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TALE: An Arbitrary Lagrangian-Eulerian Approach to Fluid-Structure Interaction Problems[1]

P. Randall Schunk(*) and R. B. Secor(**)

(*) Multiphase Transport Processes Department
Sandia National Laboratories
Albuquerque, NM, 87185-0834

(**)3M Company
St. Paul, NM 55144

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Introduction

A Total Arbitrary Lagrangian Eulerian (TALE) technique is developed specifically for the finite element-based solution of coupled mechanics problems with fluid-solid interactions. Specifically, the method bridges non-material and arbitrary reference frames in solid and fluid regions with conforming finite element meshes. The formulation allows for substantial interaction of stresses between the solid and fluid materials, leading to potentially large solid deformations. One critical element of the development is a generalized kinematic boundary condition which helps unify the concepts underpinning ALE techniques so that they are applicable to all classes of materials (i.e., fluids and solids). Other features of the development include a complete mathematical description of a generalized fluid-structural interaction system, a theoretical foundation for the modeling of moving three-phase liquid/solid/gas contact lines on a deformable solid, and integration of material displacement fields and mesh displacement fields together with the proper transformations to account for changing/varying solid-stress reference state. The technique will be demonstrated with several simple coating-flow problems involving flexible webs and substrates and more challenging deformalble-roll coating problems.

Background and Approach

Of all the available methods of solving free and moving boundary problems in computational mechanics, the most accurate are those based on free boundary parameterization. In this class are discretization techniques which employ a connected mesh capable of adapting/moving with free boundaries. When constrained to conform to free and moving surfaces, a mesh provides a mathematical description endowed with a functional representation of the normal-tangent vector basis with sufficient smoothness to allow for application of curvature-dependent boundary conditions. Finite element and finite volume approaches to solving partial differential equations are in this class, but

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only if equipped with proper constraints or distinguishing conditions forcing mesh conformation to free or moving boundary shapes. The advent of so-called arbitrary Lagrangian Eulerian (ALE) formulations for free and moving boundary problems greatly extends the robustness and generality of mesh-based approaches. Even though this class of techniques has been the mainstay of coating flow analysis for decades (cf. Kistler et al. 1996 for references), there is still room for improvement. This paper addresses a significant extension of the classical ALE techniques to coupled problems characterized by fluid and structural mechanics dominating different but connected regions or materials.

ALE methods as applied to coating flow problems have been advanced for nearly three decades. Beginning with the spine method of Kistler and Scriven (1983), improvements in generality and robustness were realized with the elliptic grid generation approach of Christodoulou and Scriven (1988) and the pseudo-solid mesh motion approach of Sackinger et al. (1996). The main alternatives to ALE have been so-called Eulerian formulations, like level-set and volume-of-fluid techniques. These and all other methods have been deployed to various levels of success (see Christodoulou et al. 1997 for a review of the subject). In this work we build on the ALE concept with an advance that allows a general application to fluid/structural interaction problems. Before we present a description of the method, however, it is important to acknowledge the work of Madasu and Cairncross (2004). These authors advanced what amounts to the same algorithm as that described here but focused on an application of dynamic contact line motion on flexible substrates. Our work casts the algorithm in a general ALE framework and broadens its application to more complicated coating flow configurations.

The kinematics of moving structures and fluids in compatible reference frames forms the basis of our approach. Here we give a brief description; the details of the algorithm and formulation can be found in a report by Schunk (2000). The kinematic boundary condition, the mainstay of ALE techniques, is a statement of mass conservation and amounts to assuring that the mesh motion and material motion are compatible at the boundaries of the domain.. Conservation laws in fluids are most conveniently treated with Eulerian material variables, i.e., velocities. That is, the fluid material motion is tracked with its local velocity. These velocities are then referenced to the mesh velocity which serves as the reference frame on which all conservation laws (mass, momentum, etc.) are derived. At free boundaries of the fluid we apply a mass-balance relative to this frame, which can be stated mathematically as:

$$\underline{n} \bullet (\underline{v} - \underline{v}_s) = 0,$$

Here $\underline{v}$ is the fluid velocity with respect to a laboratory frame, and $\underline{v_s}$ is the mesh velocity measured in the same frame, and $\underline{n}$ is a vector normal to free surface. In essence, this condition states that the mesh surface and the material (or fluid surface) coincide. The condition is applied to the moving mesh equations (whether they be algebraic equations like spines, elliptic mesh generation equations, or equations of a nonlinear elastic "pseudo-solid" material) to "distinguish" the free surface motion; it is often called the "kinematic boundary condition".

For moving structures wetted by fluids (e.g. as occurs in deformable roll, blade and membrane coating), the mesh must move to accommodate the motion of both materials. In contrast to fluids, mechanics of solids are most conveniently treated with so-called "Lagrangian" variables, i.e., deformed positions or displacements. The most convenient reference state is the material position in a stress-free state, often denoted by the vector $\underline{X}$. A displacement is defined as the difference between the current material coordinates $\underline{x}$ and this material state $\underline{X}$, viz. $\underline{d} = \underline{x} - \underline{X}$. In a pure Lagrangian formulation, the deformed coordinates $\underline{x}$ are tracked as nodal values in a finite element mesh, and the material coordinates are simply the initial state of the mesh. The purpose of an ALE formulation is to alleviate the requirement that the mesh follows the motion of the material in the bulk. If we allowed these motions to be independent, as in the fluid case, then we must introduce an additional diplacement field for the solid material, viz. $\underline{d_m} = \underline{x_m} - \underline{X}$. Here the deformed mesh coordinates $\underline{x}$ and the deformed material coordinates $\underline{x_m}$ are allowed to be independent, except at the boundaries where we must satisfy mass conservation, or that the material boundaries and mesh boundaries coincide. To accommodate this, we derive the mathematical equivalent to the equation above for solid materials by integrating over "material-time":

$$n \cdot (\underline{d}_m - \underline{d}_m^0) - n \cdot (\underline{d} - \underline{d}^0) = 0$$

This condition is applied to all boundaries of solid materials in the domain. It too is a "kinematic boundary condition".

Sample Results and Discussion:

These concepts together with proper extensions of all relevant conservation laws and boundary conditions have been implemented in the code Goma 5.0 (Schunk et al. 2006). The motivation for the development of TALE came largly from the poor performance of Lagrangian solid/ALE fluid formulations in coating flows. Particularly challenging are those problems which require wetting line motion on deformable solid materials and drastically changing fluid-region shape due to squeeze-like metering, as in deformable roll or blade coating configurations. The TALE algorithm allows for arbitrary mesh motion in both fluids and solids, which in turn allows for large variations in fluid-bead position relative to a coating nip (as shown in the film-split example below) or for wetting line motion on a flexible solid, as demonstrated by Madasu and Cairncross (2004). Another possible way to circumvent the need for this arbitrary motion would be to disconnect the mesh at fluid/solid bounadries. In that case, one would need to solve the contact problem, which is feasible, but not in a fully-coupled framework. Coupled with remeshing and remapping techniques, TALE offers much rapid convergence with much greater accuracy.

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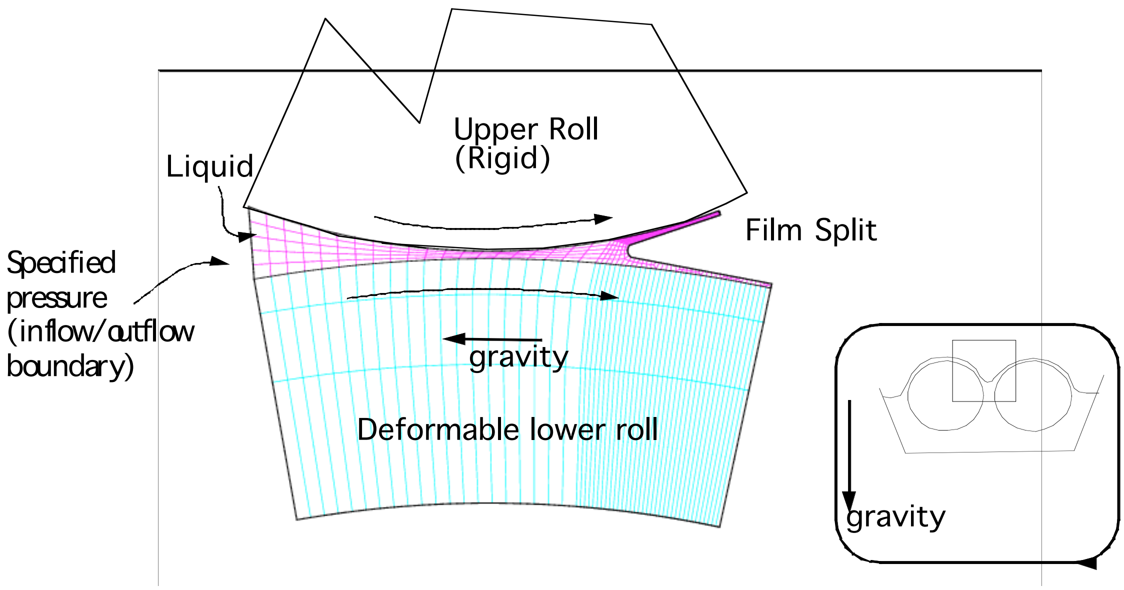
Upper Roll
(Rigid)
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Film Split
Specified
pressure
(inflow/outflow
boundary)
gravity
Deformable lower roll
gravity