

An Approach for In-Situ Scan Impedance Characterization of Phased Antenna Arrays

8th July 2013

IEEE APS 2013, Paper #2873

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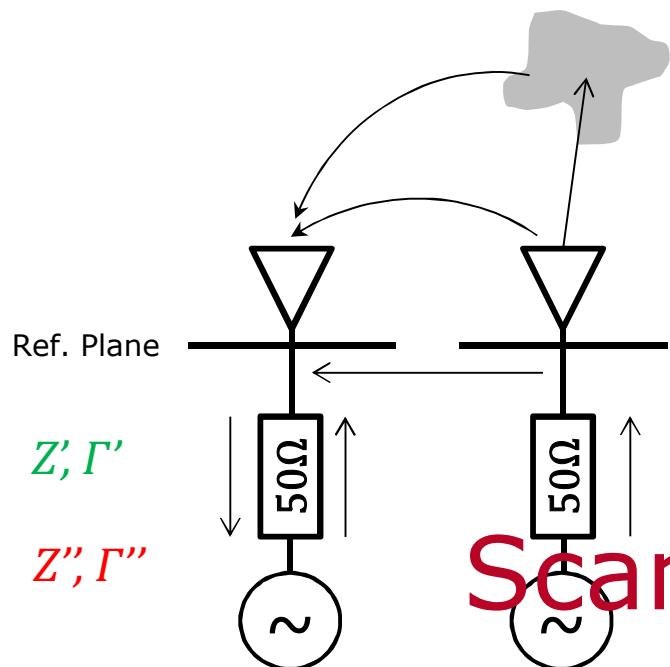
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Objectives of this Study

- **Understand** mutual coupling and its effects on scan/active impedance, and subsequent array excitation
- **Develop** an automated in-situ instrumentation system to measure mutual coupling between four planar array elements, but adaptable to an arbitrary number, or non-planar elements
- **Formulate** a mathematical algorithm to predict active scan impedance variations via measured mutual coupling



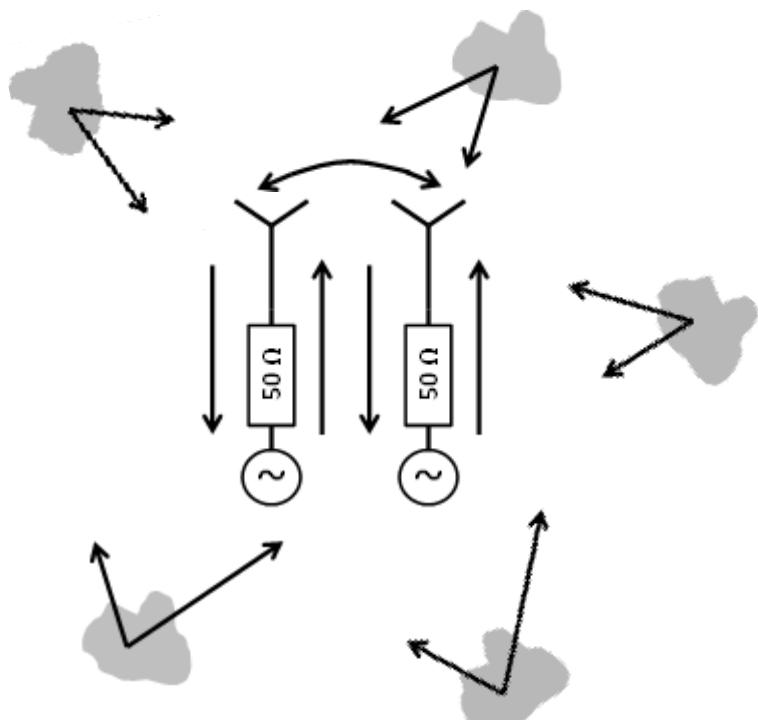
$$Z' = Z_o \frac{1 + \Gamma'}{1 - \Gamma'}$$

$$Z'' = Z_o \frac{1 + \Gamma''}{1 - \Gamma''}$$

Scan Impedance

Primary Mechanisms of Mutual Coupling

- Coupling between elements
- Coupling from within the array
- Scattering from nearby objects





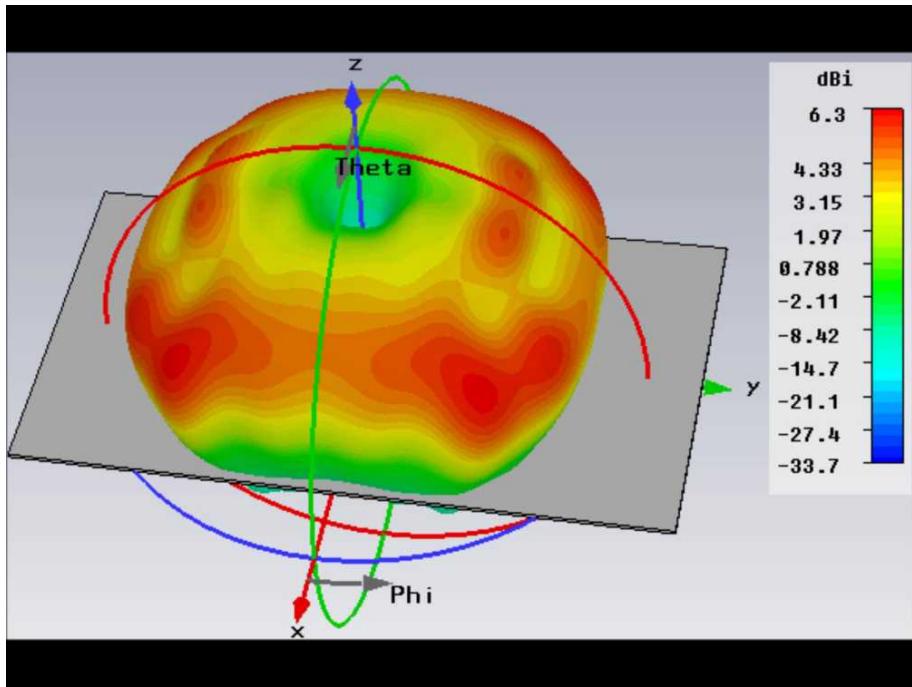
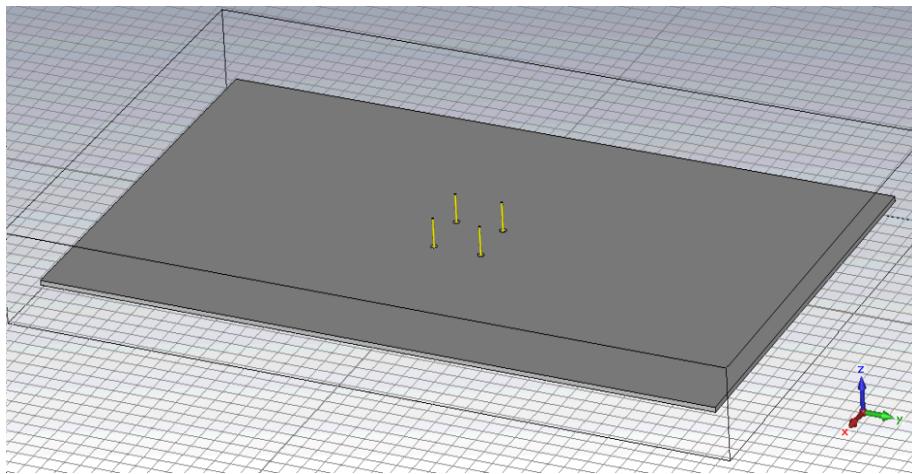
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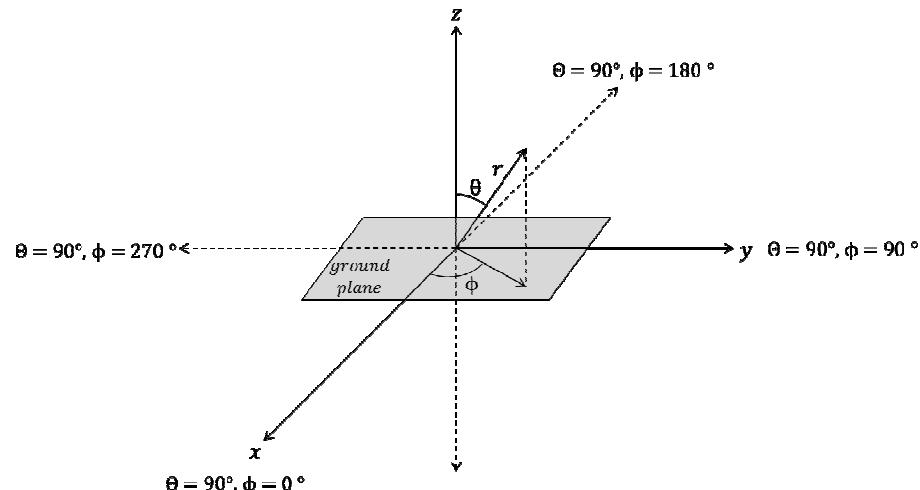
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Spherical Coordinate System



$$E_\theta \simeq j\eta \frac{kI_o le^{-jkr}}{4\pi r} \sin \theta [2 \cos(kh \cos \theta)], \quad z \geq 0$$
$$E_\theta = 0, \quad z < 0$$

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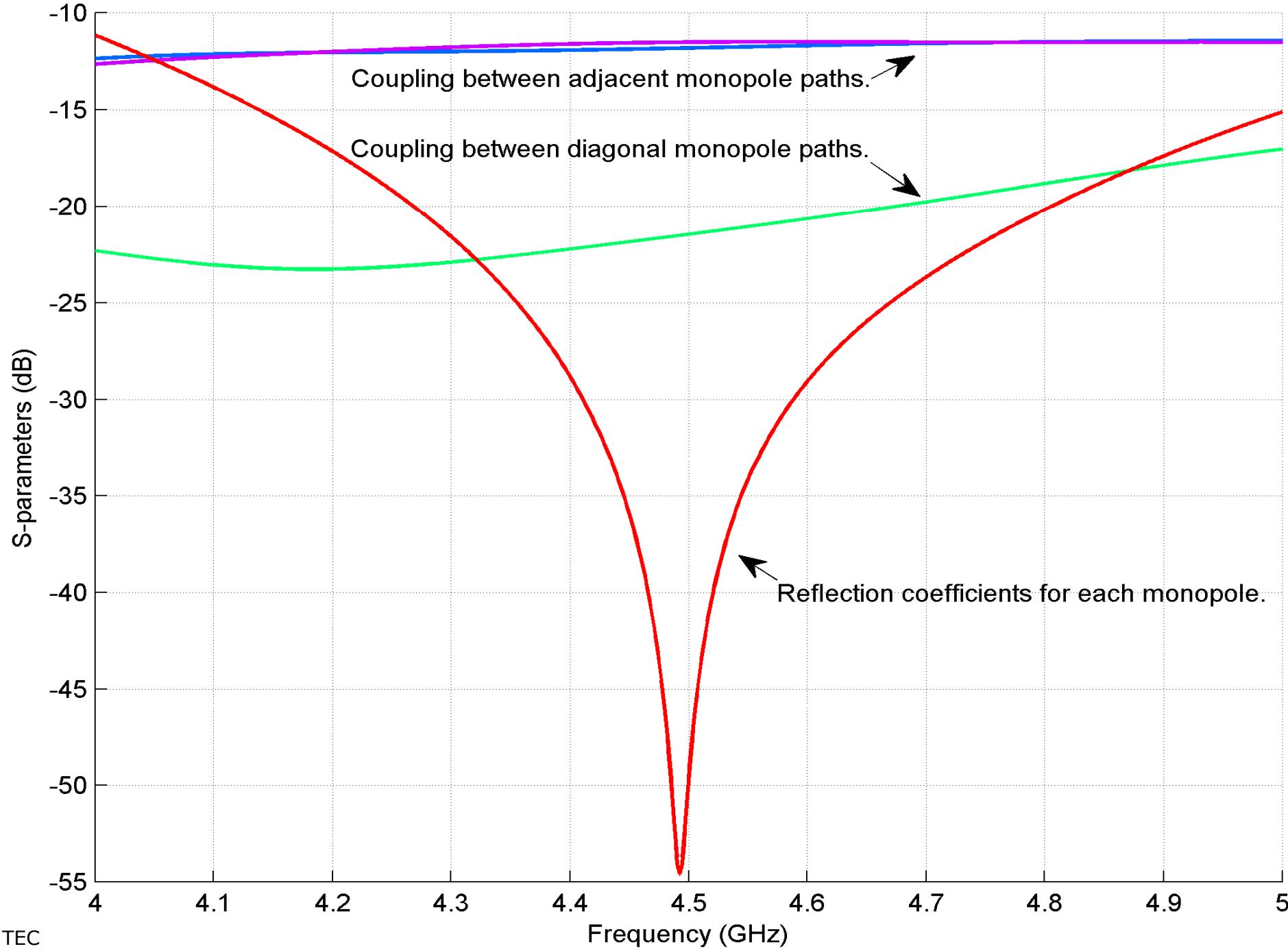
Scattering Parameters

- Represent transmission and reflections of an N-port network. Perfect for representing mutual coupling of an antenna array.
- Can easily be measured with a network analyzer

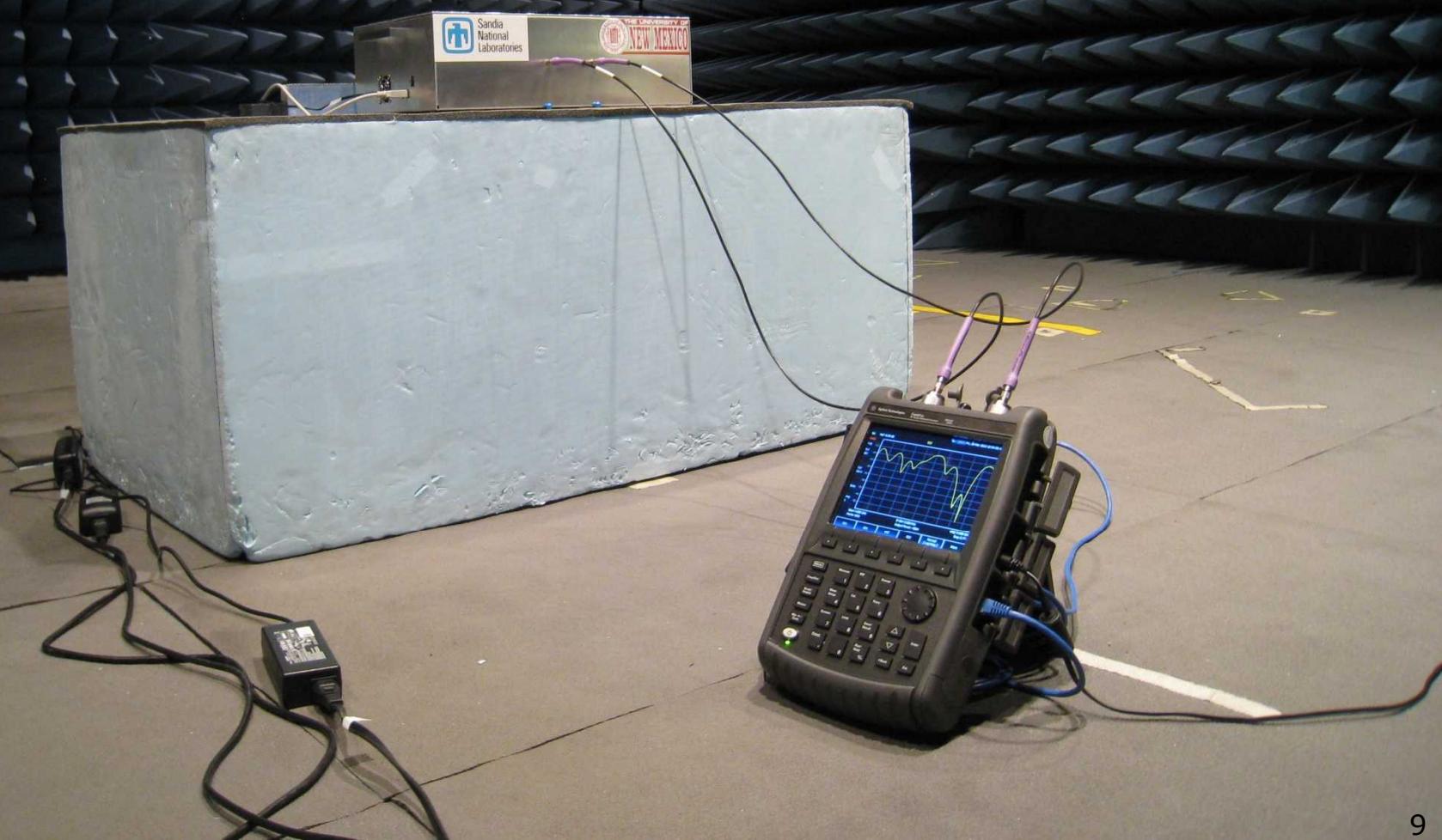
$$\begin{bmatrix} b_1 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & \cdots & S_{1N} \\ \vdots & \ddots & \vdots \\ S_{N1} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix}$$

$$a_n = \frac{V_n^+}{\sqrt{Z_{0n}}} \quad b_n = \frac{V_n^-}{\sqrt{Z_{0n}}} \quad S_{ij} = \left. \frac{b_i}{a_j} \right|_{a_k=0, for\ k \neq j}$$

CST MWS Scattering Matrix for 2x2 Monopole Array

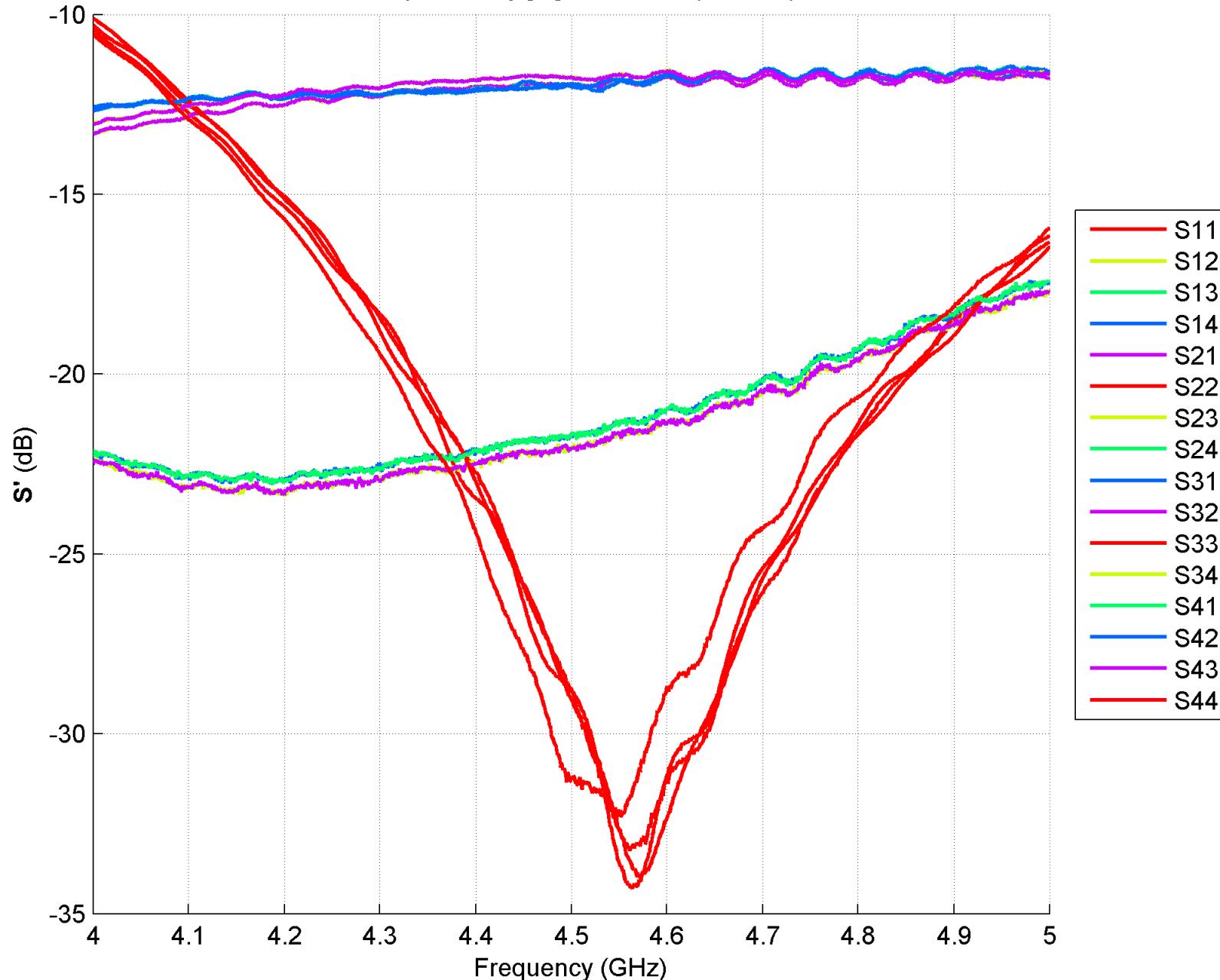




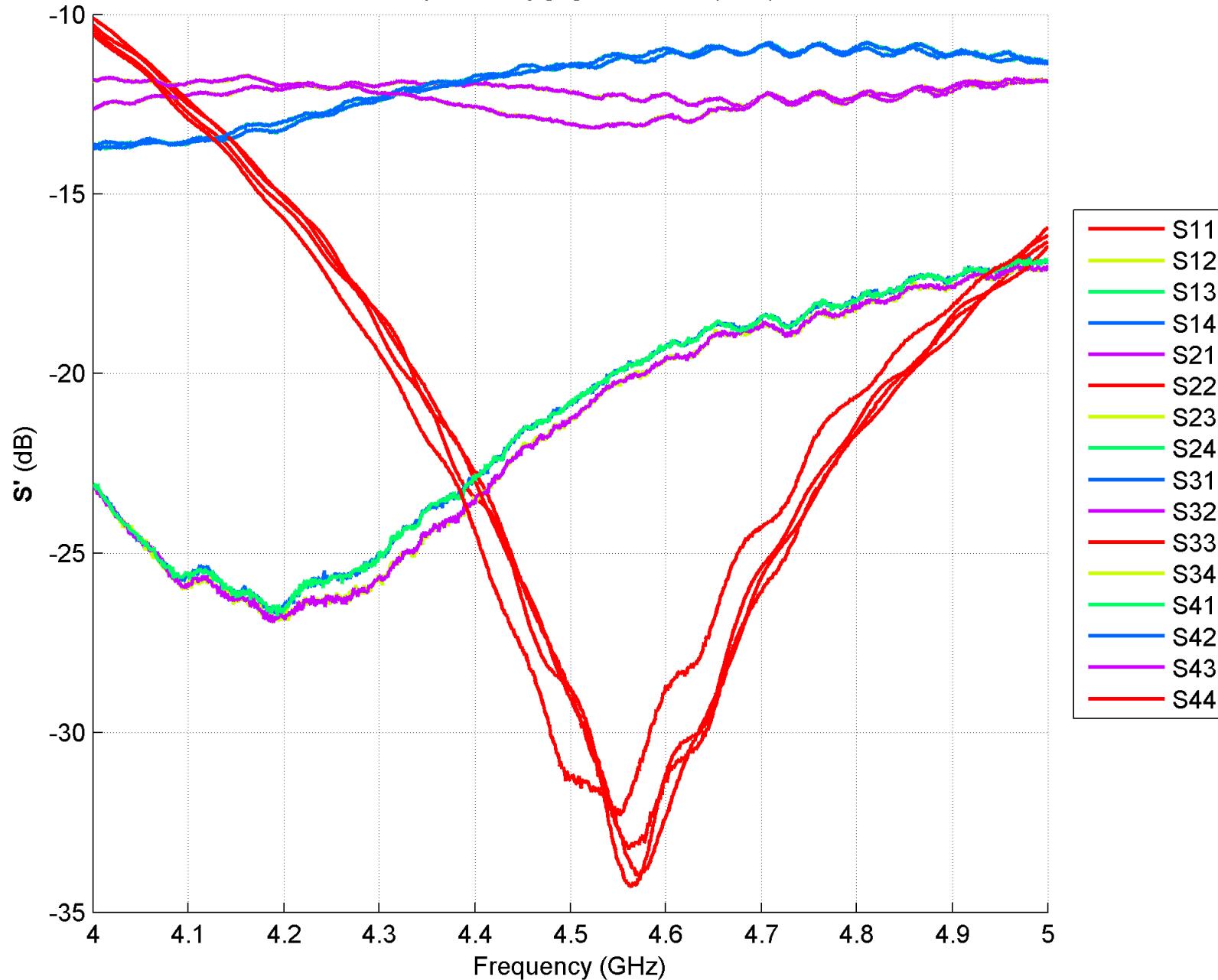


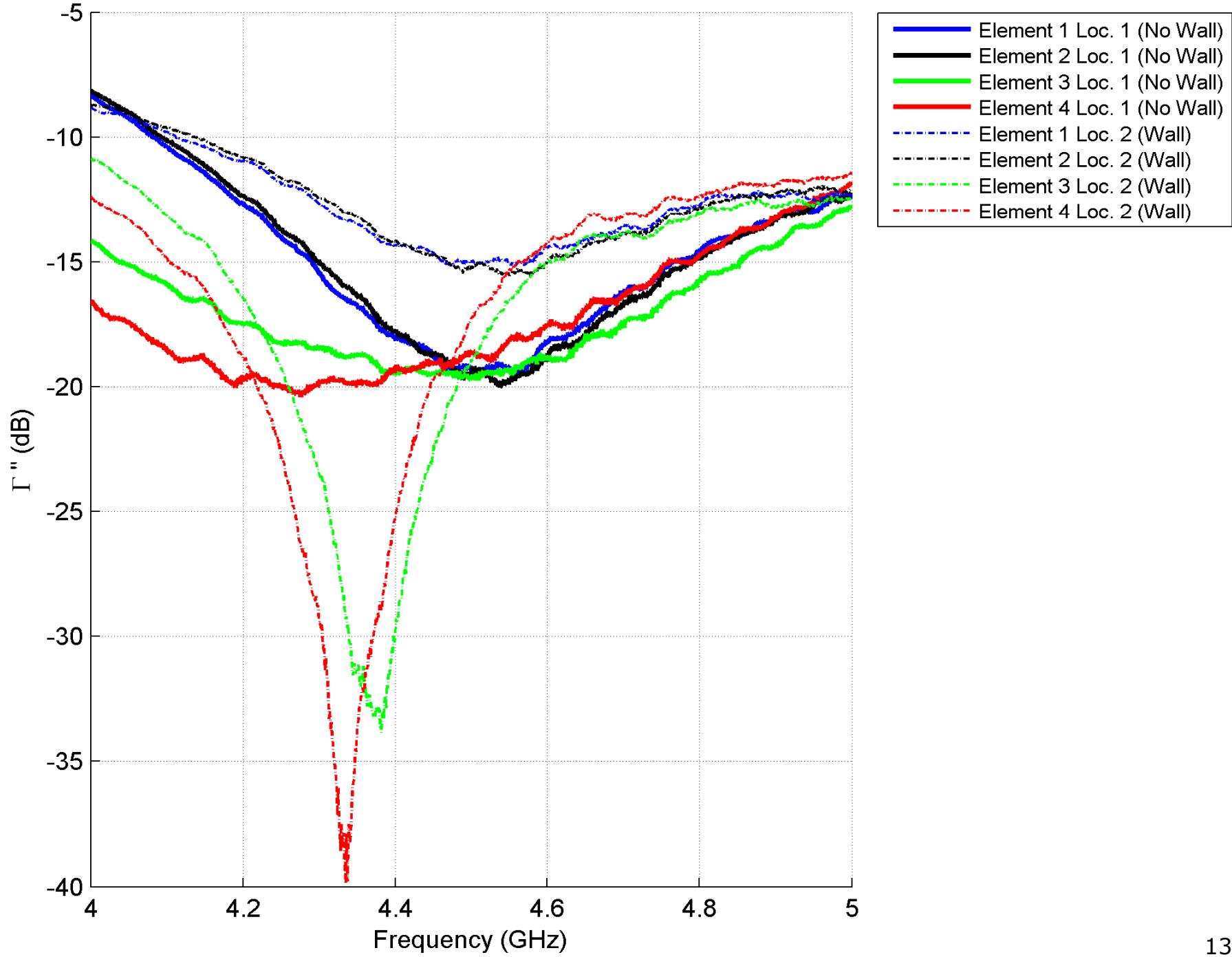


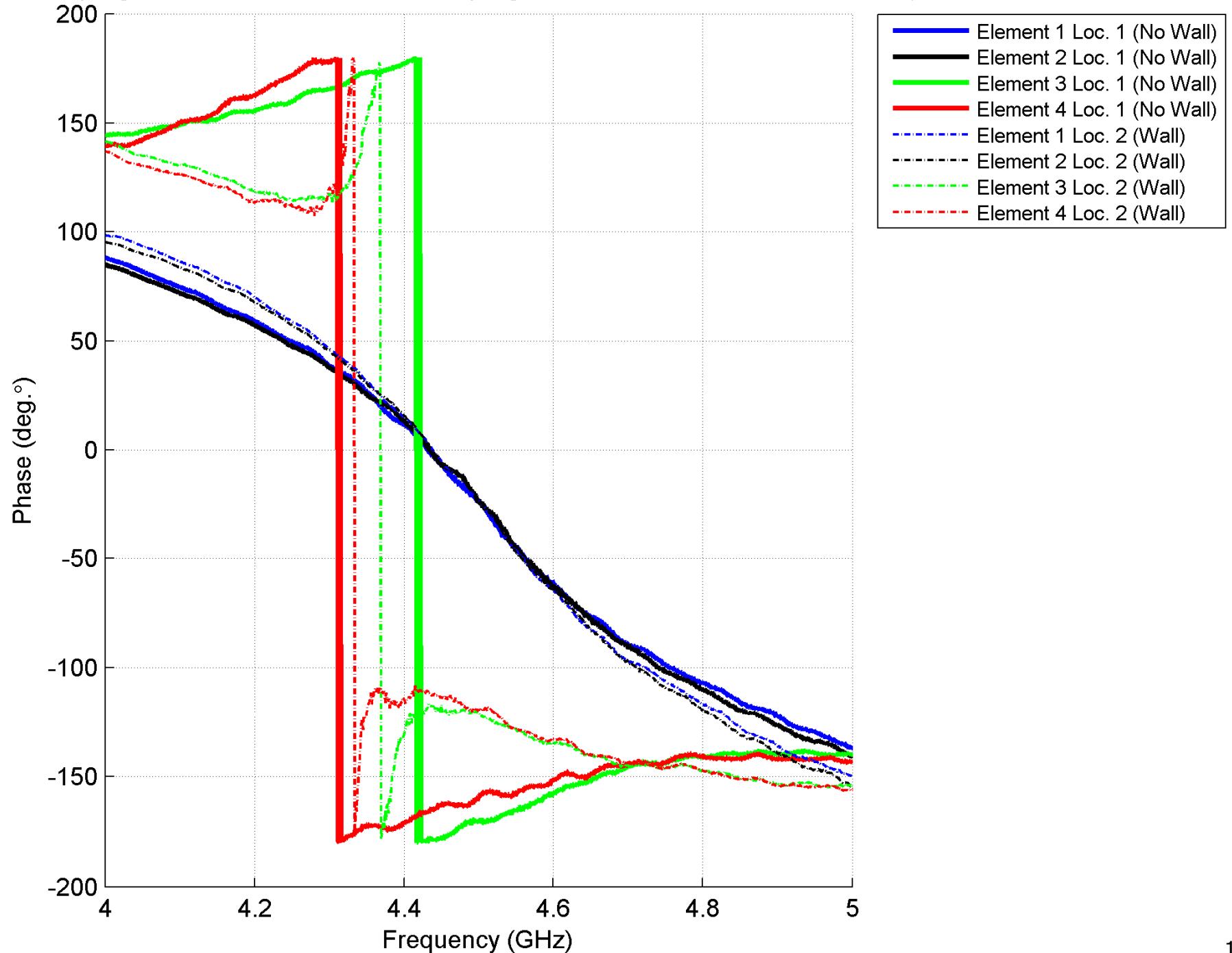
Monopole Array [\mathbf{S}'] Location 1 (No Wall)



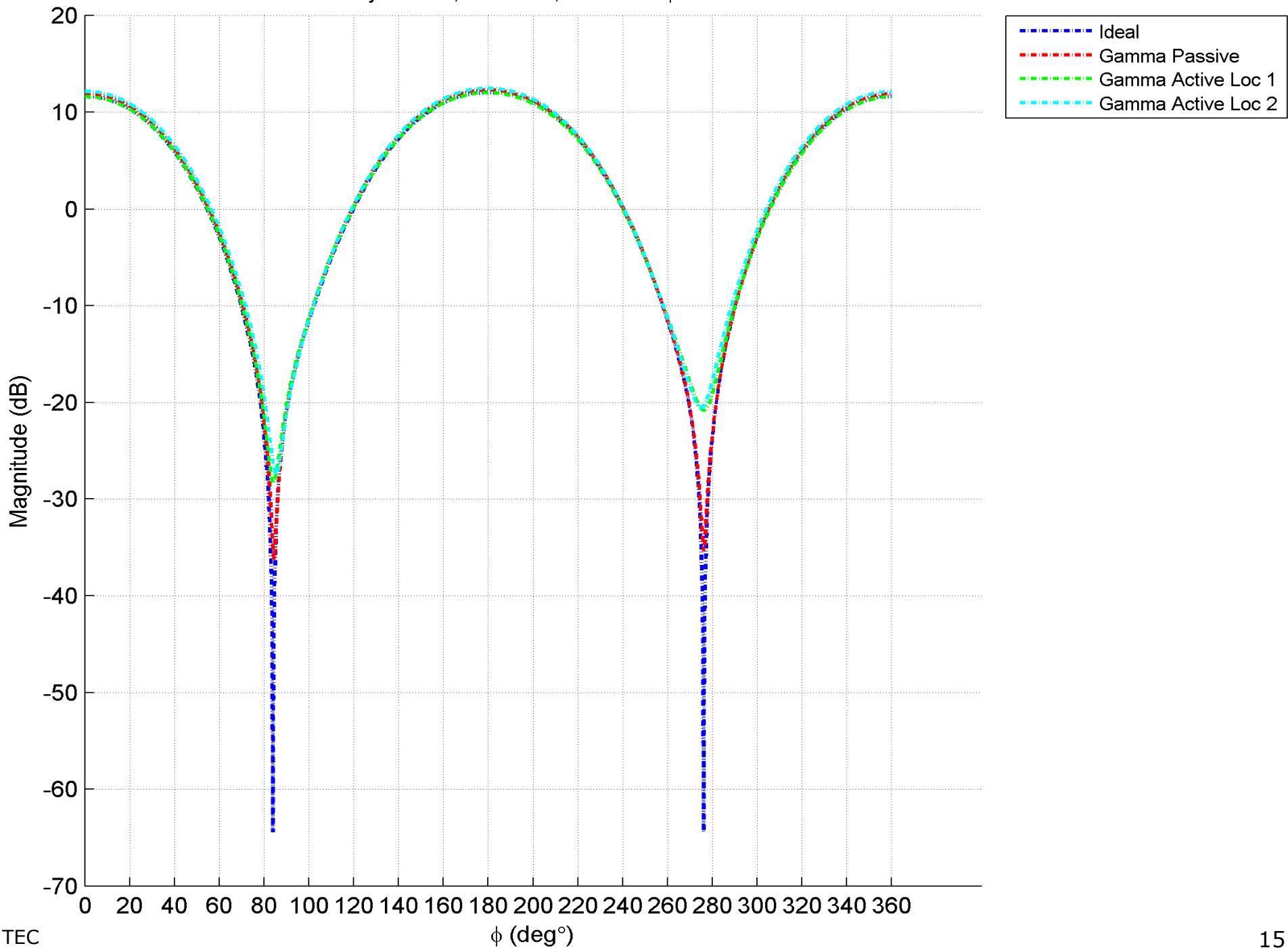
Monopole Array [S'] Location 2 (Wall)

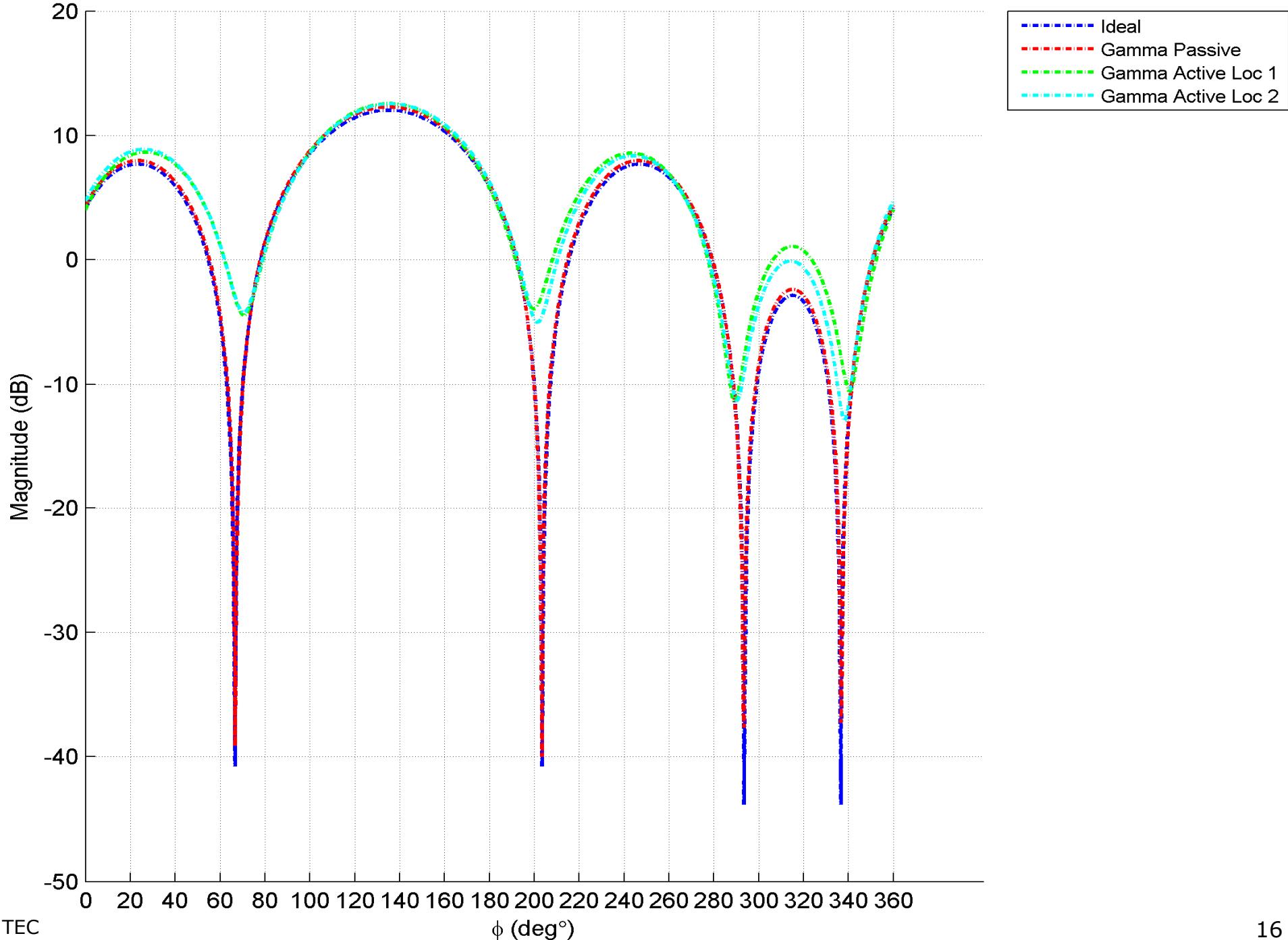


Mutual Coupling Environment 1 vs. Mutual Coupling Environment 2, Γ'' at $\theta = 90^\circ$ and $\phi = 180^\circ$ 

Mutual Coupling Environment 1 vs. Mutual Coupling Environment 2, Γ'' at $\theta = 90^\circ$, and $\phi = 180^\circ$ 

Array Factor, 4.5 GHz, at $\theta = 90^\circ$ $\phi = 180^\circ$



Array Factor, 4.5 GHz, at $\theta = 90^\circ$ $\phi = 135^\circ$ 



Scan Impedance Theory

$$Z_{mn}(\theta, \phi) = Z(0,0) \frac{1 + \Gamma_{mn}(\theta, \phi)}{1 - \Gamma_{mn}(\theta, \phi)}$$

$$\Gamma_m = \frac{b_m}{a_m}$$

$$\begin{bmatrix} b_1 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & \cdots & S_{1N} \\ \vdots & \ddots & \vdots \\ S_{N1} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix}$$

Each element is active during scanning according to an excitation defined by the array factor which forms [a]

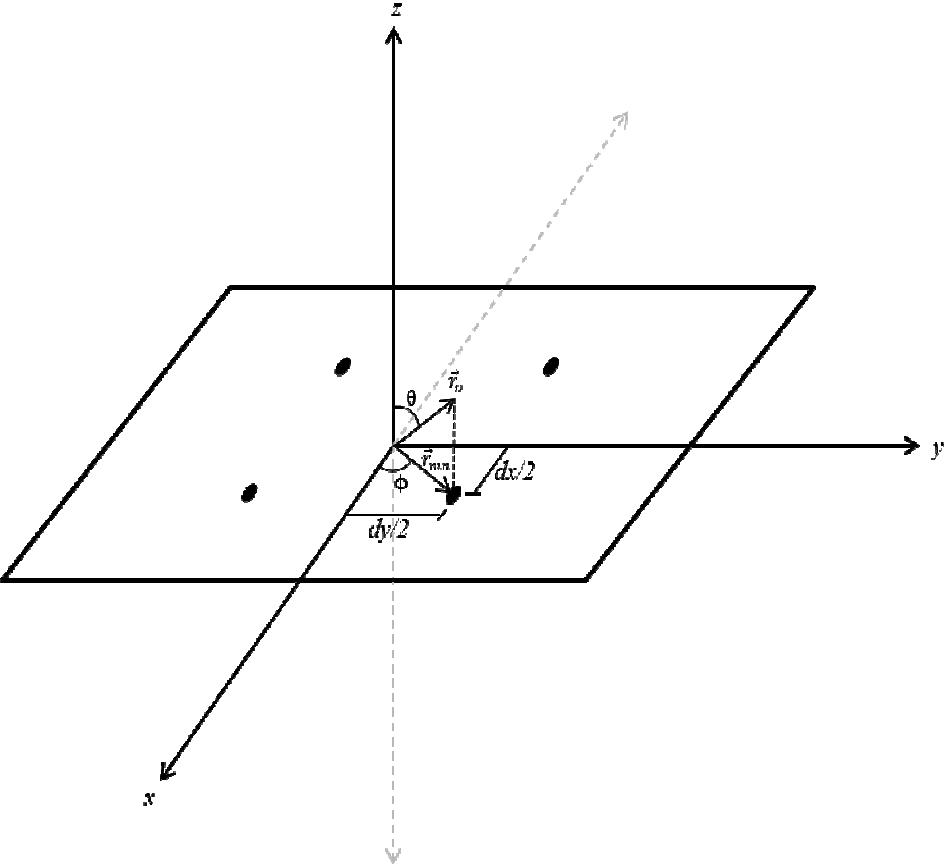
$$\begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + S_{14}a_4 \\ b_2 &= S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + S_{24}a_4 \\ b_3 &= S_{31}a_1 + S_{32}a_2 + S_{33}a_3 + S_{34}a_4 \\ b_4 &= S_{41}a_1 + S_{42}a_2 + S_{43}a_3 + S_{44}a_4 \end{aligned}$$

scan reflection coeff. for port N:

$$\Gamma_N = \frac{b_N}{a_N} = \frac{S_{N1}a_1 + S_{N2}a_2 + \cdots + S_{NN}a_N}{a_N}$$

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How is column vector [a] determined?



$$AF(\theta, \phi) = \sum_1^M \sum_1^N A_{mn} e^{-j\beta \hat{r} \vec{r}_{mn}} e^{j\alpha_{mn}}$$

Excitation for Scanning

$$\hat{r} = \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z}$$

$$\vec{r}_{mn} = x_{mn} \hat{x} + y_{mn} \hat{y} + z_{mn} \hat{z}$$

$$\alpha_{mn} = \beta \hat{r}_o \cdot \vec{r}_{mn}$$

$$\alpha_{mn} = \omega \sqrt{\mu_0 \epsilon_0} \hat{r}_o \cdot \vec{r}_{mn}$$

$$\alpha_{mn} = \omega \frac{\hat{r}_o \cdot \vec{r}_{mn}}{c} \quad \tau_{mn} = \frac{\hat{r}_o \cdot \vec{r}_{mn}}{c}$$

$$a_N = A_N e^{-j\alpha_N} \quad [a] = \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix}$$

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Scan Reflection Coefficient Solution

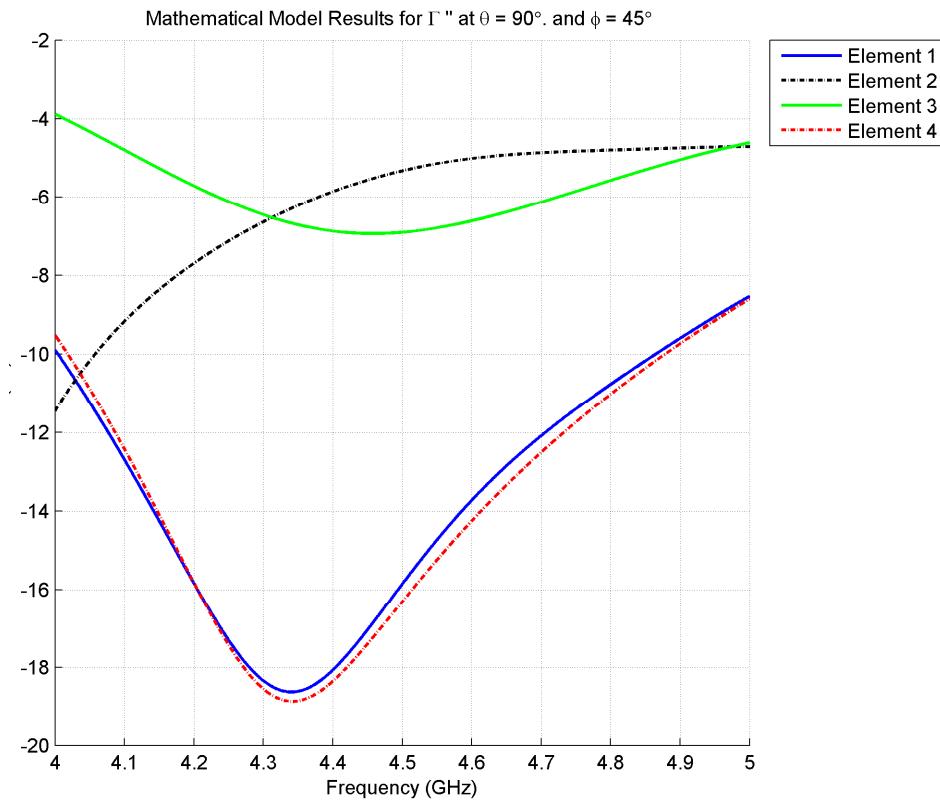
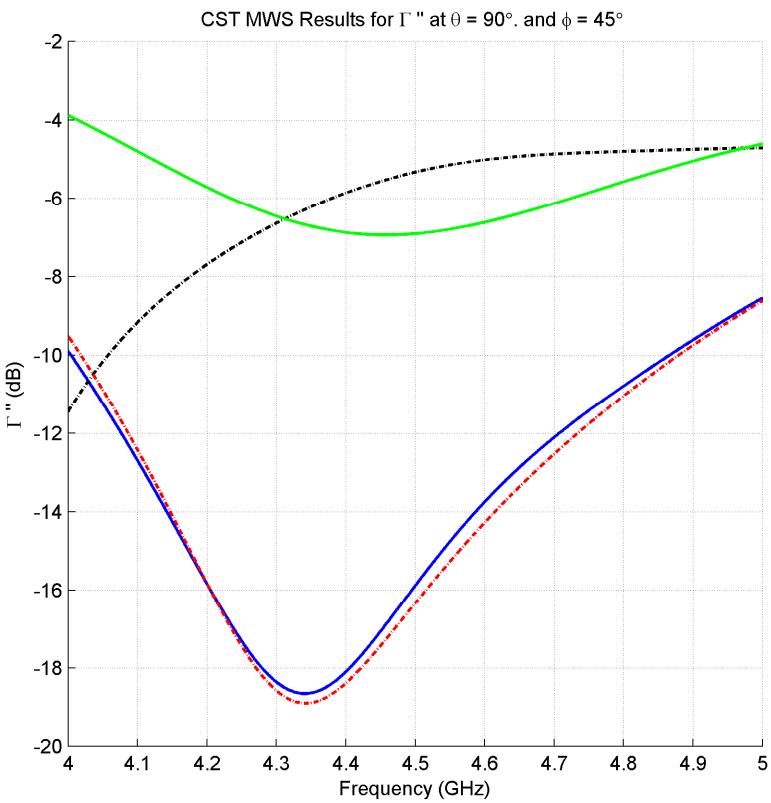
$$\begin{bmatrix} b_1'' \\ \vdots \\ b_N'' \end{bmatrix} = \begin{bmatrix} S'_{11} & \dots & S'_{1n} \\ \vdots & \ddots & \vdots \\ S'_{N1} & \dots & S'_{NN} \end{bmatrix} \begin{bmatrix} a_1'' \\ \vdots \\ a_N'' \end{bmatrix}$$

$$\begin{bmatrix} \Gamma_1'' \\ \vdots \\ \Gamma_N'' \end{bmatrix} = \begin{bmatrix} b_1'' \\ \vdots \\ b_N'' \end{bmatrix} ./ \begin{bmatrix} a_1'' \\ \vdots \\ a_N'' \end{bmatrix}$$



Element Wise Division

Validation of Scan Impedance Theory



- 8 Comparisons completed in increments of 45° around phi



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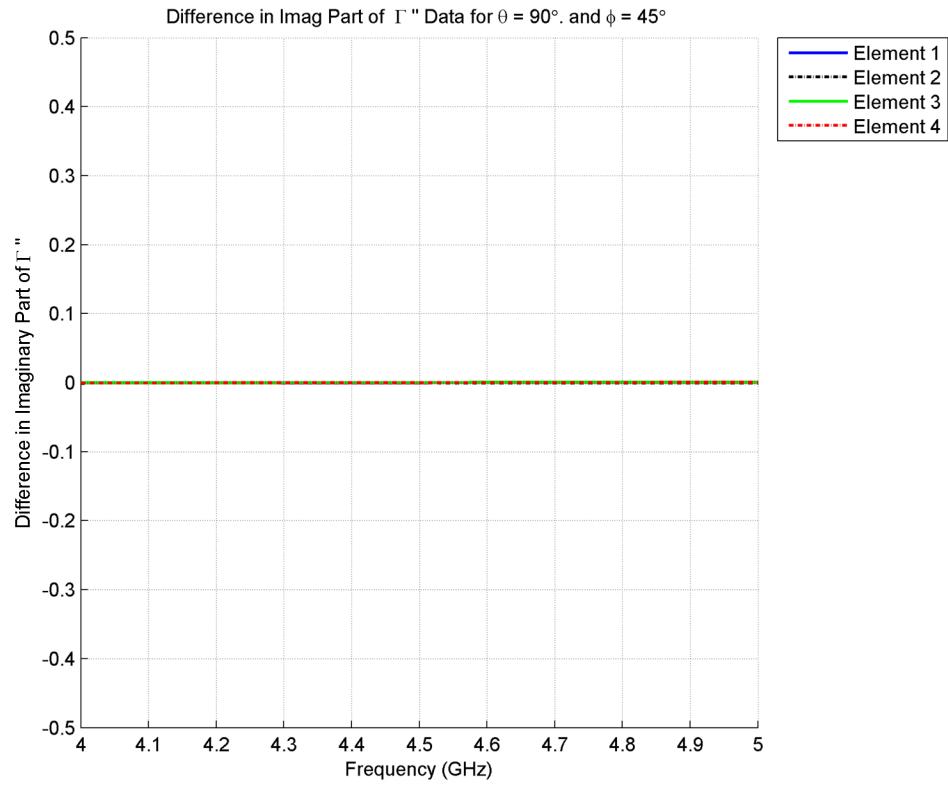
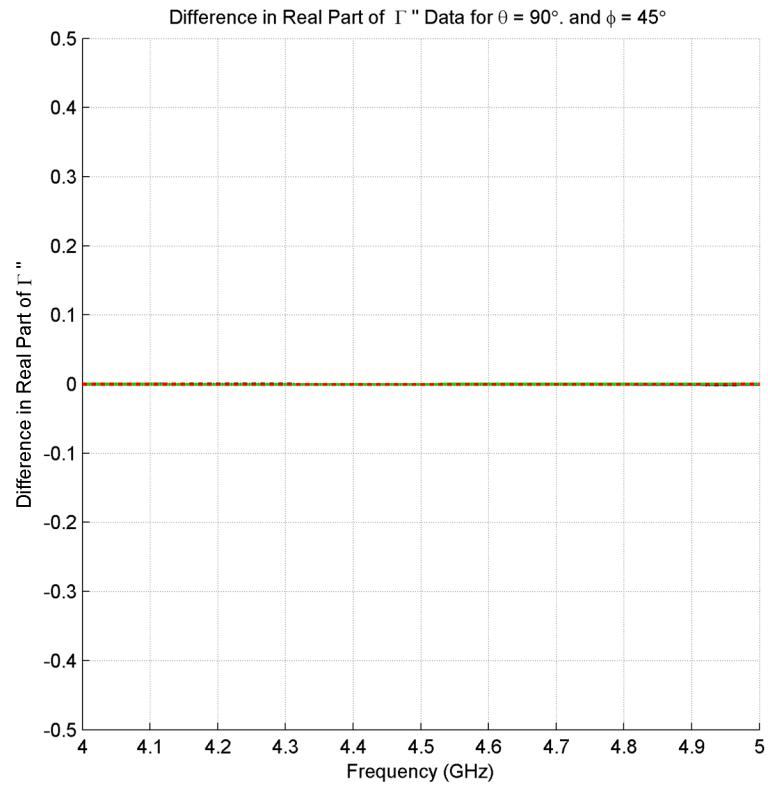
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Validation Continued...

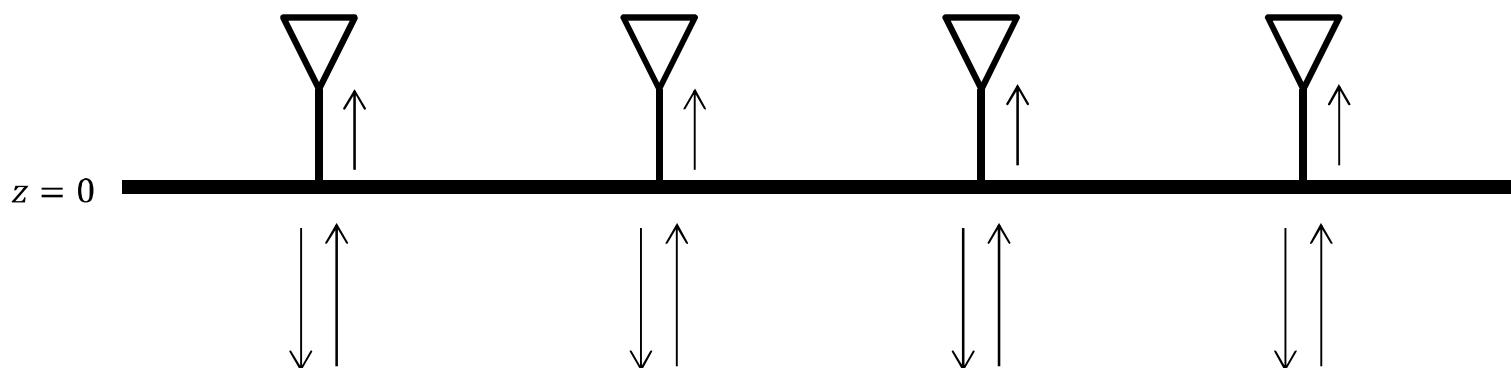




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Change In the Array Factor

$$(1 + \Gamma_1''(\theta, \phi))a_1e^{-j\phi_1} \quad (1 + \Gamma_2''(\theta, \phi))a_2e^{-j\phi_2} \quad (1 + \Gamma_3''(\theta, \phi))a_3e^{-j\phi_3} \quad (1 + \Gamma_4''(\theta, \phi))a_4e^{-j\phi_4}$$



Applied Excitations \rightarrow $a_1e^{j\phi_1}$ $a_2e^{j\phi_2}$ $a_3e^{j\phi_3}$ $a_4e^{j\phi_4}$

$$a_1 = a_2 = a_3 = a_4$$

$$V(z) = V_o^+ (e^{-j\beta z} + \Gamma e^{+j\beta z}), \quad z < 0$$

$$T = 1 + \Gamma$$

$$V(z) = V_o^+ T e^{-j\beta z}, \quad z > 0$$

$$V(z) = V_o^+ (1 + \Gamma) e^{-j\beta z}$$



In-Situ System Design

- Two ports required to measure mutual coupling between two elements
- For N elements, N ports are needed or two ports can be used to measure all permutations, however this requires access to all elements; doing this manually is impractical, and buying an N port PNA is expensive
- But we have a solution...



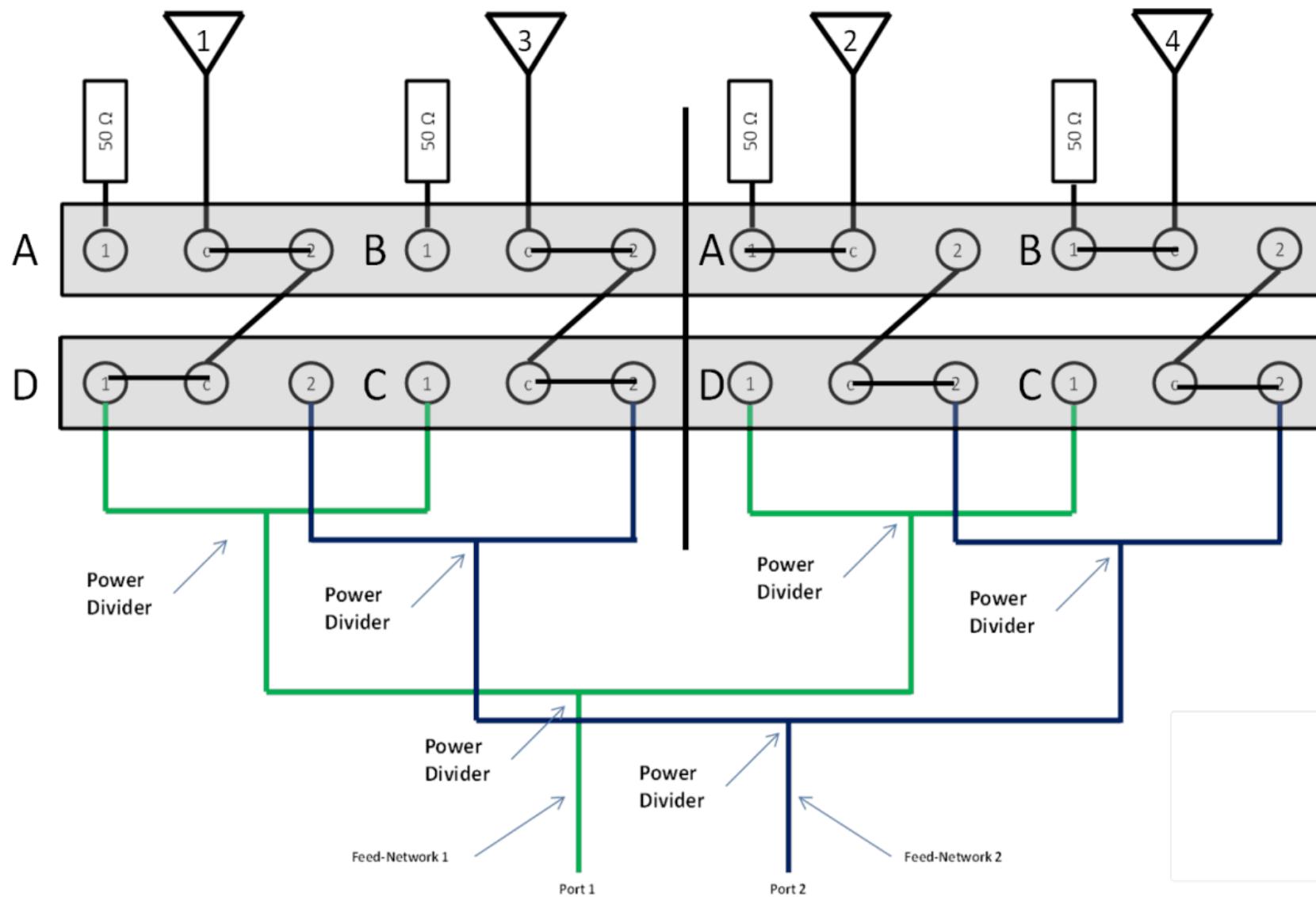
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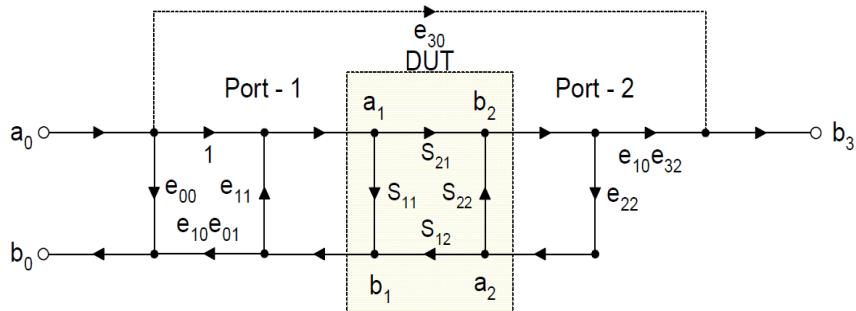




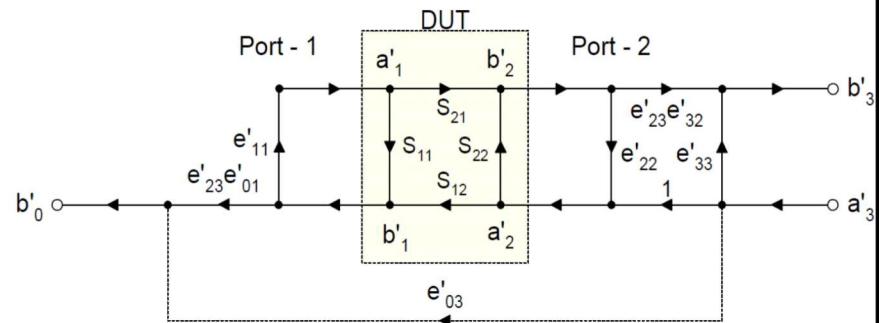
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Error Correction

FORWARD MODEL



REVERSE MODEL



$$S_{11M} = \frac{b_0}{a_0} = e_{00} + \frac{e_{10}e_{01}(S_{11} - e_{22}\Delta s)}{1 - e_{11}S_{11} - e_{22}S_{22} + e_{11}e_{22}\Delta s}$$

$$S_{21M} = \frac{b_3}{a_0} = e_{30} + \frac{e_{10}e_{32}S_{21}}{1 - e_{11}S_{11} - e_{22}S_{22} + e_{11}e_{22}\Delta s}$$

$$S_{22M} = \frac{b_3}{a'_3} = e'_{33} + \frac{e'_{23}e'_{32}(S_{22} - e'_{11}\Delta s)}{1 - e'_{11}S_{11} - e'_{22}S_{22} + e'_{11}e'_{22}\Delta s}$$

$$S_{12M} = \frac{b_0}{a'_3} = e'_{03} + \frac{e'_{23}e'_{01}(S_{12})}{1 - e'_{11}S_{11} - e'_{22}S_{22} + e'_{11}e'_{22}\Delta s}$$



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S-Parameter Actualls

$$S_{11} = \frac{\left(\frac{S_{11M} - e_{00}}{e_{10}e_{01}}\right) \left[1 + \left(\frac{S_{22M} - e_{33}'}{e_{23}'e_{32}'}\right) e_{22}'\right] - e_{22} \left(\frac{S_{21M} - e_{30}}{e_{10}e_{32}}\right) \left(\frac{S_{12M} - e_{03}'}{e_{23}'e_{01}'}\right)}{D}$$

$$S_{21} = \frac{\left(\frac{S_{21M} - e_{30}}{e_{10}e_{32}}\right) \left[1 + \left(\frac{S_{22M} - e_{33}'}{e_{23}'e_{32}'}\right) (e_{22}' - e_{22})\right]}{D}$$

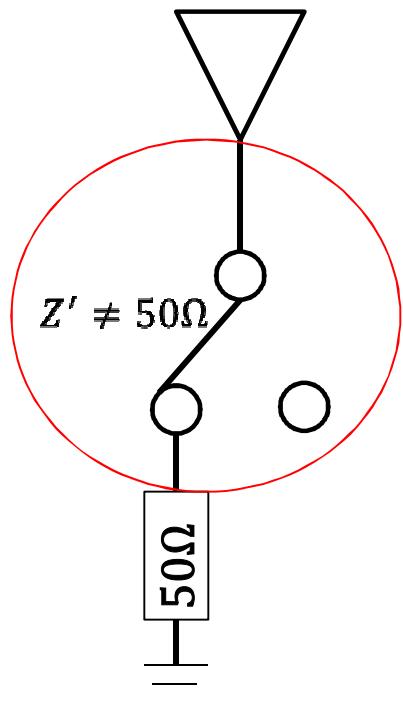
$$S_{22} = \frac{\left(\frac{S_{22M} - e_{33}'}{e_{23}'e_{32}'}\right) \left[1 + \left(\frac{S_{11M} - e_{00}}{e_{10}e_{01}}\right) e_{11}'\right] - e_{11}' \left(\frac{S_{21M} - e_{30}}{e_{10}e_{32}}\right) \left(\frac{S_{12M} - e_{03}'}{e_{23}'e_{01}'}\right)}{D}$$

$$S_{12} = \frac{\left(\frac{S_{12M} - e_{03}'}{e_{23}'e_{01}'}\right) \left[1 + \left(\frac{S_{11M} - e_{00}}{e_{10}e_{01}}\right) (e_{11}' - e_{11})\right]}{D}$$

$$D = \left[\left[1 + \left(\frac{S_{11M} - e_{00}}{e_{10}e_{01}}\right) e_{11}\right] \left[1 + \left(\frac{S_{22M} - e_{33}'}{e_{23}'e_{32}'}\right) e_{22}'\right] - \left(\frac{S_{21M} - e_{30}}{e_{10}e_{32}}\right) \left(\frac{S_{12M} - e_{03}'}{e_{23}'e_{01}'}\right) e_{22} e_{11}' \right].$$

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Impedance Renormalization



- During S-parameter measurements, unused ports should be terminated with ideal matched loads
- The unused antenna ports in the in-situ measurement system are not ideal, and thus add unwanted reflection back into the measurement
- This can be accounted for by using the impedance renormalization transform to put measurement reference back to a 50 ohm system.

$$S' = (I - S)^{-1}(S - \Gamma)(I - S \cdot \Gamma)^{-1}(I - S)$$



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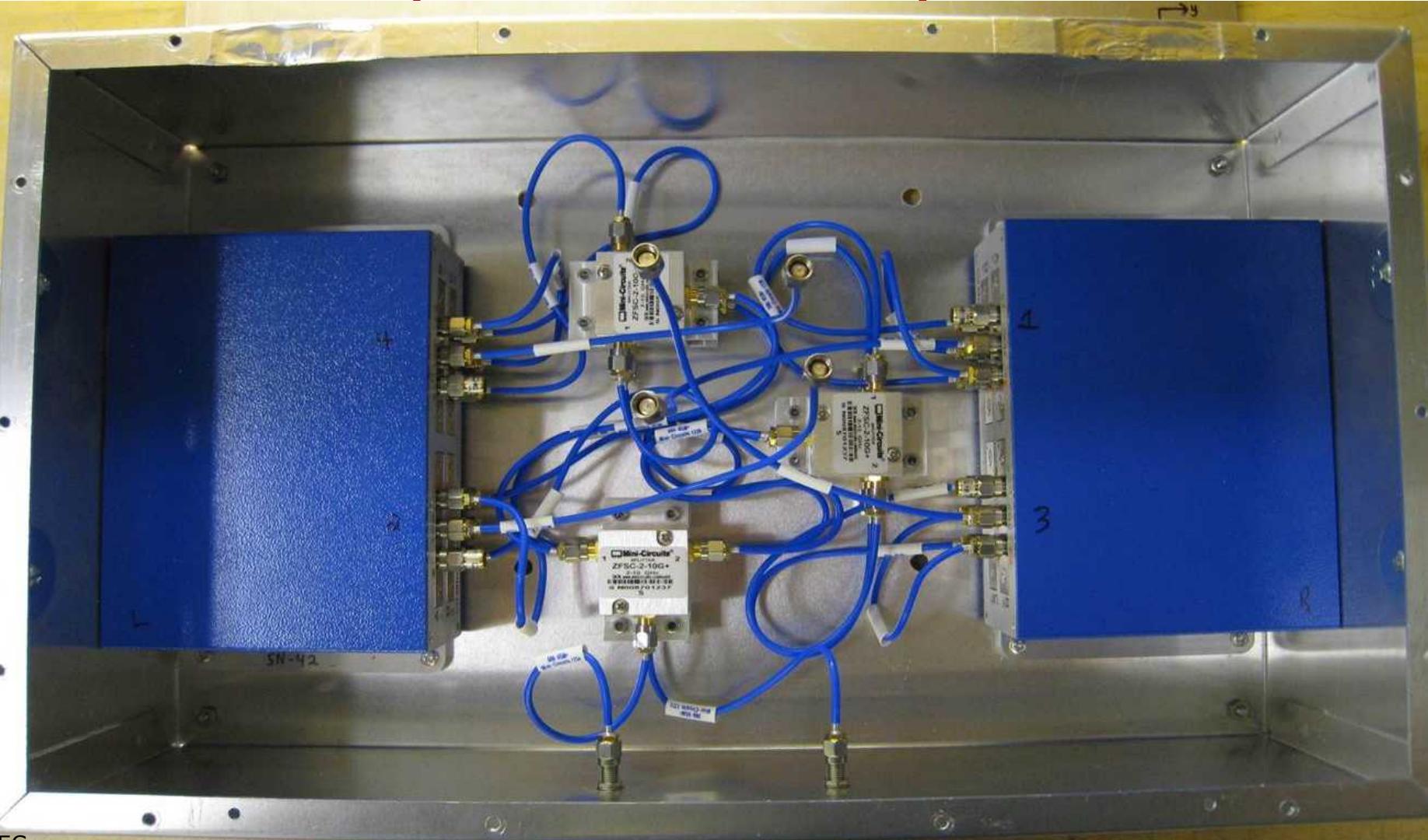
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Implemented System



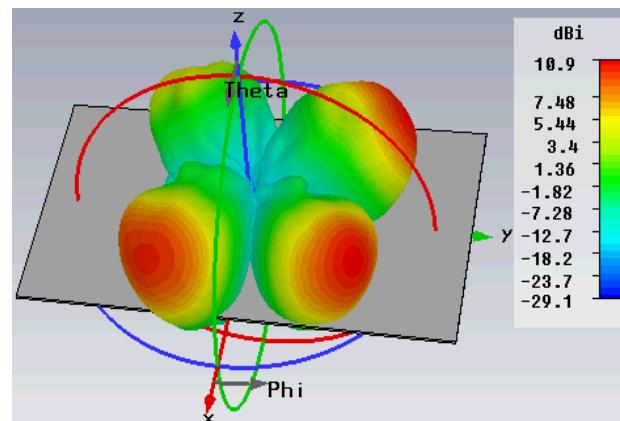
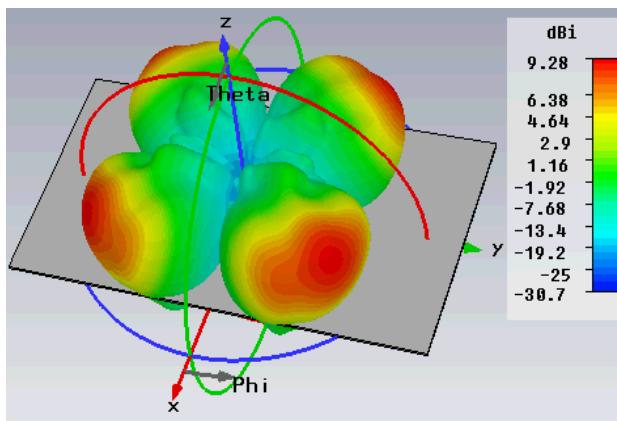
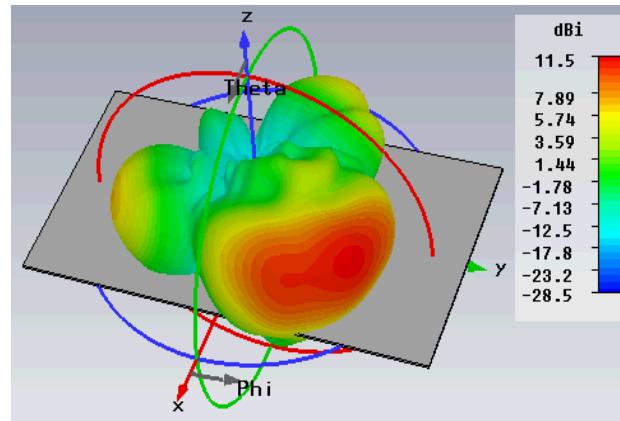
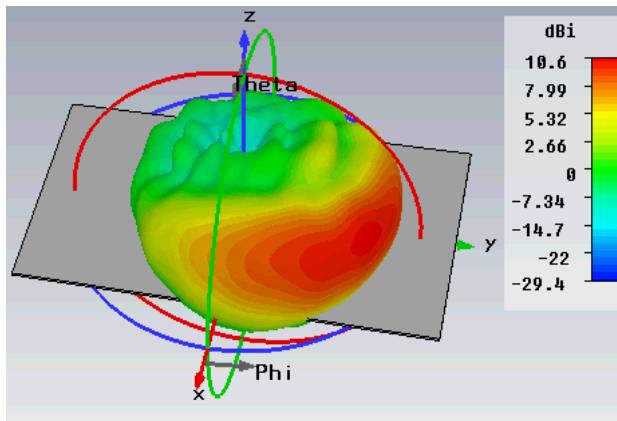


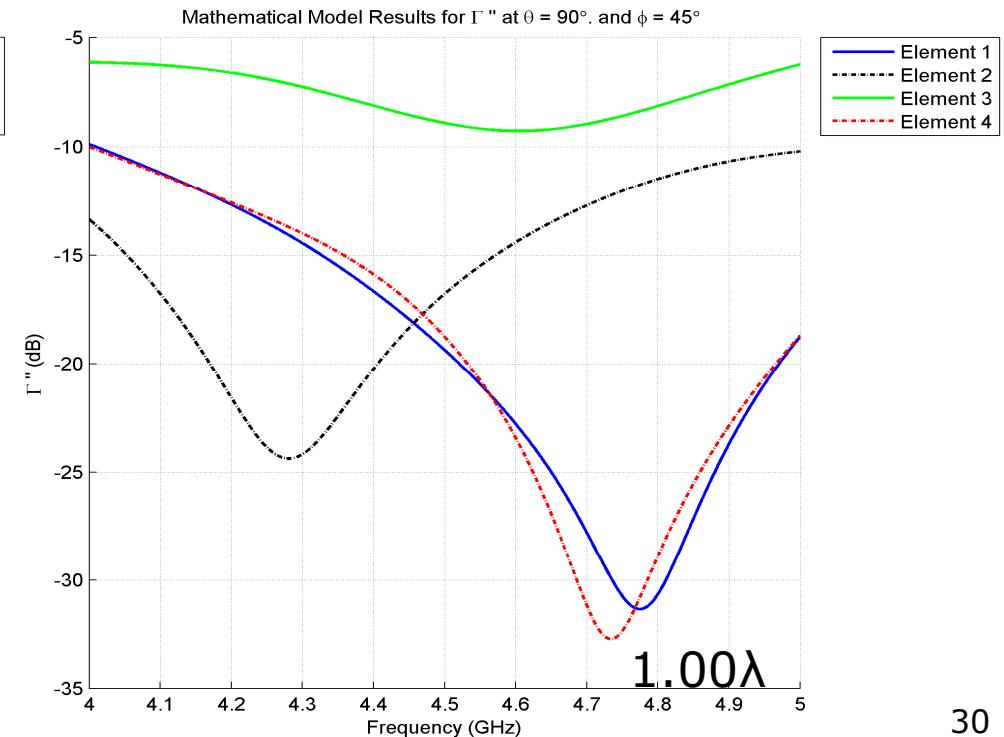
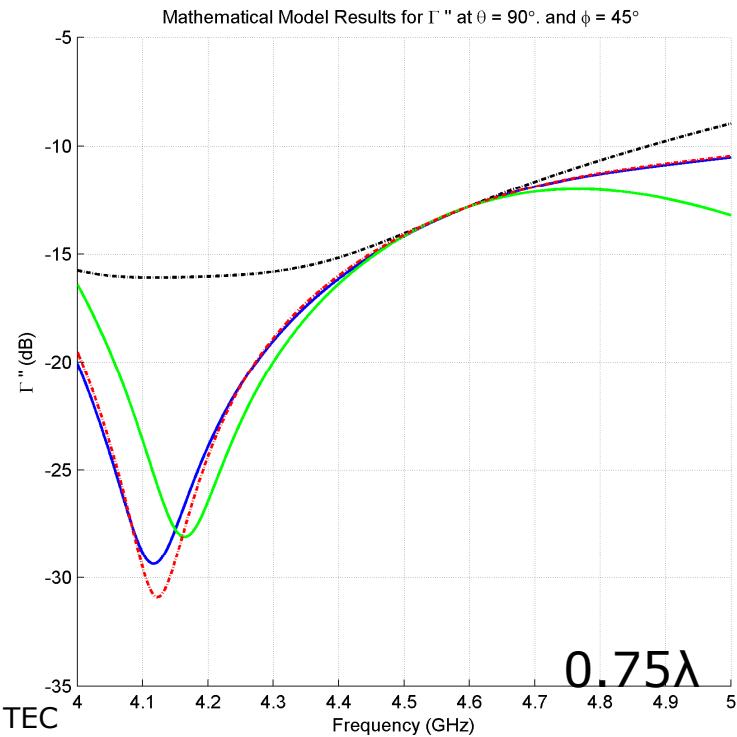
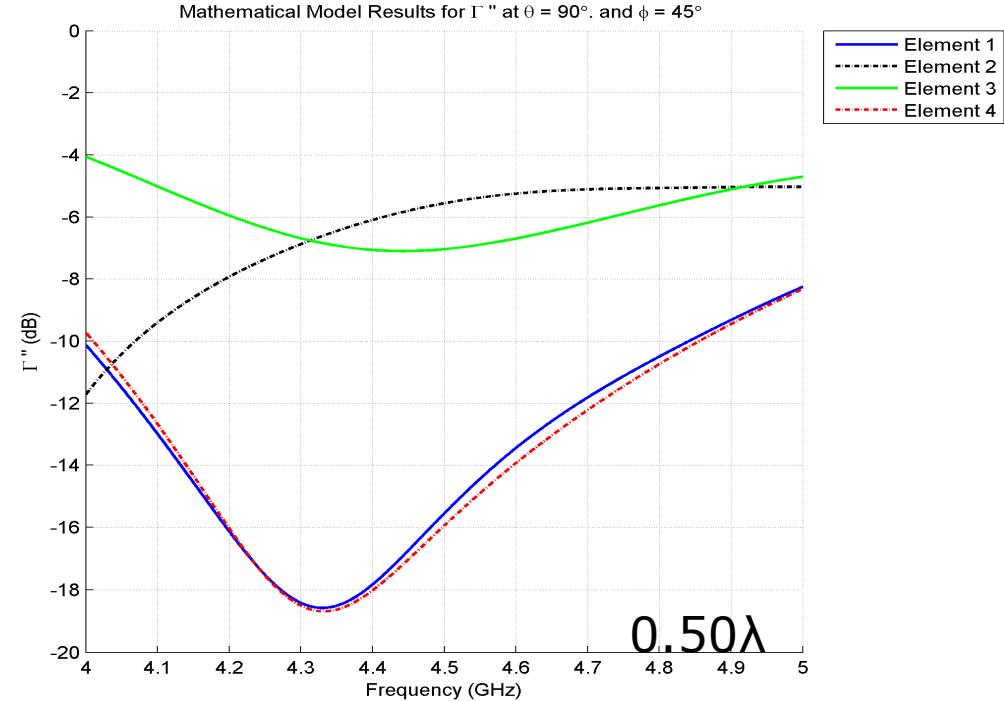
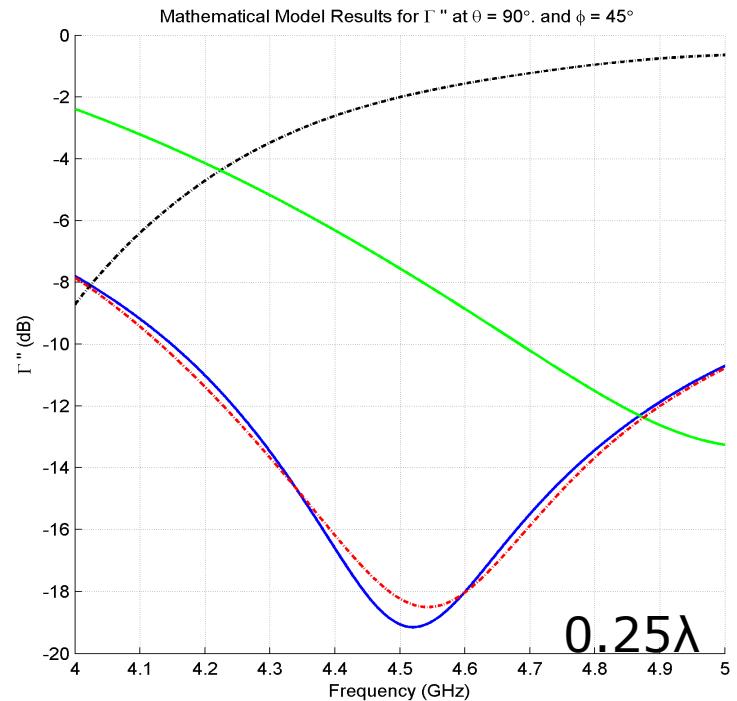
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Element Spacing Study

Element Spacings				
Wavelength Spacing	0.25λ	0.50λ	0.75λ	1.00λ

$$\Theta = 90^\circ, \phi = 45^\circ$$







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Conclusions

- In-Situ scan impedance characterization is needed for phased antenna arrays that can be placed in diverse locations in the field.
- If mutual coupling changes, so does scan impedance.
- An approach and implementation for in-situ mutual coupling measurements has been given and implemented in hardware.
- The mathematics that exists for scan impedance/reflection calculation has been expanded upon and formulated into two elegant matrix equations.



Future Work

- Expansion of this system into a compact microstrip array utilizing IC packages or dye packaging to accomplish rf switching. Also incorporation of true time delay units or phase shifters as well as attenuators.
- This study has set up the fundamental theory necessary to begin research into mitigation techniques to account for the effects of scan impedance.
 - Perhaps a tuning system could be implemented
 - Direct mitigation technique in the array fabrication to eliminate or dampen out the effect of mutual coupling
 - Potentially a new form of array synthesis could be developed to account for this issue



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Questions?



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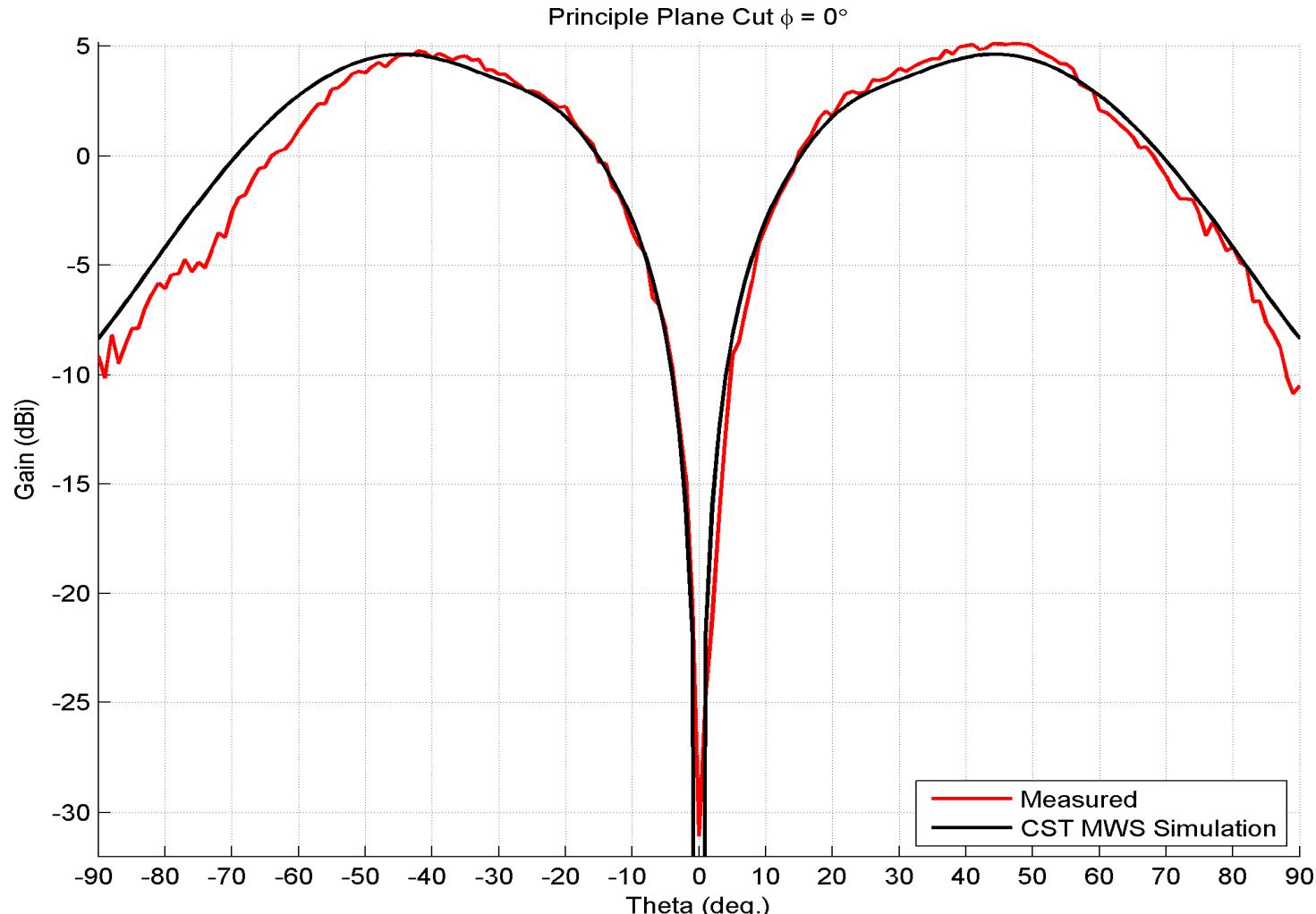
Radiation Pattern Measurement



Principle Plane Cut for $\phi = 0^\circ$

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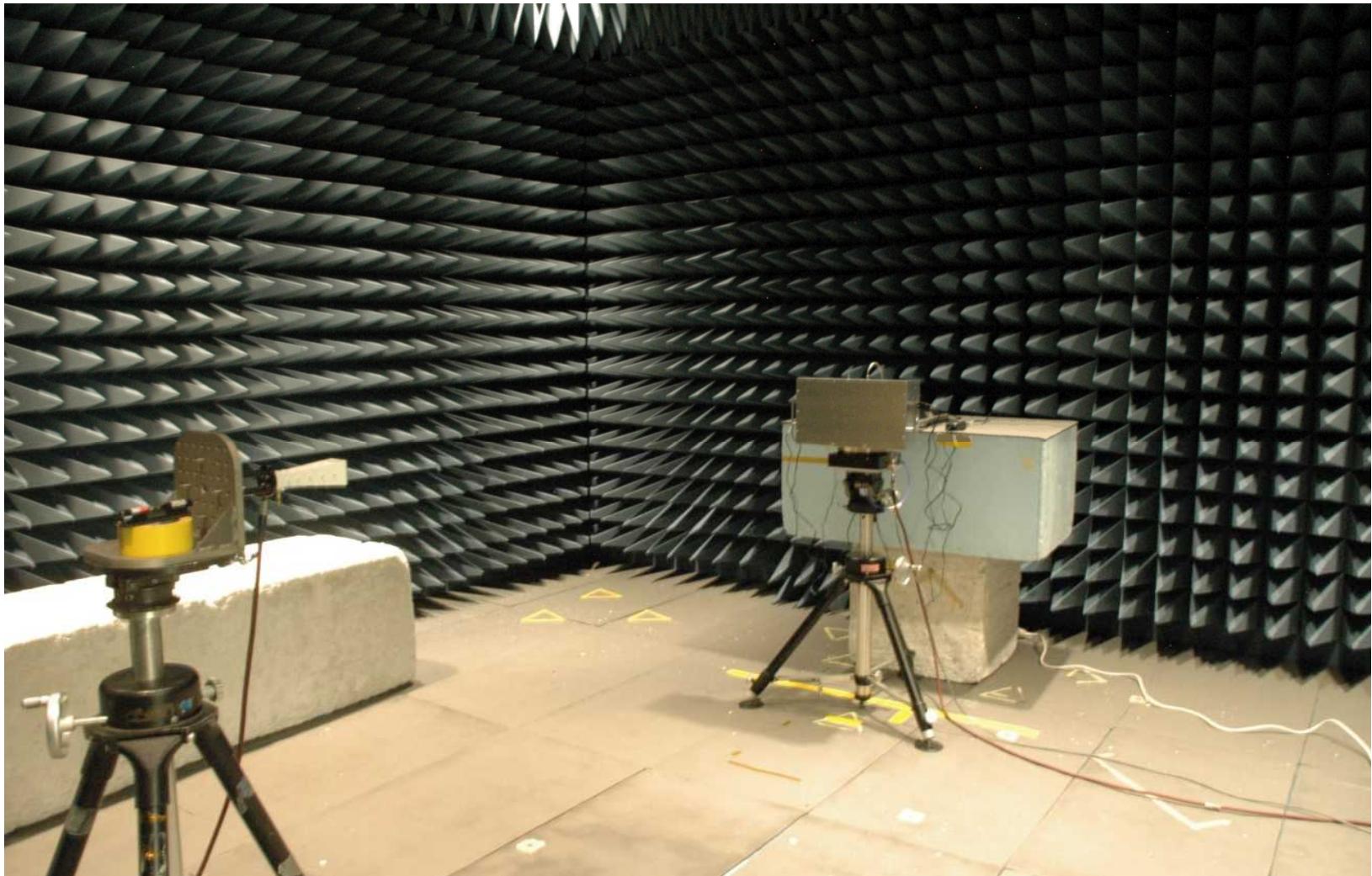
Radiation Pattern Measurement



Principle Plane Cut for $\phi = 0^\circ$

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Radiation Pattern Measurement



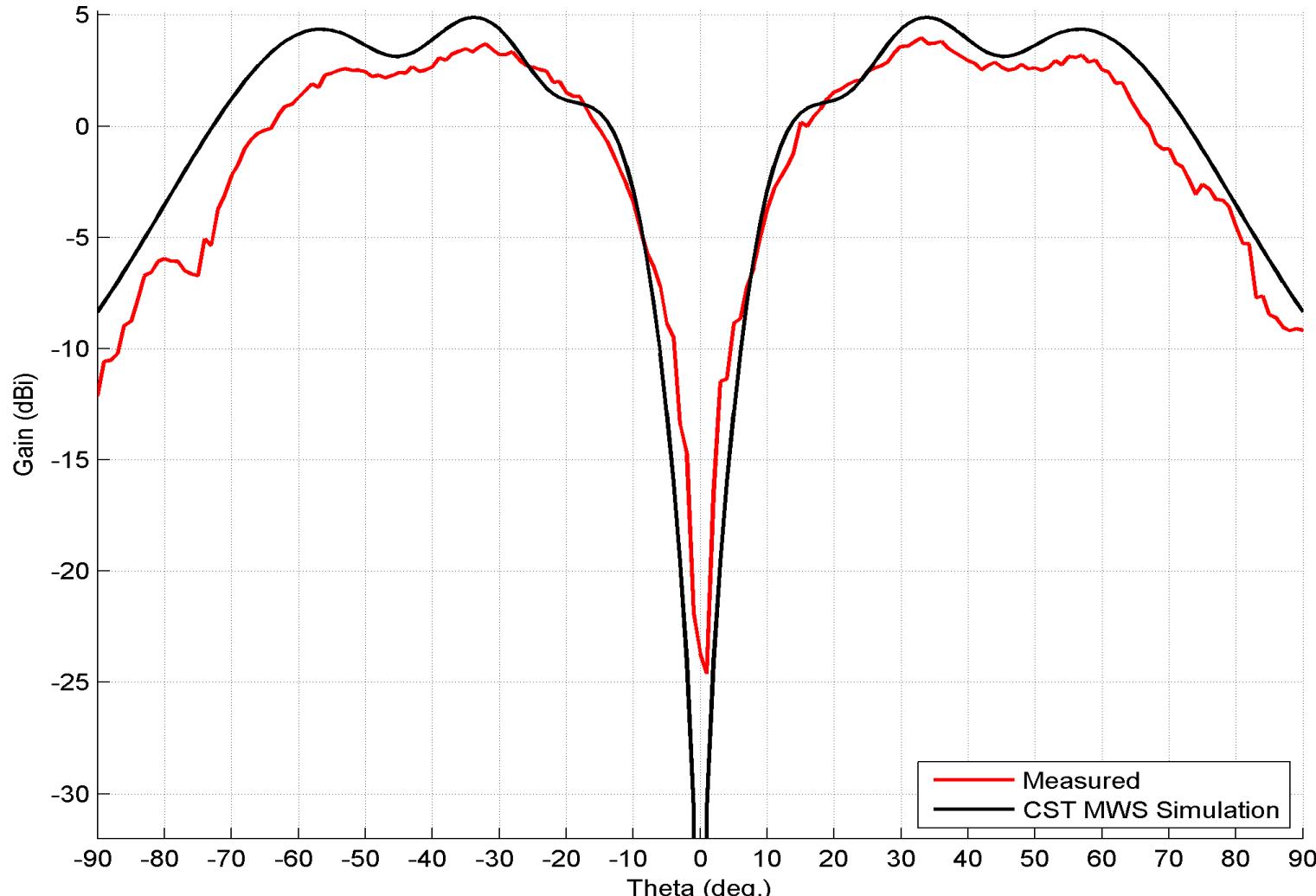
Principle Plane Cut for $\phi = 90^\circ$



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Radiation Pattern Measurement

Principle Plane Cut $\phi = 90^\circ$

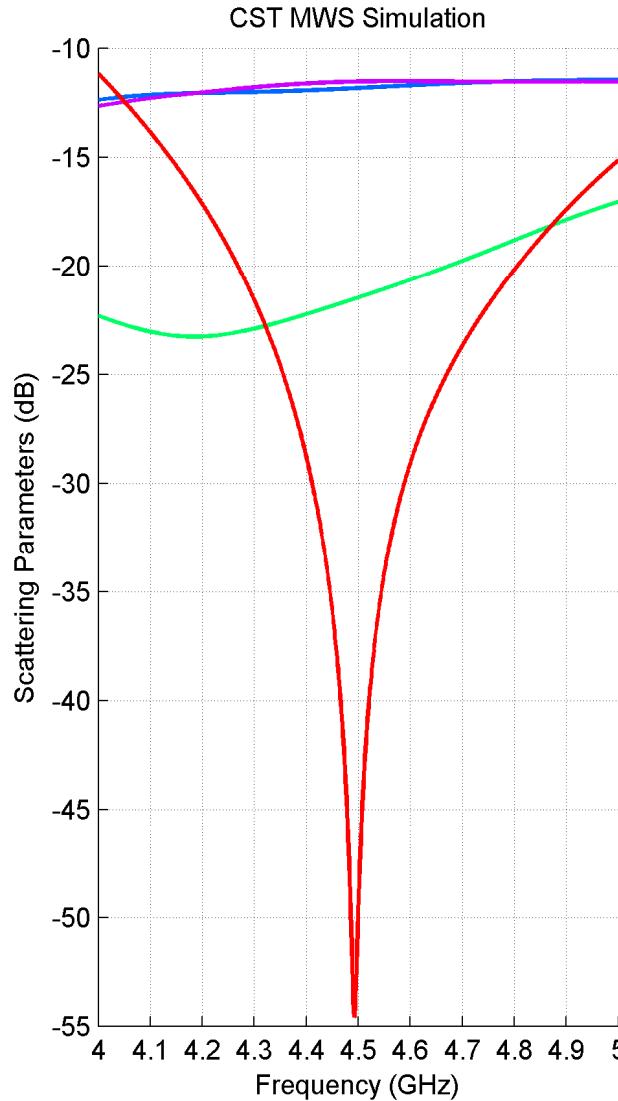
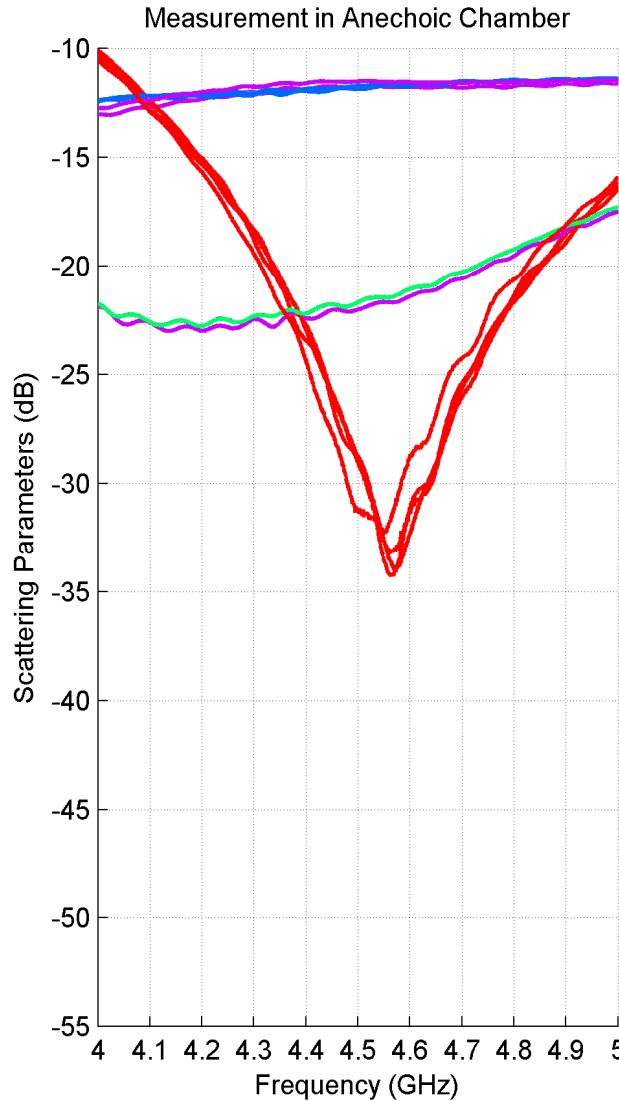


Principle Plane Cut for $\phi = 90^\circ$



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Side by Side Comparison



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Scan Reflection Matrix Expanded

$$\Gamma_1'' = \frac{S_{11}|a_1|e^{j\xi_1} + S_{12}|a_2|e^{j\xi_2} + S_{13}|a_3|e^{j\xi_3} + \dots + S_{1n}|a_m|e^{j\xi_m}}{|a_1|e^{j\xi_1}}$$

$$\Gamma_2'' = \frac{S_{21}|a_1|e^{j\xi_1} + S_{22}|a_2|e^{j\xi_2} + S_{23}|a_3|e^{j\xi_3} + \dots + S_{2n}|a_m|e^{j\xi_m}}{|a_2|e^{j\xi_2}}$$

⋮

$$\Gamma_m'' = \frac{S_{m1}|a_1|e^{j\xi_1} + S_{m2}|a_2|e^{j\xi_2} + S_{m3}|a_3|e^{j\xi_3} + \dots + S_{mn}|a_m|e^{j\xi_m}}{|a_m|e^{j\xi_m}}$$

$$a_m'' = |a_m|e^{j\xi_m}$$

Where ξ is the
respective phasing at
element m.



System Caveat

- Self-Reflection Coefficient measurements are currently not 100% accurate
- Currently, manual measured self-reflection coefficients are being used.
- All transmission coefficients are considered to be in good agreement with manual measured scattering parameters and account for 75% of the total scattering matrix and contain the effects of mutual coupling
- Extensive debugging has occurred and some simple simulations of the automatic system have been completed and shown to be in agreement with the error correction theory
- Potential reason could be an unaccounted for hardware issue