SAND2007-0856C

Comparison of Methods for the Propagation of Component Modeling Uncertainty

Timothy Hasselman and G. Wije Wathugala
ACTA Incorporated

Angel Urbina and Thomas L. Paez
Sandia National Laboratories

IMAC XXV
February 19, 2007

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under contract DE-AC04-94AL85000.

Sandia National Laboratories

Outline

- Motivation
- Introduction
  - Definitions of Bottom-Up and Top-Down UQ
- Development of UQ Procedures via Practical Example
- Comparison of Results at Preliminary Level then Following Propagation
- Conclusion

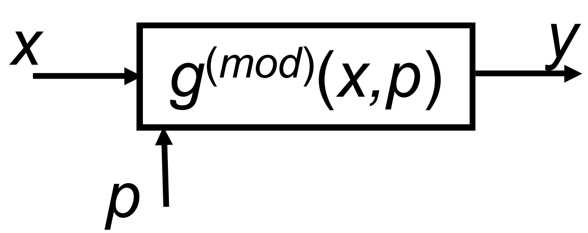
Motivation

- Uncertainty Quantification (UQ) – Activity whose goal is to characterize random (and other sources of) variability in a phenomenon
- Nondeterministic systems subjected to nondeterministic excitations (and BC, IC, etc.) have nondeterministic responses – Behaviors subject to UQ
- Reasons for UQ
  - Assessment of system reliability
  - Simulation of behaviors and environments
  - Validation
  - Etc.

Whenever there is a UQ requirement, there is a need for techniques to perform UQ

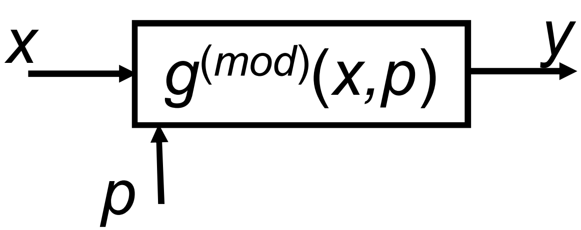
Introduction

- Probabilistic modeling/analysis - type of UQ to be considered
- Traditional probabilistic analysis usually
  - Starts with probability model of system parameters (excitations, etc.), then
  - Propagates/transforms that information through the model, $g^{(mod)}(x,p)$, to probability distribution of measure of system response



Introduction

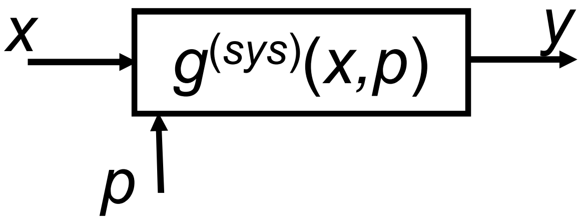
- Shortcomings of the traditional probabilistic approach
  - In practical situations (where physical system is being modeled) inadequacy of model form usually ignored
  - Noise on all measurements usually ignored
  - Therefore, probability model of response is incorrect – does not capture all correct features of real system response - and usually contains too little variation



This approach is known as "Bottom-Up"

Introduction

- Alternate approach creates probabilistic model of response, directly (Can be more accurate than bottom-up modeling – It reflects true system behavior)
- (Sometimes variability of parameters is inferred via inversion of $g^{(mod)}(x,p)$- not usually)
- Variability of response includes components due to
  - Parameter variability
  - Input variability
  - Measurement noise



This approach is known as “Top-Down”

Introduction

A comparison is to be performed

- Probabilistic behavior of subsystem temporal response is modeled via bottom-up and top-down approaches
- Modeling performances compared
- Probabilistic models of subsystem propagated through more complex system model to establish probabilistic behavior of temporal response of second system
- Performances compared

Illustrative Example

- Project objectives: Create model to predict response behavior of aerospace component and assess stochastic behavior
- To test modeling capabilities, consider simple system
  - Mass supported on three legs that are integral part of base
  - Connection is lap joint - nonlinear
  - Excitation imparted via base
- Probabilistic behavior of mass response built from probabilistic behavior of joint

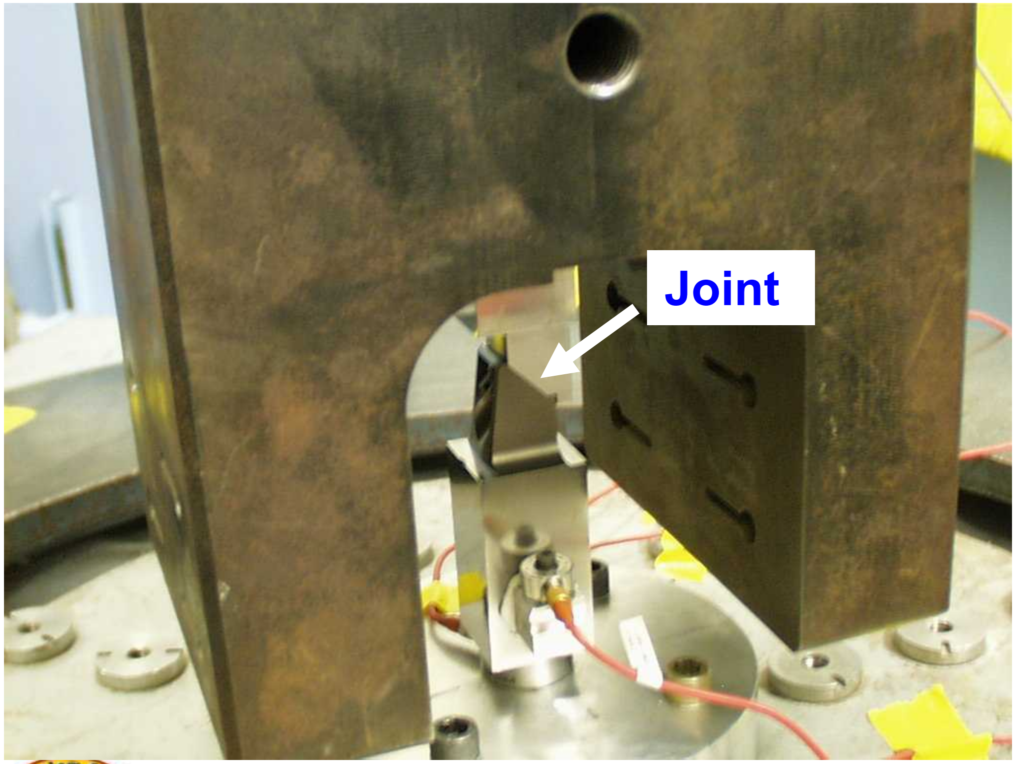
Illustrative Example (Continued)

- Experiments performed on joint to establish its behavior

- Connection is lap-joint
- No macro-slip - only micro-slip
- Three nominally identical “tops” and three nominally identical “bottoms” available
- All nine combinations tested
- Test on each combination repeated five times
- Tests excite harmonic motion

Illustrative Example (Continued)

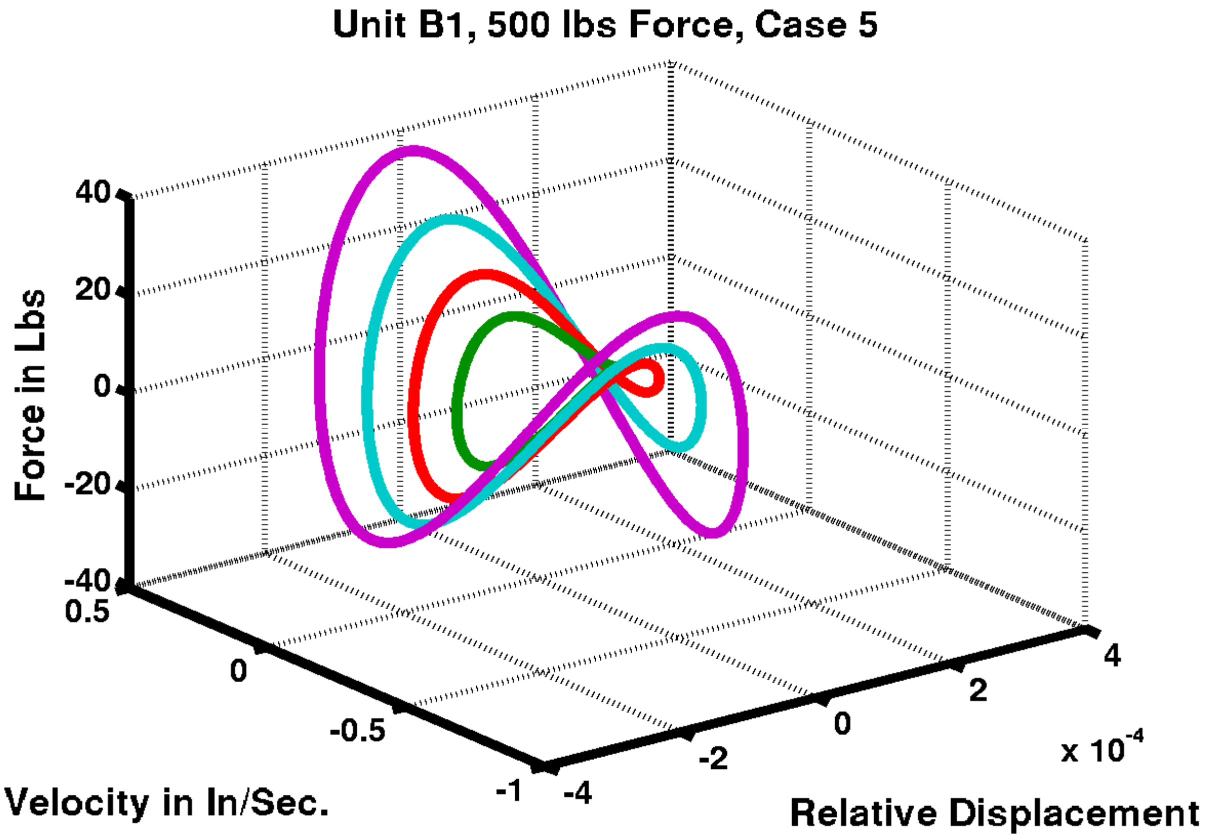
- Experiments performed on joint to establish its behavior



- Excitation generated via electro-dynamic shaker
- Excitation type – swept sine
- Data collected at resonance
- Force across joint measured
- Accelerations at joint-ends measured
- Relative displacement across joint inferred
- System nonlinear – therefore, much force in higher harmonics
- System behaves differently in tension and compression – therefore, fundamental nonlinearity is quadratic

Illustrative Example (Continued)

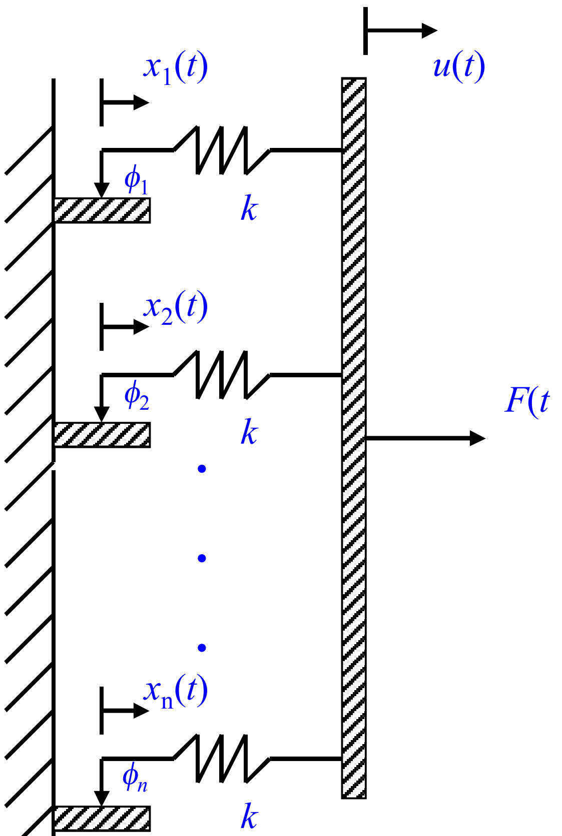
- Plot of force versus relative displacement and relative velocity with linear components removed - experiment



Sandia National Laboratories

Illustrative Example (Continued)

- Iwan model used for joint behavior

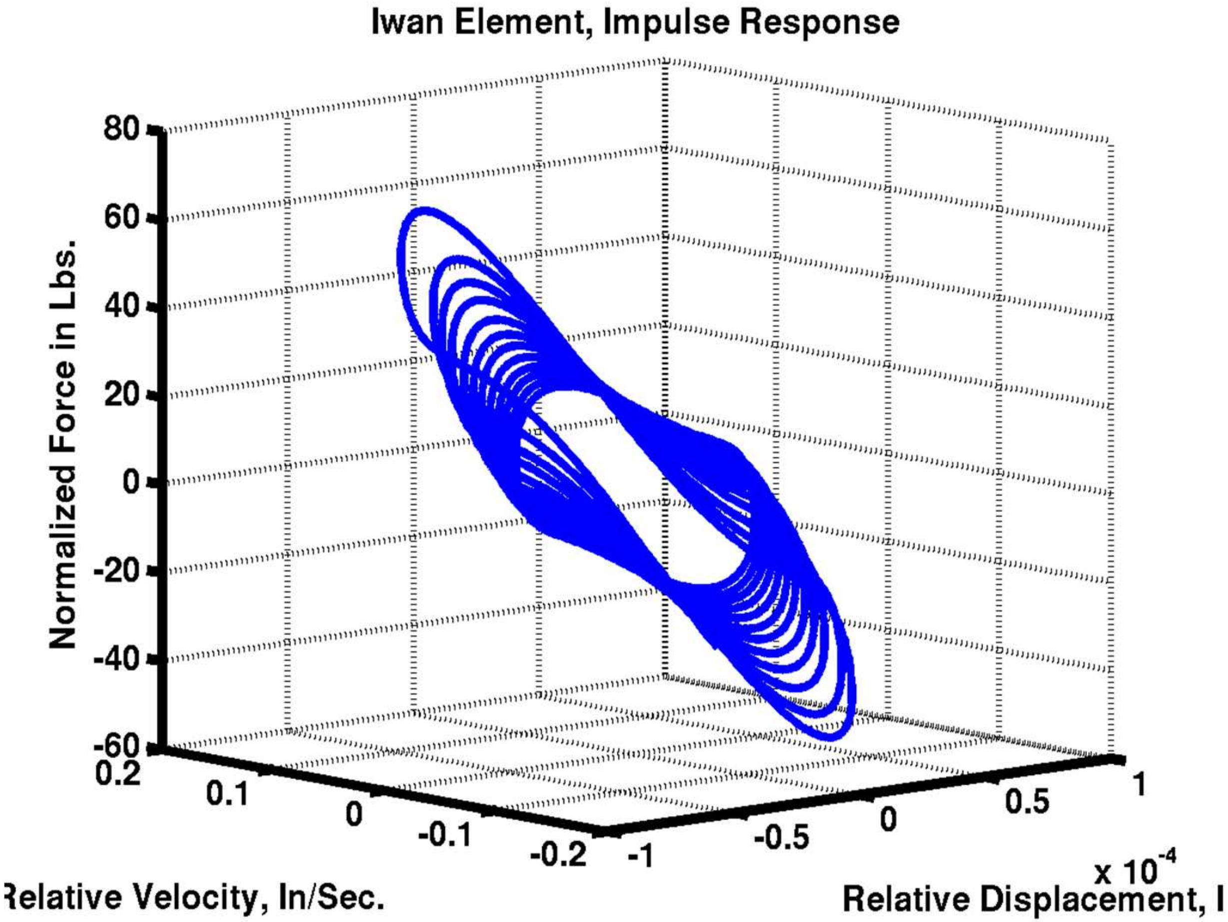


- Model consists of sequence of "Jenkins" elements
- Behavior characterized by stiffnesses and slip displacements
- Hysteretic behavior of Iwan model characterized by distribution of slip displacements in Jenkins elements
- Distribution of slip displacements of sliders is the same in tension and compression
- Therefore, hysteresis loops are anti-symmetric – fundamental nonlinearity is cubic

Sandia National Laboratories

Illustrative Example (Continued)

- Plot of force versus relative displacement and relative velocity – Iwan model



Illustrative Example (Continued)

- **Top-Down model for time domain response – Fourier series**

$$\boldsymbol{d}^{(test)}(t) \cong \sum_{k=1}^{n} a_k^{(test)} \cos(k\omega t) + b_k^{(test)} \sin(k\omega t)$$

$$\boldsymbol{d}^{(mod)}(t) \cong \sum_{k=1}^{n} a_k^{(mod)} \cos(k\omega t) + b_k^{(mod)} \sin(k\omega t)$$

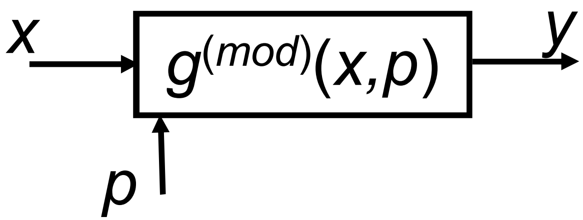
- **Generic covariance matrix of total uncertainty**

$$\boldsymbol{S}_{\tilde{\boldsymbol{B}}\tilde{\boldsymbol{B}}} = \frac{1}{N} \sum_{j=1}^{N} \{\boldsymbol{\Delta}\tilde{\boldsymbol{B}}\}_j \{\boldsymbol{\Delta}\tilde{\boldsymbol{B}}\}_j^T$$

where $\{\boldsymbol{\Delta}\tilde{\boldsymbol{B}}\}_j,\ j = 1,\ldots,N$**, are vectors of normalized differences between test and model Fourier coefficients**

Illustrative Example (Continued)

- **Bottom-Up model for time domain response**



where $g^{(mod)}(x,p)$ **is finite element model of system, including joint modeled as Iwan**

- **Covariance matrix of Iwan model parameters**

$$\boldsymbol{S_{pp}} = \frac{1}{N}\sum_{j=1}^{N}\{\boldsymbol{p}\}_j\{\boldsymbol{p}\}_j^T$$

where $\boldsymbol{p}_j,\ j=1,\ldots,N$ **, are vectors of experimentally-derived realizations of Iwan model parameters**

Illustrative Example (Continued)

- **Uncertainty of both Bottom-Up and Top-Down models for time domain response can be approximately, linearly propagated to covariances of displacement time histories**

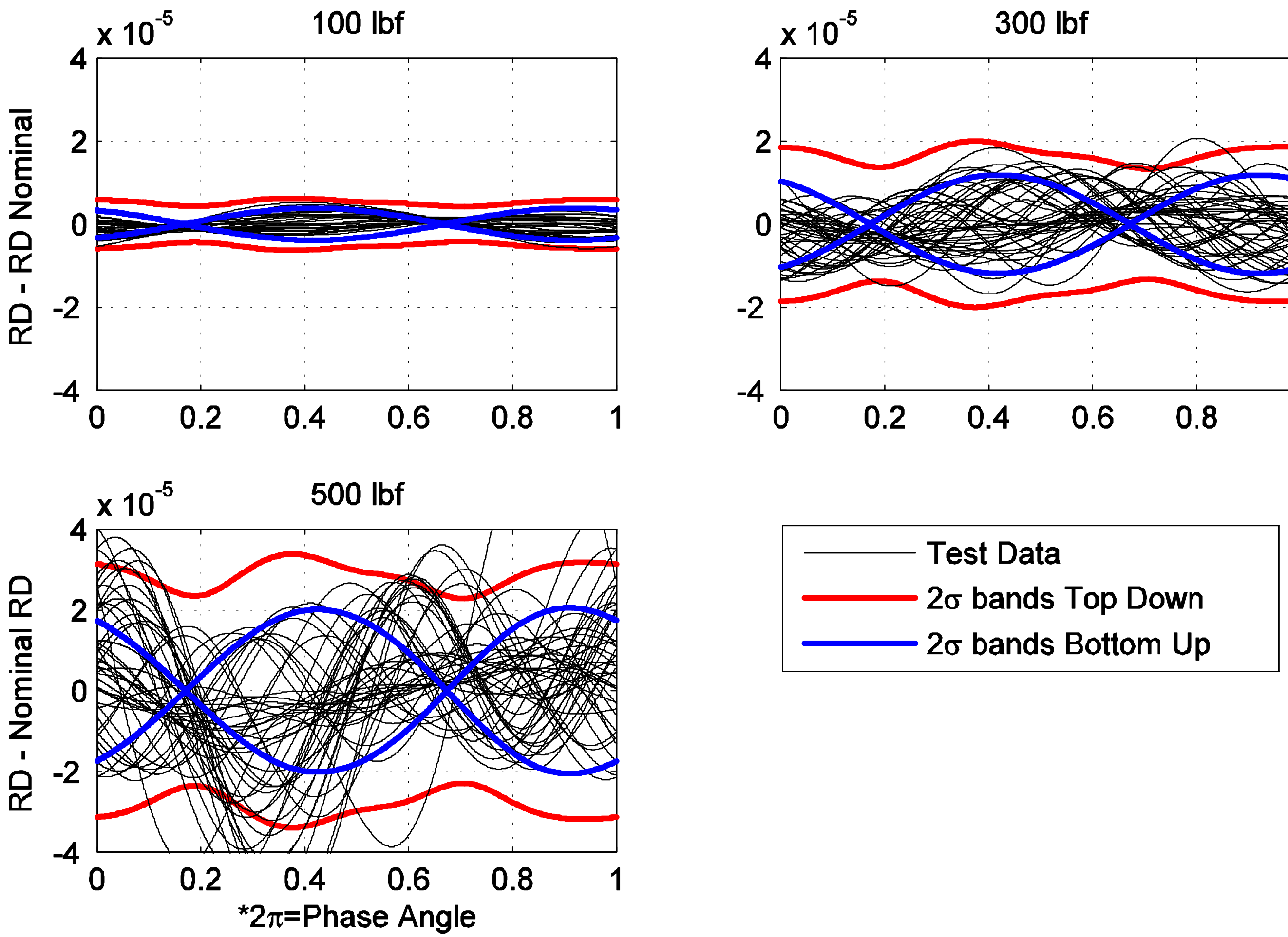
$$(S_{dd})_{\tilde{B}} = T_{d\tilde{B}} S_{\tilde{B}\tilde{B}} T_{d\tilde{B}}^{T}$$

$$(S_{dd})_{p} = T_{dp} S_{pp} T_{dp}^{T}$$

where $T_{d\tilde{B}}$ is the sensitivity matrix relating normalized Fourier coefficients to displacement time history, and T_{dp} is the sensitivity relating Iwan parameters to displacement time histories

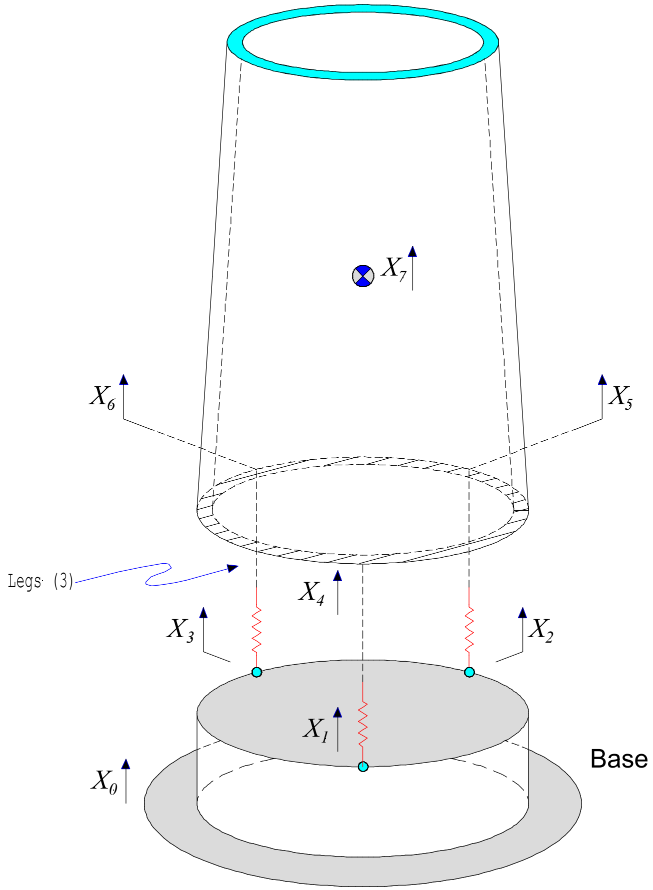
Illustrative Example (Continued)

- **Result**



Illustrative Example (Continued)

- **Next step – Propagate uncertainty of joint element to uncertainty of three-legged system**

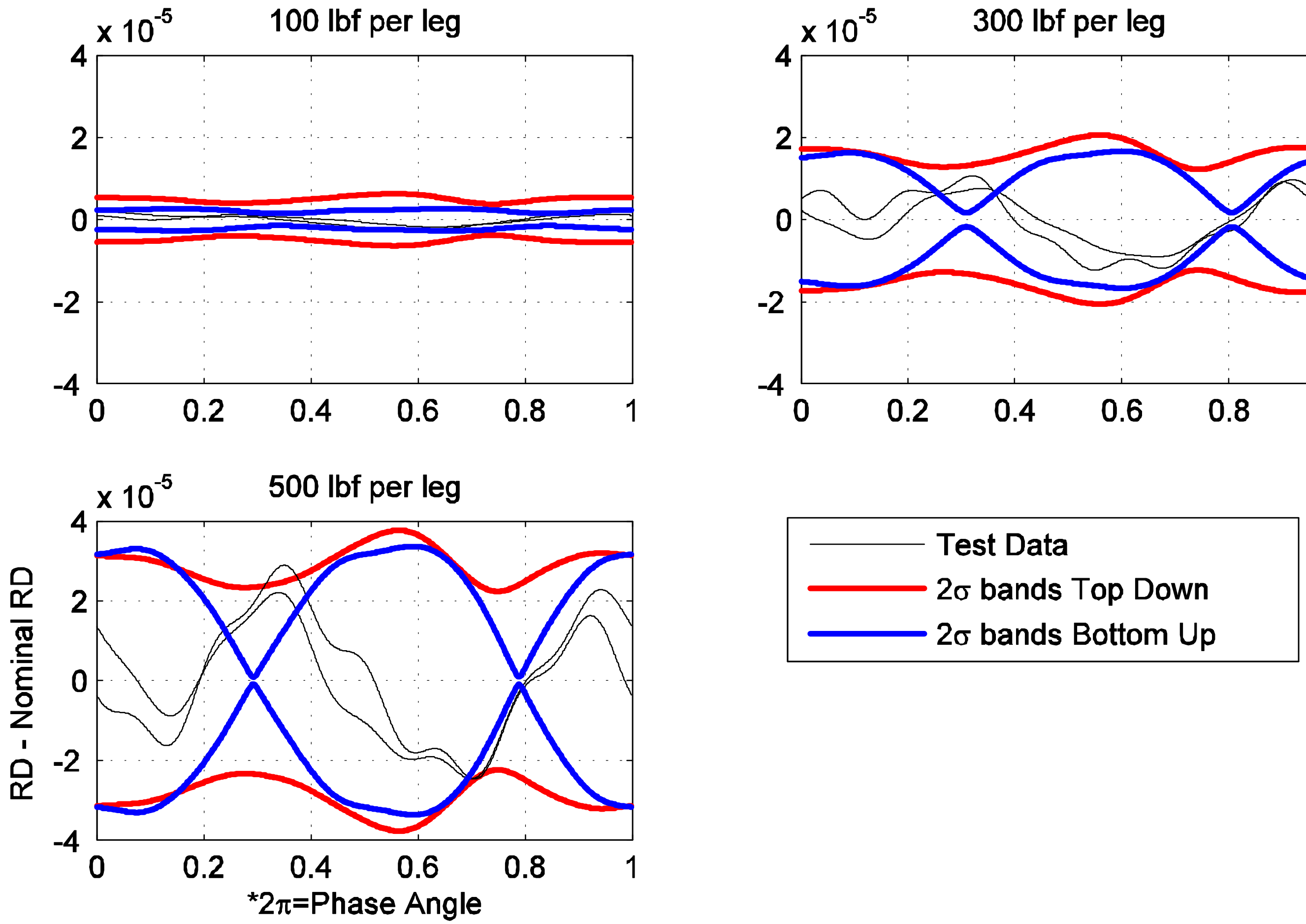


How is this accomplished?

- In this case, uncertainty associated with individual leg deformations propagated to center-of-gravity motion
- Mass assumed rigid
- Various assumptions made regarding correlations among leg motions

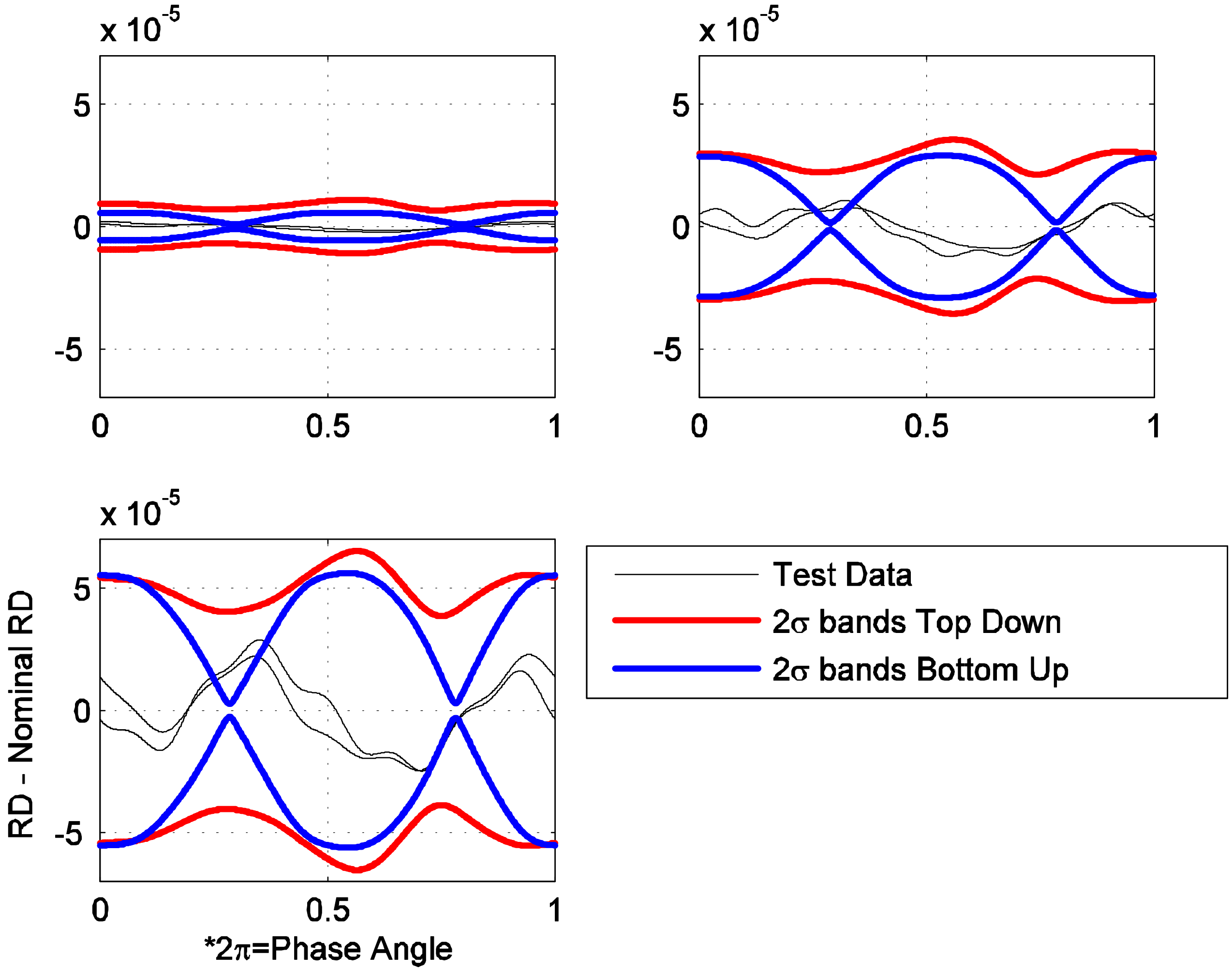
Illustrative Example (Continued)

- **Results – Uncorrelated single-leg UQ parameters**



Illustrative Example (Continued)

- Results – Perfectly correlated single-leg UQ parameters



Conclusions

- Bottom-Up and Top-Down UQ analysis approaches compared
- Top-Down approach
  - Shown to more accurately reflect response variability – form and level – for time history of response
  - Propagations use linear form – rely on sensitivity and covariance information, direct to perform
- Bottom-Up approach
  - Provides framework for relating system or component parameter variability to response variability
  - Relies on system model, at least, for first propagation, therefore, less accurate than Top-down

Sandia National Laboratories