

The Derivation of Random Vibration Specifications from Field Test Data for Use With a Six Degree-of-Freedom Shaker Test*

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**Jerome S. Cap
Member of the Technical Staff (SNL)**

**Dr. D. Gregory Tipton
Member of the Technical Staff (SNL)**

**David O. Smallwood
Consultant**



Background

- **Multi-Degree-of-Freedom (MDOF) shaker testing has the potential for being both more realistic and faster than traditional Single-Degree-of-Freedom (SDOF) shaker testing.**
- **It is rare that there are sufficient channels of field test data for use in developing MDOF test specifications.**
- **Sandia has recently completed a study aimed at developing a practical set of component test specifications for use with a 6-DOF shaker using a combination of flight test data and finite element analysis.**

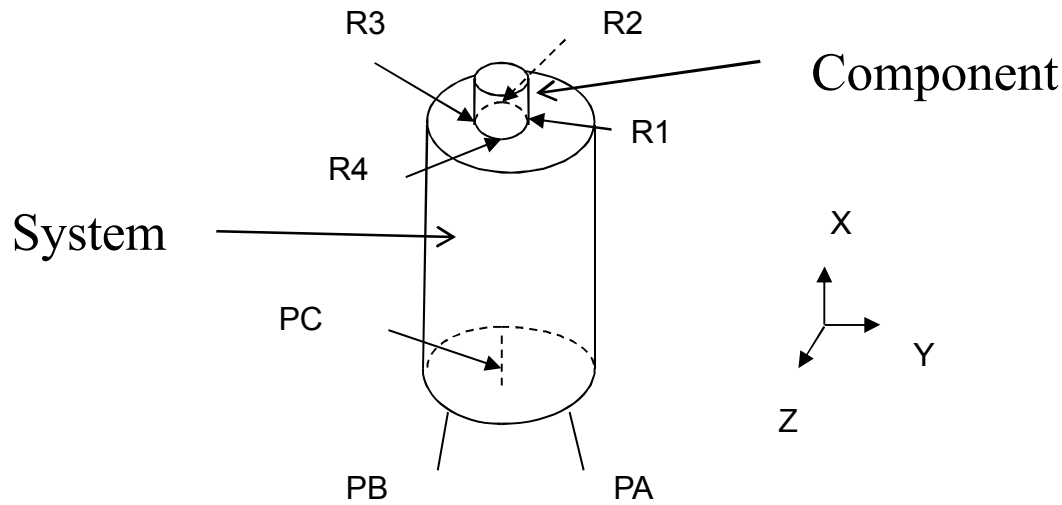


Outline

- **System Description**
- **Derivation of Transmissibility Response Functions**
- **FEA Model**
- **Derivation of System Inputs**
- **Derivation of Component Inputs**
- **Test Results**
- **Lessons Learned**

System Description

- The system inputs are PA, PB, and PC.
- The component inputs are R1, R2, R3, and R4.



Transmissibility Response Functions

The A/A Transmissibility Response Functions (TRFs) were defined using equation {1}. The application of this equation for Spectral Densities is defined in equation {2}.

$$\begin{Bmatrix} R1X \\ R1Y \\ R1Z \\ R2X \\ R2Y \\ R2Z \\ R3X \\ R3Y \\ R3Z \\ R4X \\ R4Y \\ R4Z \end{Bmatrix} = \begin{bmatrix} H_{R1XPAX} & \cdots & H_{R1XPCZ} \\ \vdots & \ddots & \vdots \\ H_{R4ZPAX} & \cdots & H_{R4ZPCZ} \end{bmatrix}_{size\ 12 \times 9} \begin{Bmatrix} PAX \\ PAY \\ PAZ \\ PBX \\ PBY \\ PBZ \\ PCX \\ PCY \\ PCZ \end{Bmatrix} \quad \text{Eqn \{1\}}$$

$$S_{RR(12 \times 12)} = H_{RP(12 \times 9)} S_{PP(9 \times 9)} H'_{RP(12 \times 9)} \quad \text{Eqn \{2\}}$$



FEA Model

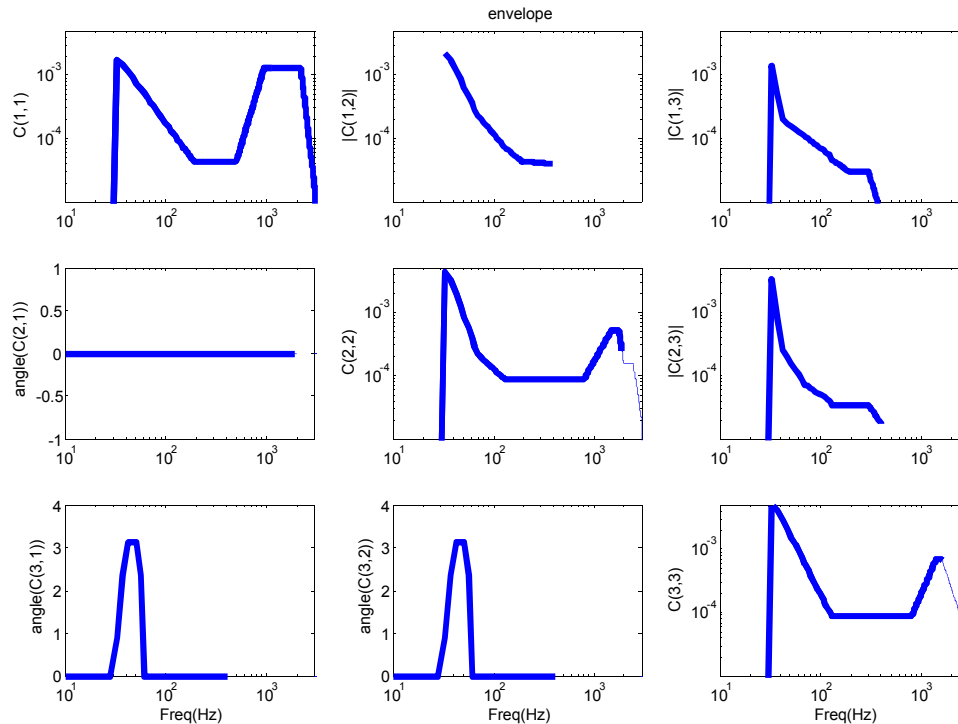
- **The TRFs were derived using a Finite Element Model.**
- **A force was applied sequentially to the base of each pedestal in the model with a large inertial mass attached to that base.**
 - **In this manner we obtained the input acceleration.**
 - **TRFs were defined for each of the 12 component input points/directions with respect to each of the pedestal (system) input points/directions.**



Derivation of System Inputs

- Data were available from two flight tests.
 - One flight with a tri-axial accelerometer located at one of the pedestals.
 - One flight with 3 uni-axial accelerometers measuring axial acceleration at each of the pedestals.
 - Only one accelerometer common to both flights
 - The levels were quite different for the two flights
 - The data have been normalized so that the highest response is 1 Grms
- In theory we could derive 5 rigid body modes from this data
 - Missing is rotation about axial direction
- Major problems
 - How do you combine data from different flights?
 - Subject of future study, but not included in this paper
 - Some of the data was quite noisy
 - Making the determination of cross spectra phase relationships difficult

Final System Input



1. Enveloped ASD's
2. Enveloped coherence
3. Simplified phase
4. Constructed SDM
(Spectral Density Matrix)
from ASD and coherence
5. Checked to make sure
result was positive definite

Frequency axis 10-3000Hz; Vertical axis $1e-5$ - $5e-3$ g²/Hz
Diagonal ASD: upper triangle, magnitude of CSD;
lower triangle, phase (0-4) radians



Derivation of Component Inputs

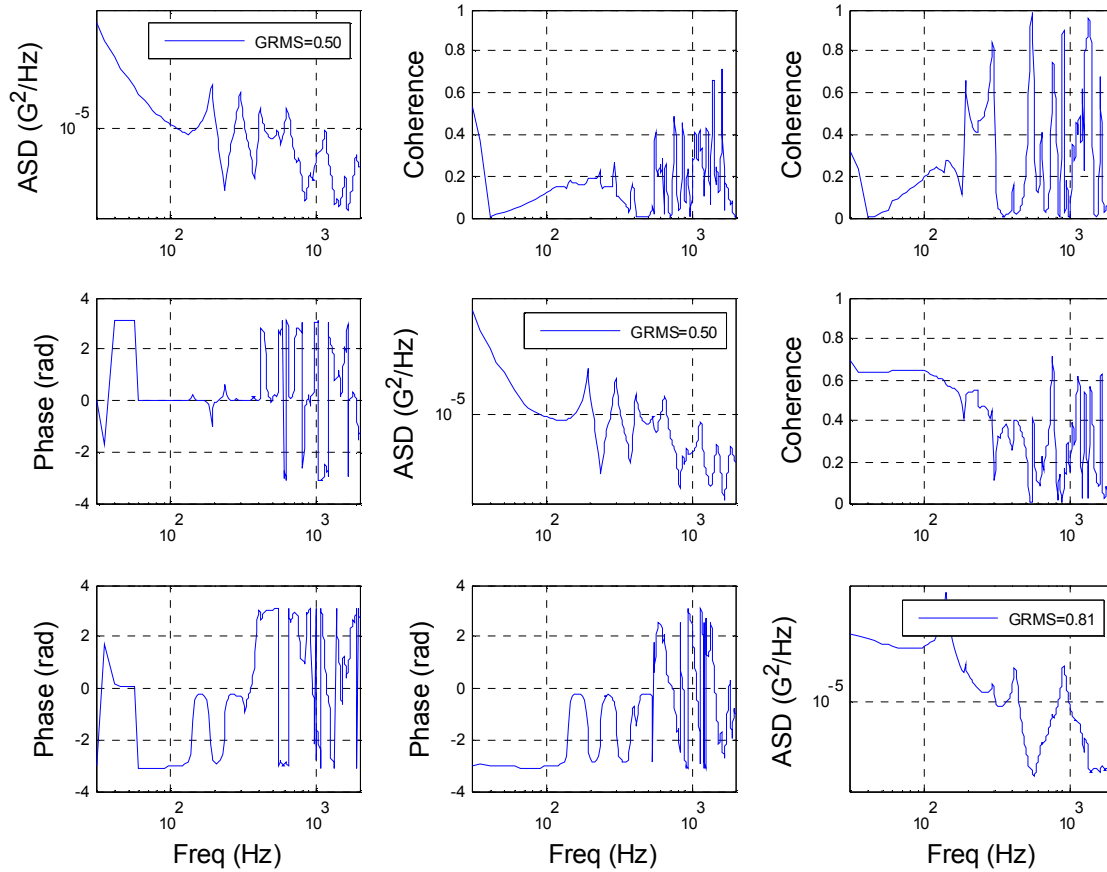
- Realized that given only 3 independent DOF for system input we could not generate 12 component inputs.
- Collapsed TRFs from a 12x9 matrix to a 12x3 matrix by summing columns associated with like input orientations, thus yielding equation {3}.

$$S_{RR(12 \times 12)} = H_{RP(12 \times 3)} S_{PP(3 \times 3)} H'_{RP(12 \times 3)} \quad \text{Eqn \{3\}}$$

- Collapsed 12x12 component input matrix to a 3x3 matrix using equation {4} where H_T is a transformation matrix used to average the four in-axis responses.

$$S_{RR(3 \times 3)} = H_T S_{RR(12 \times 12)} H'_T \quad \text{Eqn \{4\}}$$

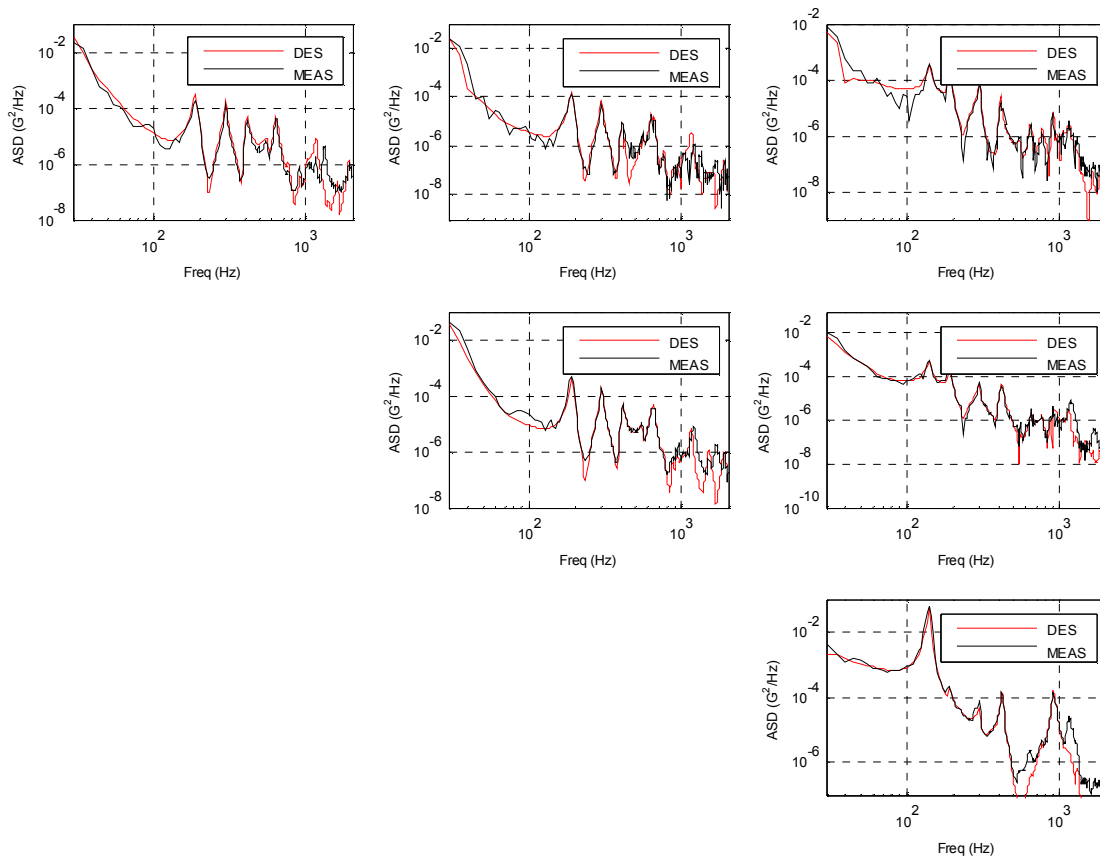
Component Inputs





Test Results

These plots compare the desired and achieved 6-DOF test control spectra.





Lessons Learned

- **You can't specify more DOF for the component inputs than are present in the system inputs.**
 - With additional work, it may be possible to incorporate the data from the 2nd flight in order to increase the DOF from 3 to 5.
- **The order in which the system and component inputs were specified were different.**
 - While the algebra needed to adjust the CSD matrix to account for this is trivial, it does represent an issue that needs close attention.
- **The original FEA results were provided using 4 Hz resolution. The closest permissible resolution for the shaker was 5 Hz.**
 - Initial attempts to interpolate caused the resulting CSD matrix to no longer be positive definite.



Conclusions

- **Within the constraints imposed by the sparseness of the available system input data we were successful in developing a workable 3DOF component test specification.**
- **Need systematic methodology for developing compact CSD inputs.**
 - **Eliminate ragged coherence and phase while still preserving positive definiteness.**