

Magnetic-pulse-driven Rayleigh-Taylor instability in plastically deforming metals

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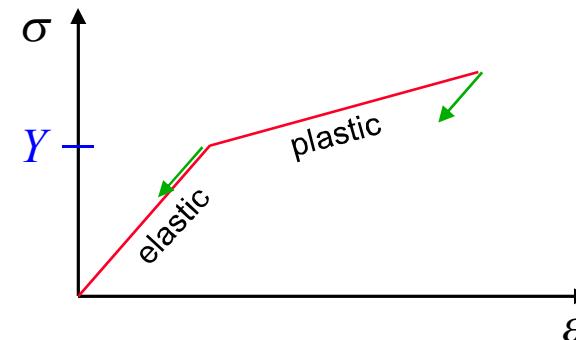
Outline

1. Rayleigh-Taylor instability in materials with strength
2. Experimental schemes for measuring strength via RT growth
3. Finite-element MHD simulations (ALEGRA)
4. Constraints and approach for experiment design using simulations
5. Parameter studies for RT growth in realizable samples
 - Aluminum-gold
 - Aluminum-copper

1. RT instability with strength: definition of and model for material strength.

Strength is the ability of a material to sustain shear stresses

Characterized by the yield stress :
stress at which deformation becomes irreversible.



A material's yield stress varies with conditions:

$$Y = f(p, T, \varepsilon, \dot{\varepsilon}, S, \dots)$$

Steinberg and Guinan* have produced a reliable model for yield stress:

$$Y_{SG} = Y_0 [1 + \beta (\varepsilon_I + \varepsilon)]^n \left[1 + A \frac{p}{\eta^{1/3}} - B (T - 300) \right]$$

Tabulated: Y_0 β ε_I n A B

1. RT instability with strength:

material strength slows RT growth in plastic flow.

Colvin et al. (2003)* consider plastic flow of an isotropic solid: analogous to viscous fluid flow, with

$$\nu_{\text{eff}} = \frac{Y}{\sqrt{6\rho} |\dot{\varepsilon}|} \quad \text{“Effective lattice viscosity”}$$

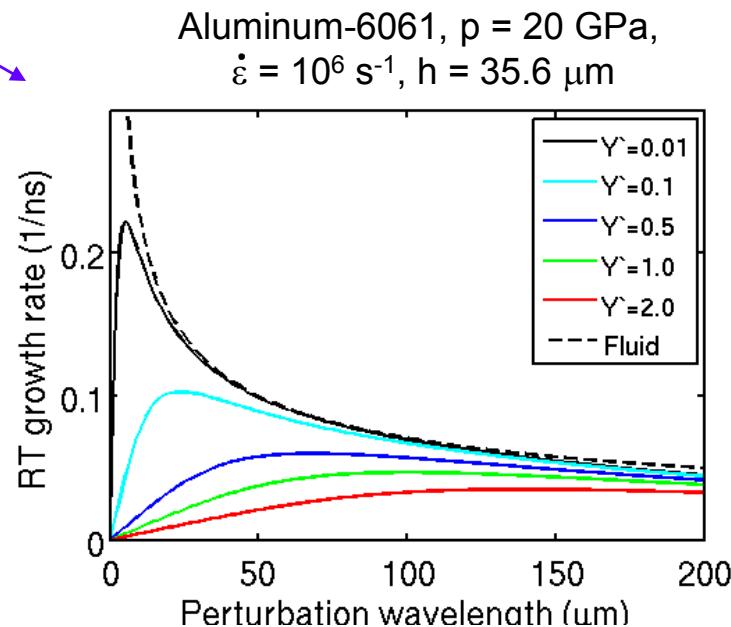
An analysis similar to Mikaelian's** produces a dispersion relation, yielding:

$$\gamma = \nu_{\text{eff}} k^2 \left(\sqrt{1 - \frac{Ag}{\nu_{\text{eff}}^2 k^3} \tanh(kh)} - 1 \right)$$

$$\eta(t) = \eta(0) e^{\gamma t}$$

Increased Y leads to decreased γ and altered sensitivity to k .

→ Basis for strength measurement technique that does not rely on shock loading: $\gamma \leftrightarrow Y$



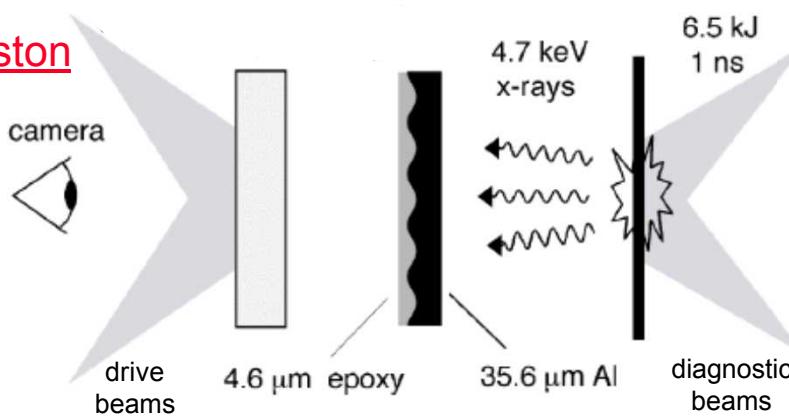
2. Experimental schemes for strength via RT: laser-driven experiments

Lorenz et al. (2005)* measured strength enhancement in Al-6061 at 20 GPa using the OMEGA laser and x-ray backlighting.

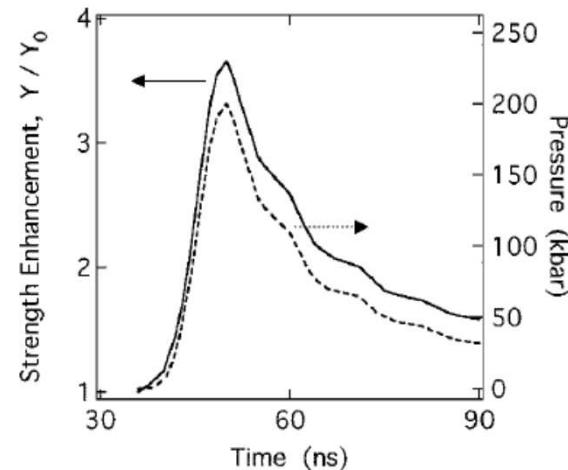
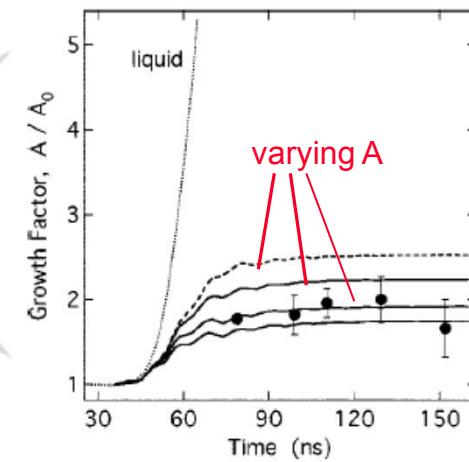
OMEGA/ plasma piston

$$\begin{aligned} p_{\max} &= 20 \text{ GPa} \\ \dot{\varepsilon}_{\max} &= 1 \times 10^6 \text{ s}^{-1} \\ \delta t &\approx 30 \text{ ns} \end{aligned}$$

$$\begin{aligned} \lambda &= 40 \mu\text{m} \\ \eta_0 &= 1.7 \mu\text{m} \end{aligned}$$



- Ramp compression wave created by “plasma piston”
- RT growth measured using variations in optical depth
- Simulations: variable strength hardening parameter, A
- 38% increase in strength at high pressure relative to nominal tabulated data



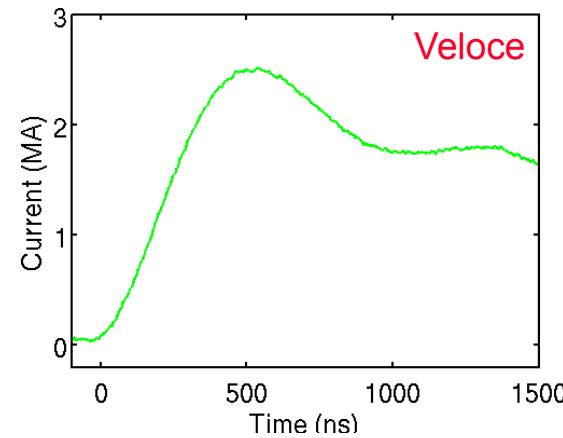
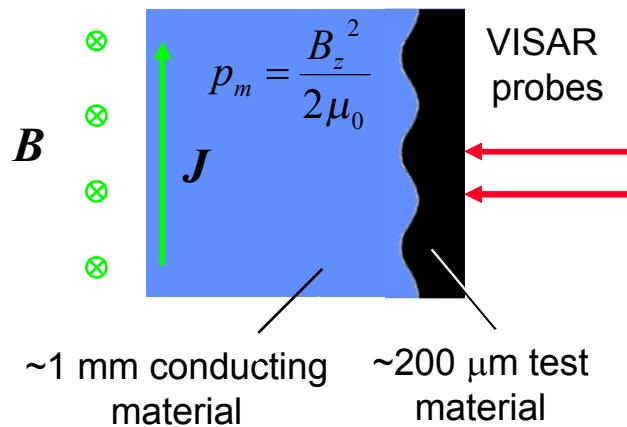
2. Experimental schemes for strength via RT: magnetically driven experiment

Experiments are planned for pulsed-power facilities at Sandia using strong currents rather than laser energy deposition as a driver.

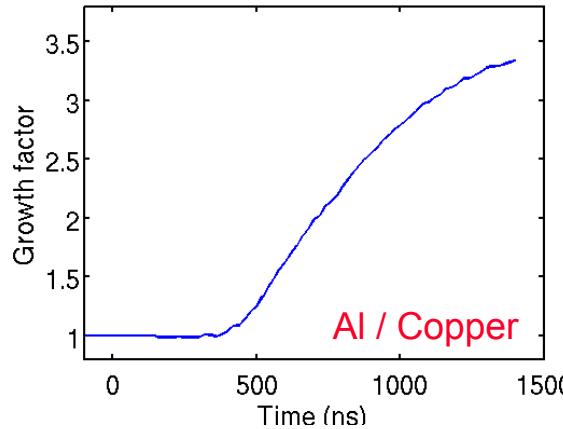
Veloce device*

$I_{max} = 2.5 \text{ MA}$
 $p_{max} = 10 \text{ GPa}$
 $\varepsilon_{max} = 5 \times 10^5 \text{ s}^{-1}$
 $\delta t \approx 2 \mu\text{s}$

$\lambda \approx 500 \mu\text{m}$
 $\eta_0 \approx 100 \mu\text{m}$



- Ramp compression wave from magnetic pressure
- RT growth measured at free surface by optical velocimetry (VISAR)
- Simulations: variable S-G parameters A , n , β
- Start with feasibility-study design for Veloce



3. Finite-element MHD simulations: purpose and scope of the present study

Mature simulation capability for magnetically-accelerated media exists at Sandia: **ALEGRA** (“ALE General Research Application”)

Purpose of the present study: use simulations in ALEGRA to design a “pilot” experiment on Veloce:

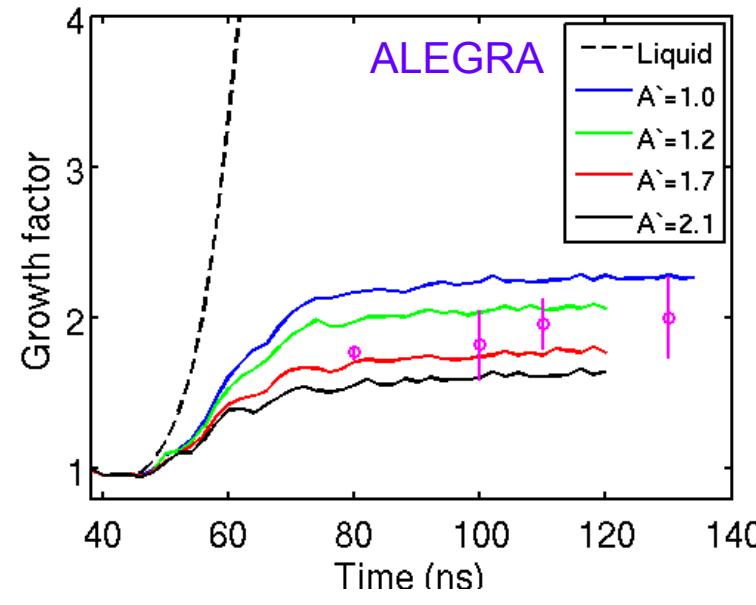
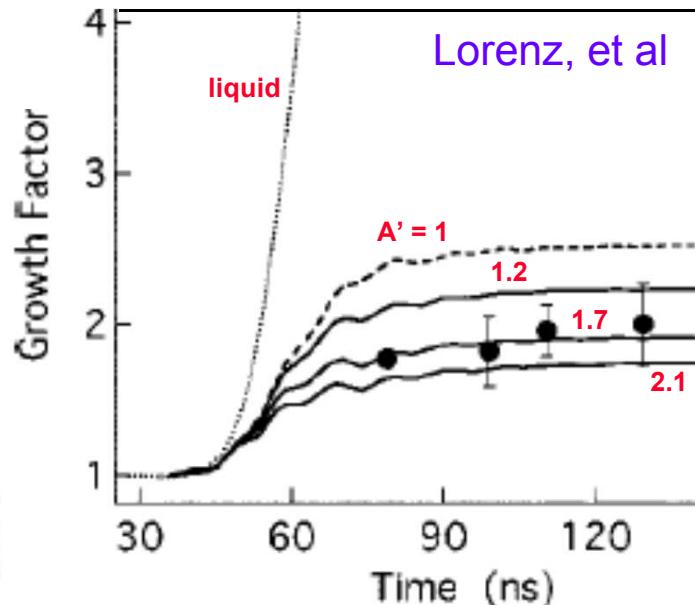
- Determine what RT growth rates are possible
- Identify ideal candidate “test” materials
- Characterize sensitivity of RT growth to strength properties
- Propose analytical techniques
- Address potential issues: release wave, realizable geometries, interface/free-surface coupling
- Verify applicability of Colvin model for this experiment

3. Finite-element MHD simulations: code and results for Lorenz problem

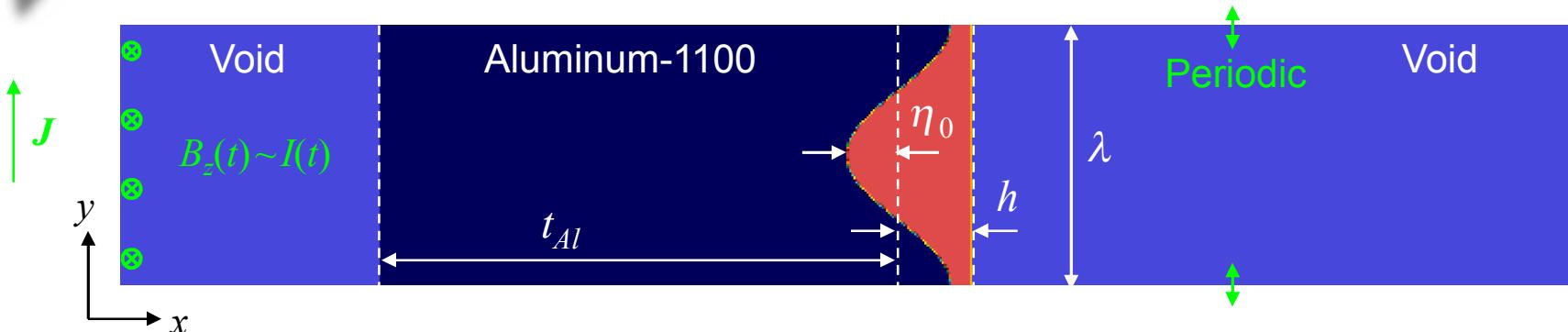
ALEGRA is a arbitrary Lagrangian-Eulerian multimaterial, multiphysics finite-element code developed at SNL since 1990:

- Operator-split: magnetic diffusion, Lagrangian step, remap
- Modern artificial viscosity, interface tracking, remeshing, and solvers
- Constitutive models for wide array of physics (EOS, strength, conductivity, fracture, etc.)
- Constrained transport to enforce divergence-free B field.
- Extensively validated using magnetically-accelerated flyer-plate experiments

ALEGRA simulations can reproduce the results of the laser-driven experiments of Lorenz et al, to within experimental error ranges:



3. Finite-element MHD simulations: problem setup



Simulation parameters

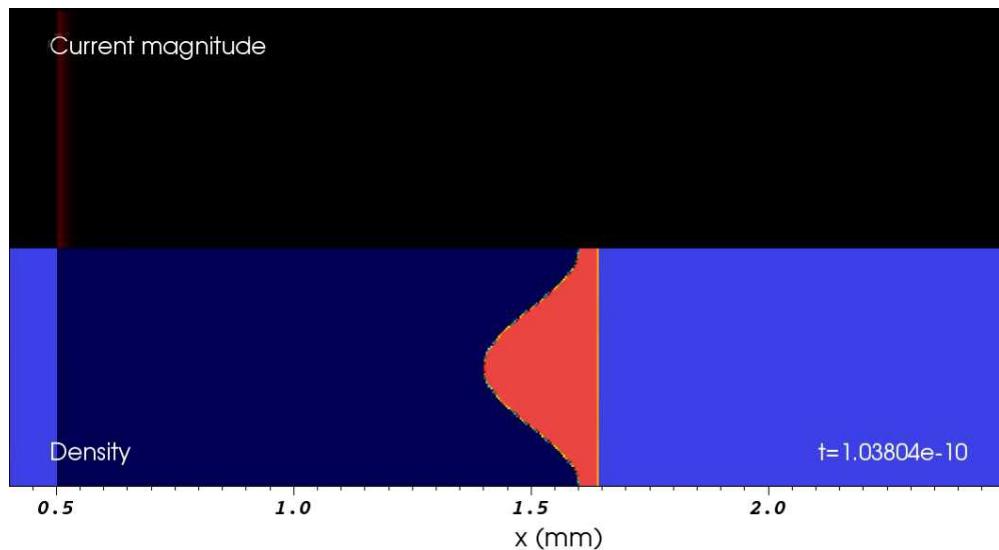
- 2D Eulerian, uniform Cartesian mesh
- Time-dependent B -field BC at left
- Elastic-, stress-wave propagation direction: $+x$
- Periodic boundaries in y
- Material models: LANL Sesame EOS, LMD conductivities, Steinberg-Guinan strength
- Test materials: V, Cu, W, Au, Pb

Steinberg-Guinan model is modified artificially to find yield-stress dependence of RT growth:

$$Y_{SG} = Y_0 [1 + \beta (\varepsilon_I + \varepsilon)]^n \left[1 + A \frac{p}{\eta^{1/3}} - B (T - 300) \right]$$

3. Finite-element MHD simulations: sample results

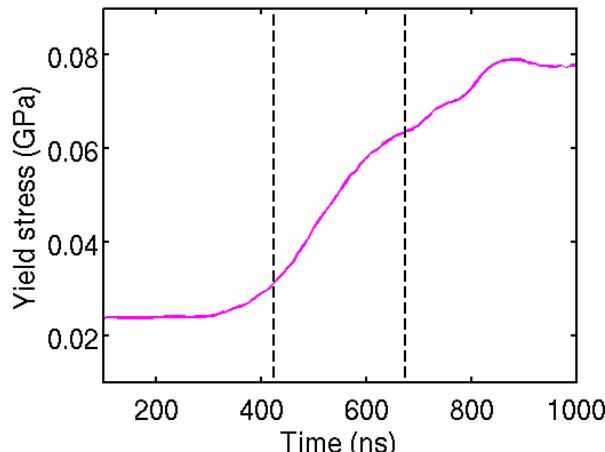
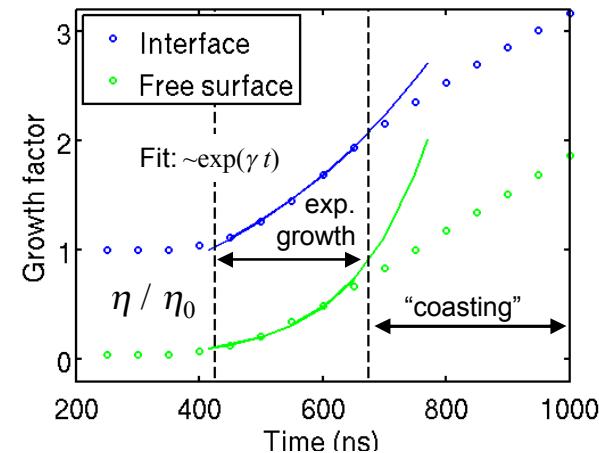
ALEGRA results for Al-1100 / gold sample:
(nominal strength parameters)



Growth rates:

- Colvin theory: $\gamma = 2.85 \mu\text{s}^{-1}$
(Max p , ambient Y)
- Al / Au interface: $\gamma = 2.80 \mu\text{s}^{-1}$
- Free surface: $\gamma = 8.43 \mu\text{s}^{-1}$

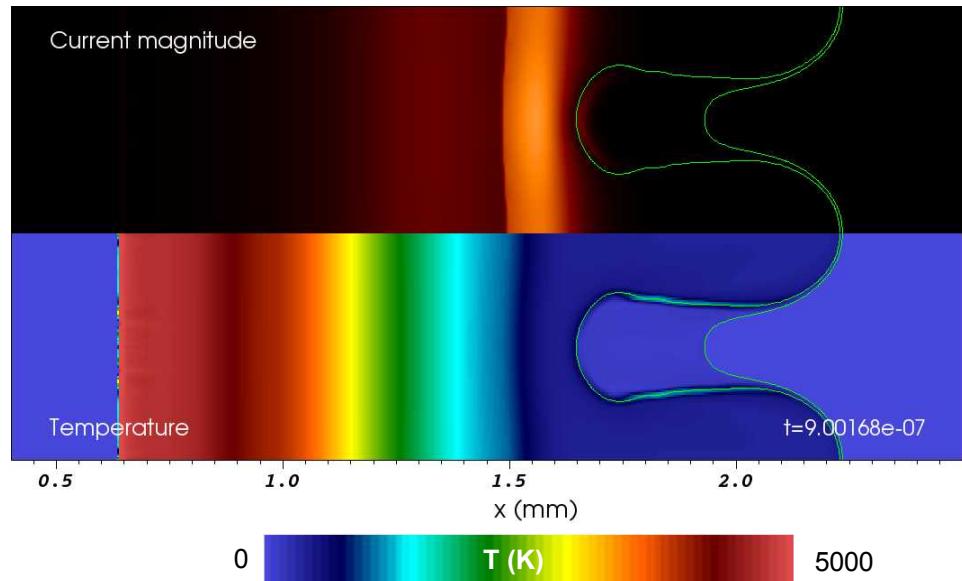
- $\lambda = 500 \mu\text{m}$ perturbation wavelength
- $h = 140 \mu\text{m}$ test material thickness
- $\eta_0 = 100 \mu\text{m}$ perturbation amplitude



4. Issues and constraints in experiment design: magnetic diffusion and interface growth speeds

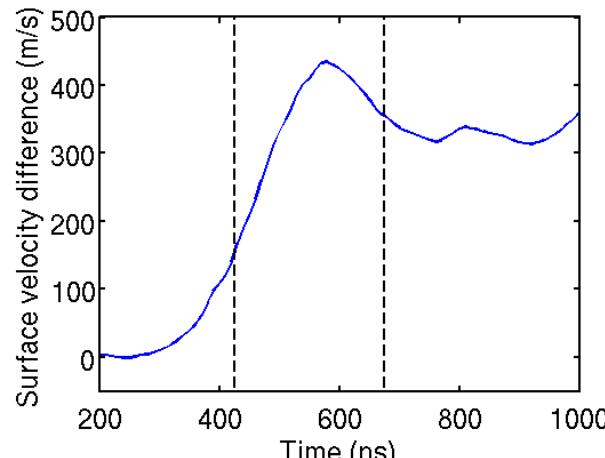
Heating associated with magnetic diffusion:

- Diffusion front propagates much more slowly than stress wave, due to high conductivity of aluminum.
- Does not reach interface until long after RT growth has occurred.



Peak-trough velocity difference at free surface:

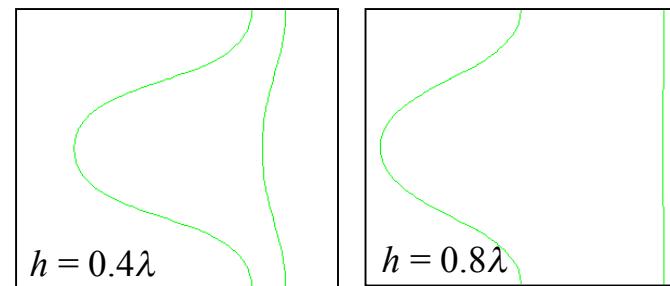
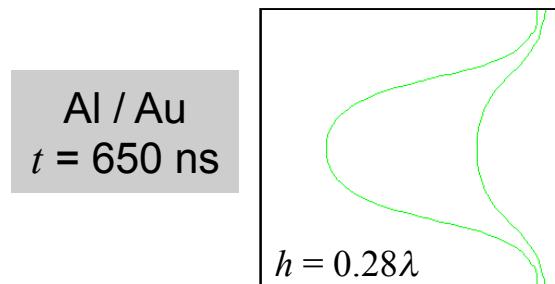
- Easily measurable using VISAR.



4. Issues and constraints in experiment design: interface-surface coupling and high pressure

Coupling of interface growth to free surface:

- If sample material is too thick, inertia will suppress RT growth at free surface.
- Thickness constraint: $h < \lambda / 2$

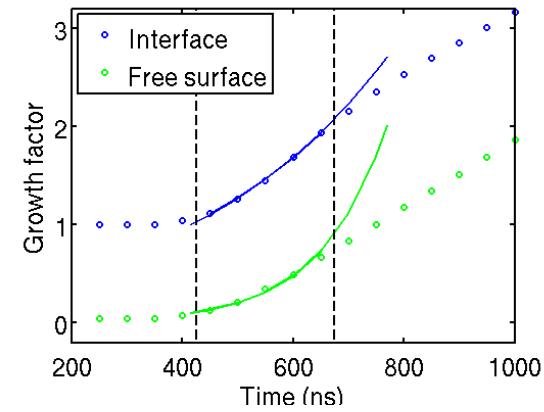
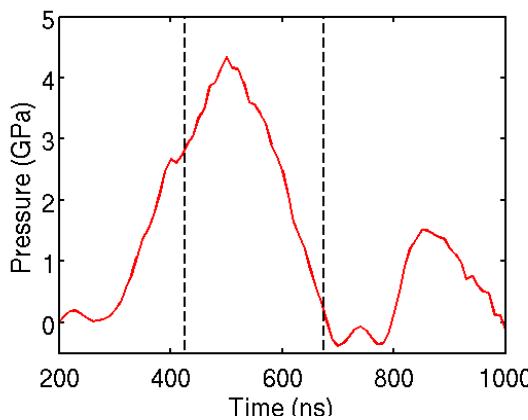


Maintenance of high pressure:

- If sample is too thin, release wave will reduce pressure too soon.
- “Coasting” phase (constant growth rate) ensues, implying $\gamma \approx 0$

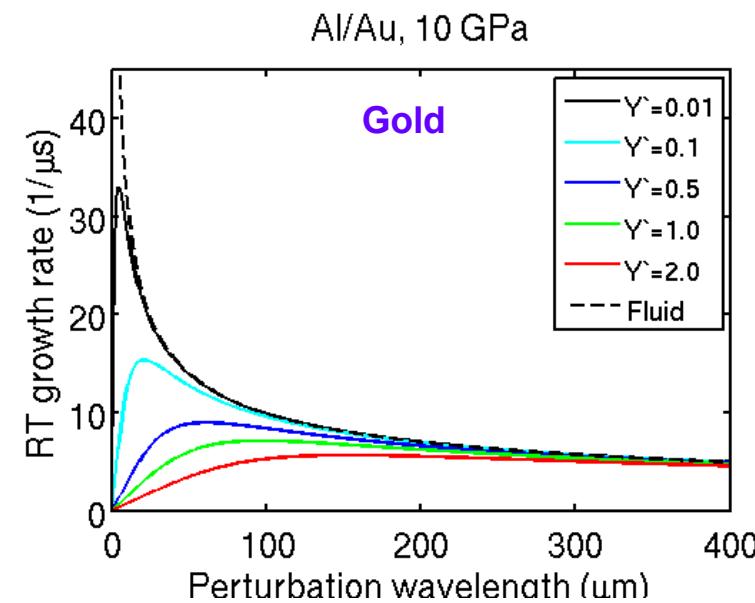
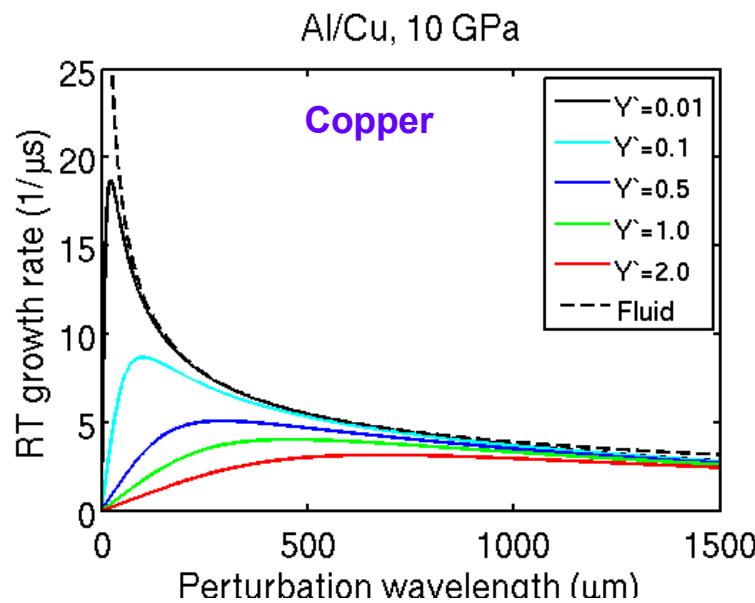


Conflicting constraints
→ Measure γ only before
onset of coasting



4. Issues and constraints in experiment design: yield-stress sensitivity

Materials and geometries must be chosen in order to maximize the sensitivity of the RT growth rate to the yield stress → use Colvin theory:



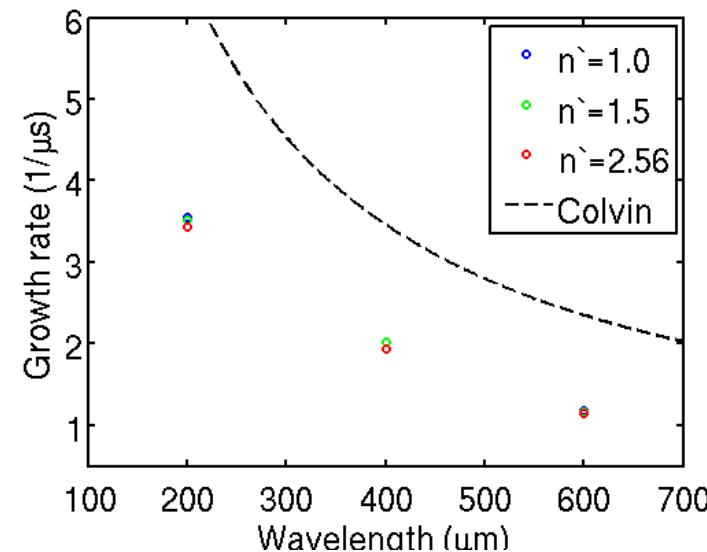
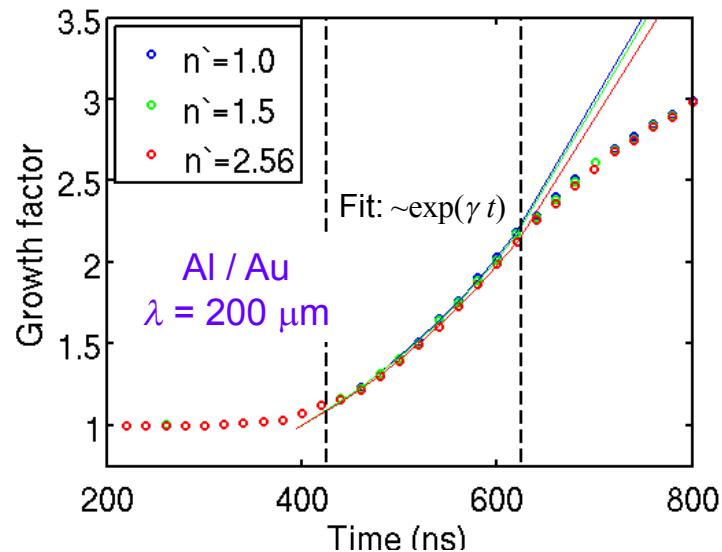
5. Parameter study: aluminum / gold samples

Al / Au parameter study

- Wavelength: $\lambda = 200, 400, 600 \mu\text{m}$
- Thickness: $h = \lambda / 2$
- Amplitude: $\eta_0 = h / 2 = \lambda / 4$

Vary Steinberg-Guinan hardening parameters:

- Pressure hardening: $A' = 1, 5, 10$
- Strain hardening (exponent): $n' = 1.0, 1.5, 1/n_0$
- Strain hardening (coefficient): $\beta' = 1, 2, 4$



- Strain-hardening exponent has greatest effect on yield strength and RT growth.
- Growth rate variability, even at 200 μm , is only a few percent.
- Factor of 2 or more difference between the ALEGRA result and Colvin prediction.

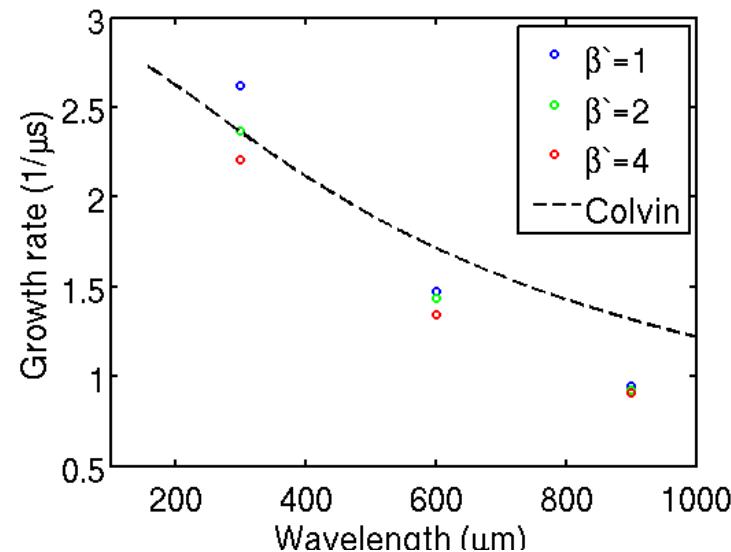
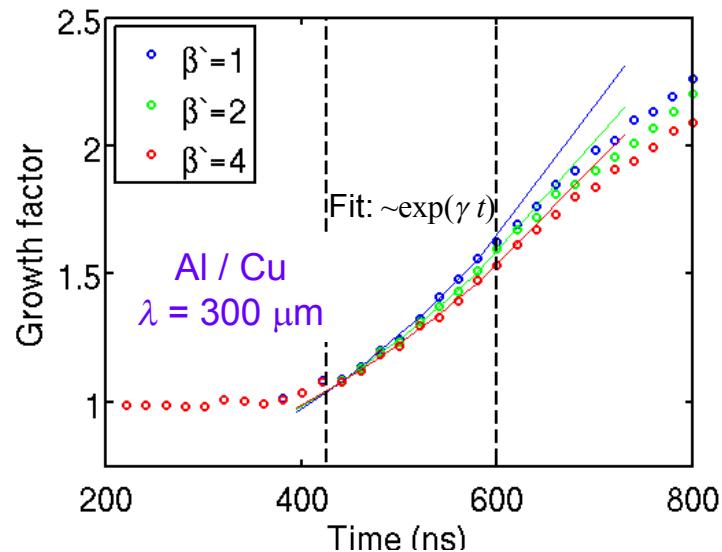
5. Parameter study: aluminum / copper samples

Al / Cu parameter study

- Wavelength: $\lambda = 300, 600, 900 \mu\text{m}$
- Thickness: $h = \lambda / 3$
- Amplitude: $\eta_0 = h / 2 = \lambda / 6$

Vary Steinberg-Guinan hardening parameters:

- Pressure hardening: $A' = 1, 5, 10$
- Strain hardening (exponent): $n' = 1.0, 1.5, 1/n_0$
- Strain hardening (coefficient): $\beta' = 1, 2, 4$



- Strain-hardening exponent and coefficient both have significant effect on yield strength and RT growth.
- Growth rate variability at 300 μm is $>10\%$ over the range in β .
- Much better agreement between Colvin theory and ALEGRA result for this case.

Concluding observations

Experimental scheme proposed and scoping calculations for proof-of-principle experiment performed.

Usefulness of ALEGRA for simulating this problem proven.

Optimal materials and geometries identified for experiment design.

Applicability of Colvin theory for this problem explored.

Several future directions suggested by the current results (in progress).

Extra slides

Analysis of Colvin, et al

Mikaelian:

$$\begin{aligned}
 & D \left[\left(\rho - \frac{\mu}{\gamma} (D^2 - k^2) \right) D W - \frac{1}{\gamma} D \mu (D^2 + k^2) W \right] \\
 & + k^2 \left[\frac{g}{\gamma^2} D \rho - \left[\frac{k^2}{\gamma^2} \sum_i T_i^{(s)} \delta(y - y_i) \right] W \right] \rightarrow \gamma^2 + 2k^2 \nu \gamma - g k A \left(1 - \frac{k^2}{k_c^2} \right) \tanh(kt) = 0, \quad (26) \\
 & - k^2 \left[\rho - \frac{\mu}{\gamma} (D^2 - k^2) \right] W + 2 \frac{k^2}{\gamma} D \mu D W = 0, \quad (3) \quad k_c = [(\rho_2 - \rho_1)g/T^{(s)}]^{1/2}
 \end{aligned}$$

Colvin:

$$\begin{aligned}
 & \frac{d}{dz} \left\{ \rho \left[1 - \frac{\nu}{\gamma} \left(\frac{d^2}{dz^2} - k^2 \right) \right] \frac{dw(z)}{dz} - \frac{1}{\gamma} \frac{d}{dz} \rho \nu \left(\frac{d^2}{dz^2} + k^2 \right) w(z) \right\} \\
 & + k^2 \left[\frac{a}{\gamma^2} \frac{dp}{dz} - \left[\frac{kG}{\gamma^2} \right] w(z) - k^2 \rho \left[1 - \frac{\nu}{\gamma} \left(\frac{d^2}{dz^2} - k^2 \right) \right] w(z) \right. \\
 & \left. + \frac{2k^2}{\gamma} \frac{d}{dz} \rho \nu \frac{dw(z)}{dz} \right] = 0. \quad \rightarrow \gamma_s^2 + 2k^2 \nu \gamma_s + k \cdot \tanh(kh) \cdot (kG/\rho - Aa) = 0. \quad (7)
 \end{aligned}$$

$$\sigma_{jk} = 2 \rho \nu \varepsilon'_{jk} \quad \leftrightarrow \quad |\sigma_{jk}|^2 = 2 Y^2/3 \quad \rightarrow \quad \nu_{ep} = \frac{Y}{\sqrt{6\rho |\varepsilon'_{jk}|}}$$

Newtonian fluid Von Mises plasticity criterion

2. Experimental schemes for strength via RT: fundamental pro's and con's

Disadvantages:

- No closed-form relationship
- Acceleration is not constant

Advantages:

- No shocks are involved
- No need for impedance-matched window.

Disadvantages of MHD drive:

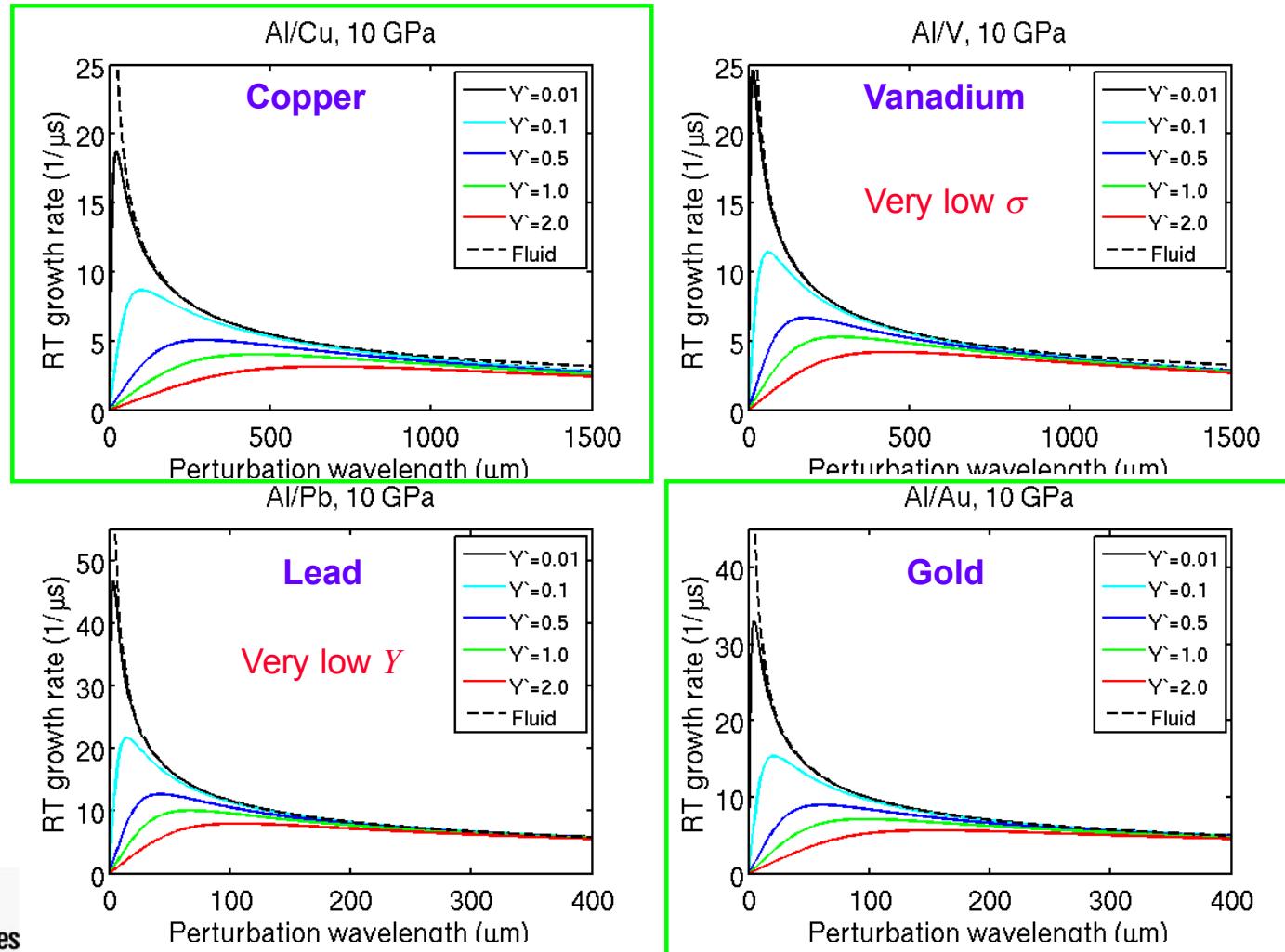
- Magnetic diffusion and Joule heating
- Fabrication issues
- Free-surface growth

Advantages of MHD drive:

- Pulse length, control
- Velocimetry rather than radiography.
- Cost and complexity

4. Issues and constraints in experiment design: yield-stress sensitivity

Materials and geometries must be chosen in order to maximize the sensitivity of the RT growth rate to the yield stress → use Colvin theory:



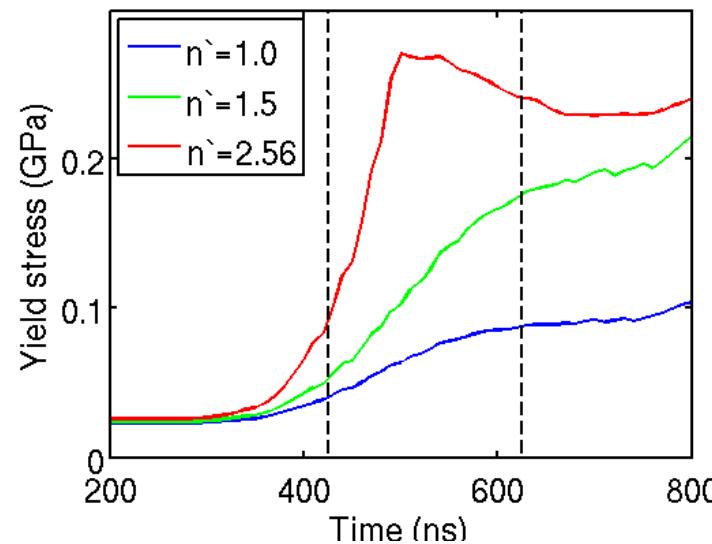
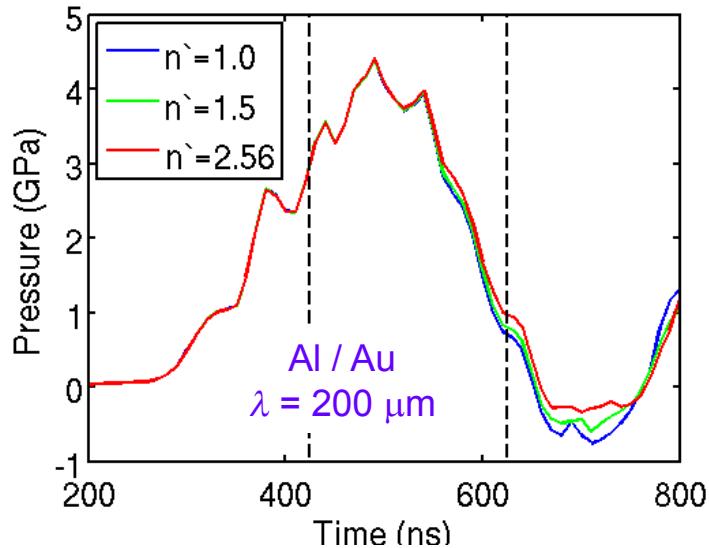
5. Parameter study: aluminum / gold samples

Al / Au parameter study

- Wavelength: $\lambda = 200, 400, 600 \mu\text{m}$
- Thickness: $h = \lambda / 2$
- Amplitude: $\eta_0 = h / 2 = \lambda / 4$

Vary Steinberg-Guinan hardening parameters:

- Pressure hardening: $A' = 1, 5, 10$
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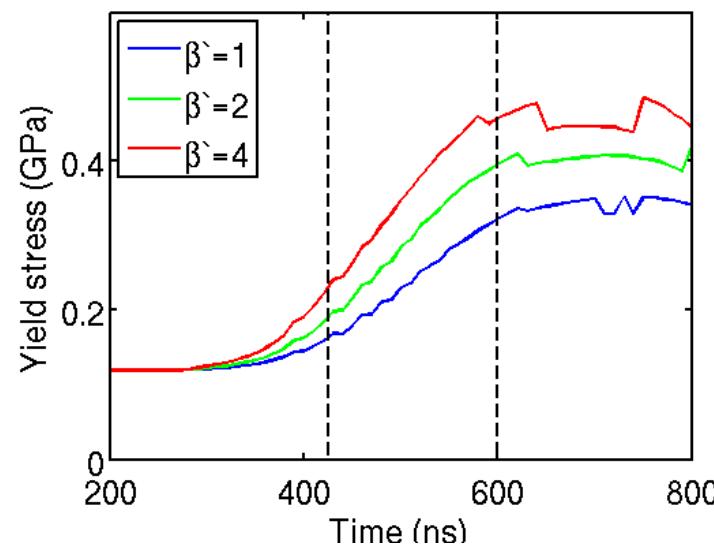
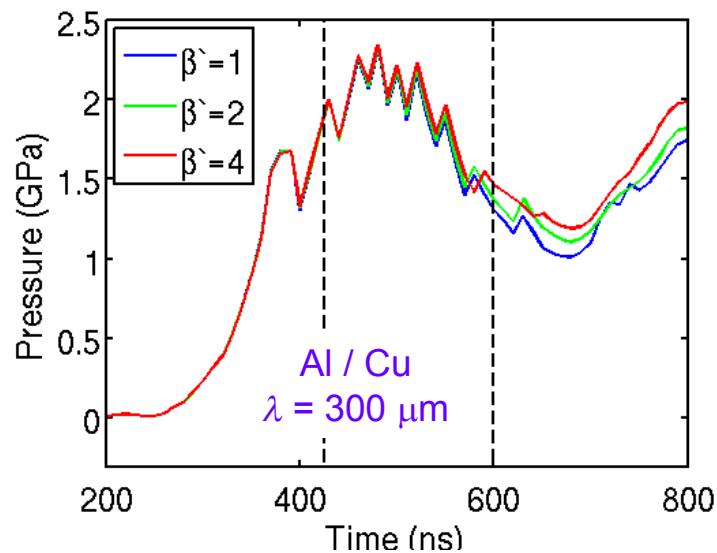
5. Parameter study: aluminum / copper samples

Al / Cu parameter study

- Wavelength: $\lambda = 300, 600, 900 \mu\text{m}$
- Thickness: $h = \lambda / 3$
- Amplitude: $\eta_0 = h / 2 = \lambda / 6$

Vary S-G hardening parameters:

- Pressure hardening: $A = 1, 5, 10$
- Strain hardening (exponent): $n = 1.0, 1.5, 1/n_0$
- Strain hardening (coefficient): $\beta = 1, 2, 4$

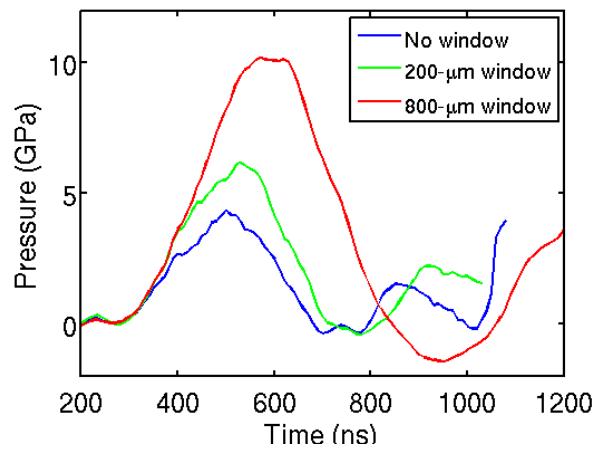
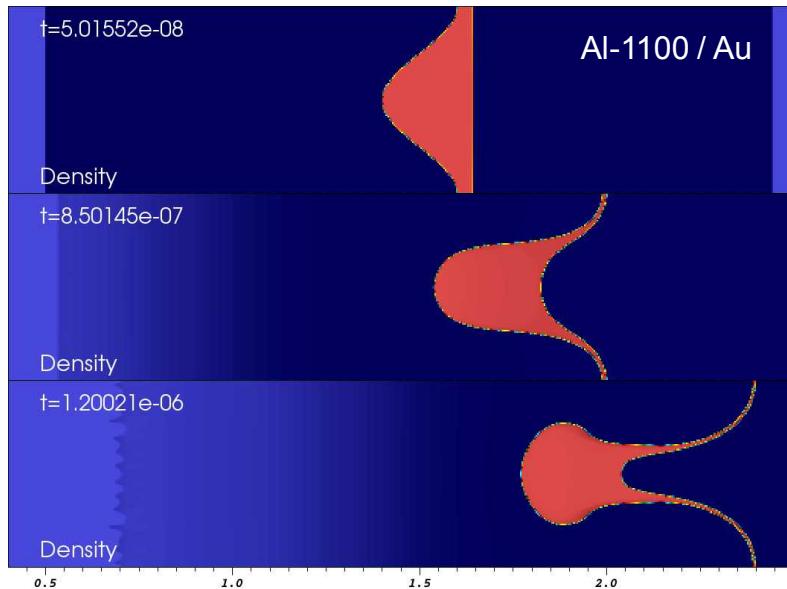
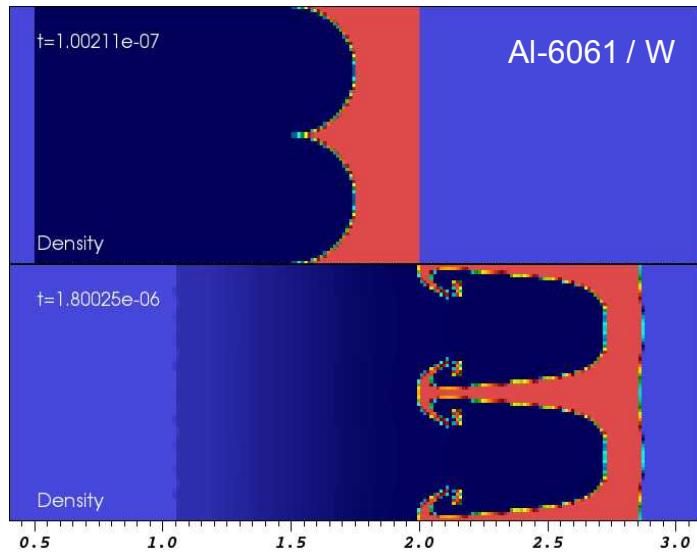


6. Additional issues:

realizable interface geometry and impedance-matched window

Further tests have indicated that:

- Use of non-sinusoidal interface shapes may be useful.
- A “window” panel may be added behind the sample material without suppressing the RT growth.



Summary and conclusions

Experimental scheme proposed and scoping calculations for proof-of-principle experiment performed.

- Experiments that expose yield stress are possible on Veloce.
- Free-surface/interior coupling imposes maximum sample thickness constraint.
- No obvious means for relating interior-interface and free-surface RT growth.
- ALEGRA useful as analytical tool – reproduces Lorenz et al results.

Optimal materials and geometries identified for experiment design.

- Tungsten, vanadium, and lead eliminated as sample materials.
- Gold and copper studied as candidates, and copper shown to have greatest sensitivity of RT growth to yield strength.
- Shorter wavelengths shown to have greater sensitivity to yield strength.

Effectiveness of Colvin theory evaluated.

- Prediction of greater sensitivity at shorter wavelength confirmed.
- Accurate prediction in some cases; error as large as factor of 2 in others.

Future directions for this work:

- Verification.
- Consequences of increasing current by order of magnitude.
- Explore possibility of using a window.
- Find relationship between interior-interface and free-surface RT growth.

Outline and content

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 - 4 • Development of Colvin formula – 2
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- 2. Laser-driven experiments
 - 5 • Setup
 - 5 • Results
- 3. Concept for magnetically driven experiments
 - 6 • Veloce
 - 6 • Targets
 - 7 • Advantages and disadvantages
 - 7 • Constraints
- 4. MHD simulation capability: ALEGRA
 - 8 • Code
 - 8 • Lorenz simulations
 - 9 • Concept for experiment and problem setup
 - 9 • Sample results
- Issues to consider in experiment design
 - 10 • Magnetic diffusion/Joule heating
 - 10 • Coupling of interface growth to free surface growth
 - 10 • Timing of growth versus release
 - 10 • Sensitivity to strength
- Approach to design/scoping simulations
 - 11 • Candidate materials
 - 11 • Exposed variables in S-G model
 - 11 • Evaluate sensitivities, feasibility
- Test results
 - 12 • 1D stress wave tests
 - 12 • Exploration: eliminate W, V
 - 13 • Sample animated results: Al/Au
 - 14 • Parameter study: Al/Au
 - 15 • Parameter study: Al/Cu
- Additional issues
 - 16 • Grooved interface
 - 16 • Strength of driver
 - 16 • Liquid window
- In progress!
- Conclusions:
 - 17 • Pilot experiment is possible: VISAR-measurable velocity differences.
 - 17 • Sensitivity is low for Veloce-level currents
 - 17 • Data extraction will depend on simulations
 - 17 • Simulations currently do not match theoretical predictions

3D view of setup

