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ROBUST, EXTENSIBLE REPRESENTATION OF COMPLEX EARTH MODELS FOR USE IN SEISMOLOGICAL SOFTWARE SYSTEMS

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ABSTRACT

Models of geophysically important properties of the Earth, such as seismic velocity, Q and density, can become large and complex when those properties vary in three dimensions within the model. We have developed a system to represent the distribution of seismic properties in the Earth that can accommodate a wide range of local to global scale 3D Earth models with spatially variable resolution. A 2D grid of nodes is tessellated using either triangles or quadrilaterals and a profile is defined at each 2D grid node that extends from the center of the Earth to the surface. The surface of the model corresponds with the topographic/bathymetric surface of the Earth, which is referenced to the surface of the GRS80 ellipsoid. Each profile can be separated into a number of layers defined by interfaces across which geophysical properties may be discontinuous. Within the layers between interfaces, the vertical distribution of geophysical properties may be defined by a number of continuous sublayers, by arbitrary order polynomials or by various types of splines. Layer thicknesses can vary laterally and zero thickness layers and layer pinch-outs are accommodated. The distribution of nodes is very flexible, allowing model resolution to vary over a wide range in 3 dimensions. In this paper, we present detailed descriptions of the software algorithms used to construct, store and interpolate these models.

OBJECTIVE

Complex models representing the three dimensional distribution of seismological properties within extensive areas of the Earth are becoming ever more common in seismological endeavors. Notable examples include the global Crust 2.0 model (Laske et. al., 2008), the broad regional WENA model (Western Eurasia and North Africa; Pasyanos et al. 2004), and WINPAK3D model (India Pakistan; Reiter et al, 2001). The prevalence of such models has come about in part as a result of the dramatic improvement in computational capabilities in recent years. Computer systems able to handle large (gigabyte), geographically extensive models that represent properties from deep in the Earth to its surface are now commonly available.

Unfortunately, the level of sophistication of model representation has not kept pace with the models themselves. Many models are still based on regularly spaced grids that lead to increasing densities of nodes towards the poles and deep within the Earth. These simplistic representations are employed not because they match the structure of the Earth, but because they are easy to implement and use. However, these representations are inherently inefficient and they are particularly ill-suited to global scale models. To develop global models that can be accessed and used quickly, a model representation is needed that is designed specifically to fit the real ellipsoidal, laterally and radially heterogeneous structure of the Earth. In this paper we describe an Earth model representation that enjoys global coverage in both radial and geographic dimensions, and variable resolution that reflects the state of knowledge of underlying parameters rather than ease of interpolation. As described in detail below, our representation involves a two dimensional tessellation of triangles on the surface of a unit sphere, with radial profiles of Earth properties defined at points on the sphere where three or more triangles intersect. We also describe the types of information that must be stored in order to completely represent the model, and methods for interpolating information from the model.

COORDINATE SYSTEM

Before we can describe the model representation, we first define the Earth-centered coordinate system used to define the positions of points relative to the Earth. Points are defined by a unit vector, $\mathbf{x} = \{ x_0, x_1, x_2 \}$, with its origin at the center of the Earth, and radius, r, measured in km from the center of the Earth. As illustrated in Figure 1, this coordinate system is oriented such that x_0 points from the center of the Earth towards the point on the surface with latitude and longitude 0°, 0°; x_1 points toward latitude, longitude 0°, 90° and x_2 points toward the north pole.

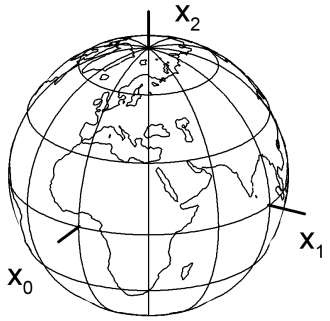

Figure 1 – Earth centered Cartesian coordinate system.

EARTH MODEL REPRESENTATION

The Earth model representation we describe here consists of a 2 dimensional, multi-level tessellation of triangles which completely covers the surface of a unit sphere without gaps or overlaps. The highest level of the tessellation consists of large triangles which are subdivided into ever smaller triangles at lower levels of the tessellation. At the vertices of the triangles, which all reside at the surface of the unit sphere, Earth properties are defined along radial profiles that extend from the center of the Earth to its surface. We first describe how the tessellation is constructed, and then how to find an estimate of the value of an Earth property at an arbitrary point in the Earth by interpolating values stored in the model.

Definitions

To facilitate description and manipulation of the tessellation, the following terms are defined:

- If vertex V is a corner of triangle T, then V is a *member* of T and T *contains* V.
- Given vertex V_i that is a member of triangle T, then V_j is the *next* vertex in T if one arrives at V_j by traversing the edge of T that leaves V_i in a clockwise direction as viewed from outside the unit sphere.

- Triangle T has *edges*, E_i, i=1,3, where E_i connects vertices V_j and V_k, where V_j is the next vertex in T after V_i and V_k is the next vertex in T after V_j. Note that given a vertex V_i, E_i is the edge of T that does not contain V_i.
- Triangle N_i is the i'th *neighbor* of triangle T if N_i is the triangle on the other side of edge E_i.

Each vertex should maintain a *triangle membership list*, i.e., a list of the triangles of which it is a member.

Each triangle T should maintain the following information:

1. references to the 3 vertices which are members of T
2. references to its 3 neighbors
3. references to three new vertices. These should default to null, but may be set to instantiated vertices if T is subdivided, as will be described shortly.
4. a reference to a descendent triangle, if there are any. If T is subdivided, as described later, the reference can refer to any of the triangles that are descendents of T.
5. a Boolean flag indicating whether or not T is 'marked'. All triangles start out 'unmarked'.

Tessellation Construction

Construction of each level in the tessellation involves

1. identifying the vertices involved in the given level of the tessellation,
2. connecting the vertices together to define triangles that completely span the surface of the unit sphere,
3. for each vertex, populating its triangle membership list,
4. for each triangle, establishing which triangles are its neighbors.

The first level of the tessellation consists of a regular icosahedron, which is a convex regular polyhedron composed of twenty congruent, equilateral triangles that meet at twelve vertices (Figure 2). Using the coordinate system defined earlier, the coordinates of the twelve vertices of a regular icosahedron are given in Table 1.

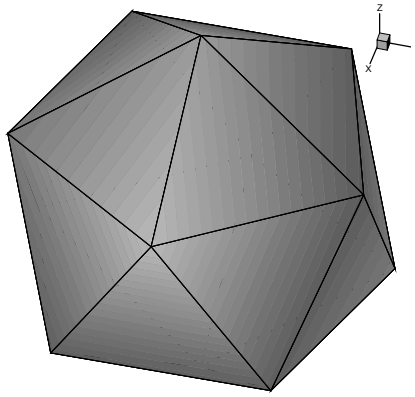

Figure 2 – A regular icosahedron consisting of 12 vertices and 20 congruent equilateral triangles.

Table 1 – Vertices of a regular icosahedron.

Vertex	v_0	v_1	v_2
0	0.000000000000000	0.000000000000000	1.000000000000000
1	0.894427190999916	-0.000000000000000	0.447213595499958
2	0.276393202250021	0.850650808352040	0.447213595499958
3	-0.723606797749979	0.525731112119134	0.447213595499958
4	-0.723606797749979	-0.525731112119134	0.447213595499958
5	0.276393202250021	-0.850650808352040	0.447213595499958
6	0.723606797749979	-0.525731112119134	-0.447213595499958
7	0.723606797749979	0.525731112119134	-0.447213595499958
8	-0.276393202250021	0.850650808352040	-0.447213595499958
9	-0.894427190999916	0.000000000000000	-0.447213595499958
10	-0.276393202250021	-0.850650808352040	-0.447213595499958
11	0.000000000000000	0.000000000000000	-1.000000000000000

These vertices are connected together to form 20 congruent, equilateral triangles as shown in Figure 3. The 20 triangles are numbered from 0 to 19 and the order in which the vertices are connected together is given in Table 2.

Table 2 – Connectivity of initial tessellation.

Triangle	Vertex 0	Vertex 1	Vertex 2
0	0	2	1
1	0	3	2
2	0	4	3
3	0	5	4
4	0	1	5
5	2	7	1
6	3	8	2
7	4	9	3
8	5	10	4
9	1	6	5
10	1	7	6
11	2	8	7
12	3	9	8
13	4	10	9
14	5	6	10
15	7	11	6
16	8	11	7
17	9	11	8
18	10	11	9
19	6	11	10

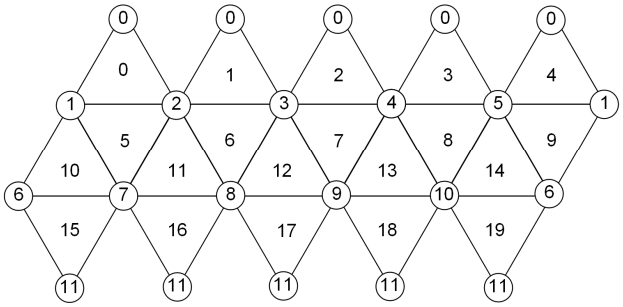

Figure 3 – Icosahedral net illustrating one possible node and triangle numbering scheme.

As each triangle listed in Table 2 is instantiated, a reference to the triangle is added to each of its vertex's triangle membership list.

Triangle Neighbor Identification

Once a set of vertices have been combined into a set of triangles at a given level of the tessellation, and the vertices have updated their triangle membership lists, it is necessary to discover, for each triangle T at the current level of the tessellation, references to the triangles N_i, i=1,3, that are the neighbors of T. This is accomplished by performing the following operations for each vertex V_i in T (see Figure 4):

1. Identify the other two vertices in T, V_j and V_k, such that V_j is the next vertex after V_i and V_k is the next vertex after V_j.
2. Mark each triangle in V_j's triangle membership list that resides at the same level of the tessellation as T.
3. Search through the triangles in V_k's triangle membership list. Exactly two triangles will be marked: the original triangle T, and the neighbor of T that lies across the edge E_i, thereby allowing N_i to be identified.
4. Unmark all the triangles which were marked in step 2.

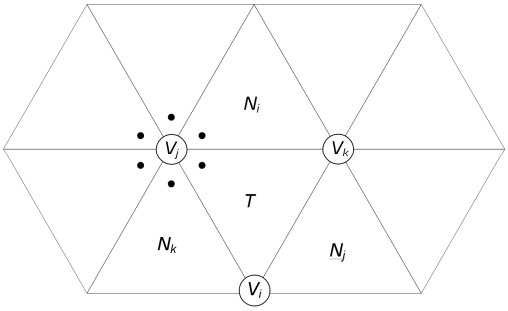

Figure 4 – Algorithm for identifying triangle T's i'th neighbor, N_i. Triangles containing a black dot are triangles that have bee marked.

Descendent Level Construction

From a given level of the tessellation, a lower level can be constructed by subdividing some or all of the triangles of the given level into smaller triangles. Variable resolution is achieved by only subdividing a subset of the triangles at the higher level of the tessellation. Construction of a new, lower level from a given higher level is a 3 step process.

Step 1

Visit each triangle T at the lowest level of the tessellation. If it is determined that T should be subdivided, then visit each edge E_i, of T. If there is not already a new vertex positioned in the middle of E_i, then

1. instantiate a new vertex, V_n, in the middle of E_i,
2. add a reference to V_n to T's list of new vertices

3. add a reference to V_n to T's i'th neighbor, N_i's list of new vertices.

After all the triangles at the current level of the tessellation have been visited, each triangle will have had 0, 1, 2 or 3 new vertices added to its list of new vertices.

Step 2

Each triangle T at the current level of the tessellation is visited a second time and the number of new vertices added to its edges is identified. If 3 new vertices were added, then T is divided into 4 new triangles, as illustrated in Figure 5a. If 2 new vertices were added, then there are two distinct ways that T can be divided into 3 new triangles, as illustrated in Figure 5c and 5d. The choice can be made arbitrarily. If a single new vertex was added, then T is divided into 2 new triangles as illustrated in Figure 5b. If no new vertices were added to T, then a new triangle is instantiated that has the same three vertices as T. It is necessary to create this new triangle, even though it has the same vertices as T, because the new triangle will ultimately reside on a different level of the tessellation and have different neighbors than T.

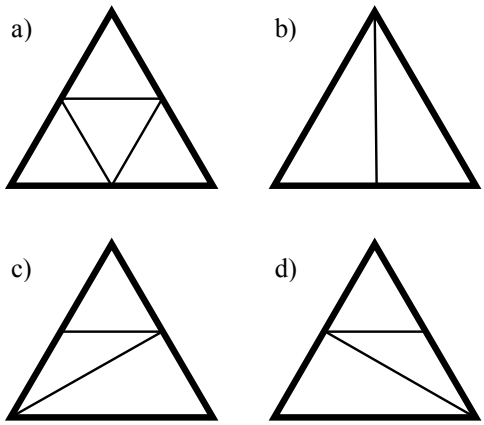

Figure 5 – An illustration of some of the ways in which a triangle can be subdivided into smaller triangles. a) 3 new vertices, b) 1 new vertex, c and d) 2 new vertices.

One of the newly created triangles must be identified as the descendent of T. If T was divided into 3 or 4 new triangles, then the center one should become T's descendent. Otherwise there is either no choice to make, or the choice is arbitrary.

Once all the new triangles have been created and each old triangle's descendent identified, then all the new triangles are added to a new level of the tessellation. The third and final step of the creation of the new level of the tessellation is to determine each triangle's three neighbors as described above.

PROPERTY PROFILES AT VERTICES

At each vertex of the tessellation, a radial property profile is defined which is comprised of some number of major layers. All property values within a major layer must be radially continuous but property values may be discontinuous across major layer boundaries. Examples of major property discontinuities within the Earth are sedimentary layer boundaries in the upper crust; boundaries between the upper, middle and lower crust; the Moho, the 410 km discontinuity, the 660 km discontinuity, the core-mantle boundary (CMB) and the inner core boundary (ICB). The radial property distribution within a major layer can be constant, or it can vary. Currently, variations can be defined by a number of sublayers, by a polynomial in radius, or by a cubic spline. Support for other distributions could easily be added.

Property values within each layer, and the radii of the layer boundaries, may be different along profiles at adjacent tessellation vertices, and layers may pinch out to zero thickness. Since linear interpolation between adjacent profiles is employed, as described next, property values and interface radii will be laterally continuous.

MODEL INTERPOLATION

Determining estimates of the property values and layer radii at an arbitrary position X in the Earth consists of the following steps:

1. finding the triangle T in the tessellation such that X resides in T and T has no descendents
2. determining the interpolation coefficients in geographic dimensions

3. computing the major layer radii along an interpolated profile at the geographic position of X using the interpolation coefficients computed in step 2
4. identifying the layer in the interpolated profile in which the radius of X resides
5. for each vertex V_i of T, interpolating the value of the desired property at the radius of X, but constrained to be in the layer identified in step 4
6. computing the interpolated property values at X from the three property values calculated in step 5.

These steps are now described in more detail.

Triangle Walking Algorithm

To find the triangle in which X resides, a triangle walking algorithm is implemented (Lawson, 1977). This method is the standard approach of point searching a well behaved, convex-everywhere, 2D mesh. We initialize T to be an arbitrary triangle at the highest level of the tessellation. For each edge E_i in T we compute the scalar triple product $s_i = (V_k \times V_j) \bullet X$, where V_j and V_k are the first and second vertices of E_i, respectively. X resides in T if $s_i \geq 0$ for $i = 1,3$. If s_i is negative for any E_i in T, then T is set equal to the neighbor that resides on the other side of edge E_i and the search continues. When T is identified such that $s_i \geq 0$ for $i = 1,3$, T is checked to see if it has a descendent. If it does, then T is set equal to the descendent and the search continues. The search ends when X resides in T and T has no descendents.

The interpolation coefficients to be applied to the three vertices V_i, i=1,3 are

$$c_i = \frac{s_i}{S}, \quad i = 1,3 \tag{11}$$

where

$$S = \sum_{j=1}^{3} s_j \tag{12}$$

Next, we identify the 3 property profiles P_i at vertices V_i, and instantiate an interpolated profile, I, at the geographic position of point X. Then we compute interpolated values for the radii of all the major layer boundaries in I using

$$r_j = \sum_{i=1}^{3} c_i r_{ij} \tag{13}$$

where r_j and r_{ij} are the radii of the j'th major layer boundaries in I and P_i, respectively. We then identify m, the index of the major layer in I in which the radius of X resides using bisection. Next, we interpolate property values v_i at the radius of X in profiles P_i using whatever interpolation algorithms are appropriate for the property distributions in layer m of profiles P_i. The interpolations should be constrained to layer m, i.e., if the radius of X is greater than the radius of the top of layer m in P_i, or less than the radius of the bottom of layer m in P_i, then v_i is set to the value of the desired property at the top or bottom or layer m in P_i, as appropriate. Finally, we compute the interpolated value of the property at X

$$v_X = \sum_{i=1}^{3} c_i v_i \tag{14}$$

EXAMPLE MODEL

To assess some of the advantages and disadvantages of the unstructured tessellations described here relative to more commonly encountered structured grids, we have constructed an unstructured version of the Crust 2.0 Model of Laske, et. al. (2008), which is represented with uniform, 2º latitude and longitude spacing. We built the unstructured tessellation in the manner described above, with the criterion that triangles were only subdivided if they:

1. contained at least two of the original nodes from the Crust 2.0 Model, and
2. the geophysical parameter values at any two of the original nodes were different, and
3. the triangle edges were larger than 2º.

Our triangles are no smaller than 2º and at low latitudes they either contain no original nodes, or all of the original nodes that they do contain are identical to each other. At high latitudes, where the Crust 2.0 model is very highly sampled, many triangles do contain original nodes that are not identical in order to avoid violating the 2º minimum triangle size constraint. Figure 6 shows an orthographic projection of the globe, centered on the boundary between the Pacific Ocean and South America, illustrating the economies achieved over the oceans where the crustal structure is constant over broad areas.

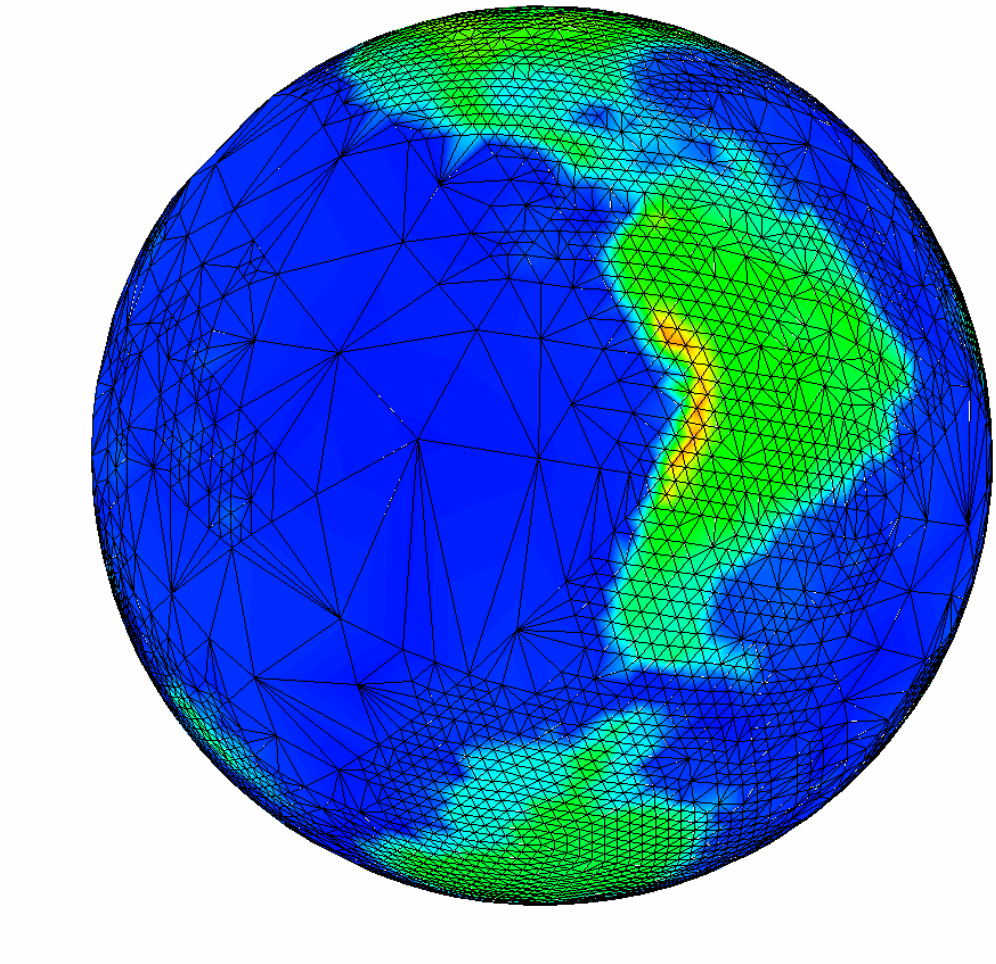

Figure 6 – Orthographic projection of crustal thickness in the Eastern Pacific and South America.

Figure 7 is an azimuthal equal angle projection of the area near the pole, illustrating that the triangular cells maintain approximately constant area, even at high latitudes, while the cells of the regularly spaced, 2º × 2º grid are characterized by very high aspect ratios.

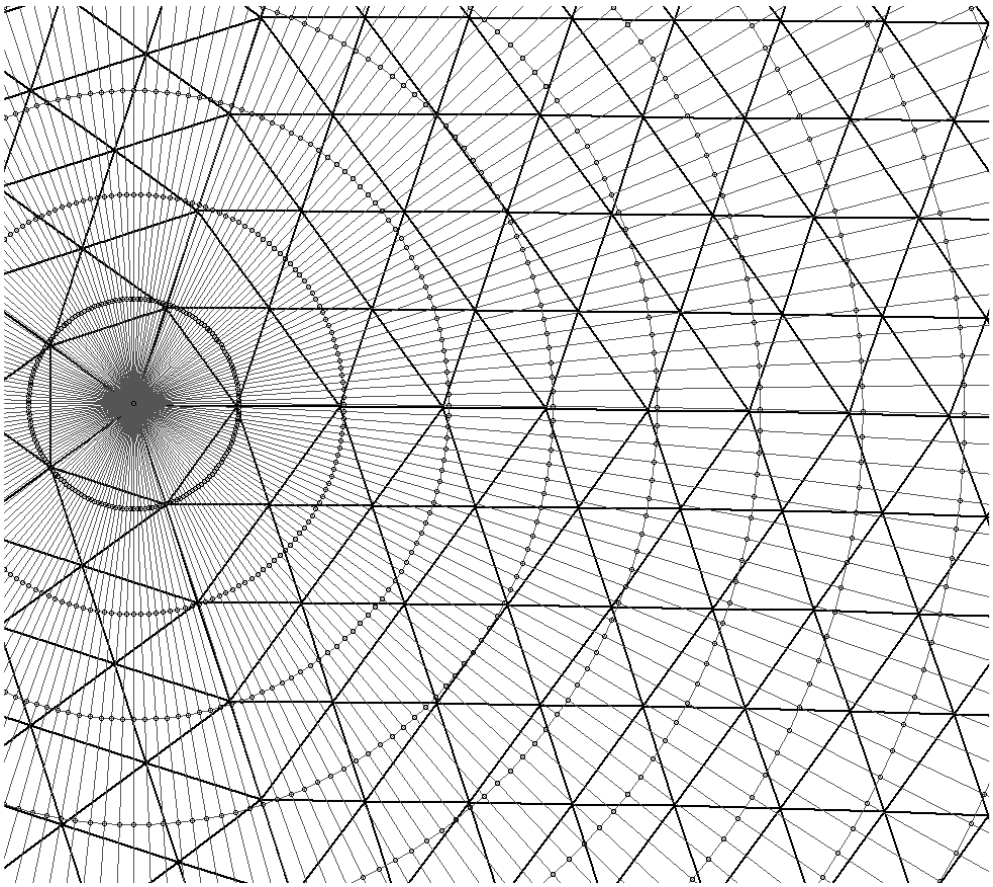

Figure 7 – Comparison of the unstructured triangular tessellation and the regularly spaced 2º × 2º original grid at high latitude. The North Pole is near the left edge of the image

CONCLUSION

We have described a method for representing complex 3D Earth models based on a multi-level triangular tessellation of a unit sphere. At each triangle vertex, geophysical properties are defined along a radial profile that extends from the center of the Earth, through the triangle vertex and out to the topographic/bathymetric surface of the Earth. Construction of the tessellation starts with an icosahedron and proceeds by triangular subdivision. Variable resolution is accomplished by subdividing triangles into smaller triangles only where the distribution of the geophysical properties represented in the model warrants additional resolution. Unlike structured, latitude-longitude grids, resolution need not increase arbitrarily at high latitudes, making this method of model construction particularly suitable for global scale models.

Efficient interpolation of geophysical properties at arbitrary points in the model is accomplished by implementation of a walking triangle algorithm which starts at the highest level of the multi-level tessellation and proceeds down through successively deeper levels until the desired position is reached.

REFERENCES

Laske, G., G. Masters and C. Reif, CRUST 2.0: A New Global Crustal Model at 2x2 Degrees, http://igppweb.ucsd.edu/~gabi/rem.dir/crust/crust2.html

Lawson, C.L., 1977. Software for C^1 surface interpolation, in Mathematical Software III, ed. Rice, J., Academic Press, New York.

Pasyanos, M. E., W. R. Walter, M. P. Flanagan, P. Goldstein and J. Bhattacharyya (2004), Building and testing an a priori geophysical model for western Eurasia and North Africa, Pure and Applied Geophysics, 161, 235-281.

Reiter, D., W. Rodi, M. Johnson, C. Vincent, and A. Rosca (2001). A new regional 3-D velocity model of the India-Pakisan region, Proc. 23rd Seism. Res. Rev., 305-314.