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# A Study of the Conservatism of Maxi-Max ASDs in the Analysis of Transient Random Environments Using Rainflow Fatigue Analysis

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# Background

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- Autospectral Densities (ASDs) are used to estimate the spectral content and amplitude of random vibration environments.
- Theoretically the ASD will only be correct if the underlying signal is not changing with time.
- Since the environments associated with missile flights are changing rapidly with time the concept of a Maxi-Max ASD [1] was developed.
- The concern with this approach is that a Maxi-Max ASD based on the highest 1-second of response is then used to define a 60 second long component test.



# Objective of This Study

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- We chose to use 5-second long segments of data to generate ASDs for a Sandia launch vehicle.
- The conservatism of this practice was challenged by our customer.
- Therefore, a method was sought that would provide an unbiased estimate of the fatigue damage of the flight data with respect to the corresponding test specification.
- The rainflow cycle counting algorithm [2] was chosen because it is a recognized industry standard for the measure of fatigue damage.



# Rainflow Method

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- The rainflow cycle counting method was developed to sort random signals into “bins” associated the number of cycles of a given amplitude.
- The fatigue damage,  $D$ , associated with each bin is then calculated using the following equation

$$D = \sigma^{A\eta^2} N$$

- Where  $\sigma$  represents the stress amplitude,  $N$  represents the number of cycles at that amplitude,  $A$  is the fatigue scaling coefficient for the material of interest, and  $\eta$  is a correction term associated with the component damping [3].

# Estimating the Stress Levels



- Since the data available for flight are in the form of acceleration signals it was necessary to use a model to transform those signals into an equivalent stress.
- Our approach was to excite a series of Single-Degree-of-Freedom (SDOF) oscillators using the flight data.
- The relative displacement of the oscillator spring was assumed to be proportional to the strain associated with a resonant mode of the component centered at the natural frequency of that oscillator.
  - We could have used the relative velocity as well, but this would not have changed the results for this analysis.



# Estimating the Stress Levels



- The stress is then computed as the product of the strain and Young's modulus.
- The total damage potential as a function of frequency is then computed as the superposition of the damage for a series of 1/12<sup>th</sup> octave SDOF oscillators.
- The resulting spectrum will be called the “Fatigue Damage Spectra” or FDS for the remainder of this presentation.



# Analysis

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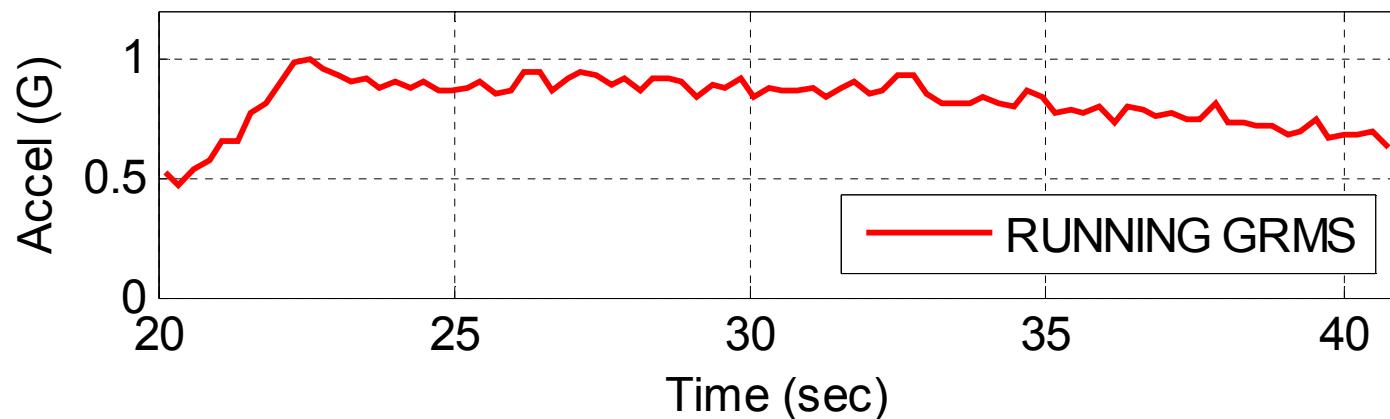
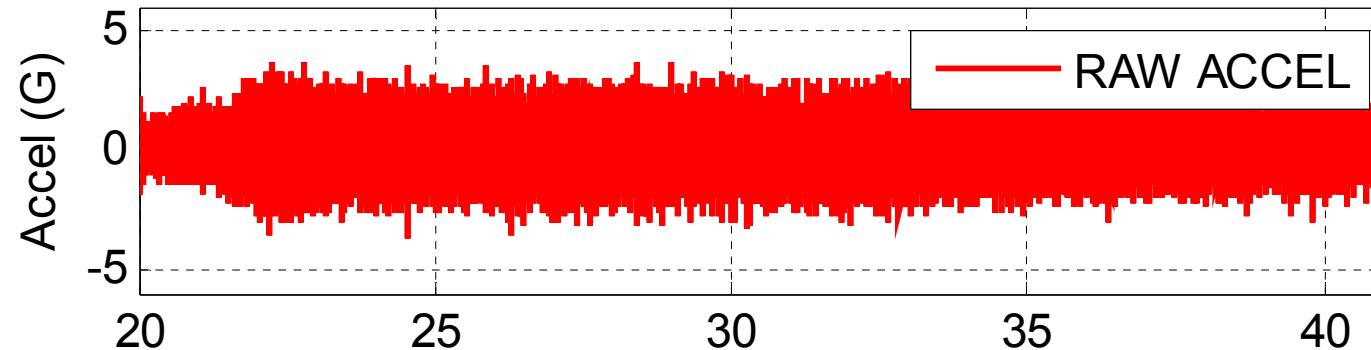
- The remaining slides in this presentation will show the results from an analysis using the response data measured on a single channel during an actual flight.
- The FDS was computed for the raw acceleration signal to serve as a baseline or “base” measure of the damage.
- Maxi-Max ASDs using the traditional 1-second segments and 5-second segments were then generated from the raw signals.

# Analysis

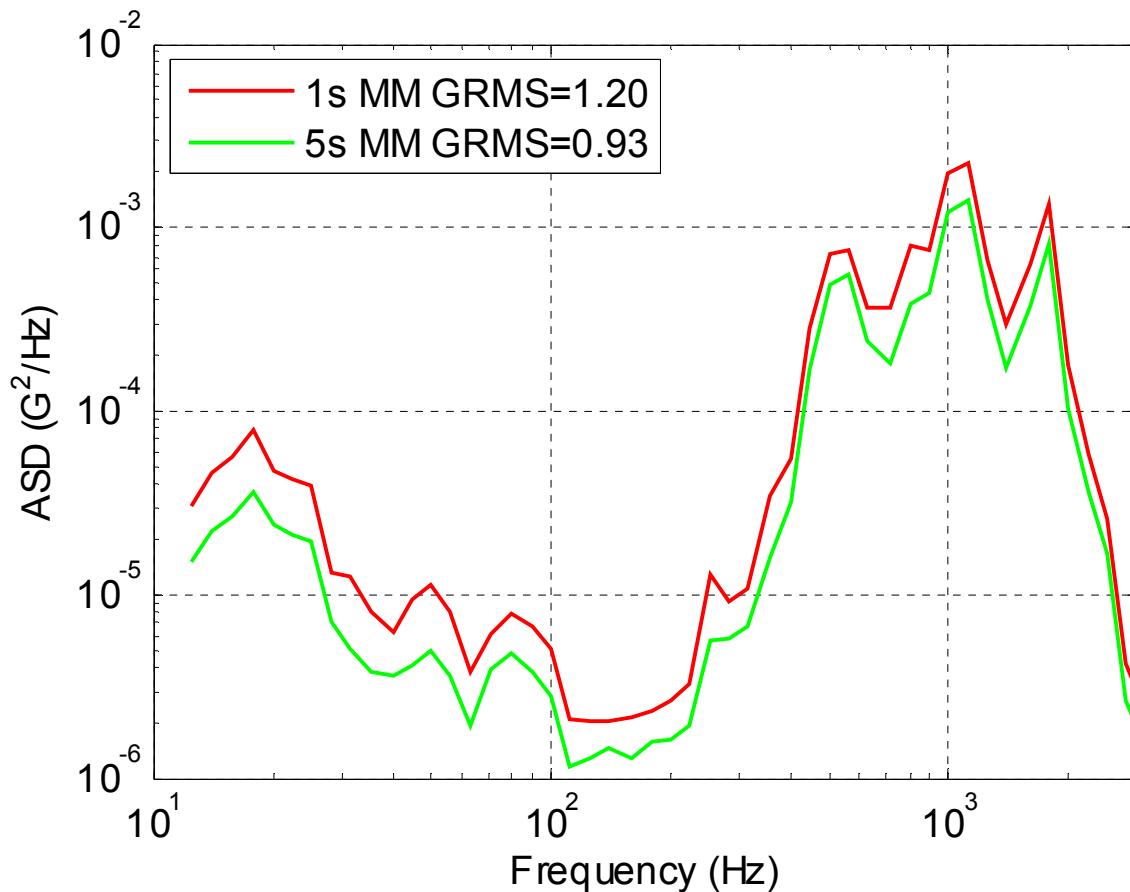


- FDS were then computed using realizations of the different ASDs.
- In order to be sure to address a representative range of component responses the FDS were computed for three SDOF quality factors ( $Q=10, 16, \text{ and } 50$ ) and two extreme values of the fatigue scaling coefficient ( $A\eta^2=6.66 \text{ and } 20$ ).
  - $A\eta^2=6.66$  represents a ductile material in an encapsulated assembly.
  - $A\eta^2=20$  represents a brittle material in a bolted assembly.
  - The values for  $Q$  are not independent of  $A\eta^2$  but no attempt was made to identify an exact relationship.
  - The results for the extreme cases will be presented in the following slides.

# Raw Acceleration Signal



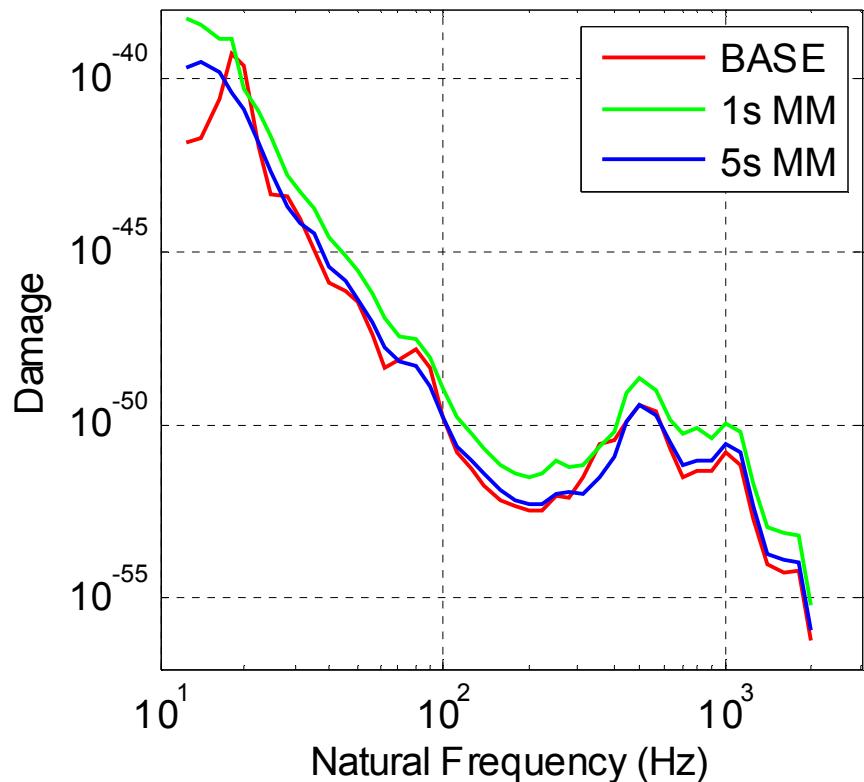
# Maxi-Max ASDs



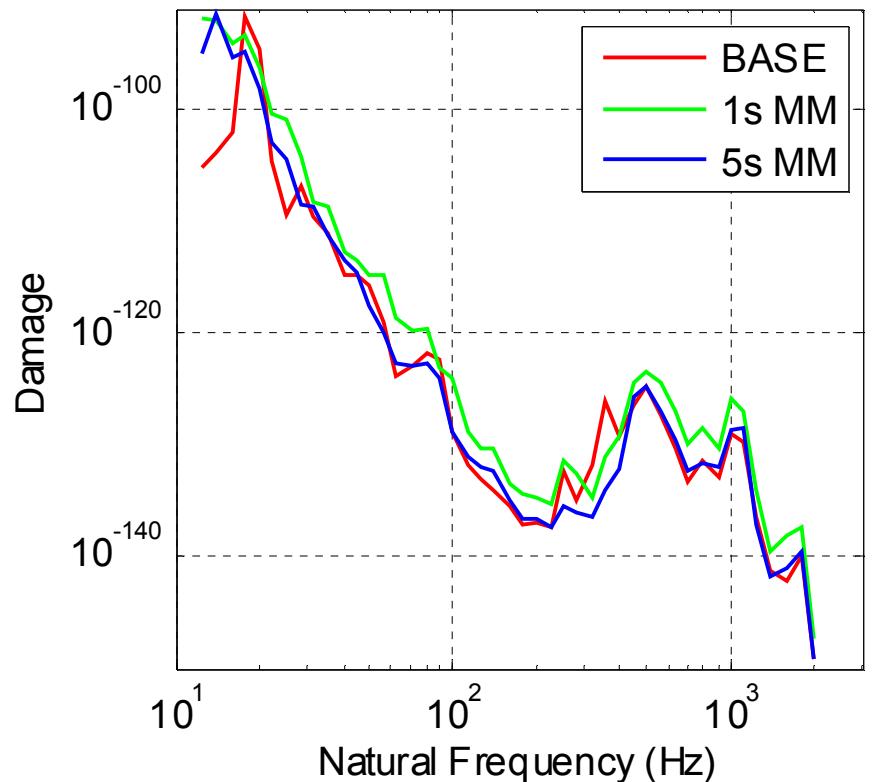
# Raw Fatigue Damage Spectra



$Q=10, A\eta^2=6.66$



$Q=50, A\eta^2=20$



# Normalized FDS

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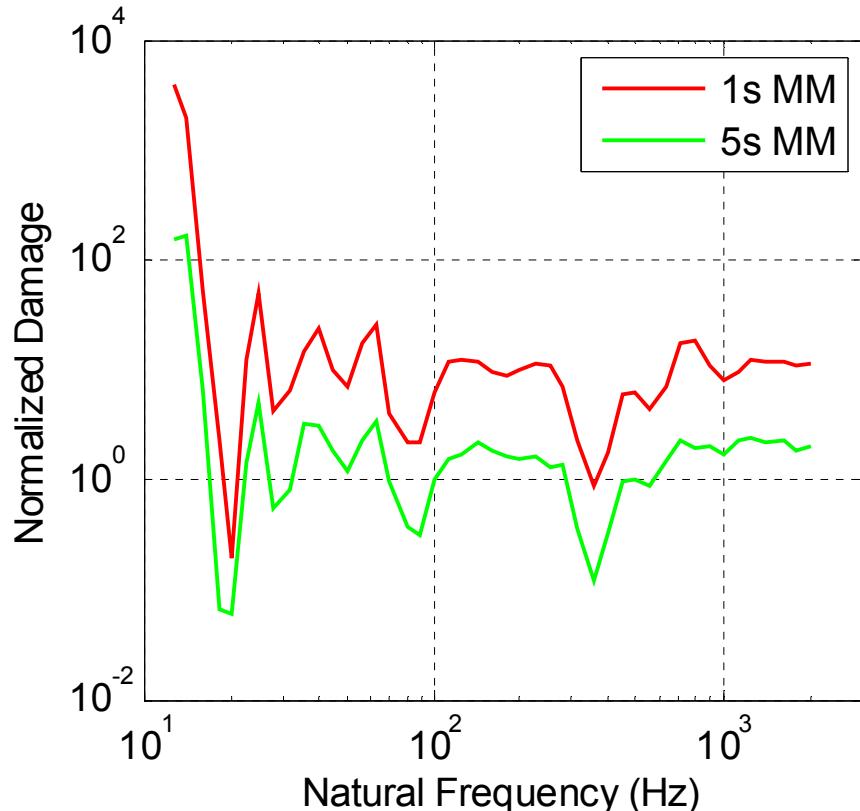
- Since the FDS span many decades on a log-log plot it is difficult to resolve subtle differences.
- Therefore, the FDS for the 1-second and 5-second Maxi-Max ASDs were divided by the baseline FDS for easier evaluation.
  - A value of  $< 1$  for the ratio denotes that the resulting Maxi-Max ASD is not conservative relative to the baseline FDS.



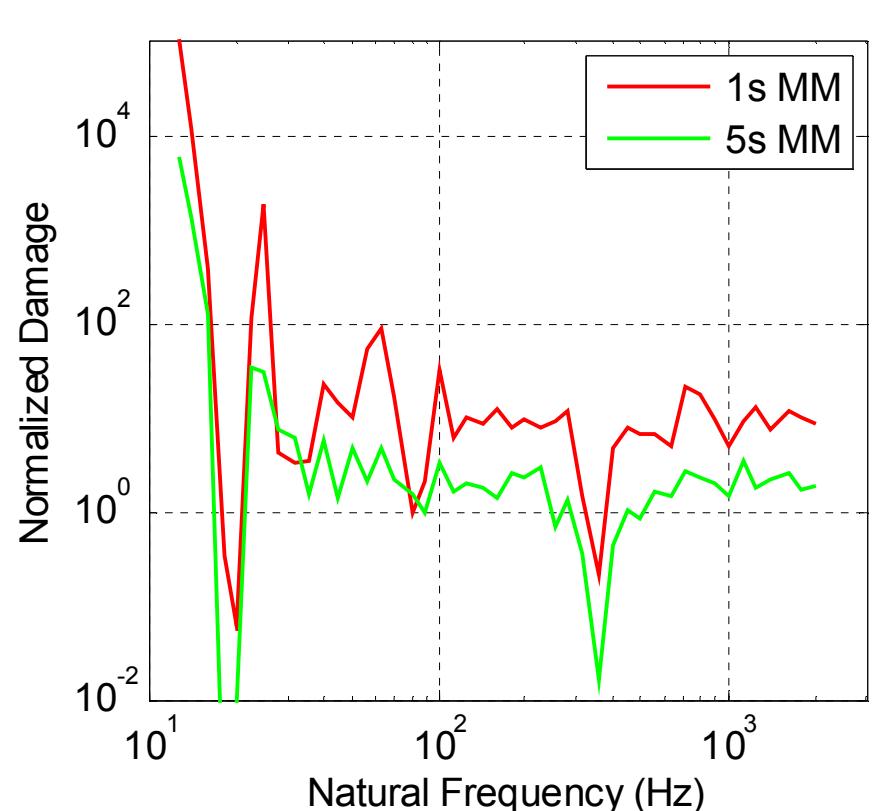
# Normalized FDS ( $A\eta^2=6.66$ )



$Q=10$



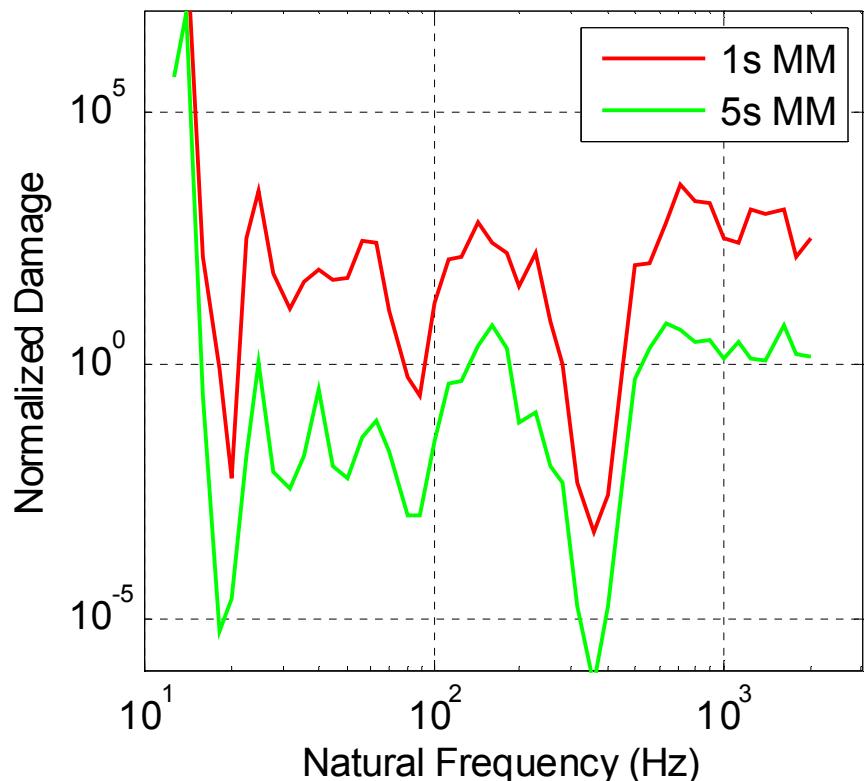
$Q=50$



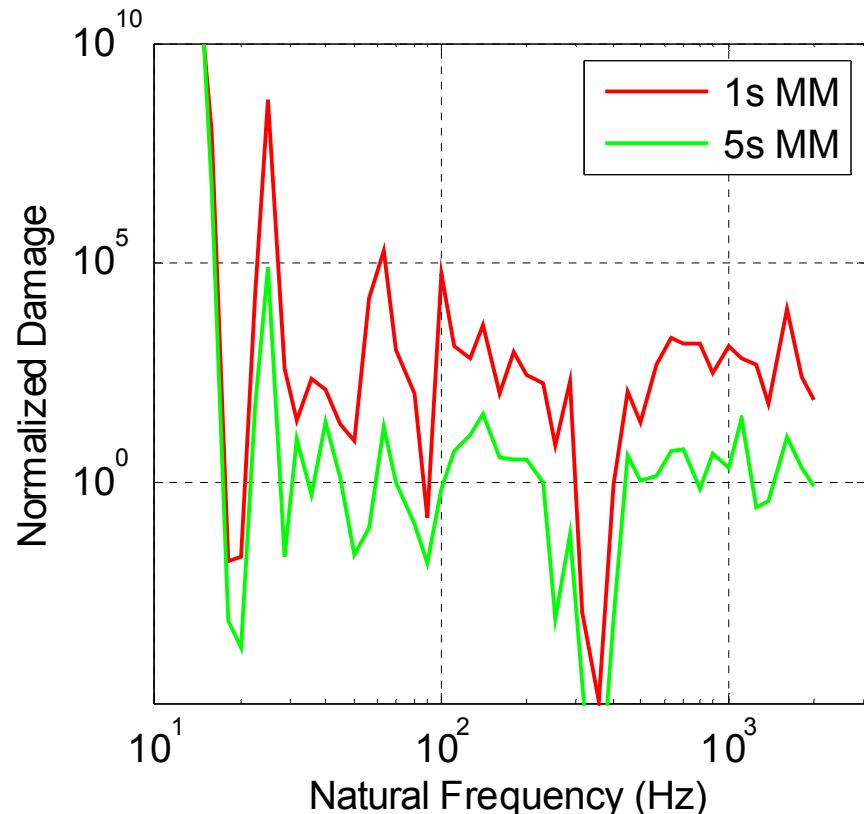
# Normalized FDS ( $A\eta^2=20$ )



$Q=10$



$Q=50$



# Summary



- The relative severity of the 1-second and 5-second Maxi-Max ASDs were exactly as expected.
- The normalized FDS for the 5-second ASDs were generally  $> 1$  for ductile materials, but less so for brittle materials.
- What was not expected was the fact that neither ASD was conservative for all frequencies.
  - The deep notch in the FDS between 300 and 400 Hz will require further study. This particular frequency range may contain non-Gaussian content.

# Future Work

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- It might be possible to define the Maximum Predicted Environment (MPE) using FDS and then tailor the ASD test specification to have an FDS with the desired conservatism for all frequencies and a reasonably wide range of material properties.
  - The MPE will be calculated in terms of the flight specific FDS.
  - We will need to formalize the procedures for converting back and forth from ASDs to FDS.



# References

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- [1] NASA-HDBK-7005, December 4, 2000; “Dynamic Environmental Criteria.”
- [2] ASTM Standard E 1049-85; “Standard Practice for Cycle Counting in Fatigue Analysis.”
- [3] R. G. Lambert, General Electric Company, Aircraft Equipment Division; “Criteria for Accelerated Random Vibration Tests with Non-Linear Damping.”