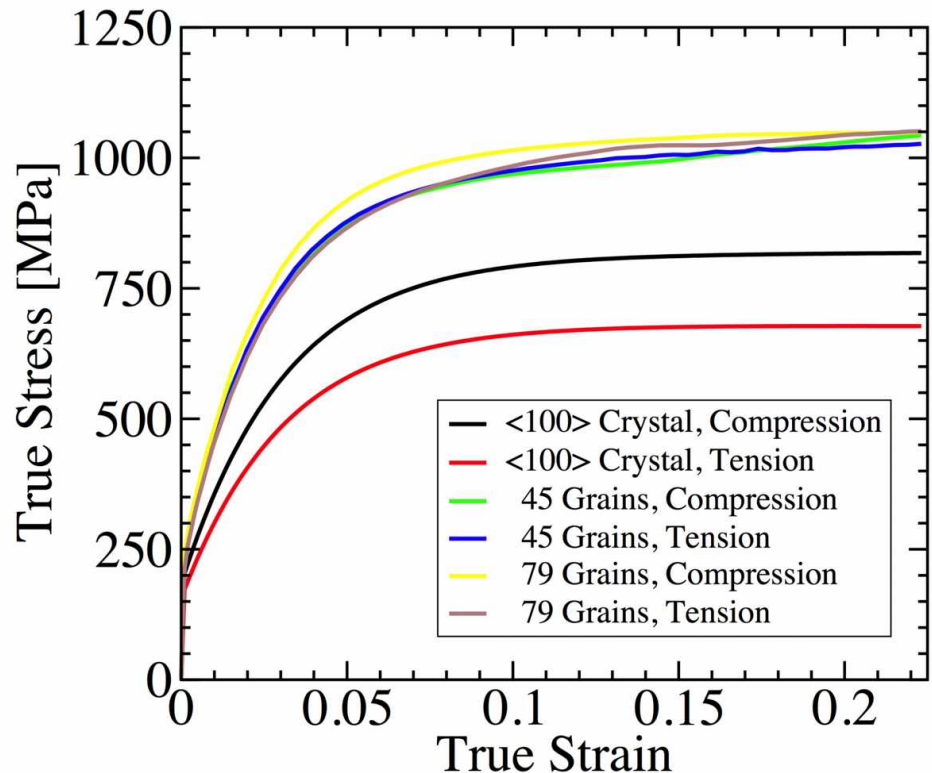




## Microstructural Results: Continuum response of BCC polycrystals

---

- In plasticity simulations of single- and polycrystalline Mo:
  - *Single crystal and polycrystal response differ considerably.*
  - *Single crystals show considerable tension/compression asymmetry.*
  - *Polycrystals do not exhibit tension/compression asymmetry.*
  - *There is no grain size dependence in this model.*

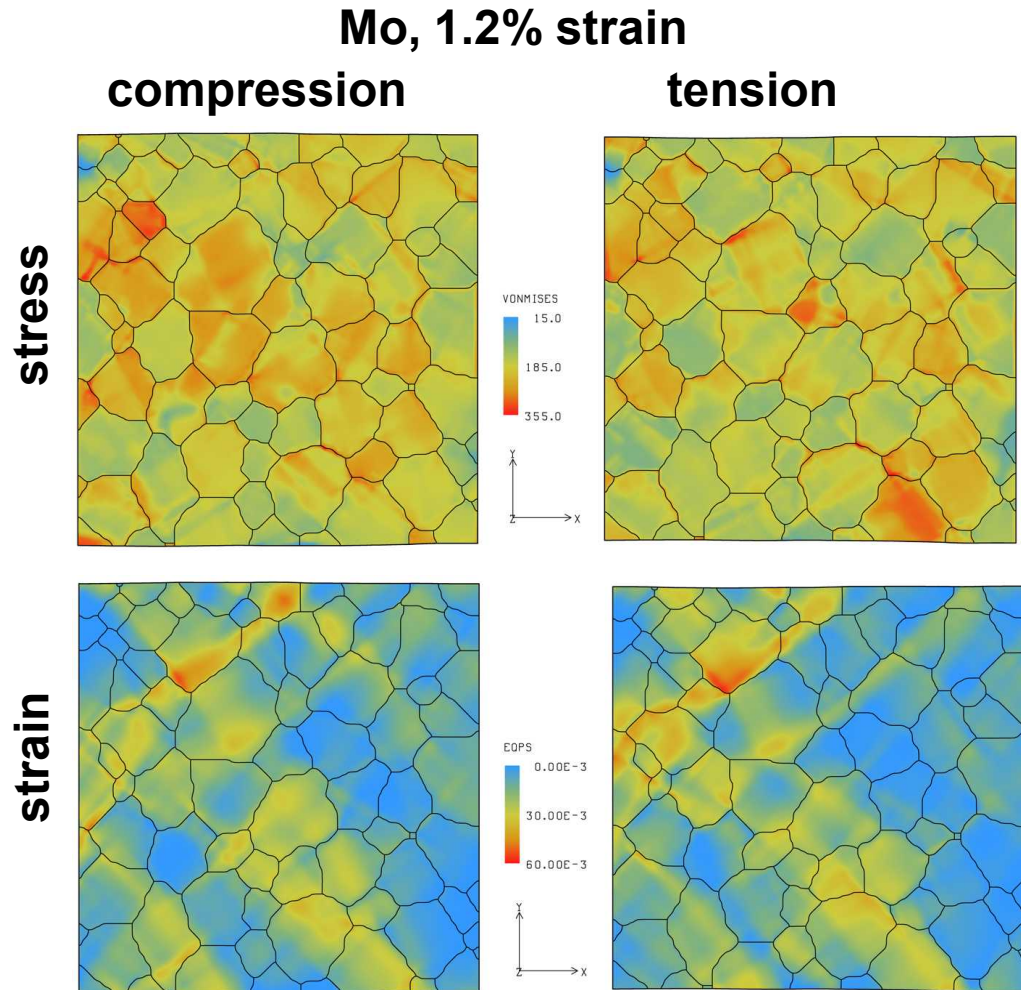




# Microstructural Results: Grain scale stress and strain partitioning

- At the grain scale, tension/compression asymmetry affects local stress distribution.
- Grain structure influences the distribution of local strains, but tension/compression asymmetry does not.
- Local strains are partitioned to accommodate global deformation.
- Grain-scale stresses adjust to produce the required local strain.

**Polycrystal plasticity reveals the complex interdependence of local stress and strain in BCC metals.**





## Summary and Conclusions

---

- **BCC plasticity differs fundamentally from FCC plasticity**
- **A physically-based model captures key elements of BCC plasticity**
  - *Tension/compression asymmetry and yield surfaces*
  - *Differences in Taylor factor, slip system activity, crystal rotation*
- **Polycrystal plasticity reveals how single-crystal properties interact in realistic grain structures**
  - *Tension/compression asymmetry is maintained*
  - *Significant stress concentrations occur*



## *Supplementary Slides*

---

# Polycrystal plasticity model

Cauchy stress resolved on each slip system:

$$\tau = \sigma : (\mathbf{s} \otimes \mathbf{m})$$

Cauchy stress  
from FE solver

→ Slip rate on each system:

$$\dot{\gamma} = \dot{\gamma}_o \left| \frac{\tau}{\tau_{CRSS}} \right|^{\frac{1}{m}} \text{sign}(\tau)$$

Plastic velocity gradient:

$$\hat{\mathbf{L}}_p = \dot{\gamma} (\hat{\mathbf{s}} \otimes \hat{\mathbf{m}})$$

Plastic deformation gradient:  
(Cayley-Hamilton theorem<sup>1</sup>)

$$\dot{\mathbf{F}}_p = \hat{\mathbf{L}}_p \cdot \mathbf{F}_p \Rightarrow \begin{cases} \mathbf{F}_p = \exp(\hat{\mathbf{L}}_p \Delta t) \mathbf{F}_p \\ \exp(\hat{\mathbf{L}}_p \Delta t) = \mathbf{I} + \frac{\sin \phi}{\phi} \hat{\mathbf{L}}_p \Delta t + \frac{1 - \cos \phi}{\phi^2} (\hat{\mathbf{L}}_p \cdot \hat{\mathbf{L}}_p) \Delta t^2 \\ \phi = \Delta t \sqrt{\frac{1}{2} (\hat{\mathbf{L}}_p : \hat{\mathbf{L}}_p)} \end{cases}$$

Elastic deformation gradient and strain:

$$\mathbf{F}_e = \mathbf{F} \cdot (\mathbf{F}_p)^{-1} \Rightarrow \hat{\mathbf{E}}_e = \frac{1}{2} [(\mathbf{F}_e)^T \cdot \mathbf{F}_e - \mathbf{I}]$$

2nd Piola-Kirchhoff stress (hyper-elasticity):

$$\hat{\sigma}_{PK2} = \mathbf{C} : \hat{\mathbf{E}}_e$$

Updated Cauchy stress:

$$\sigma = \frac{1}{J^{\mathbf{F}_e}} [\mathbf{F}_e \cdot \hat{\sigma}_{PK2} \cdot (\mathbf{F}_e)^T]$$

Cauchy stress  
to FE solver

Updated crystallographic orientation:

$$\mathbf{F}_e = \mathbf{U}_e \cdot \mathbf{R}_e \Rightarrow \mathbf{R}_{lattice} = \mathbf{R}_e \cdot \mathbf{R}_h$$

Effective plastic strain:

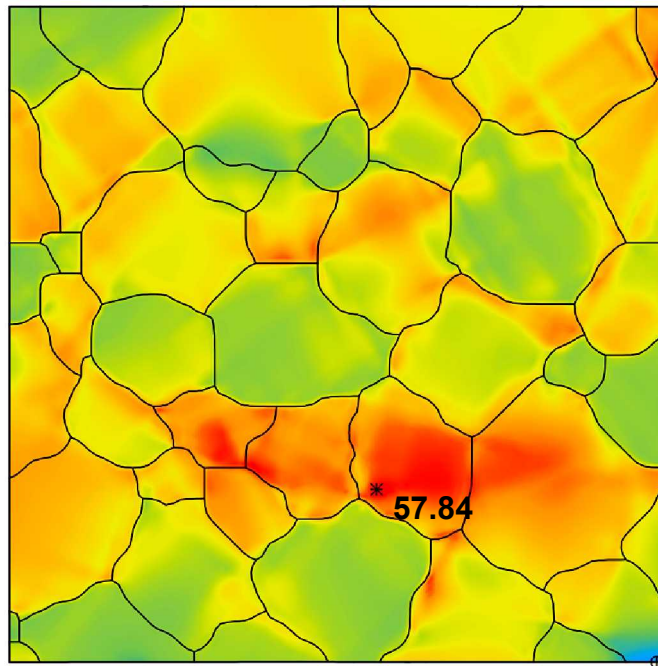
$$\bar{\varepsilon}_p = \sqrt{\frac{2}{3} \mathbf{E}_p : \mathbf{E}_p}$$

Updated “hardness” (CRSS):

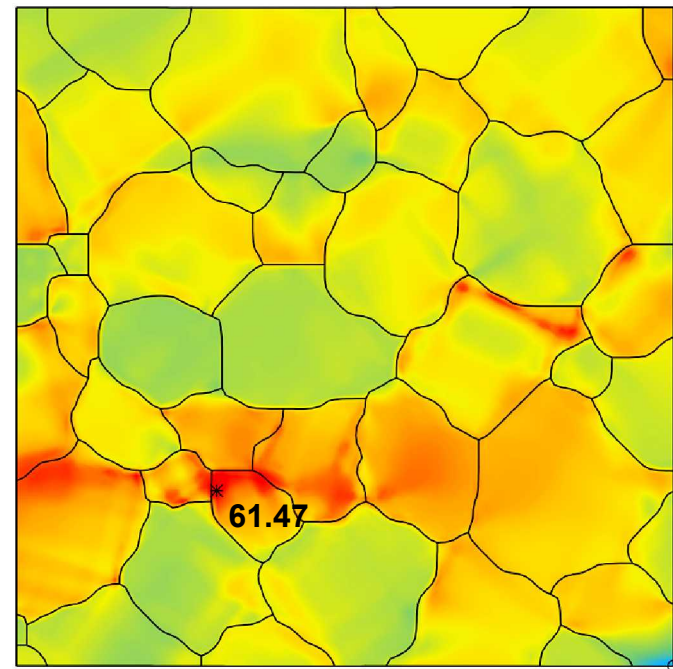
$$\tau_{CRSS} = \tau_o + A \exp\left(-\frac{n}{A} \bar{\varepsilon}^p\right)$$

*Local stress concentrations are  
more severe prior to yielding*

### Von-Mises Stress Distribution in Mo (0.1% Strain)



**Tension**



**Compression**