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Validation of a Viscoelastic Model for Foam Encapsulated Component Response Over a Wide Temperature Range

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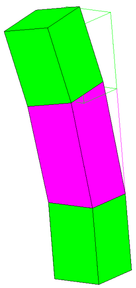
Overview

- Validation Strategy
- Viscoelastic Foam Model
- Validation Process
- Validation Results
- Summary

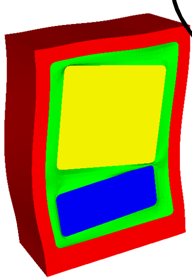
Validation Strategy
Foam/Component Mechanical Modeling
Rigid Epoxy Foam, 20 lbs/ft3 density

DMA Constitutive Test Articles

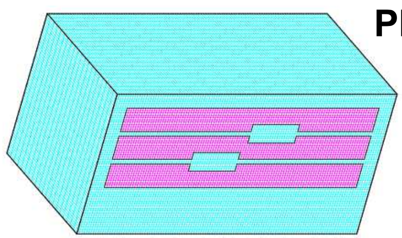
1) Rm Temperature
2) Full Temp Range



Phase I Test

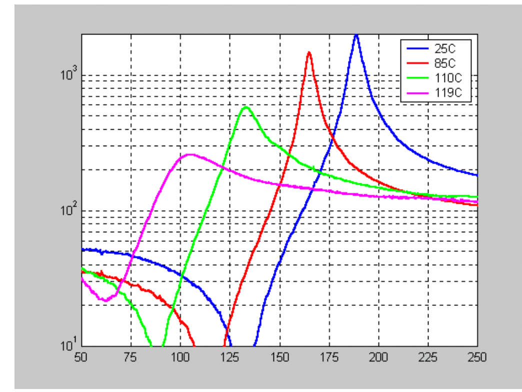


Phase II

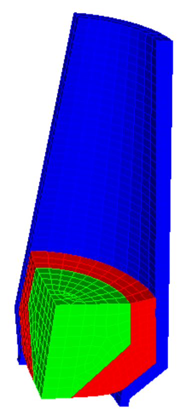


Phase III

Performance Measure



Frequency Response Functions



System Mass Mockup

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Calibration Procedure for the Viscoelatic Model in the Salinas Code

Dynamic Mechanical Analysis (DMA) Tests provides:

• estimates shear modulus vs. frequency and temperature

• basis for fitting Prony Series

• estimates of material loss factor

• still need second elastic constant

• works best near the glass transition temperature (95 deg C)

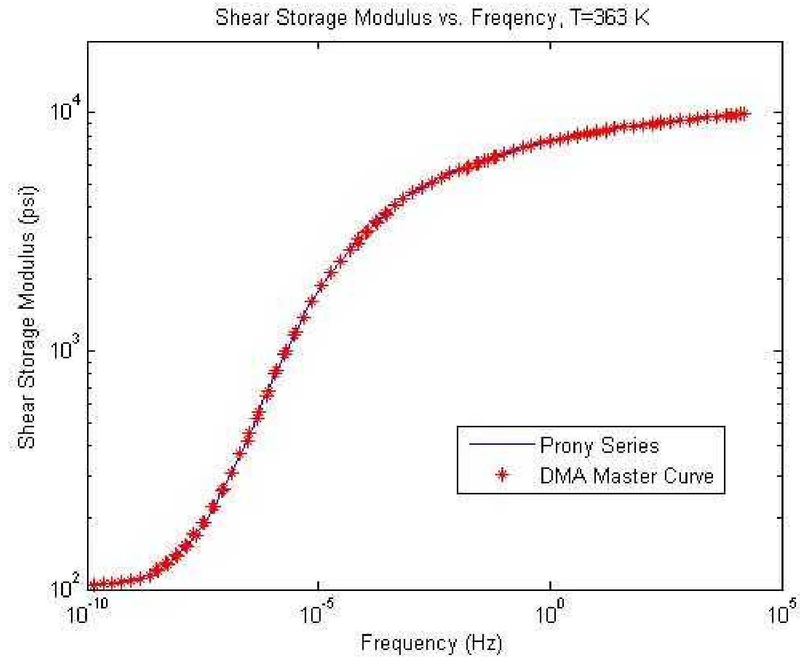
Prony Series

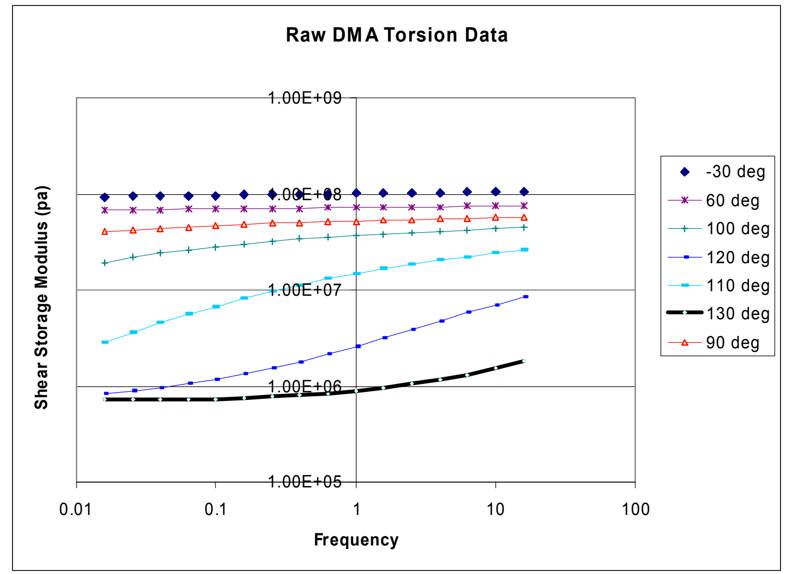
Viscoelastic Model

Phase I Modal Tests
• provides modal frequencies and damping
• analytically back out E and G with Salinas by matching test modes

Young's Modulus, Shear Modulus and damping

Shear Modulus Master Curve with 20 term Prony Series Fit
(at 90 deg. C)





DMA data has been shifted into the Master Curve

Frequency sweeps from 15.9e-3 to 15.9 Hz

Temperatures: -30 to 130 deg C

$$G(t,T) = Gr + (Gg - Gr)\sum_{j=1}^{N} m_j \exp\left(-\frac{t}{aT(T) * \tau_j(T_{ref})}\right)$$

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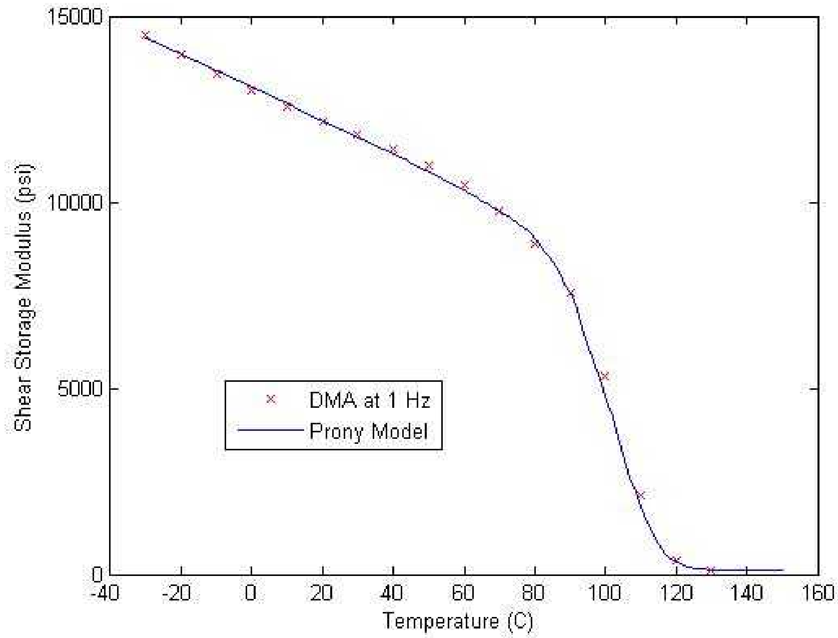
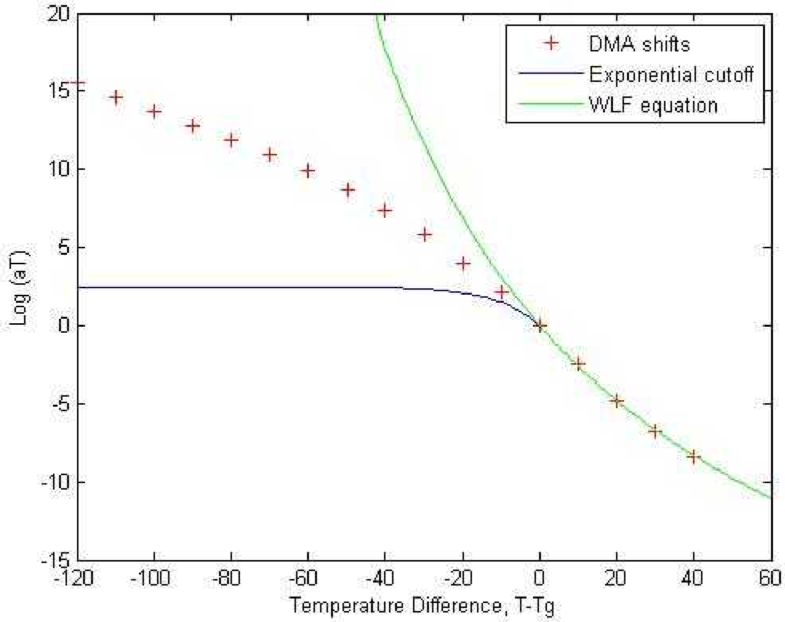
Shear Storage Modulus Dependence on Temperature Incorporated

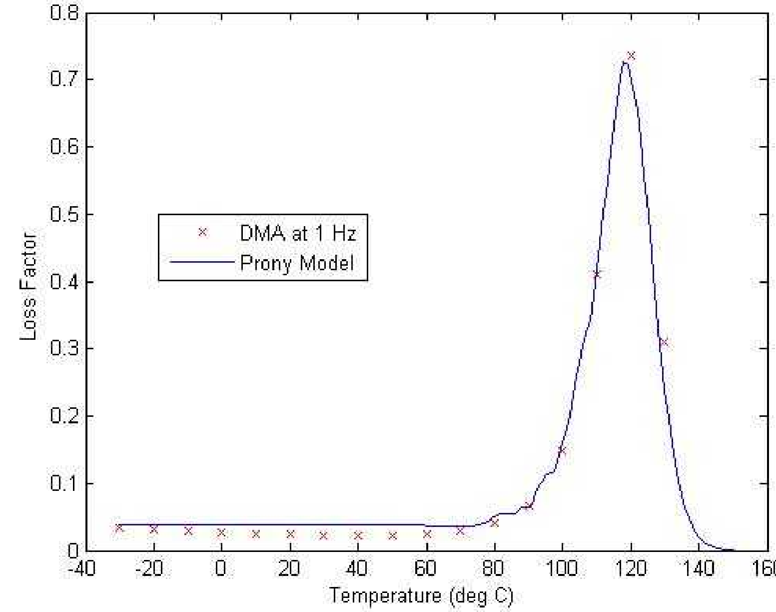


Glassy modulus as a function of temperature is fit to DMA data:

$$G(T) = 2.78 * Gg * (1 - 0.64 * T / Tg)$$ for T<Tg

Damping Dependence on Temperature





Cutoff established to transition from frequency or time shifts to temperature shifts to form the Master Curve (somewhat arbitrary)

This enables retaining some damping in the model for temperatures and frequencies far below Tg

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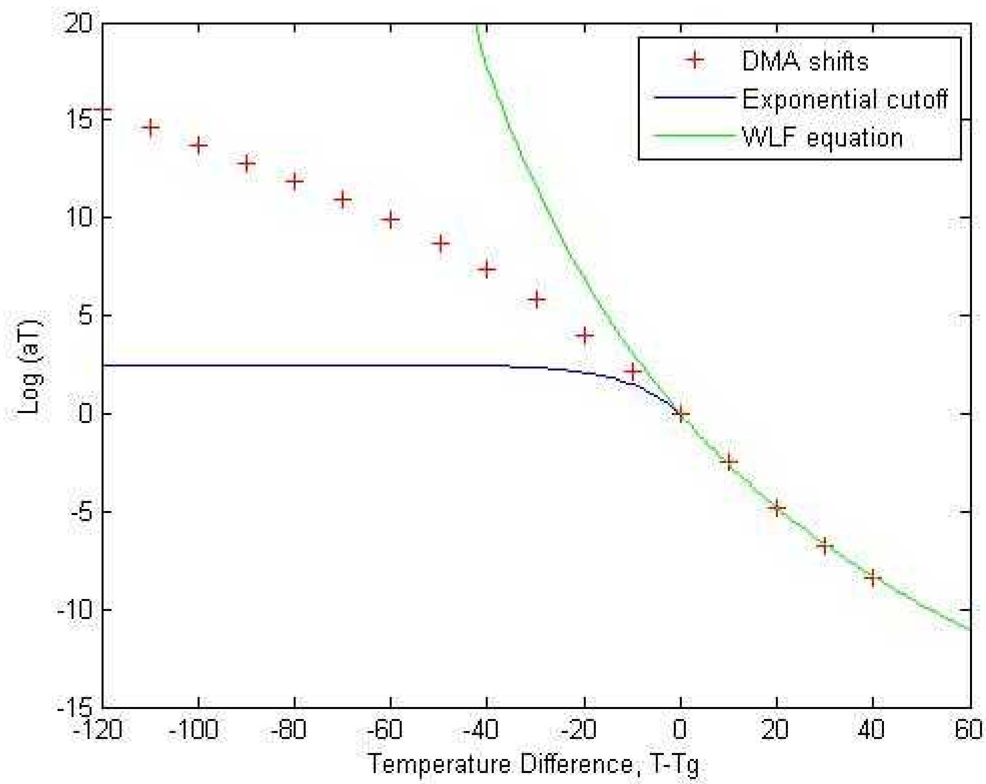
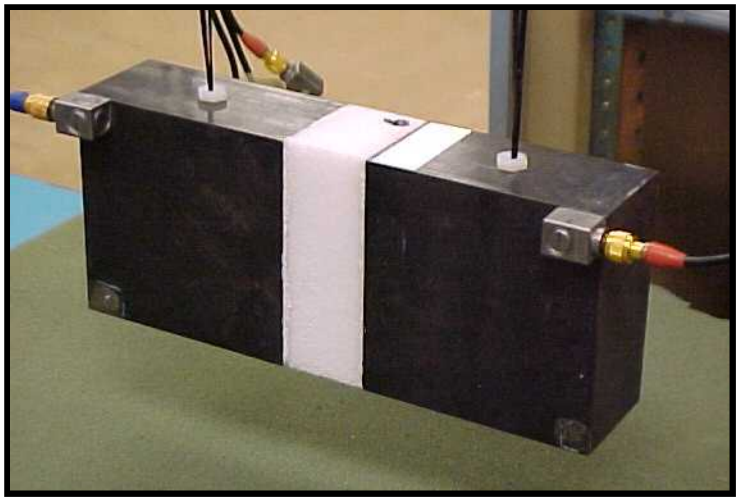
Shift Function based on forming Master Curve from DMA data



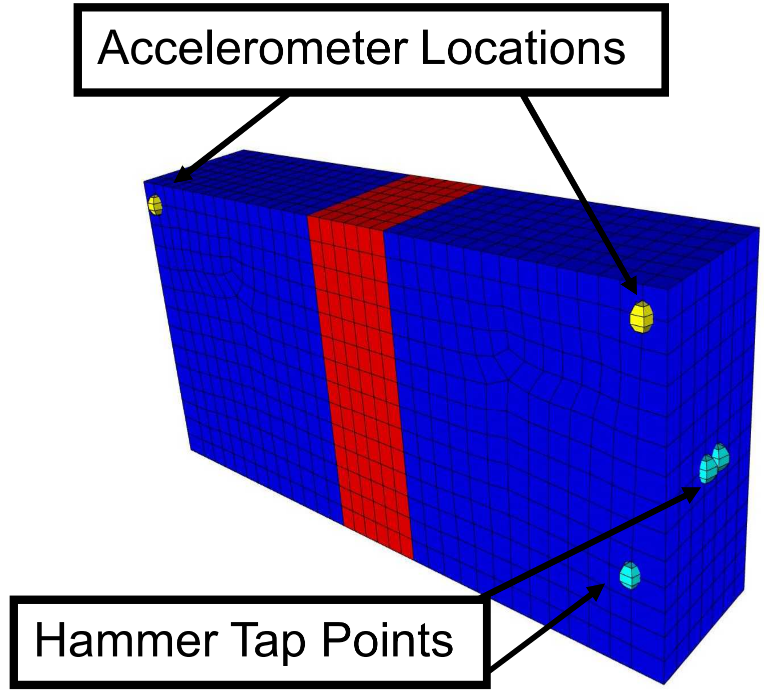
Cutoff established to transition from frequency or time shifts to temperature shifts to form the Master Curve

This also enables retaining some damping in the model for temperatures and frequencies far below Tg

Phase I Test Article

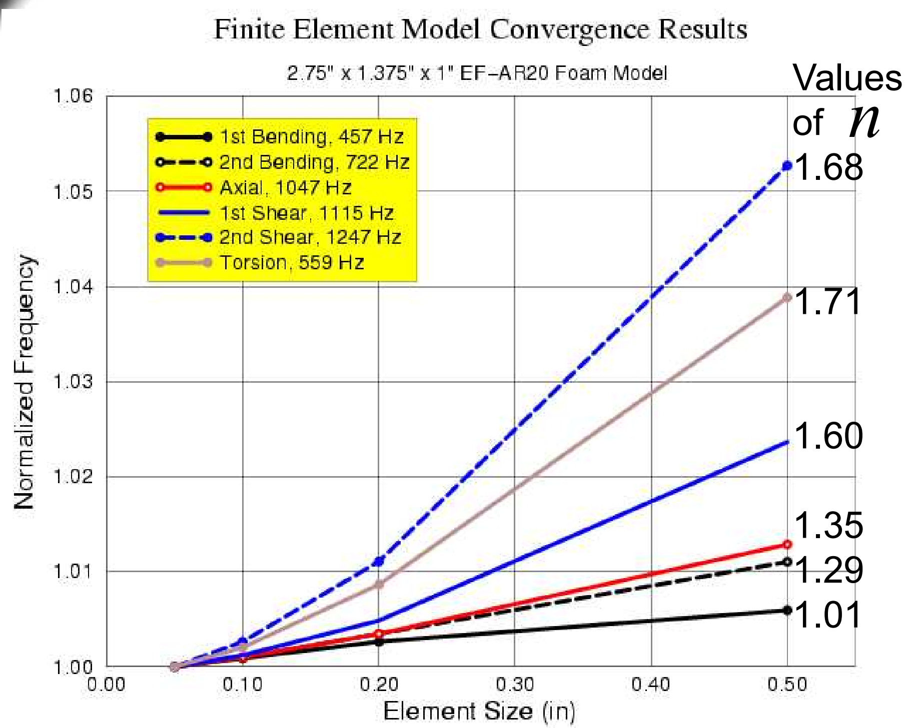


Test Hardware

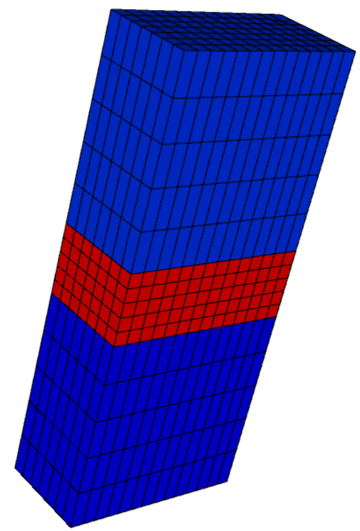


Salinas FE Model

Phase 1 Model Convergence using Modal Frequencies



0.2" element selected for computational efficiency



8 node hexahedral elements used

Richardson Extrapolation: $E(h) = E' + Ch^n$

Exponent n, has values given above, ranging from 1.01 to 1.71

Normalized error, $(E - E')/E'$, same as shown in plot at h = 0.2, **less than 1.3% error**

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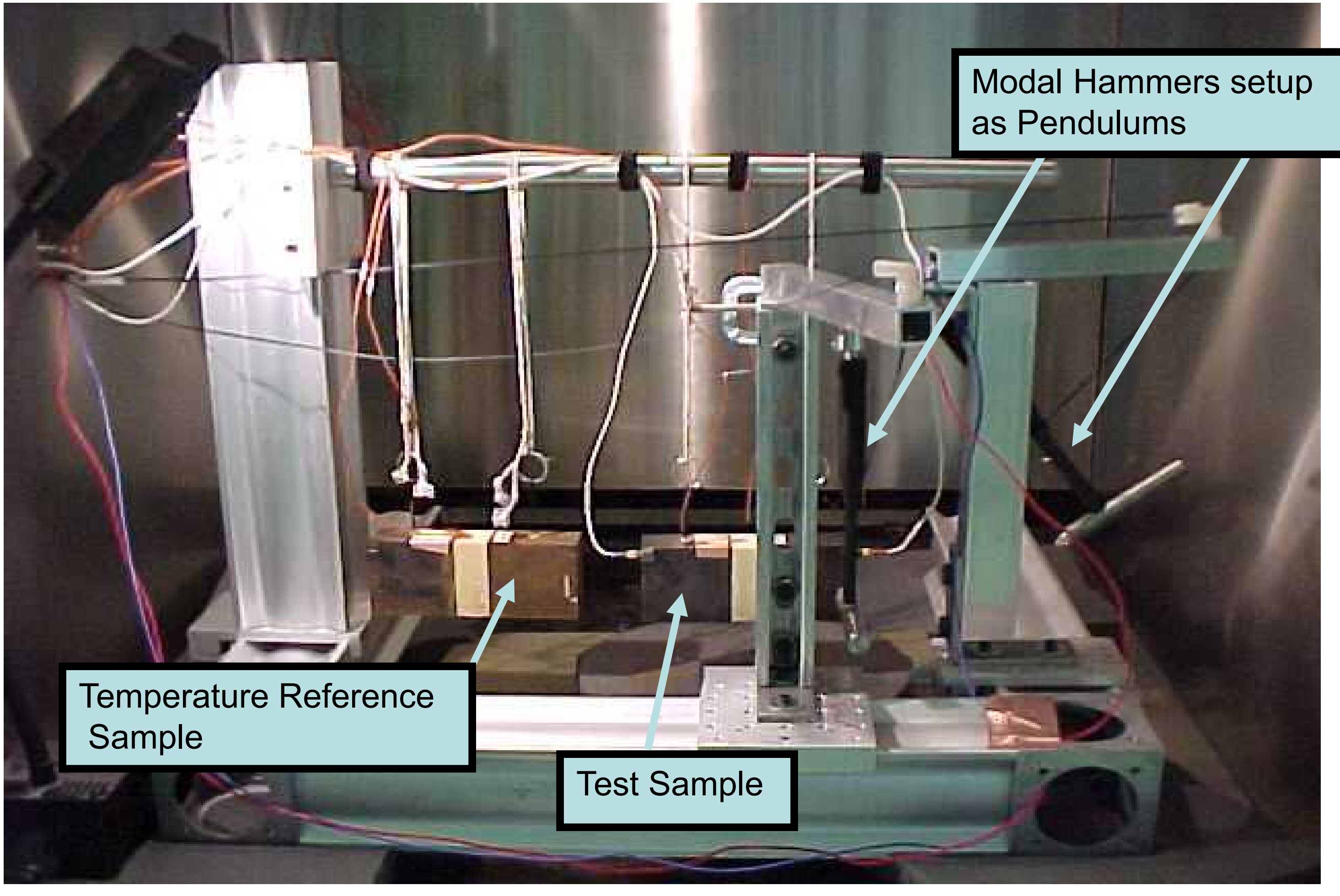
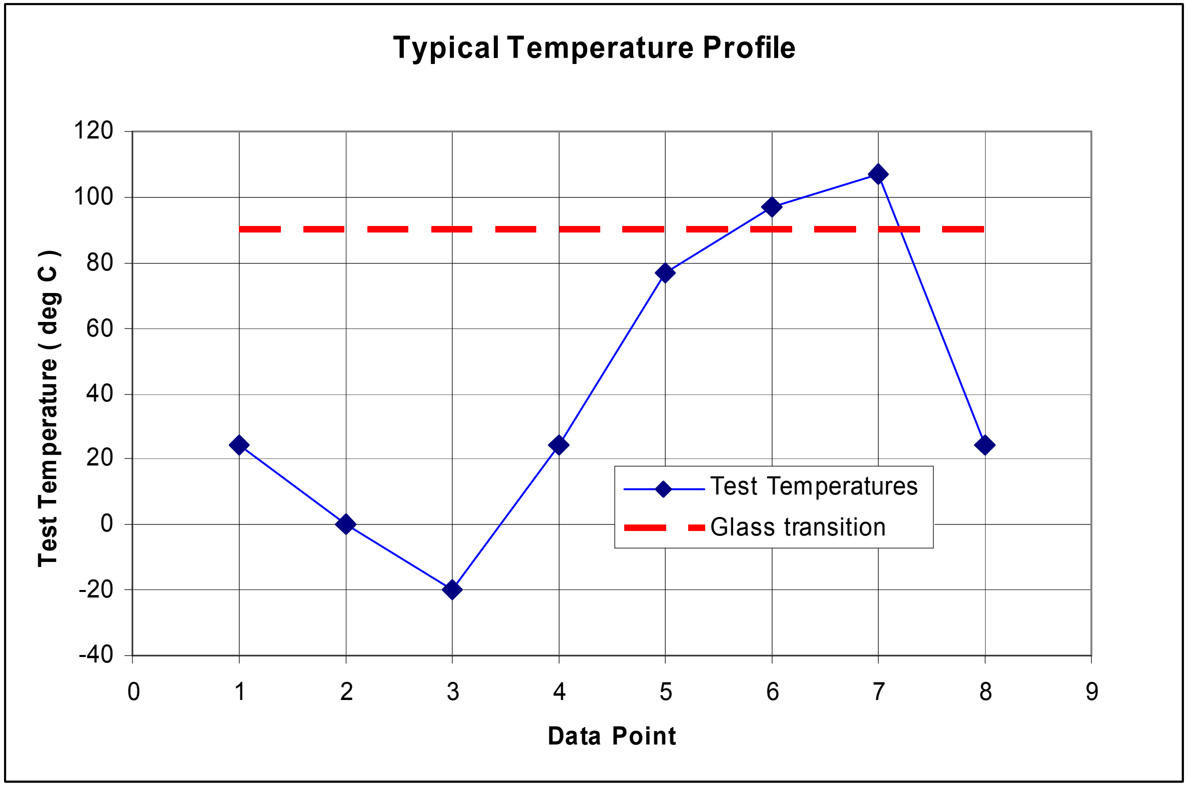
Environmental Chamber Setup



Validation Experiments



Phase I Modal Test Matrix

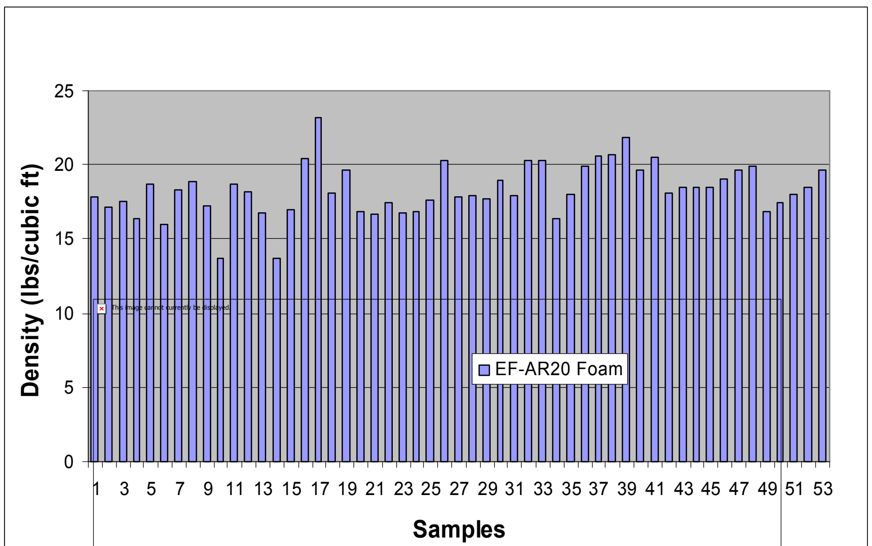
Sample	Density lbs/ft3
A	17.94
B	17.7
C	18.92
D	17.95
E	20.33
F	20.28

Test matrix included:
- variable foam densities
- 6 temperatures

Model Uncertainties Included

- Foam Density – see next slide
- Temperature range: -20 C to 110 deg C
- Variation of modulus with density

Density Variation in Foam based on several batches



$$\rho = \sigma_\rho Z_1 + \mu_\rho$$

$\rho =$ density

$\sigma_\rho =$ standard deviation $= 1.75$ pcf

$Z_1 =$ std. normal random variable

$\mu =$ mean density $= 18.32$ pcf

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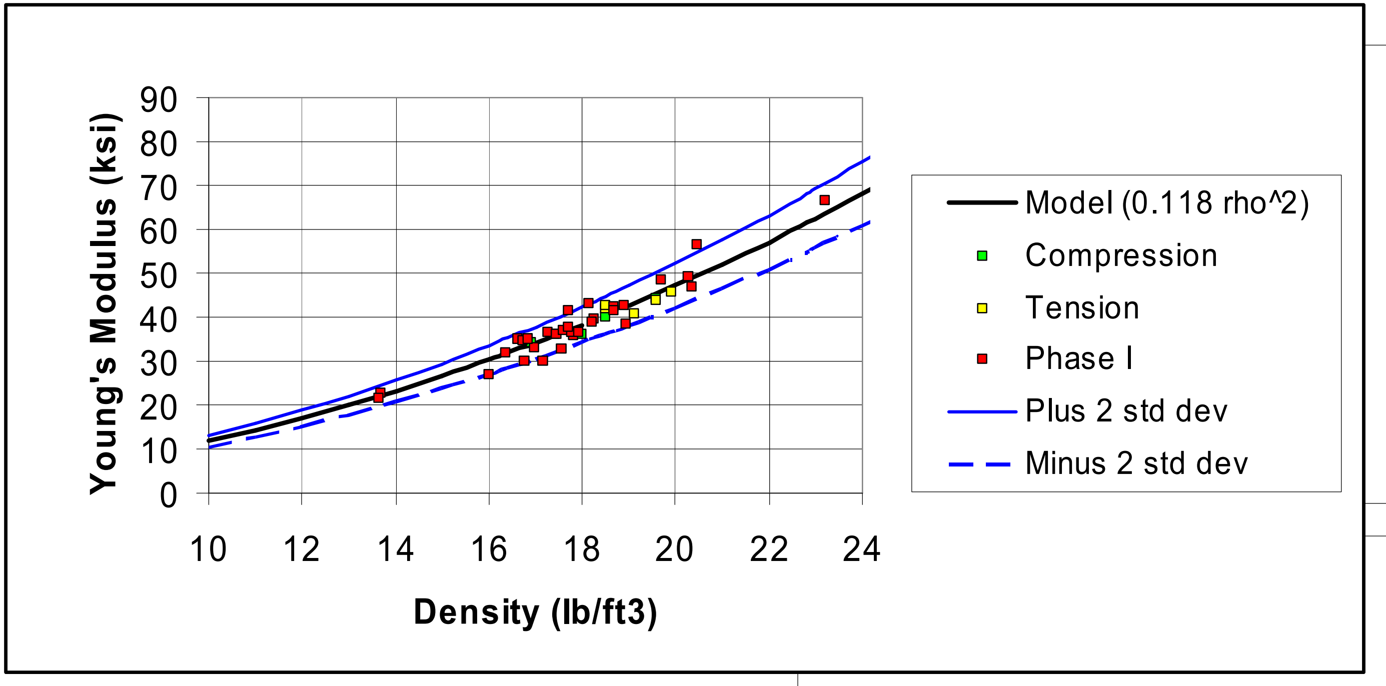
Modulus dependence on Density



$$E = 0.118\rho^2 + 0.00635\rho^2 Z_2$$

$$Z_2 = \text{std. normal random variable}$$

Upper/Lower Bound Values for Density and Modulus

Ninety-five percent probable limits for EF-AR20 foam parameters at room temperature based on a 95% probability point of a Chi Squared distribution With two degrees of freedom (Z_1 and Z_2)

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	ρ lbs/ft^3	***E*** *ksi*	***G*** *ksi*	% damping
Lower bound	14.0	23.2	9.06	0.0145
Upper bound	22.4	61.6	24.1	0.0145

Treating the average Poisson's ratio from Phase I data as a constant:

$$\nu = 0.28$$

$$G = \frac{E}{2(1+\nu)}$$

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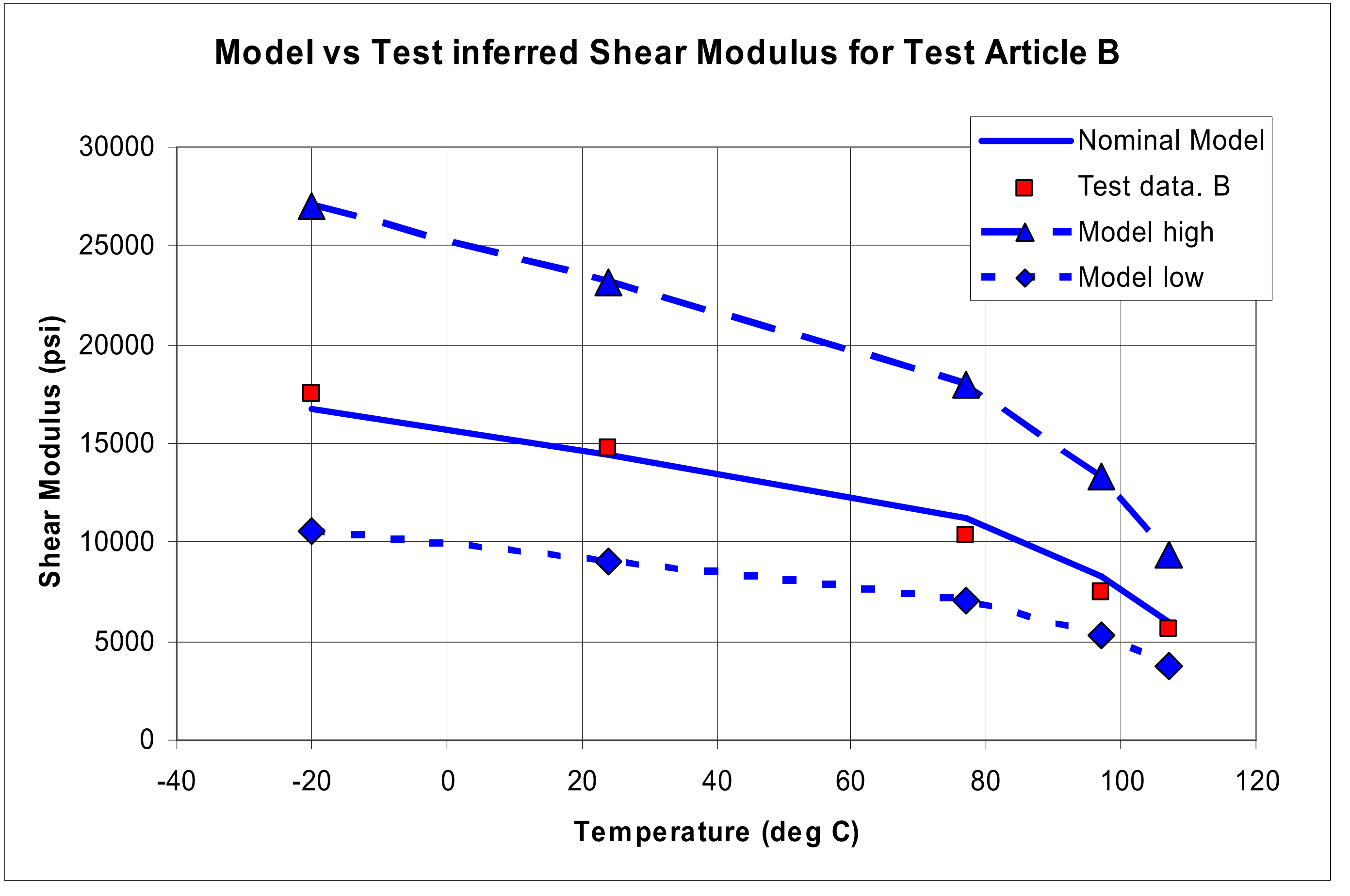
Model Validation Predictions Bounding and Deterministic



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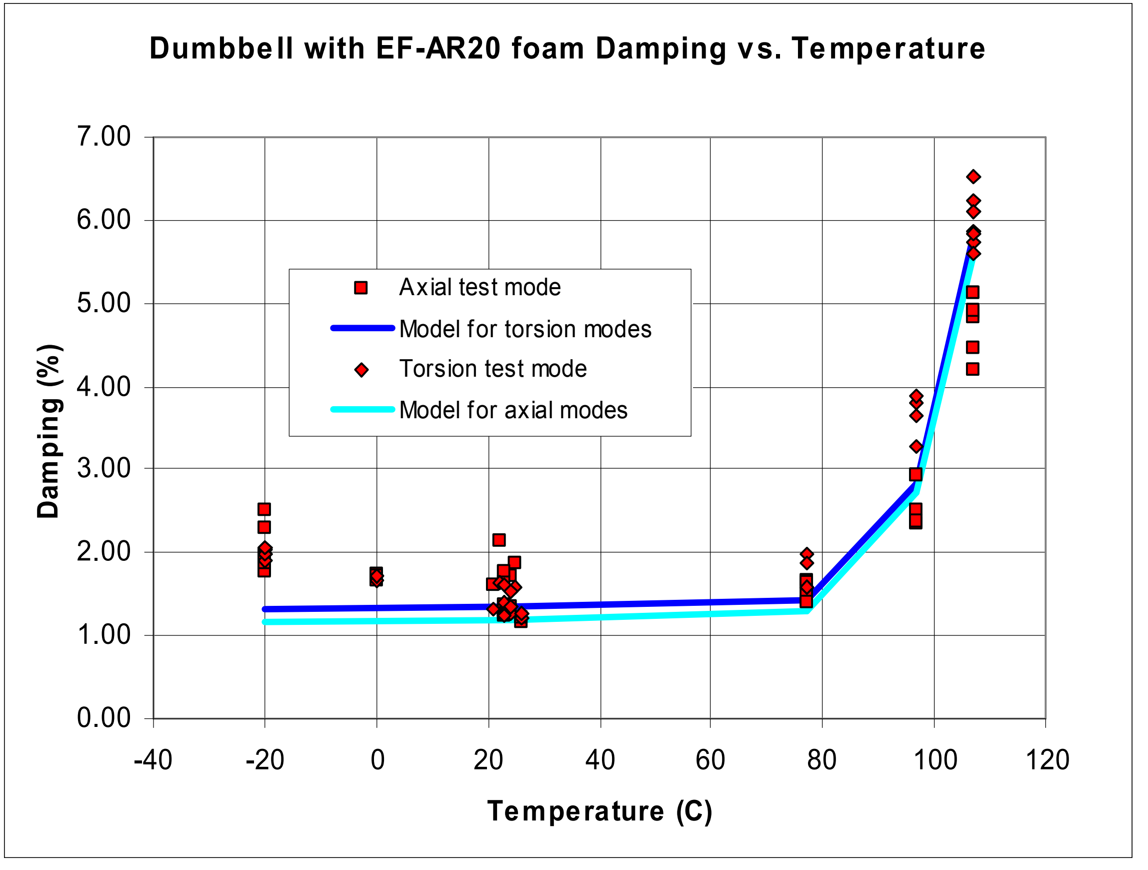
Damping predicted by model compared with Test Data



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Additional Validation Metrics to be applied

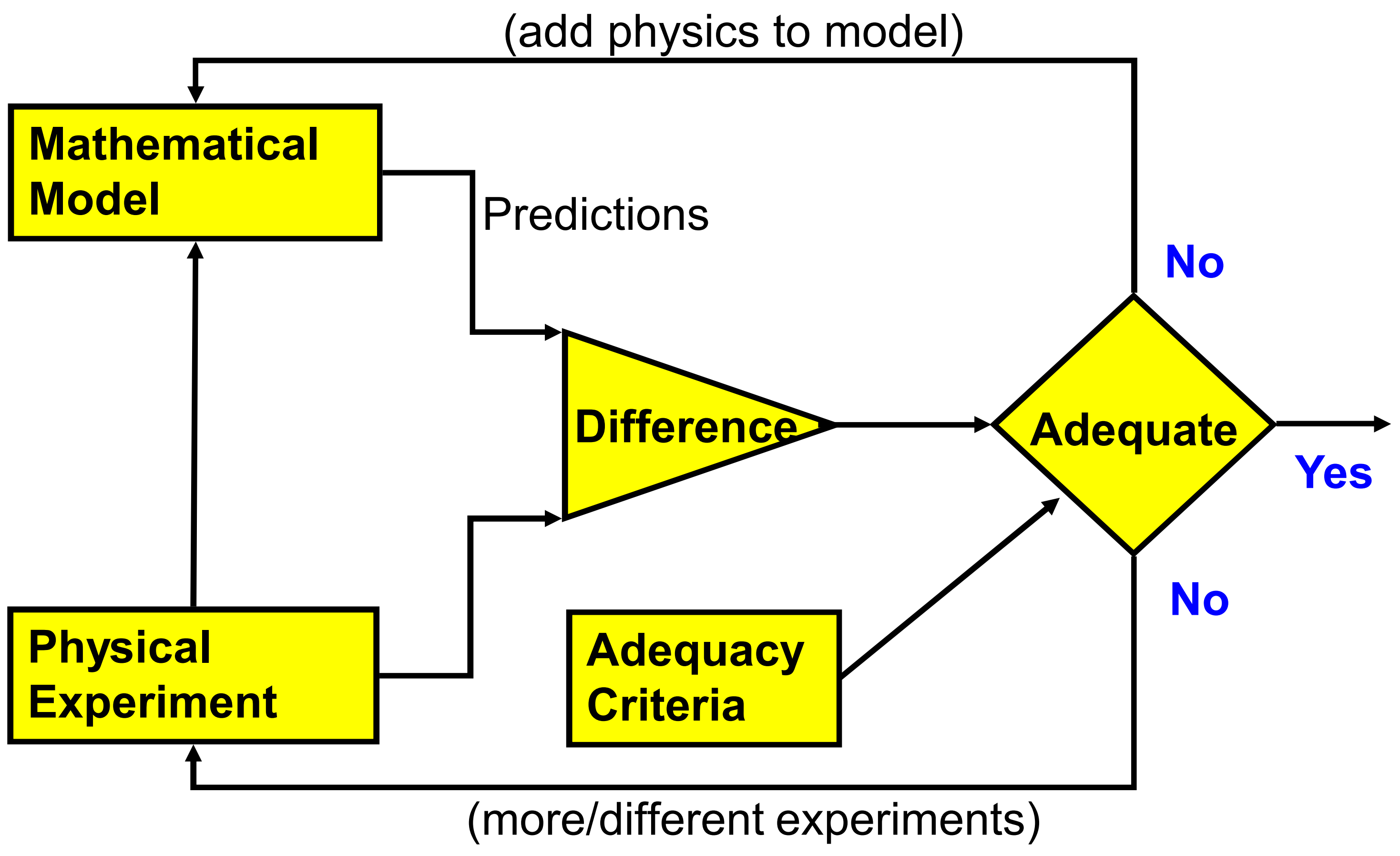
- Peak acceleration
- Windowed FRF
- Shock Response Spectrum

Summary

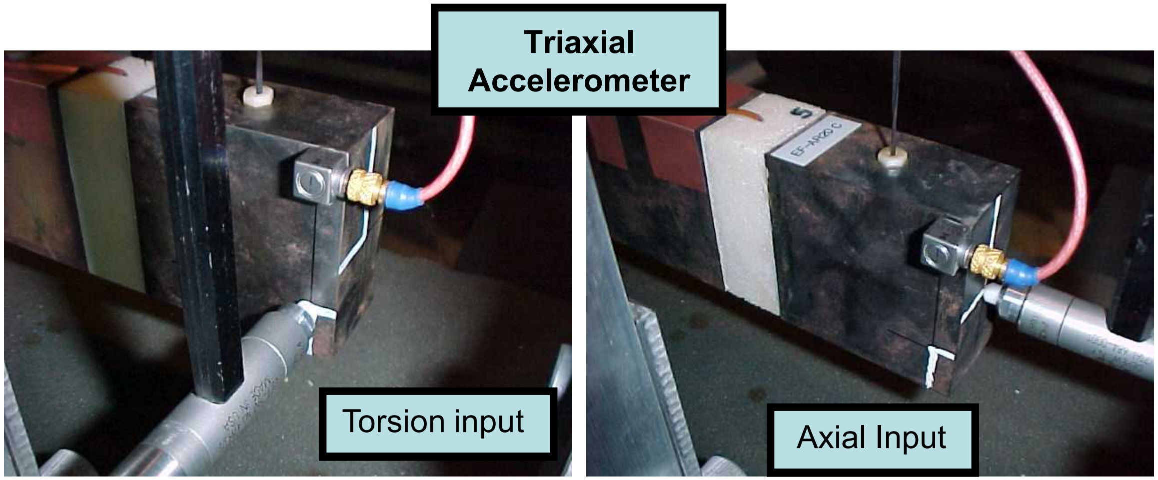
- Calibrated a linear viscoelastic model for EF-AR20 foam over Temperature span from -20 to 110 deg C
- Foam density and elastic modulus were treated as random variables
- Upper/lower bound validation approach provided confidence in model's ability to predict shear modulus values inferred from the Phase I modal tests
- Deterministic predictions also agreed well
- Model predicts conservative damping levels relative to Phase I test data
- Will be applying Peak response, Windowed FRF and Shock Response Spectra metrics

Backup Slides

Calibration/Validation Process



Axial and Lateral Excitation for Modal Tests



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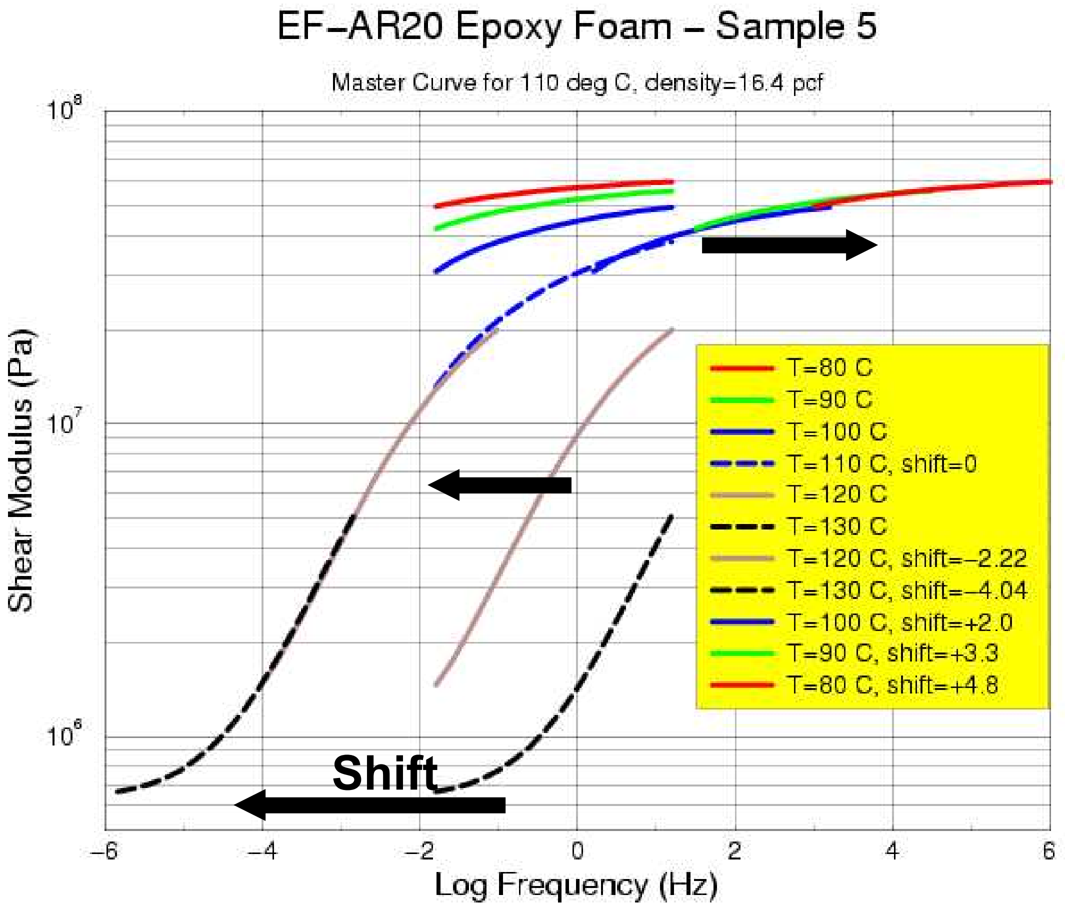
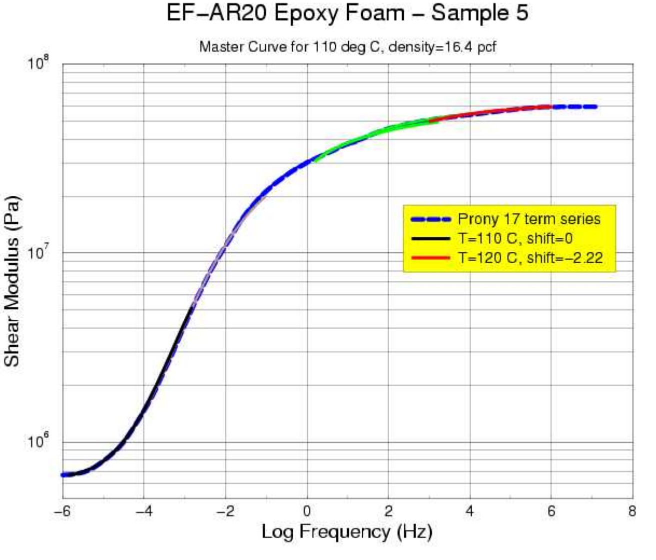
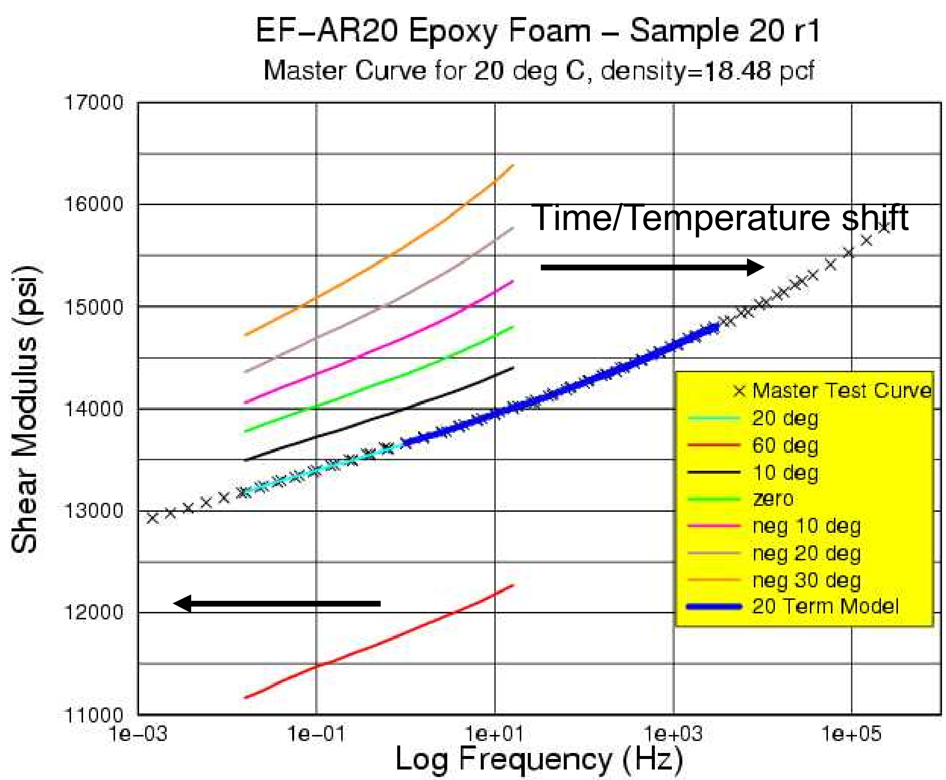
Constitutive Experiments
DMA Temperature/Frequency Shifts



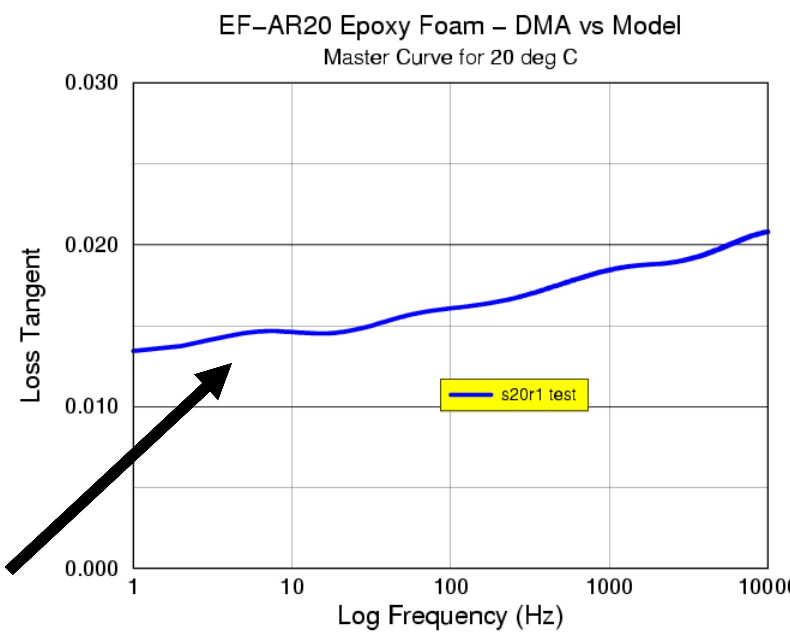
17 Term Prony Series Fit



Prony Model of Master Curve at Room Temperature



- Individual DMA curves shifted to form Master Curve
- 20 Term Prony Series fit to Master Curve (blue curve)



Resulting master curve for Loss Tangent (damping = 0.5 LT for Phase I)

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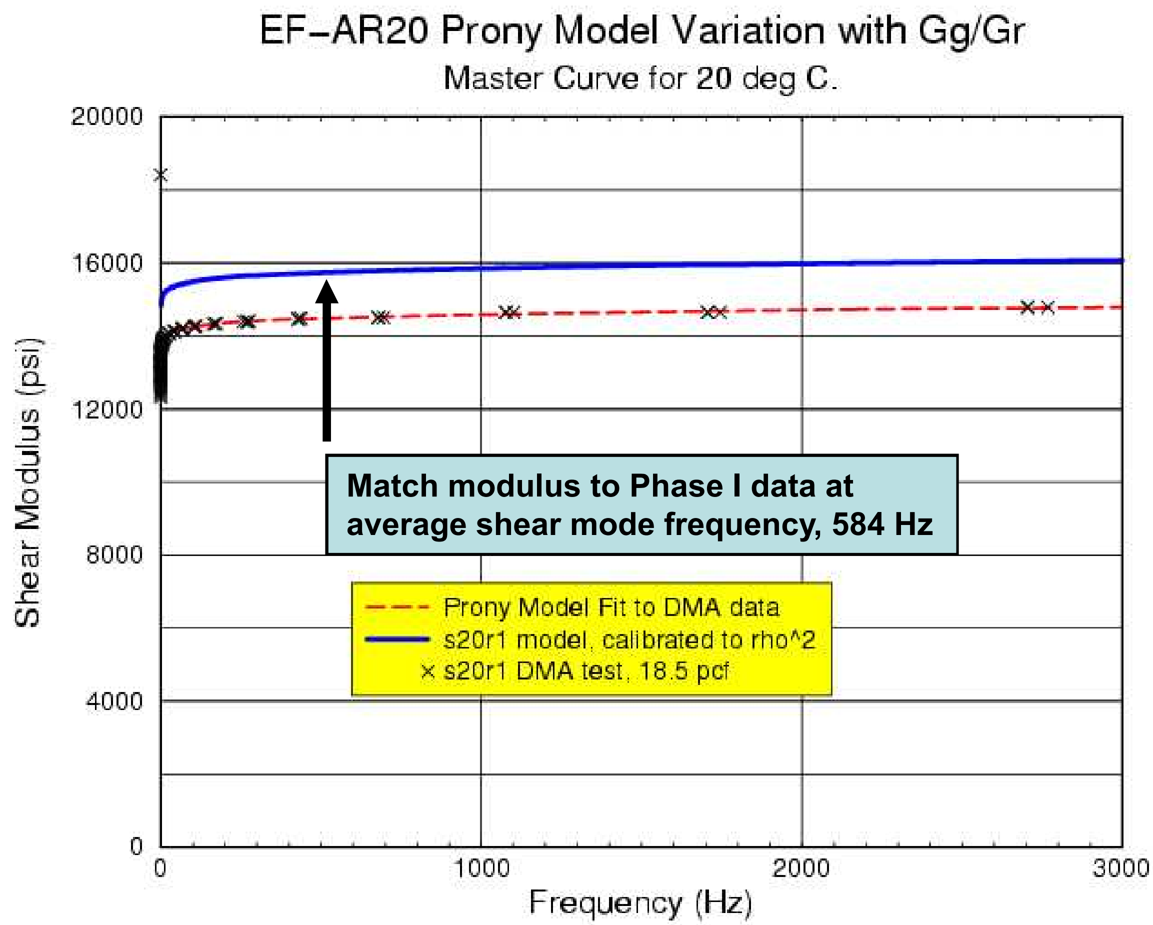
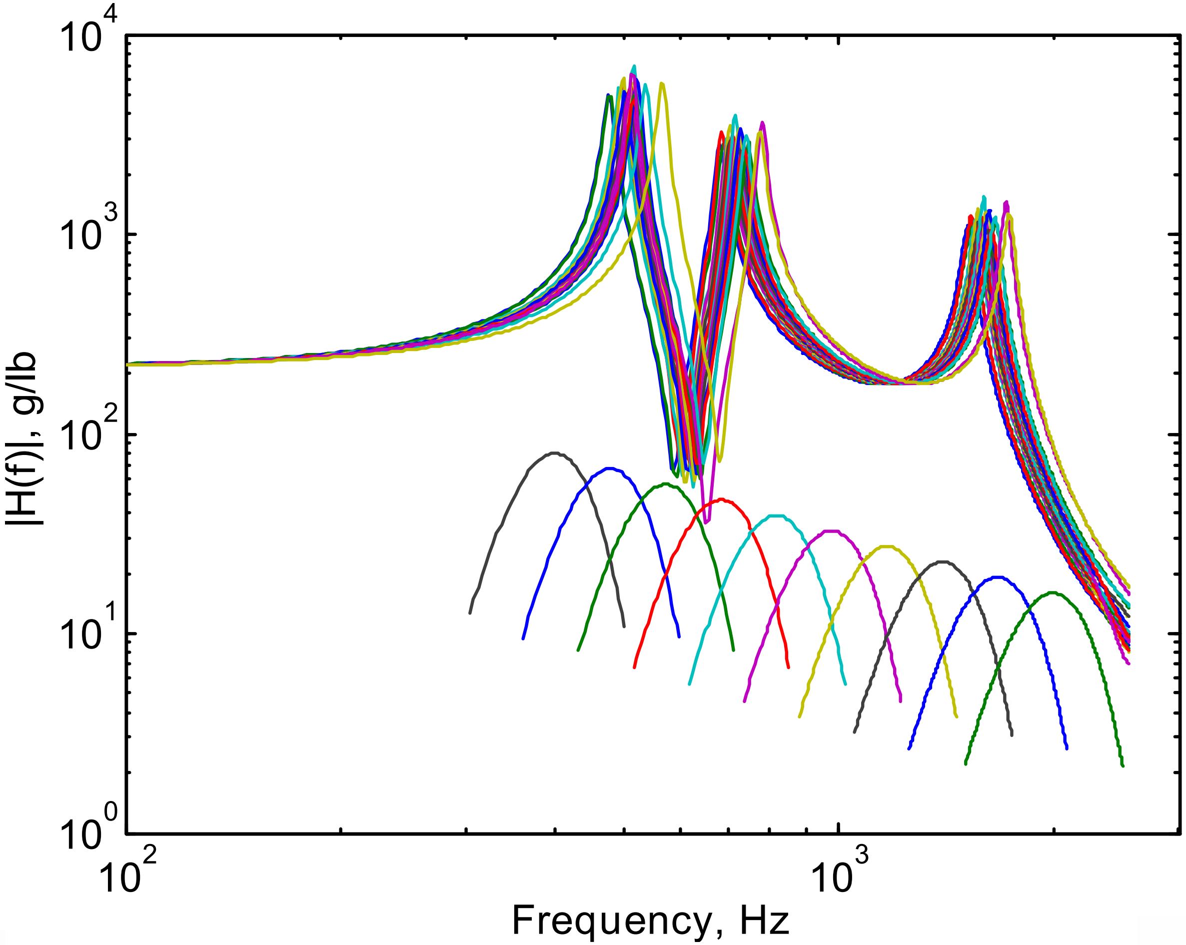
Matching Prony Series Model with Phase I Test

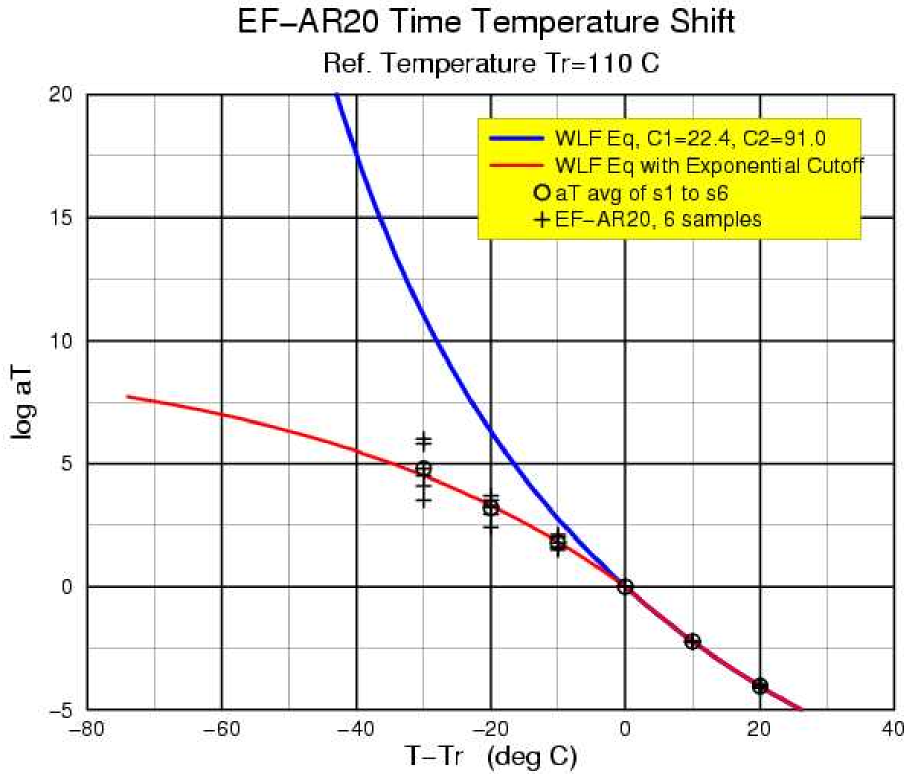


Prony Series model is matched to Phase I derived modulus values by shifting the curve with modified Gg and Gr values

Window Functions for Validation Metric



Time(Frequency)/Temperature Shift Function based on the Master Curve for EF-AR20 Foam



Time domain:

$$G(t) = G_r + (G_g - G_r)\sum_{j=1}^{N} m_j \exp(-t / a_T \tau_j)$$

Frequency domain:

$$G'(\omega) = G_r + (G_g - G_r)\sum_{j=1}^{N} \frac{m_j(\omega a_T \tau_j)^2}{(1+(\omega a_T \tau_j)^2)}$$

where:

$$a_T = t / t_r$$

t_r = **reference time**

Modified WLF Time-Temperature Shift Function

$$Log(t/t_r) = (-C_1 * (T - T_r)) / (C_2 + (T - T_r)) \qquad T > T_r$$

$$Log(t/t_r) = a_1 * (1 - \exp(a_2 * (T - T_r))) \qquad T < T_r$$

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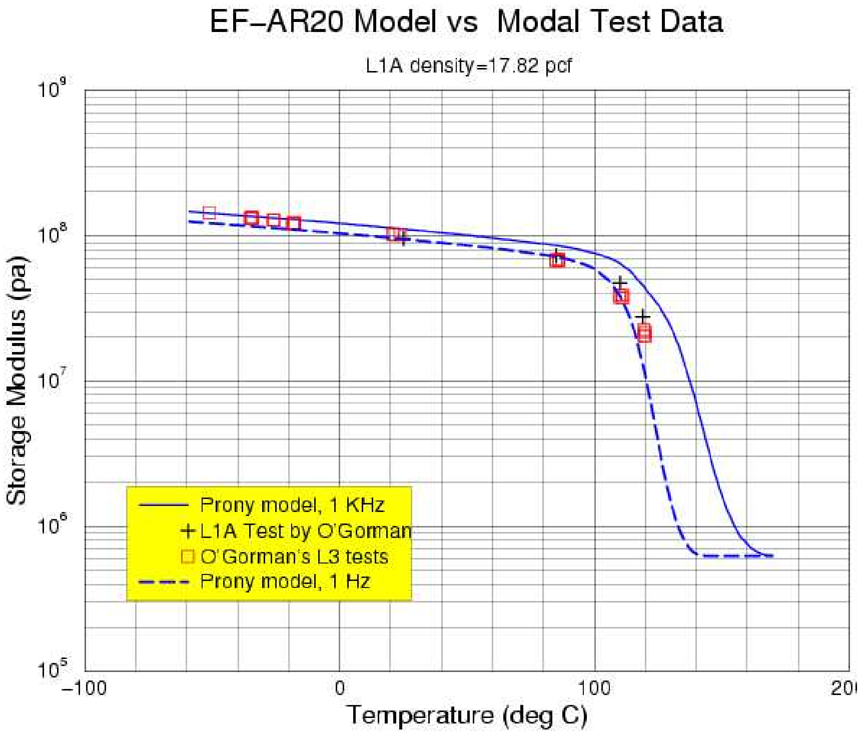
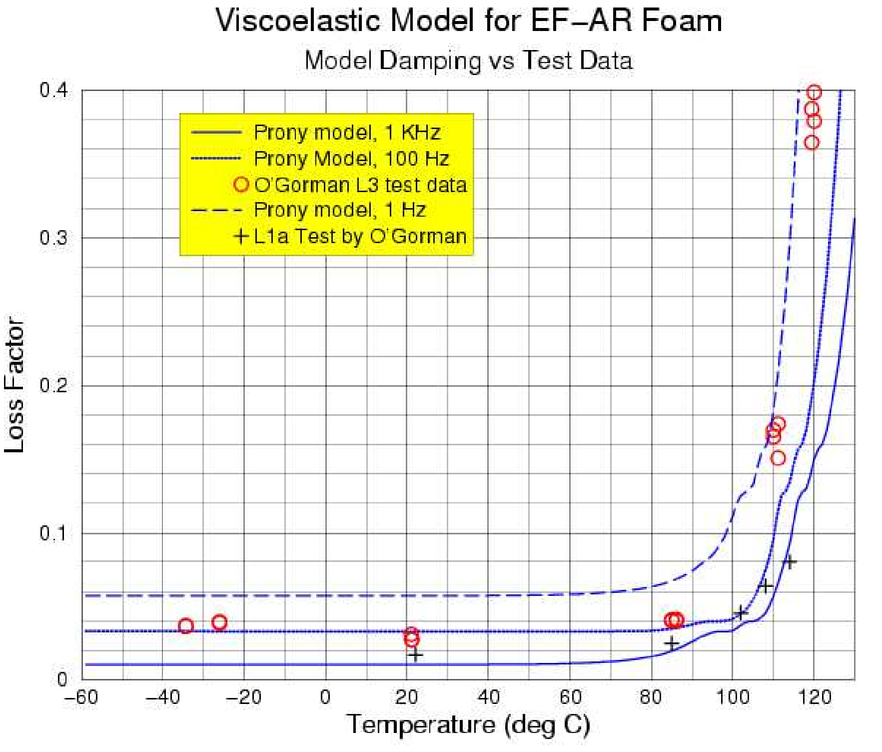
Epoxy Foam Behavior

Modulus & Damping vs Temperature
Test Data superimposed on model curves





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Linear Viscoelasticity in SALINAS

- Stress is an Integral function of strain: Convolution Integral

$$\sigma_{ij}(t) = \int_0^t \widehat{G}_{ijkl}(t-s)\frac{d\varepsilon_{kl}}{ds}\,ds$$

- Isotropy is assumed:

$$\sigma_{ij}(t) = \int_0^t 2G(t-s)\frac{d}{ds}\varepsilon_{ij}^d\,ds + \delta_{ij}\int_0^t K(t-s)\frac{d}{ds}tr(\varepsilon)\,ds$$

where $\varepsilon^d = \varepsilon - \delta_{ij} tr(\varepsilon)/3$

- Material functions $G(\ )$ and $K(\ )$ are selected to reproduce experimental data

Model Form in SALINAS Code

Measure Shear Relaxation Modulus with DMA tests and fit Prony Series:

$$G(t) = G_r + (G_g - G_r)\sum_{j=1}^{N} m_j \exp(-t/\tau_j)$$

Use same Prony Series for the Bulk modulus and estimate K_g and K_r:

$$K(t) = K_r + (K_g - K_r)\sum_{j=1}^{N} m_j \exp(-t/\tau_j)$$

A constant value of Poisson's ratio with UQ for this viscoelastic foam will be used based on best estimates from measuring E and G directly in constitutive tests and indirectly from Phase I modal tests.

Assuming Isotropic behavior, Poisson's Ratio is: $\nu = (\frac{E}{2G}) - 1$

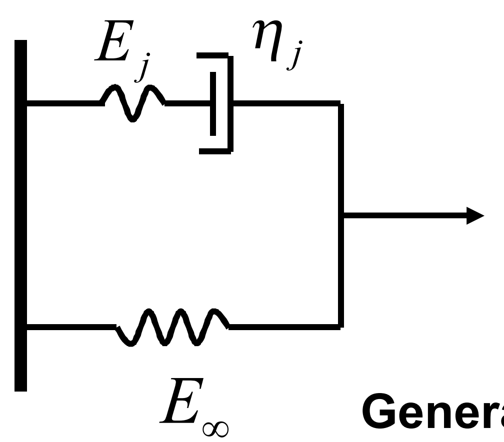
and the Bulk Modulus will be estimated as follows: $K = 2G(1+\nu)/3(1-2\nu)$

Linear Viscoelastic Constitutive Model

Stress is an Integral function of strain: Convolution Integral

$$\sigma(t) = \int_0^t E(t-s)\frac{de}{dt}ds$$

Where the Modulus is defined by a Prony Series:



$$E(t) = E_\infty + (E_g - E_\infty)\sum_{j=1}^{N} m_j \exp(-t/\tau_j)$$

$$\sum_{j=1}^{N} m_j = 1$$

Generalized Maxwell Model

and the parameters, E_∞, E_g and τ_j are based on Mat'l Property Tests

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Modulus in the Frequency Domain

For steady state conditions, the complex dynamic shear modulus is:

Shear Modulus

$$G(\omega) = G_\infty + (G_g - G_\infty)\sum_{j=1}^{N} m_j \frac{i*\omega*\tau_j}{(1+i*\omega*\tau_j)}$$

or

$$G(\omega) = G' + iG''$$

Storage Modulus

$$G'(\omega) = G_\infty + (G_g - G_\infty)\sum_{j=1}^{N} \frac{m_j\omega^2\tau_j^2}{(1+\omega^2\tau_j^2)}$$

Loss Modulus

$$G''(\omega) = (G_g - G_\infty)\sum_{j=1}^{N} \frac{m_j\omega\tau_j}{(1+\omega^2\tau_j^2)}$$

Loss Factor

$$\eta = \frac{G''}{G'} \qquad \eta = 2\zeta = \frac{\delta}{\pi} \qquad \delta = \frac{1}{N}\ln\left(\frac{X_n}{X_{n+N}}\right)$$