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Uncertainty Quantification in Model Validation of a Complex Aerospace Structure

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ABSTRACT

Modal vibration test data were measured on a complex aerospace structure, consisting of a shell structure with joints and discrete payloads, in order to validate a finite element model of the structure. Uncertainty was assessed for both the measurements and the analysis predictions and incorporated into the validation process. The uncertainty quantification on measured data included an estimate of fabrication uncertainty, an acknowledged but frequently unquantified source. The analysis uncertainty addressed mesh convergence, material modulus and joint stiffness. Model updating was performed with modal properties initially, at which point a method was developed and implemented to validate the analysis model using a metric comparing the measured and predicted frequency response functions.

NOMENCLATURE

w_{exact}	Exact Solution	*h*	Relative mesh size
w_n	Estimated Solution	*p*	Order of convergence
A	Constant	H.O.T.	Higher order terms

INTRODUCTION

The purpose of this work was to investigate uncertainty in both the test hardware and the finite element model of a complex aerospace structure, and to validate the model taking into account these uncertainties. The primary source of experimental uncertainty investigated was assembly uncertainty, because it was significantly greater than the uncertainty in the measured data. Manufacturing (or part to part) uncertainty was not investigated because only one piece of hardware was available for testing. Modeling uncertainty included material property and joint stiffness uncertainties.

The aerospace structure was modeled using the finite element method (FEM) in order to predict component responses to flight environments. Before the model could be used for this purpose it was validated using test data. The validation process includes solution verification and uncertainty quantification, and this report outlines the process used for validation of the model [1].

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[1] Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

MODEL

The aerospace structure consists of an exterior shell comprised of two stiff materials with a soft adhesive sandwiched between, interior bracing, supporting brackets and payloads, which are bolted into place. The finite element model is composed of 400,000 2^{nd} order elements, a mix of hexes, quads and beams, resulting in 5.6 million degrees of freedom. A simplified illustration of the structure is shown in Figure 1.

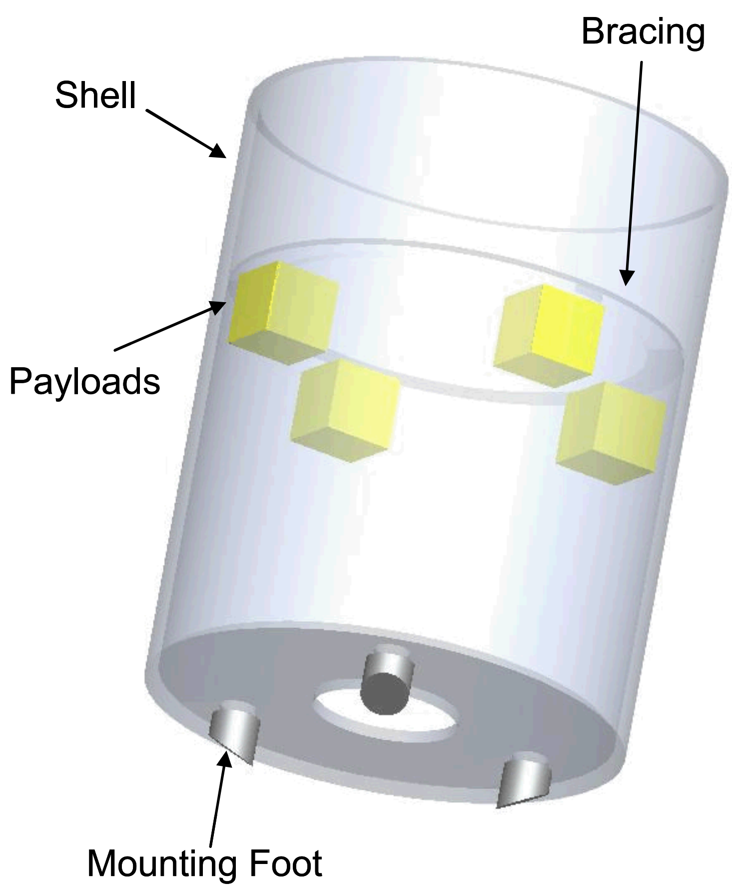

Figure 1: Simplified Illustration of Structure

The modes and frequency response functions (FRFs) of the structure were calculated using Salinas [3], a massively parallel structure dynamics code developed at Sandia National Laboratories. Each iteration of the model ran for 5 hours on 100 processors to calculate 150 modes and FRFs up to 2000 Hz.

TESTING

Testing was conducted on the structure with a free boundary condition. Energy was input into the system on the exterior shell, near one of the mounting foot locations at an angle of 30° from vertical. The structure was excited using a small modal shaker, using continuous random input with Hanning windows. Long time records were measured to minimize the effect of the Hanning window on the measured damping. The shaker input was specifically chosen to average the effects of nonlinearities at the bolted joints in the assembly and produce good FRF fits for a linear system approximation.

The structure was lightly instrumented with just a few accelerometers for the validation test where only a few FRFs were measured. The modes had previously been measured during the calibration experiments. Data was measured using tri-axial accelerometers at the input location as well as at 3 locations on the shell, 2 locations on the payload brackets and 7 locations on the payloads. The acceleration/force FRF was calculated at each instrumentation location for validating the finite element model of the structure.

MODEL VERIFICATION

Model verification consisted of a mesh convergence study for a simplified version of the model. In this case, the full exterior shell was modeled, but the interior components such as bracing, brackets and payloads were removed. The mesh convergence study was conducted with 1st order (linear) elements instead of 2nd order (quadratic) elements because the tools available for convergence studies in Salinas have been optimized for linear elements. The model was also run and errors calculated with quadratic elements since the actual model is meshed with quadratic elements. Error calculations made with quadratic elements would therefore be more representative of the errors in the actual mesh.

Five levels of refinement were created and the frequencies for the first 12 nonsymmetric modes were calculated for each mesh. Richardson extrapolation was used to estimate an exact solution and an order of convergence for each mode. The exact solution (w_{exact}) can be estimated from an estimated solution (w_n) plus a constant (*A*) multiplied by the relative mesh size (*h*) raised to the power of the order of convergence (*p*) plus some higher order terms, which can be neglected [2].

$$w_{\text{exact}} = w_n + Ah^p + \text{H.O.T.}$$

Using multiple mesh sizes, the constants (*A*) and (*p*) can be calculated and the exact solution can be estimated. The order of convergence (*p*) varied from 1.6 to 2.0 for the 12 modes, with an average value of 1.9. Only three levels of mesh refinement are required to calculate *(A)* and *(p)*, but additional levels were used to verify the order of convergence.

Since the actual model was meshed with quadratic elements, the error was then calculated for the quadratic convergence study mesh using the estimated exact solution from the linear convergence study for each mode. Errors varied between 0.05% and 1.75% with an average error of 0.51%. Figure 2 shows the mesh convergence graphically for one mode.

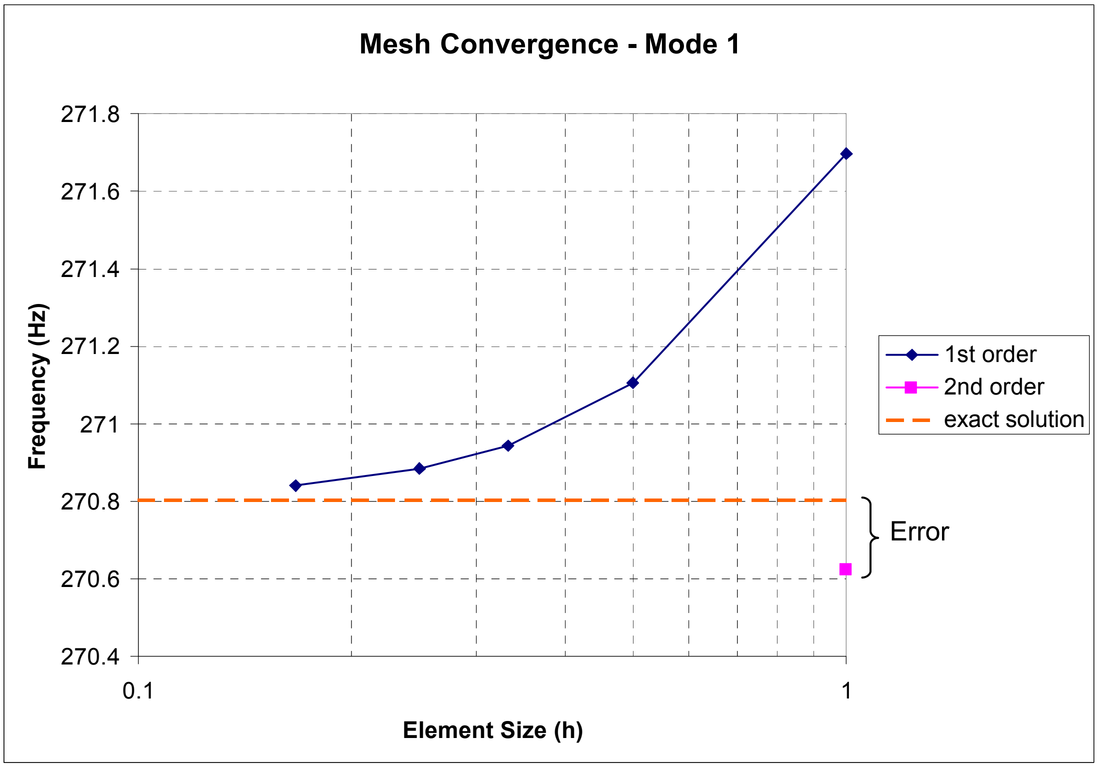

Figure 2: Mesh Convergence

UNCERTAINTY QUANTIFICATION

Two areas of uncertainty were explored for this analysis, modeling uncertainty and assembly uncertainty. Ideally, manufacturing uncertainty would be explored as well by testing several structures, but in this case only one structure was available. Testing uncertainty should also be addressed, although in this structure, the assembly uncertainty dominated the testing uncertainty.

The assembly uncertainty was observed by assembling, disassembling, and reassembling the aerospace structure by different individuals. Recognized and approved hardware and fasteners as well as applicable procedures with torque values were instituted for each buildup. The fastener tightening sequence was varied as much as possible on each buildup. Additional variations came by the way of exchanging nominally identical components and brackets to attain additional experimental data. Ten buildups were assembled and tested.

Variability in test FRFs due to the different assemblies is shown in Figure 3 and Figure 4. Figure 3 shows variation across the full frequency range. Figure 4 zooms into the first axial and lateral modes of the payload and shows a payload axial mode near 350 Hz which varies by as much as 5.5% with assembly variances.

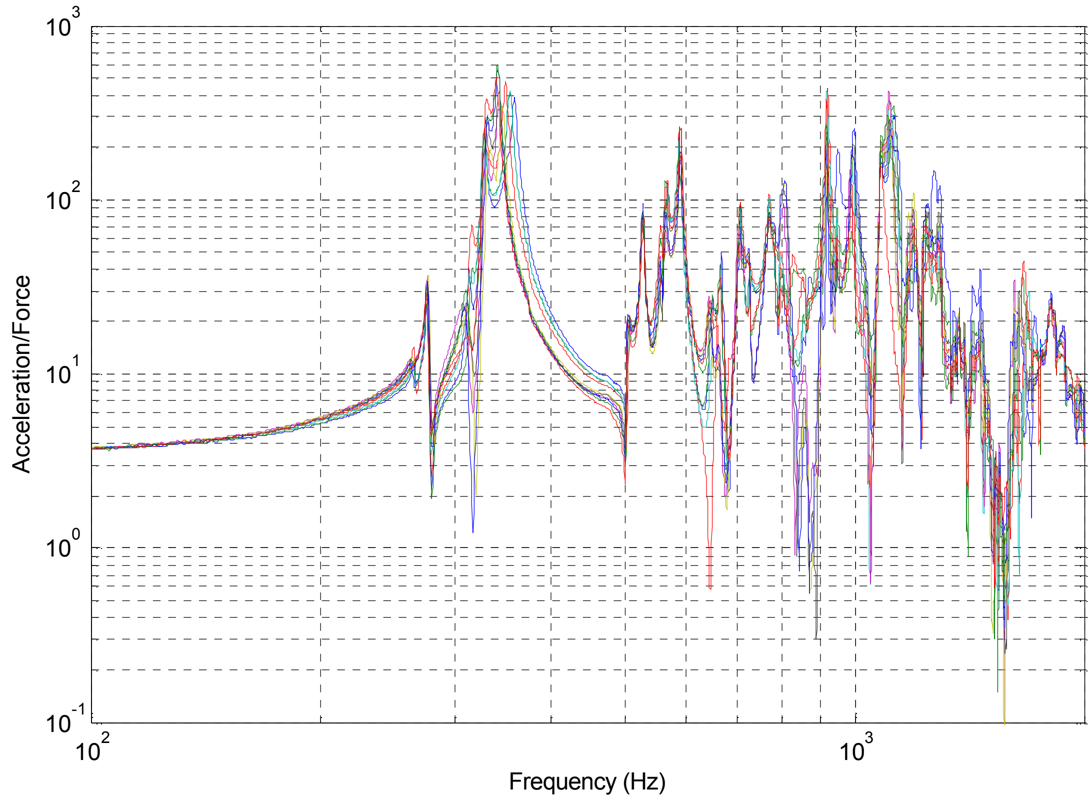

Figure 3: Assembly variability FRFs, Full Spectrum

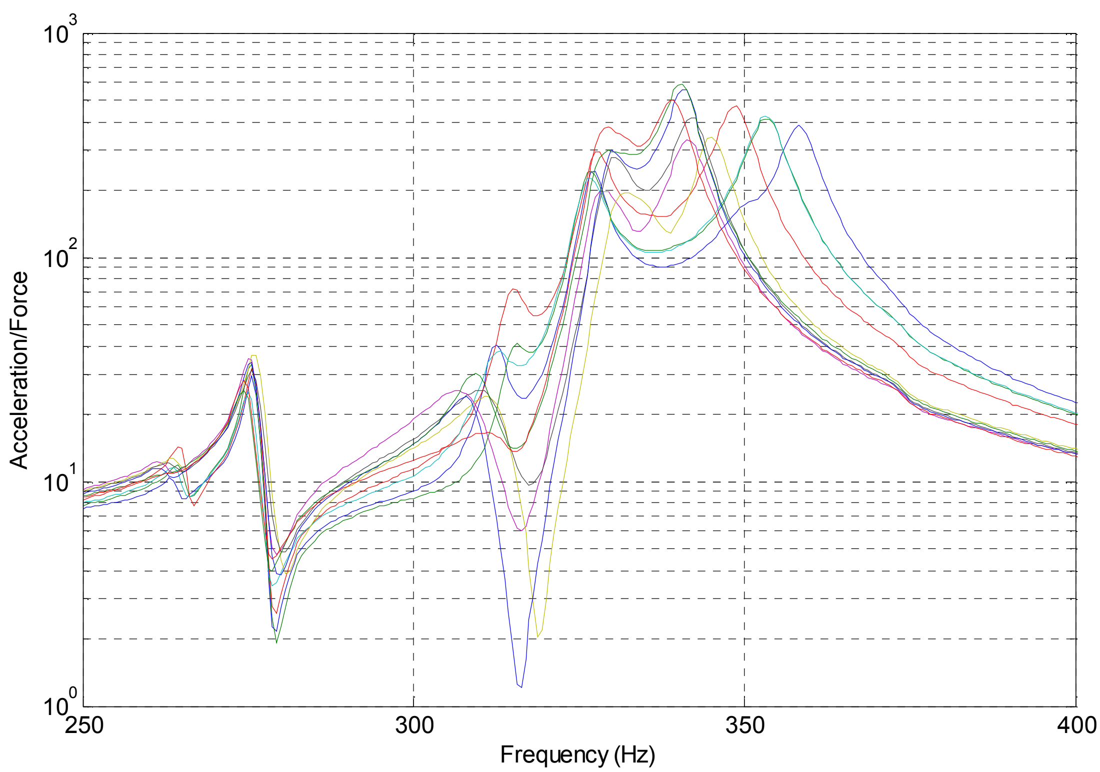

Figure 4: Assembly variability FRFs, First Lateral and Axial Modes

Modeling uncertainty was assessed by varying several properties within the model. The properties chosen for uncertainty analysis were based on the known variability or uncertainty of the property and the sensitivity of the model to variations in the property. For this model, the properties chosen were the modulus of elasticity of the adhesive between the layers of the shell, the sheet metal thickness of two sections of the interior bracing, and the stiffness of three bolted joints. Each of these properties was assigned a random variation with a mean value and a range. The mean values were set to the nominal or calibrated value of the properties derived from previous modal testing. The ranges were based on factors specific to each variable, described below.

The effective modulus of elasticity of the exterior shell adhesive was chosen as a variable in the uncertainty quantification for several reasons. First, although the modulus of the bulk material is known, there is not full adhesion between the layers that comprise the exterior shell and it is difficult to determine with accuracy the adhesion ratio and whether it varies from part to part. Second, calibration of models for other programs with similar exterior shells has shown large variability in the effective modulus of the adhesive. Third, the dynamic response of the model is quite sensitive to this property. The mean value for this variable was set to 30 ksi and the range from 10 ksi to 250 ksi. The upper limit was the calibrated value for the adhesive used in the shell. The lower limit was chosen at the analyst's discretion based on similar shell models used in other programs. Because the range was so skewed, the probability density functions for the variables were also skewed. Figure 5 shows the cumulative distribution function (CDF) and probability density function (PDF) of the random variable, on top and bottom respectively.

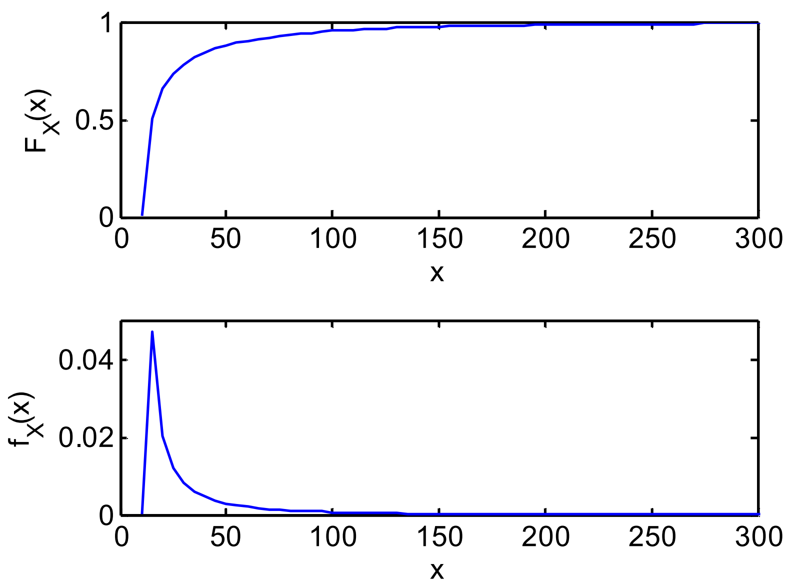

Figure 5: CDF and PDF of the Random Variable for Adhesive Modulus

The next set of properties chosen for uncertainty analysis was sheet metal thicknesses for two sections of the interior bracing. These properties were chosen because the model was quite sensitive to these properties and the measured value of the hardware was not at the nominal value on the drawing. In each case the measured value was 0.005" different from the nominal value, and ±0.005" is a standard sheet metal tolerance, so each of the variables was given a range of 0.010". The upper bracing had a measured thickness of 0.065". Figure 6 shows the cumulative CDF and PDF of the upper bracing random variable. The lower bracing had a measured thickness of 0.045". Figure 7 shows the CDF and PDF of the lower bracing random variable.

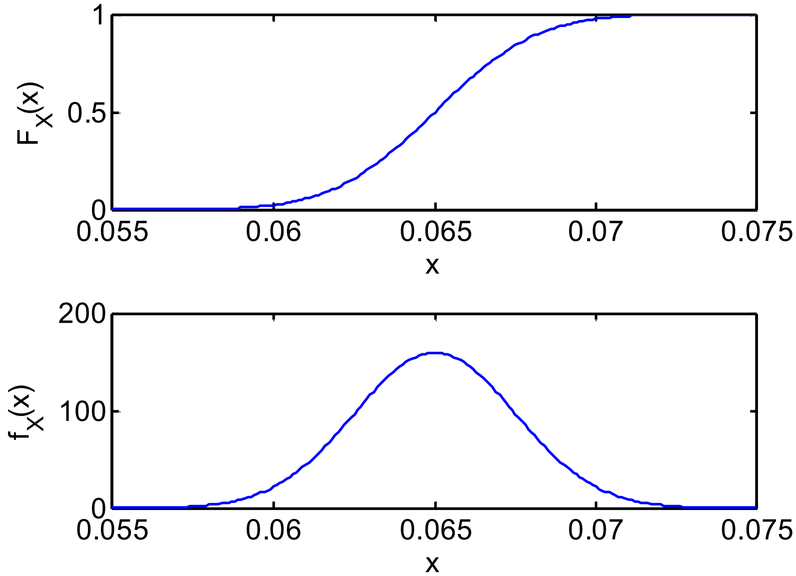

Figure 6: CDF and PDF of the Random Variable for Upper Bracing Thickness

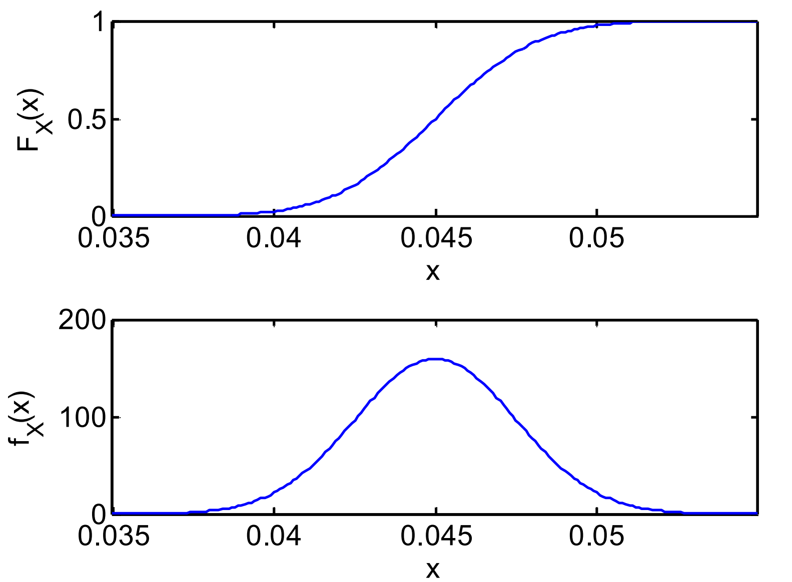

Figure 7: CDF and PDF of the Random Variable for Lower Bracing Thickness

The last set of properties chosen for uncertainty analysis was the bolt stiffnesses between the payloads and the brackets and two sets of bolts between the brackets and the bracing. Bolted joint stiffness is often an area of uncertainty in a model because each joint is unique and requires testing to calibrate the value, and the model was quite sensitive to the stiffness values chosen for each of these three joints. In this model, the joints were all calibrated to a stiffness of $5x10^5$ psi. The range was chosen as an order of magnitude on each side, based on other similar joint models. Because the logarithmic scale was chosen, the probability density function is skewed on a linear scale. Figure 8 shows the CDF and PDF of the joint stiffness random variable.

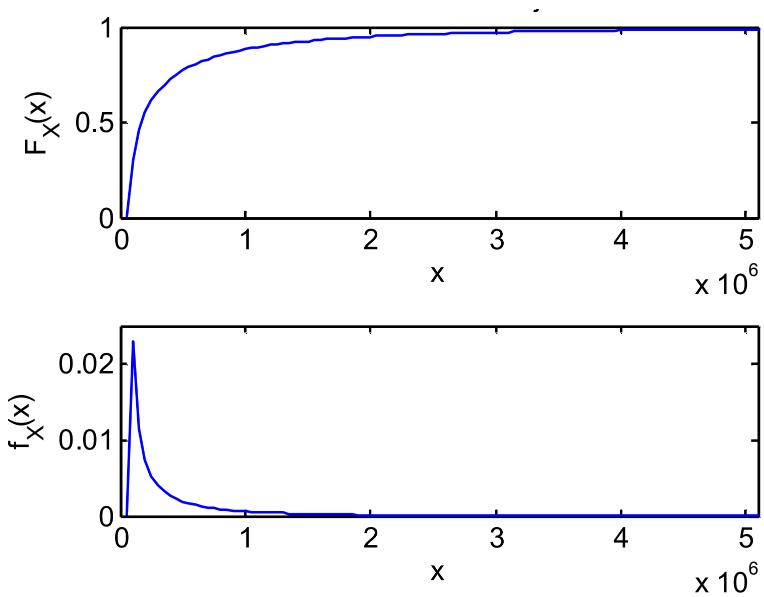

Figure 8: CDF and PDF of the Random Variables for Bolted Joint Stiffness

The next step is to use a Latin hypercube approach, described in reference [4], to generate multiple realizations of the model using the six random variables as inputs: one modulus, two thicknesses and three joint stiffnesses. In this case 50 realizations were created and run. FRFs were then calculated for each of the 50 realizations of the model and compared to the ten experimental realizations to validate the model.

MODEL VALIDATION

The validation process must make a numerical comparison between the 50 analytical FRFs and the 10 experimental FRFs, and determine whether the distribution of results could plausibly come from statistically equivalent systems. Data had been recorded in three directions at just three locations on each payload and in one location on each payload bracket. For simplification, two locations on the payloads were chosen as validation points and FRFs were calculated in three directions for a total of six validation FRFs.

For each of the six validation points, 50 analytical FRFs and 10 experimental FRFs must be compared. FRFs should be compared in frequency bands rather than at individual frequency values so that small shifts in modal frequencies don't result in rejection of valid models. To accomplish this, each FRF was multiplied by a series of windows or weighting functions, as described in reference [5]. Together the windows covered the complete frequency range of the FRFs, from 200 Hz to 2000 Hz, with overlap between windows. Figure 9 shows one series of windows used in the validation process. The windows are evenly spaced on a logarithmic scale and the window width is a function of the center frequency of the window. The area under each of the windows is constant, so the peak value of the window decreases with increasing width and frequency. The magnitude of each FRF is multiplied by each weighting function and then integrated, resulting in eight scalar numbers representing the weighted value of the FRF under each window.

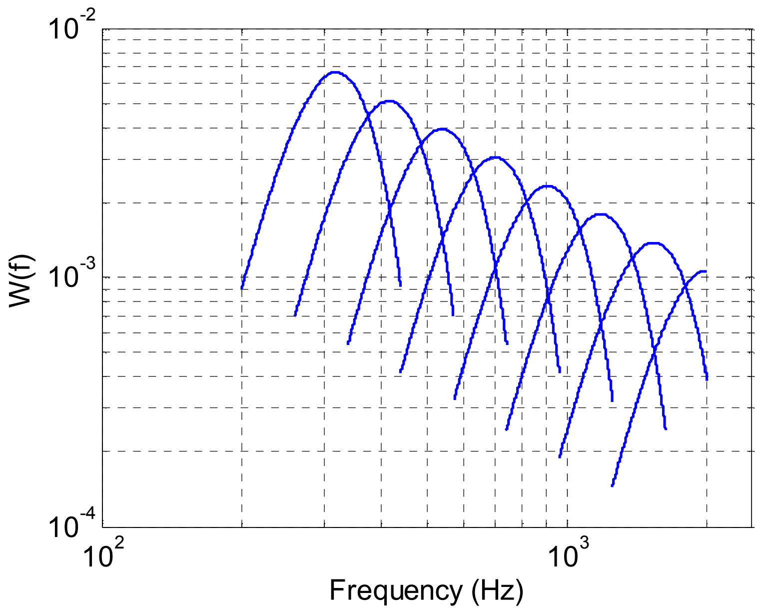

Figure 9: Weighting Functions Used for Validation Analysis

For each validation point (6 total) and each window (8 total) there are 50 model realizations and 10 experimental realizations. A probability density function can be calculated from the 50 model realizations at each validation point and window combination. The probability that each of the experimental realizations falls within a 90% probability window of the model data can then be calculated. This is a strict "accuracy assessment" for the model. Typically a looser "adequacy assessment" is made instead, historically ±6dB, or 50% - 200% nominal, indicating that a model which predicts within these bounds is adequate for the problem. Figure 10 represents this process pictorially. In this case the experimental representation would not meet the accuracy criterion but would meet the adequacy criterion.

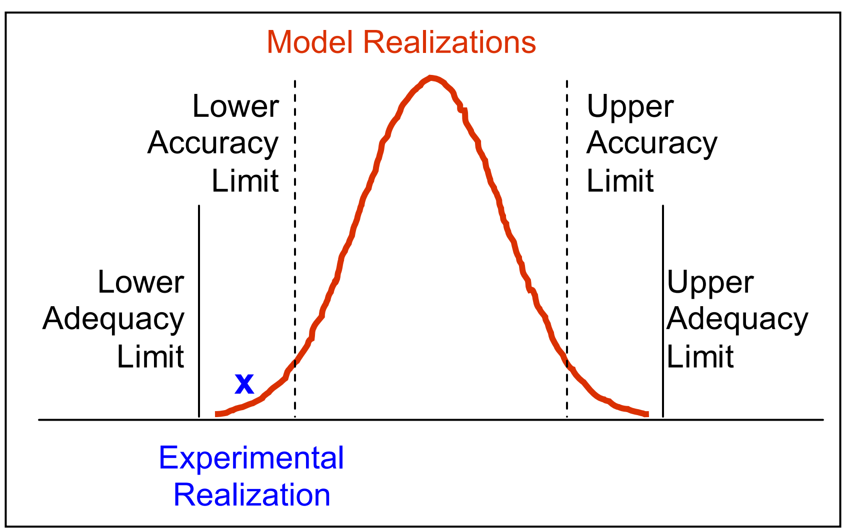

Figure 10: Pictorial Representation of Validation Process

Since each validation point (6 total) has 8 windows and 10 experimental realizations, there will be a total of 80 assessments made for each point. If the distribution of experimental data exactly matched the distribution of analytical data, then 90% or 72 of the experimental data points would fall within the 90% probability window. Since we have a limited number of data, even if the experimental distribution matched the analytical distribution, there's still a 50% chance that more than 72 of the experimental data points would fall within the probability window and a 50% chance that less than 72 of the experimental data points would fall within the probability window. So in order to assure that a valid model is not rejected, the acceptance standard must be lowered to somewhere below 90%. How much the standard should be lowered is dependant on how many data points are available, more data equals more confidence. So for this problem, if 84% or 67 of the experimental data points fall within the 90% probability window then there is only a 5% chance of rejecting a valid model. 5% is a significance level commonly used in statistical analysis.

So, in order for a point to be validated, 84% of the experimental data points must be within the adequacy limits of the model. In order for the model as a whole to be validated, all of the 6 validation points must be validated. Figure 11- 16 show the experimental realizations (blue) overlaid on the model realizations (red) at the six validation points.

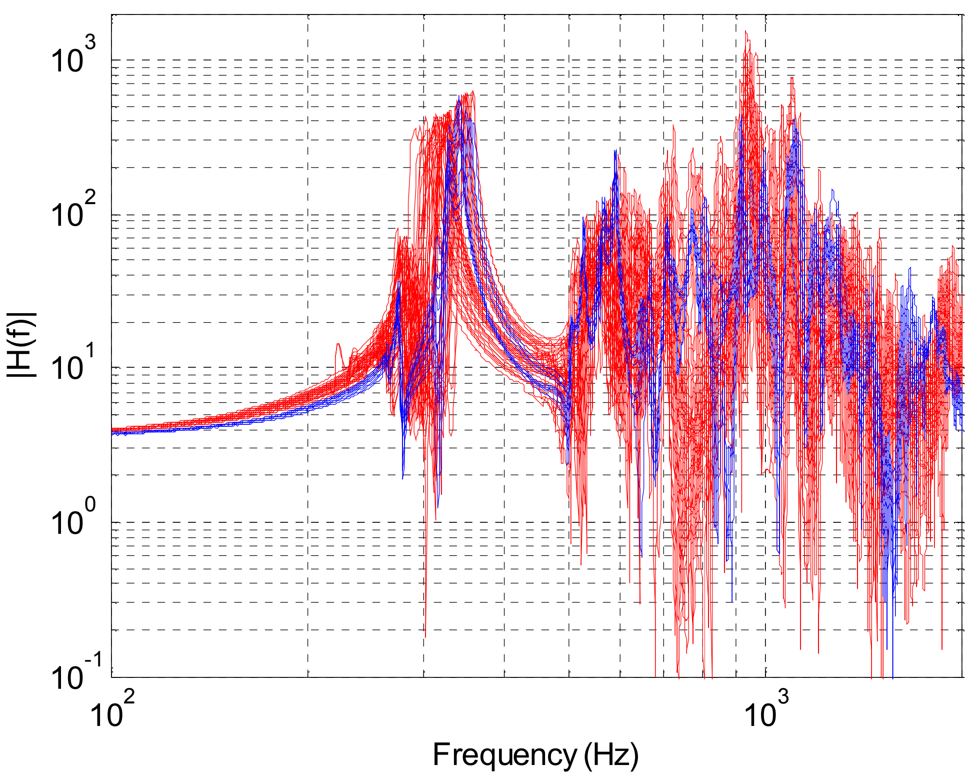

Figure 11: Validation Point 1 FRFs – Payload 1, Axial

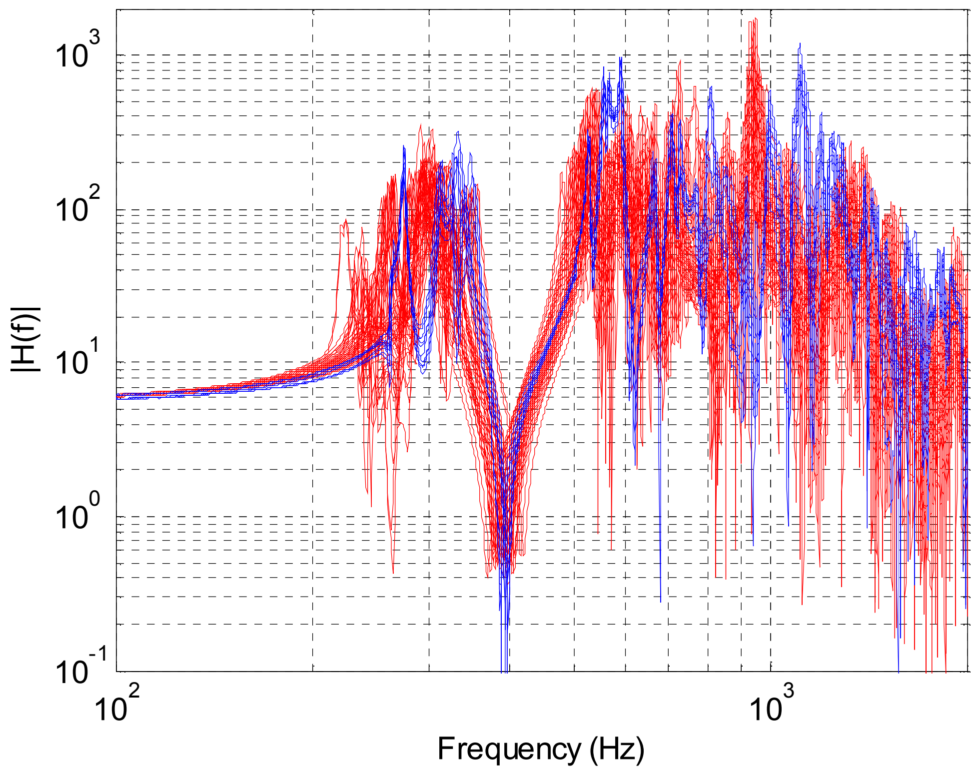

Figure 12: Validation Point 2 FRFs – Payload 1, Lateral

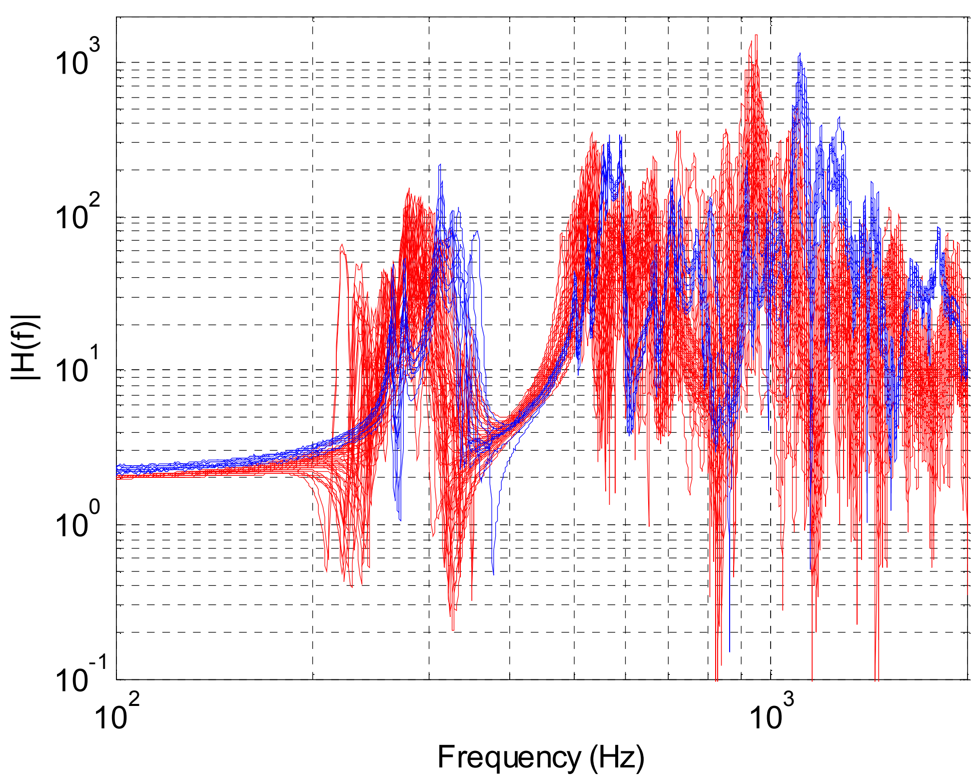

Figure 13: Validation Point 3 FRFs – Payload 1, Lateral

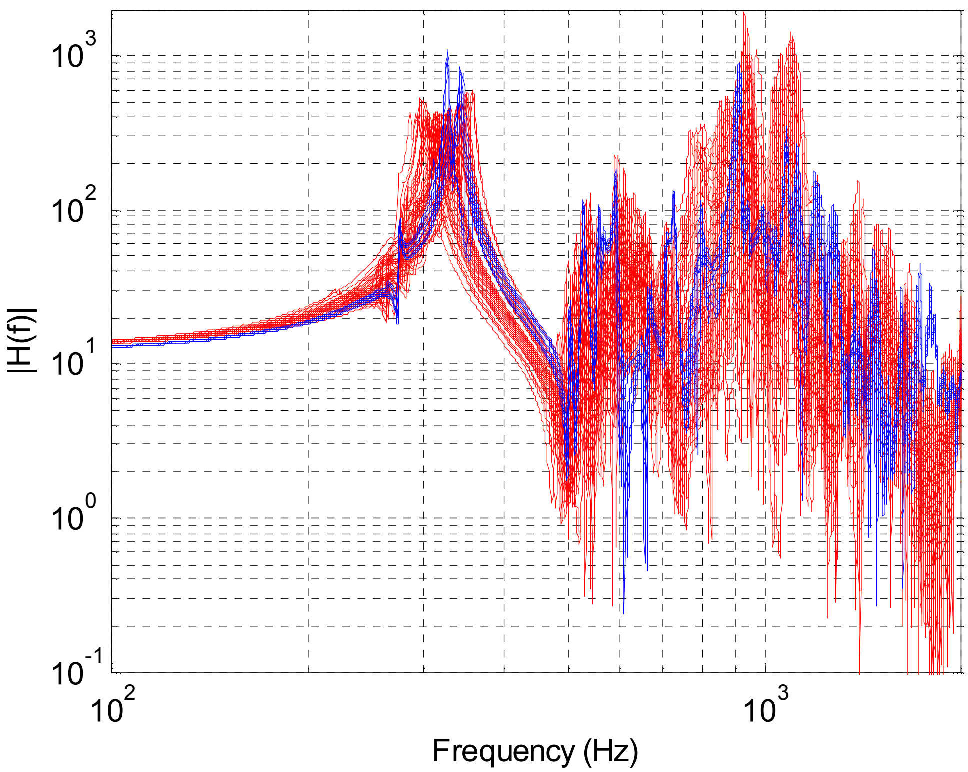

Figure 14: Validation Point 4 FRFs – Payload 2, Axial

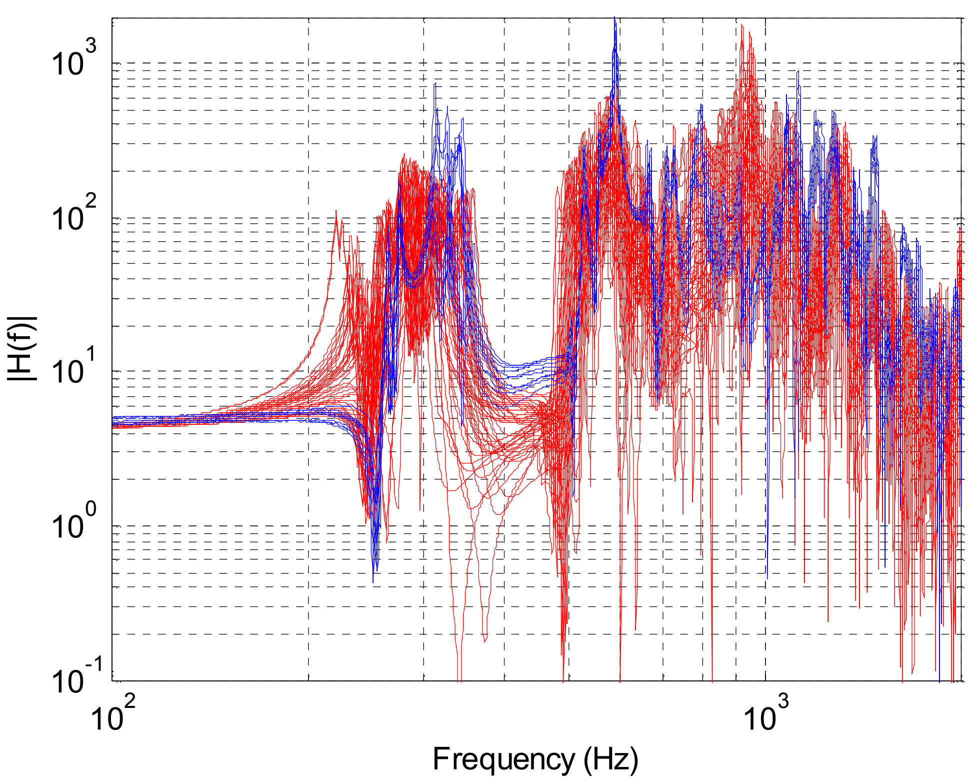

Figure 15: Validation Point 5 FRFs – Payload 2, Lateral

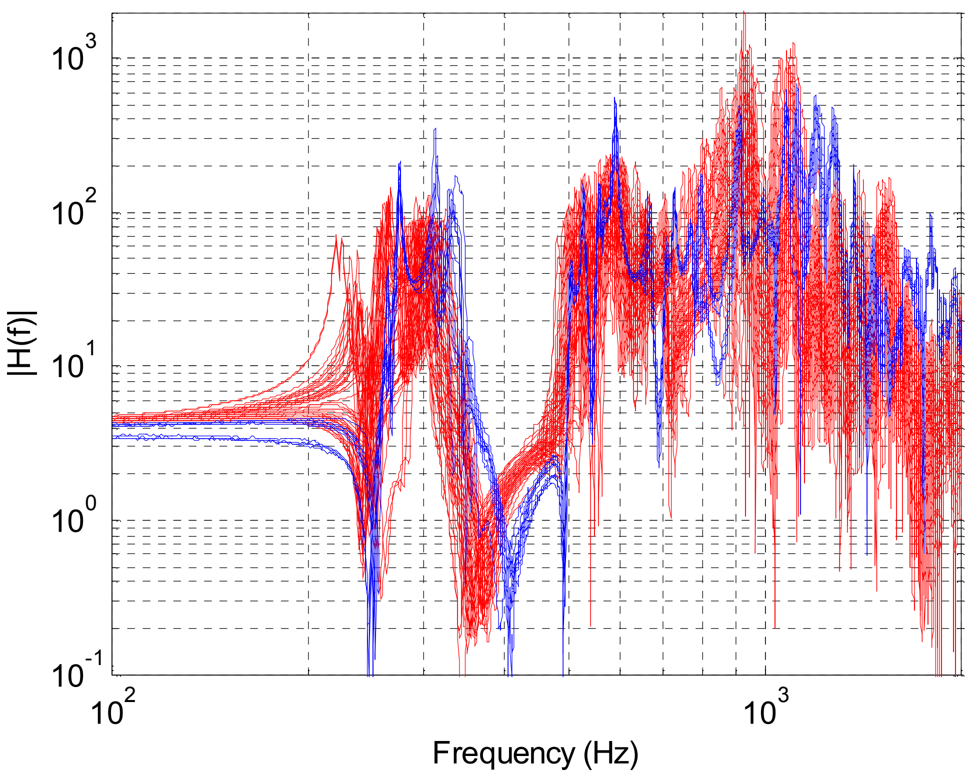

Figure 16: Validation Point 6 FRFs – Payload 2, Lateral

Each of the FRFs plotted above was multiplied by each of the eight window functions and integrated, resulting in one scalar value for each window and each FRF. In Figure 17 each of these values is plotted at the center frequency of the window with red model results offset from blue experimental results for clarity.

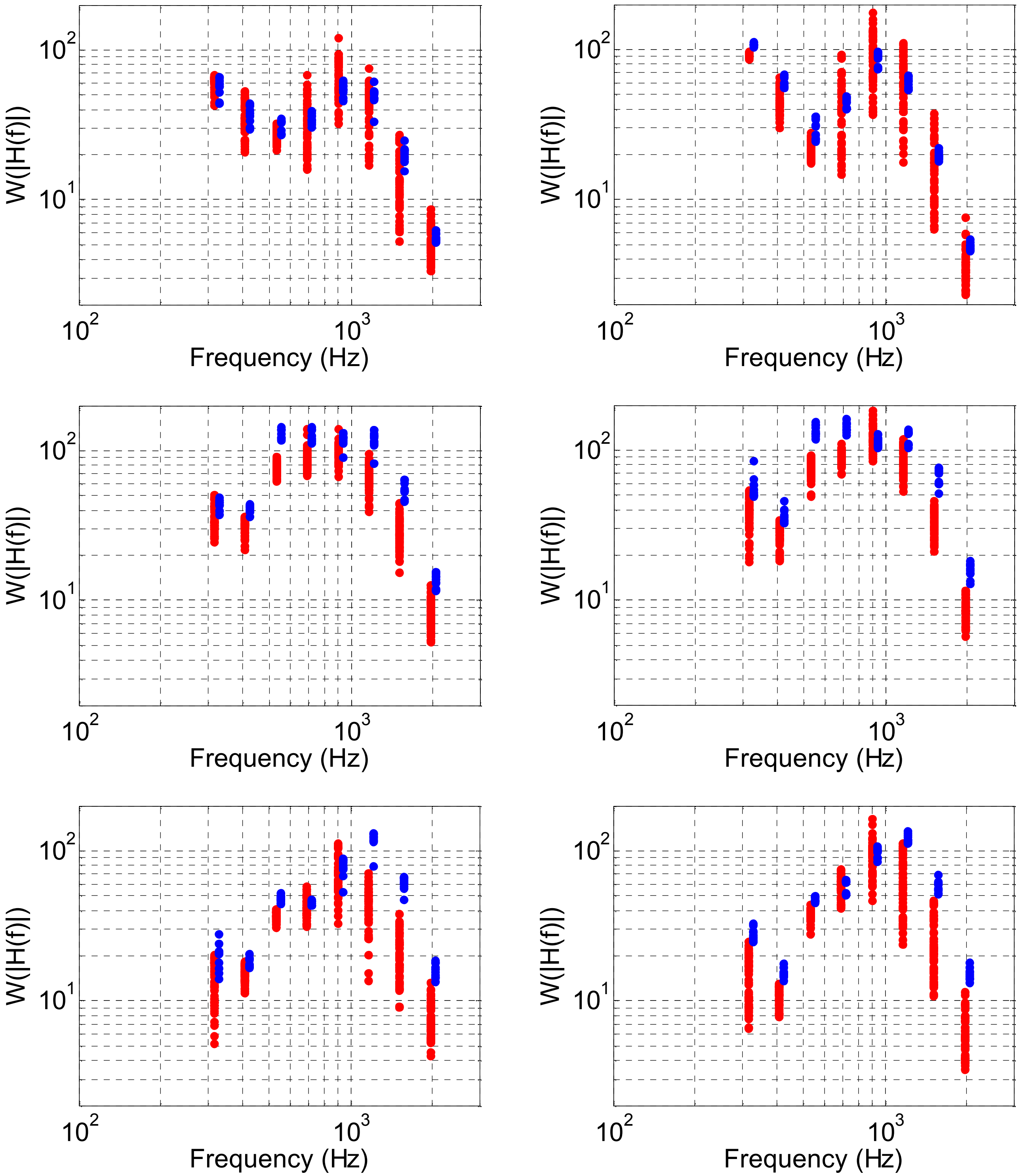

Figure 17: Measures of System Behavior for Experimental FRFs (blue) and Model FRFs (red) at Two Payload Locations and Three Directions, Payload 1 on Left, Payload 2 on Right

As depicted in Figure 10, each experimental data point must be compared to the probability density function of the model results. The accuracy criterion states that each experimental point must be within a 90% probability window of the model distribution with a 5% chance of rejecting a valid model. The adequacy criterion loosens the upper and lower limits by ±6dB, or 2X in each direction. For the model to be valid at one validation point, 84% of the experimental data at that point must meet the criterion. For a model to be valid, the validation criteria must be met at every validation point. Table 1 shows the results of this analysis using a ±6dB (2X) adequacy criterion, a ±3dB (1.5X) adequacy criterion and the accuracy criterion.

Validation Point	Location	Direction	Validated Data Points		
			Adequate ± 6dB	Adequate ± 3dB	Accurate
1	Payload 1	Axial	100%	100%	90%
2	Payload 1	Lateral	100%	89%	**26%**
3	Payload 1	Lateral	100%	**74%**	**41%**
4	Payload 2	Axial	100%	100%	**75%**
5	Payload 2	Lateral	100%	85%	**30%**
6	Payload 2	Lateral	100%	94%	**25%**

Table 1: Validation Statistics

The model meets the ±6dB adequacy criterion at all validation points and the ±3dB adequacy criterion at all but one validation point. The model meets the accuracy criterion at only one validation point. This data also indicates that the model predicts axial response better than lateral response at the payloads.

In addition to a general model validation, there are two frequency ranges that are of particular interest for this application. A second validation effort was undertaken to specifically look at these application-specific frequency windows.

As with the general model validation, weighting functions were defined for the frequency ranges of interest, displayed in Figure 18. The model will be validated separately in each window, so it is not necessary for the area under the windows to be equal.

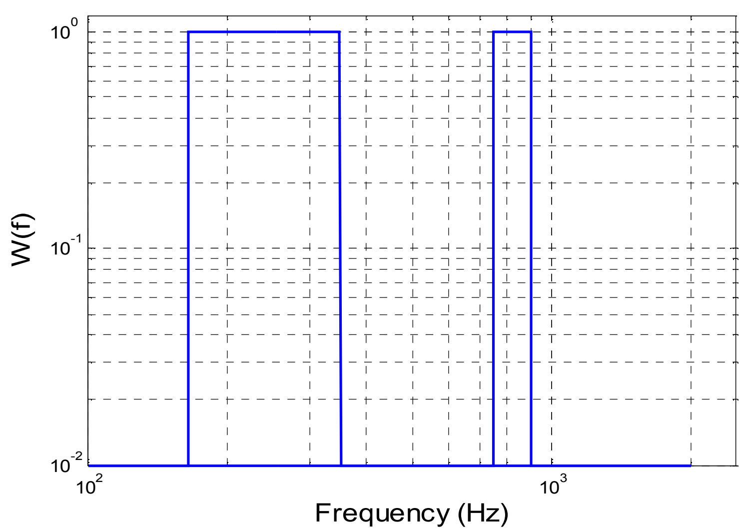

Figure 18: Weighting Functions of Environment-Specific Validation

Using the same process, the model and experimental realizations are compared for the six validation points and 2 windows, as shown in Figure 19. Each window has 10 experimental realizations for each validation point. Because there are fewer points to compare than in the full model validation, only 75% of the experimental data points must fall within the 90% probability window of the probability density function of the model data to be considered valid. This is because with fewer points, the chance of rejecting a valid model becomes higher, so the standard must be lowered to maintain a 5% chance of rejecting a valid model.

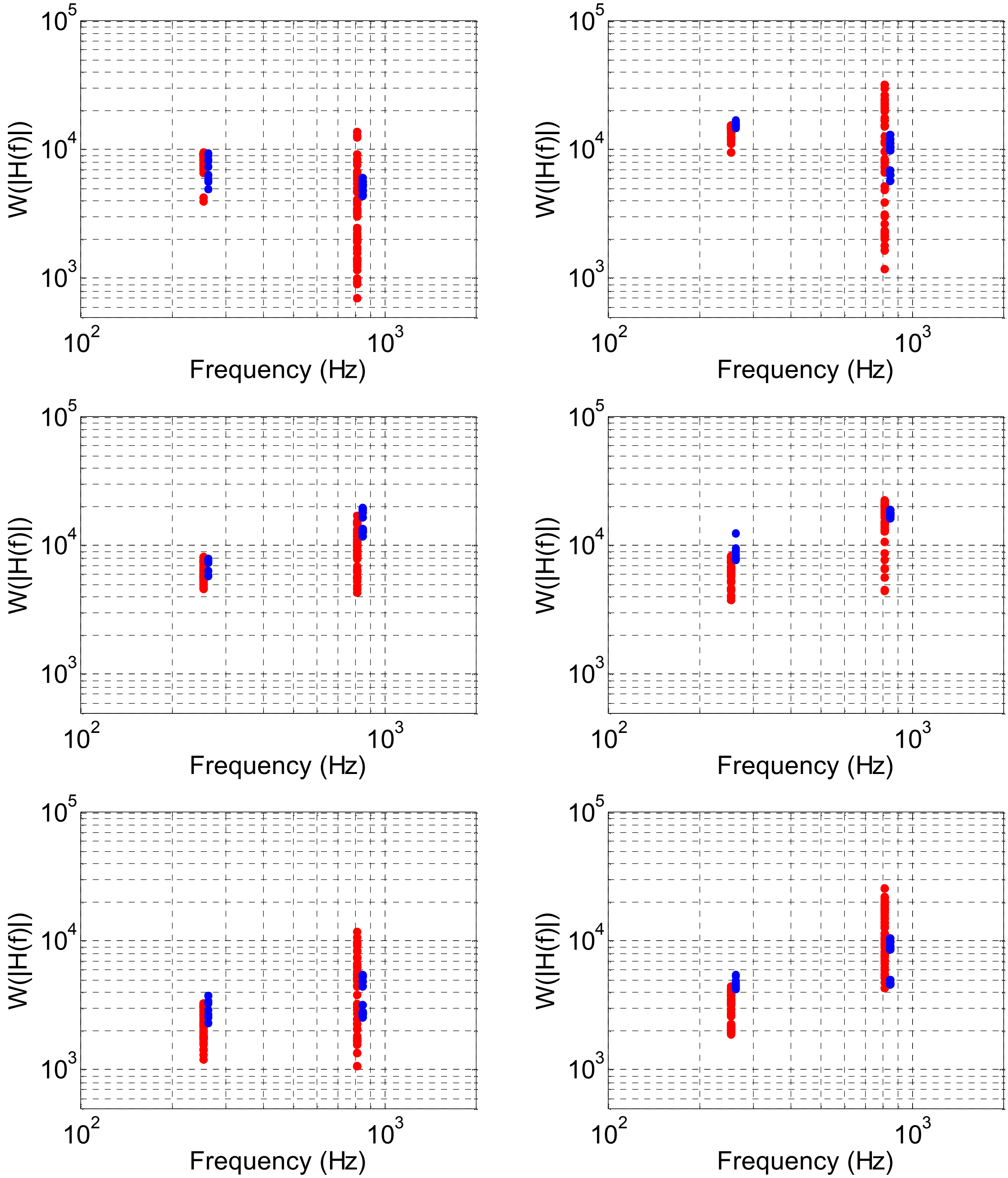

Figure 19: Measures of System Behavior for Experimental FRFs (blue) and Model FRFs (red) at Two Payload Locations and Three Directions, Payload 1 on Left, Payload 2 on Right

The validation results are separated into the two window locations for each of the validation points and shown in Table 2. Only the ±3 dB adequacy criterion is shown because the results are the same with the looser ±6 dB criterion.

Validation Point	Location	Direction	Validated Data Points Low Frequency Range		Validated Data Points High Frequency Range	
			Adequate ±3 dB	Accurate	Adequate ±3 dB	Accurate
1	Payload 1	Axial	100%	**60%**	100%	100%
2	Payload 1	Lateral	100%	100%	100%	**60%**
3	Payload 1	Lateral	100%	80%	100%	100%
4	Payload 2	Axial	100%	80%	100%	100%
5	Payload 2	Lateral	100%	**60%**	100%	100%
6	Payload 2	Lateral	100%	**20%**	100%	100%

Table 2: Validation Statistics

The results show that the model meets a ±3 dB adequacy criterion at both frequency ranges and also meets the accuracy criterion in the high frequency range at all but one validation point. The model predicts system response better at the higher frequency range than the lower one.

More details about this specific validation problem [2] or the general method [5] can be found in the referenced papers.

DISCUSSION

Looking at the data it's obvious that there is much more uncertainty or bias error in the model than in the experiments. This is expected, as there are modeling uncertainties associated with both unknown parameter values as well as expected variability of the parameters part to part. Plus the experimental uncertainty primarily includes assembly variability, not manufacturing (part-to-part) variability, which is expected to be greater. But it begs the question, if more uncertainty is added to the model, is it easier to validate? The answer is yes, but the uncertainty used in validating a model must also be carried forward into any further calculations performed using the model. So high model uncertainty means easier validation, but larger uncertainties in any subsequent simulations and predictions, creating a model that may be less useful due to the high uncertainty. So adding unnecessary uncertainty is not a good way to validate a model.

Are the uncertainties in this model reasonable? First of all, is it reasonable to include uncertainty in analytical models at all? Real structures do have variability, so any simulation of a real structure should also have variability. So yes, uncertainty must be included in a model representing a real structure. Second, are the uncertainties in this model too large? Figure 20 shows the experimental and analytical FRFs from one of the validation points on the structure. There sure seems to be a lot of "noise" in the model data. There are many closely spaced modes at higher frequencies, so adding uncertainty to the model smears the peaks into one another and it's difficult to pick out the effect of uncertainty on any specific mode. But in the lower frequencies, the variation of the first prominent peak around 350 Hz is quite apparent. Adding uncertainty to the model has added 10-15% variability to this mode. Compared to the variability of the experimental data of 5.5%, which included just one assembly (so no part-to-part variation), the model variability looks quite low. Manufacturing variability is usually much higher than assembly variability, so it wouldn't be unreasonable to expect true experimental variability to match or even exceed the model variability. So the uncertainty in this model looks quite reasonable.

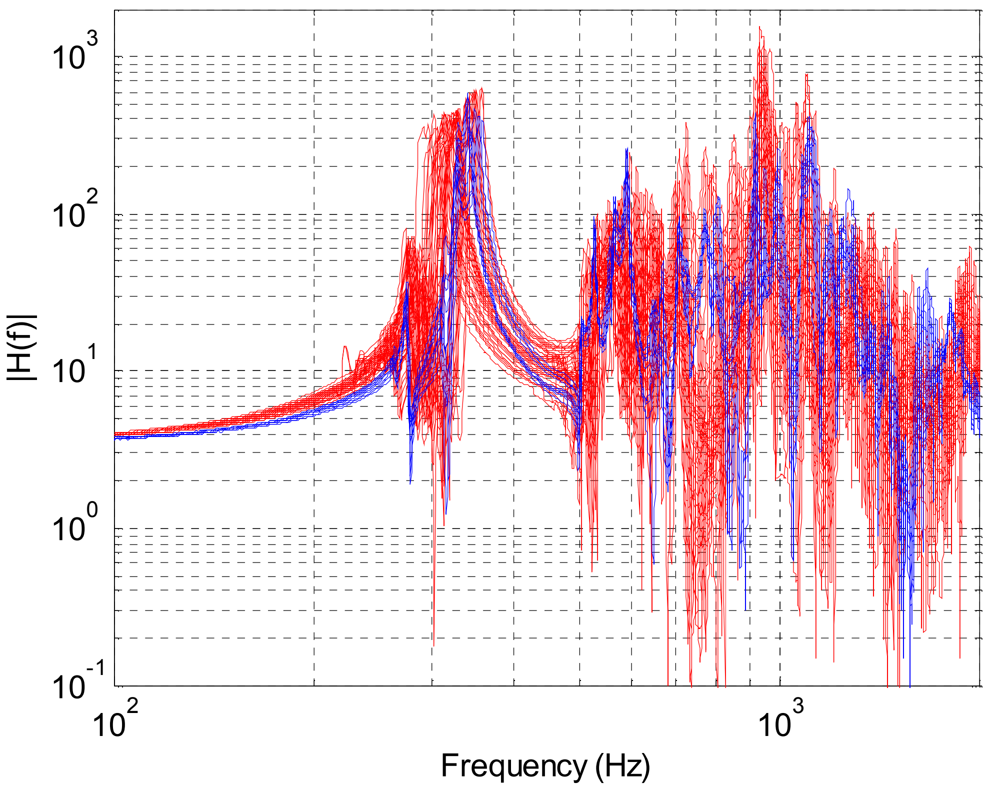

Figure 20: Validation Point 1, All Model Results (red) and Experimental Results (blue)

Another interesting point of comparison is that of the mean, or calibrated analytical model. This model uses the mean values from each parameter distribution. Figure 21 shows the calibrated model overlaid with the experimental results. These results match remarkably well. With the exception of one frequency range and a few peaks, the model data lies on top of the experimental data. But what about the frequency range in which that data doesn't match? Does that make the model inaccurate? Perhaps. Since we have test data from only one structure, there is no way to know the true variability of the assembly. Another structure could easily match in those frequencies but miss in others. Because there is uncertainty in our experimental data, uncertainty must be included to our model. And even though the mean value is a good match to our experimental data, we can't use this realization for all future analyses because the model was validated with uncertainty. To use only the mean model realization, this one model would have to be validated to the experimental data, and as stated before, this is only one copy of the structure. Variability is expected between copies.

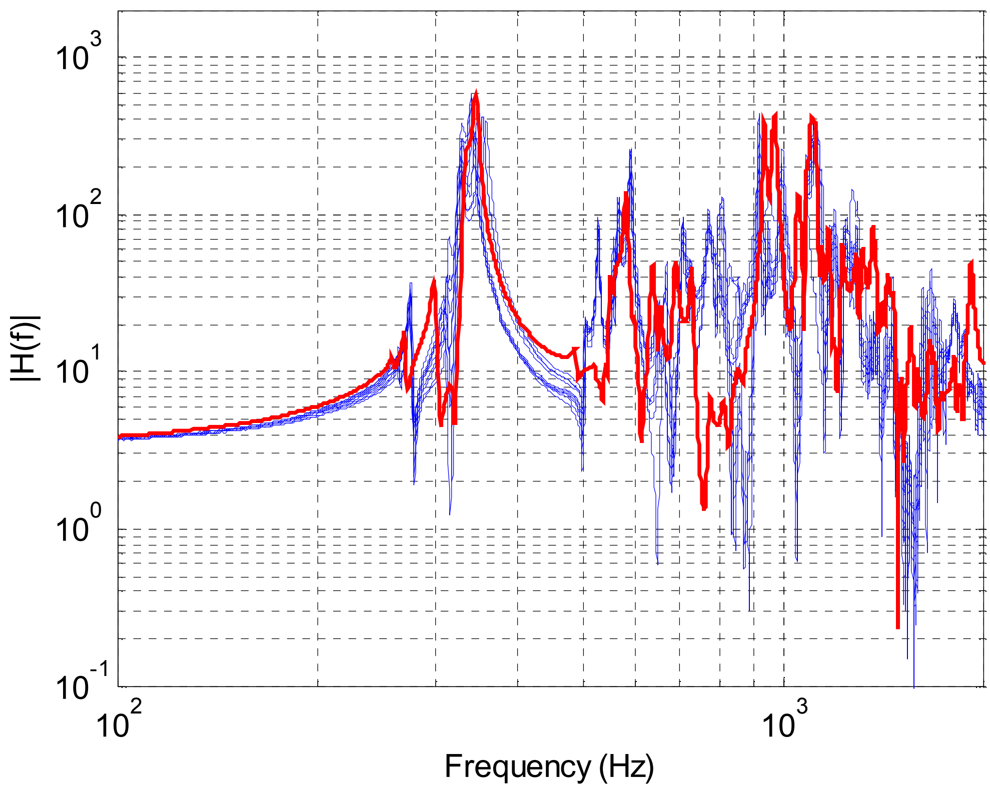

Figure 21: Validation Point 1, Calibrated Model Results (red) and Experimental Results (blue)

Although uncertainty in modeling is important, it is possible to add too much uncertainty. Some uncertainty can be reduced by more information, such as additional pieces of test data like part dimensions or material properties. This increases confidence in the data and should reduce uncertainty. But uncertainty will not be reduced completely because of inherent variability in these properties. Uncertainty was added to six properties in the model: two sheet metal thicknesses, one adhesive modulus and three joint stiffness parameters. The sheet metal thicknesses used standard tolerances, so that's a reasonable gauge of uncertainty. Both the adhesive modulus and joint stiffness uncertainties covered at least an order of magnitude. These would both be good areas in which to gather more data in order to try to reduce the magnitude of the uncertainty and then reassess the model.

Obviously, not every area of uncertainty was addressed in this model. Some areas which are more difficult to assess with a finite element model are the degree of geometric simplification and linearity. When geometry is simplified for meshing, it is difficult to take a step back and add those features back in for an uncertainty study. Also, as errors are found after geometry has been meshed, the mesh is often corrected at the node and element level, rather than going back to the geometry to correct and then remesh. These uncertainties are not addressed in this analysis. Also, an assumption of linearity is used for modal analyses such as this one. At low input levels this is a quite reasonable assumption, but as loads increase this may no longer be valid. So the experimental results from this modal test are fairly linear, but the real environmental loads may drive nonlinearities in hardware that are not captured in the model. This is a large area of uncertainty which was not captured by this analysis.

CONCLUSION

The model validation process for this complex aerospace structure included solution verification, uncertainty quantification and two validation steps comparing experimental results to model results. The level of uncertainty included in the model affects how easily the model is validated, but also affects the uncertainty in future predictions using the model, and therefore uncertainty should be minimized if possible. The model was validated to an adequacy criterion of ±6dB across the full frequency range and to a tighter adequacy criterion of ±3dB in the operational frequency ranges of interest. This model will be joined to models of the remaining structure and components to become a full system model, which in turn will need to be validated against experimental data.

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