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Total Hourglass Control: Theory, Analysis, and Examples

Jesse Thomas and Vicki Porter

Sandia National Laboratories*
Albuquerque, New Mexico, USA

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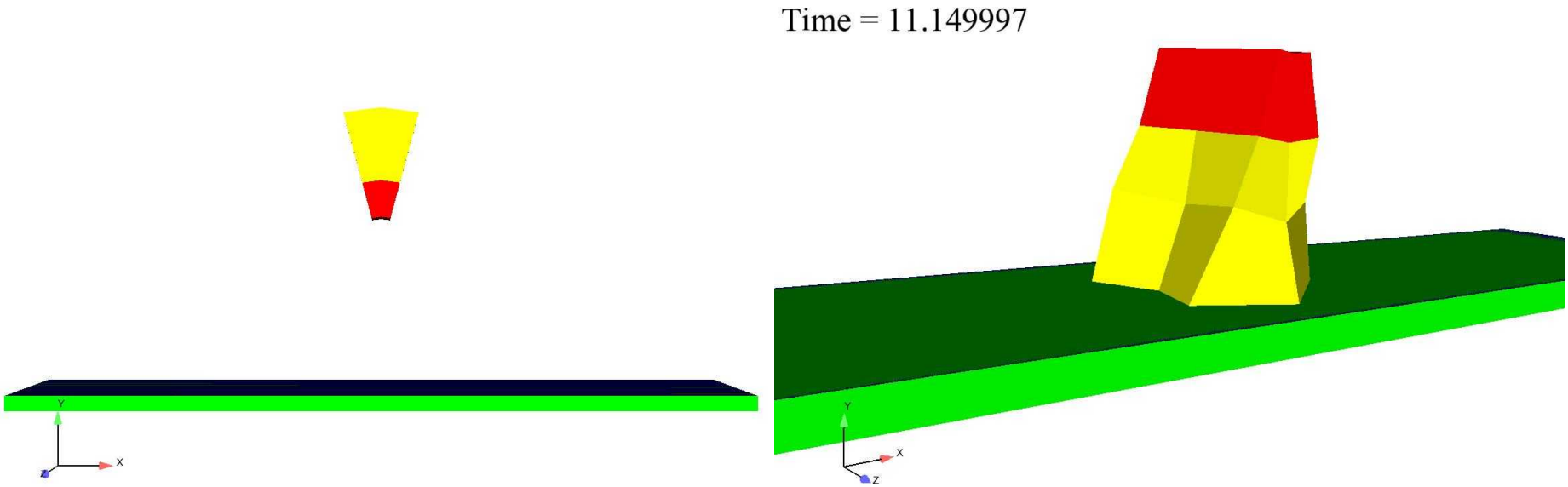
Total Hourglass Control: Motivation

- Of interest is the hourglass control in the Sandia Solid Mechanics codes Presto and Adagio for the strongly objective (SO) uniform gradient hexahedral solid element
- Hourglass control typically uses an incremental formulation (Flanagan, D.P. and Belytschko, T., IJNME, 1981) that introduces errors under large rotations and deformations

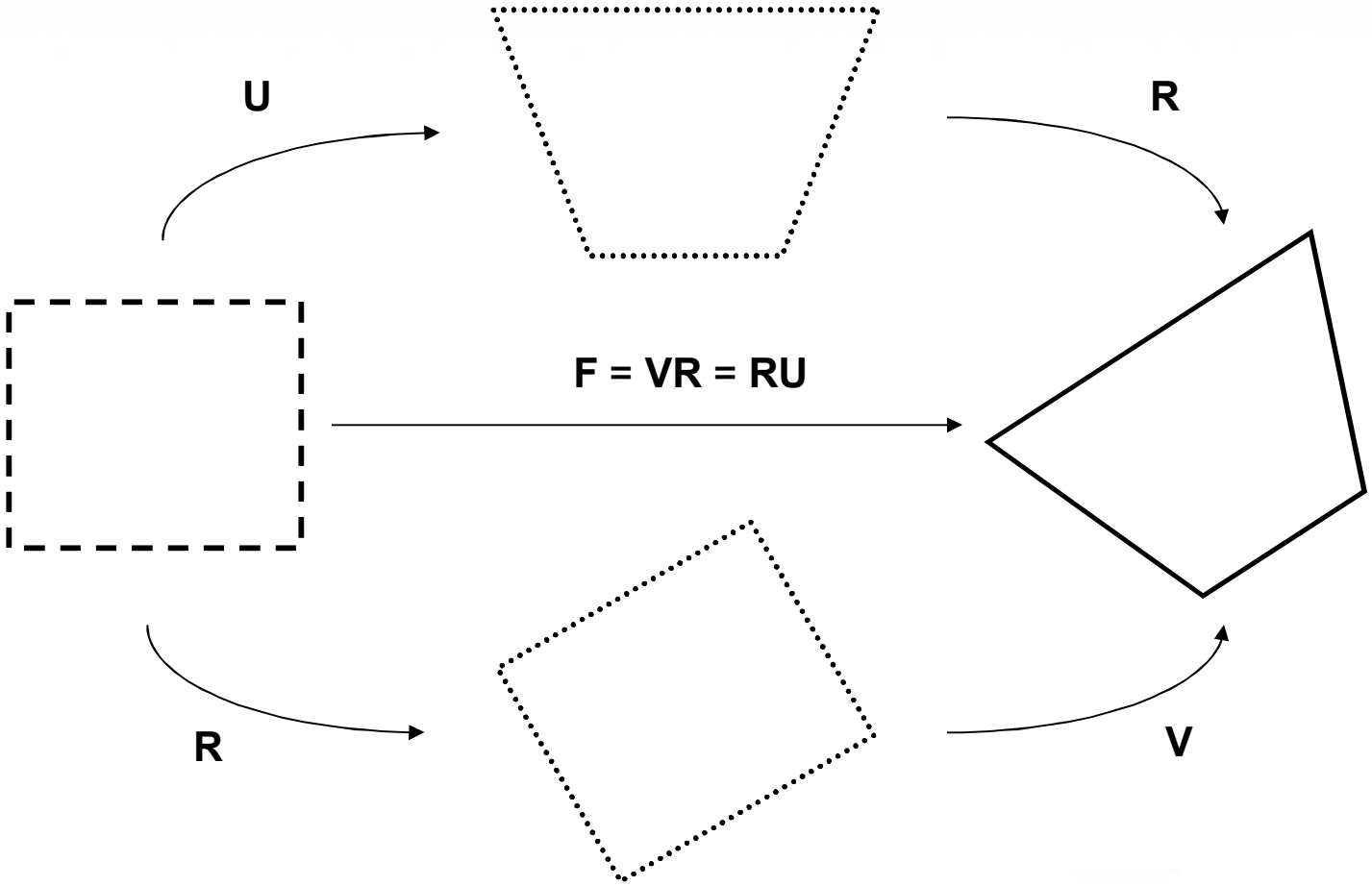


Total Hourglass Control: Motivation

- Options for hourglass control
  - Incremental hourglass formulation (Flanagan, D.P. and Belytschko, T., IJNME, 1981)
  - Total hourglass formulation
  - Reference configuration formulation (Hutter, R. et al., CMAME, 2001)
  - Selective reduced integration
  - Many others … (e.g. using subzonal pressures, explicit filtering, consistent-mass projection, moving least squares filtering, etc. See Christon, M.A., et al. 2007, LAUR-07-7514, Los Alamos National Laboratories)
- The incremental hourglass formulation is compared to the novel total hourglass formulation

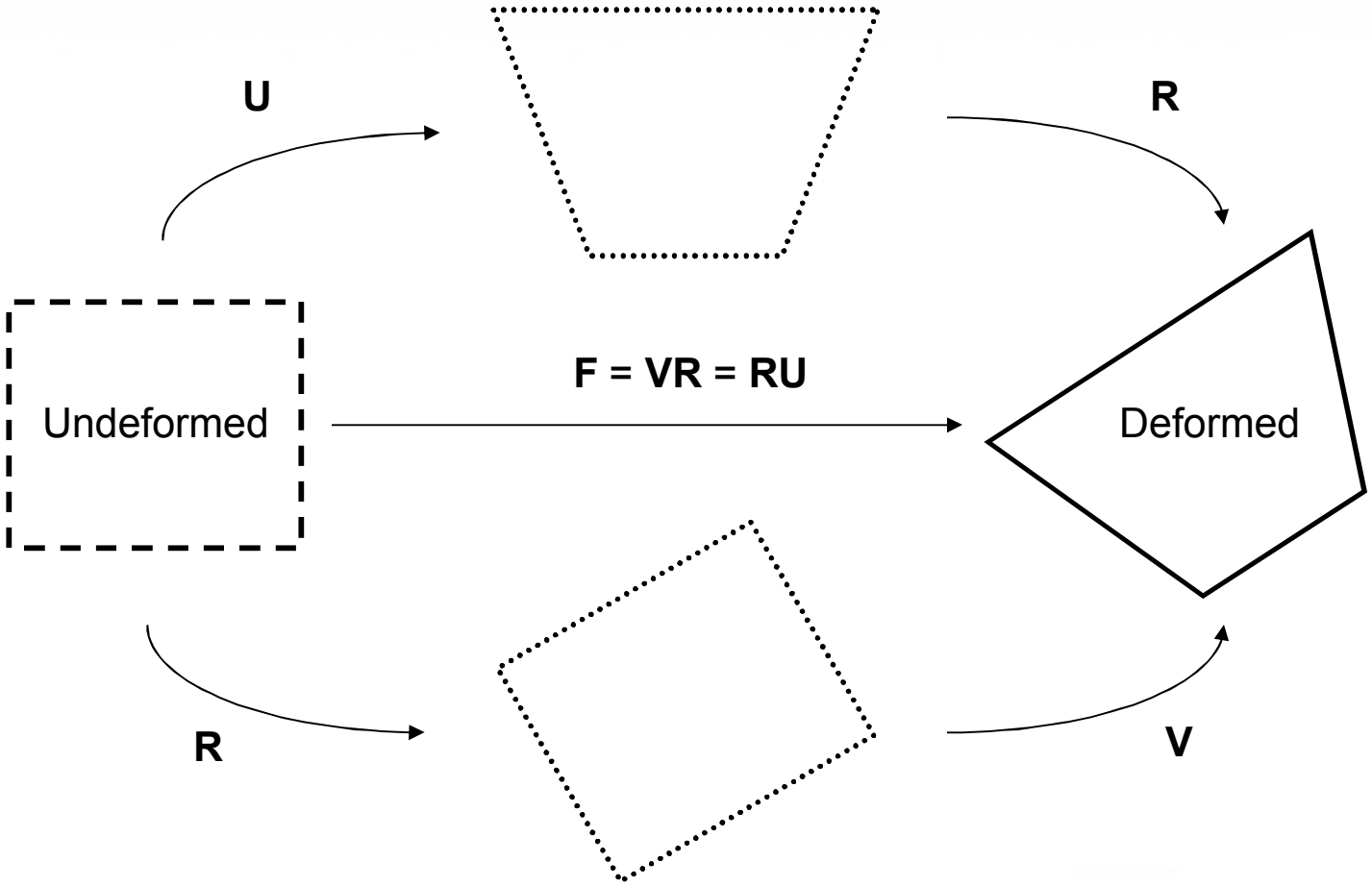
Total Hourglass Control: Theory

- Configurations



Total Hourglass Control: Theory

- Configurations



Total Hourglass Control: Theory

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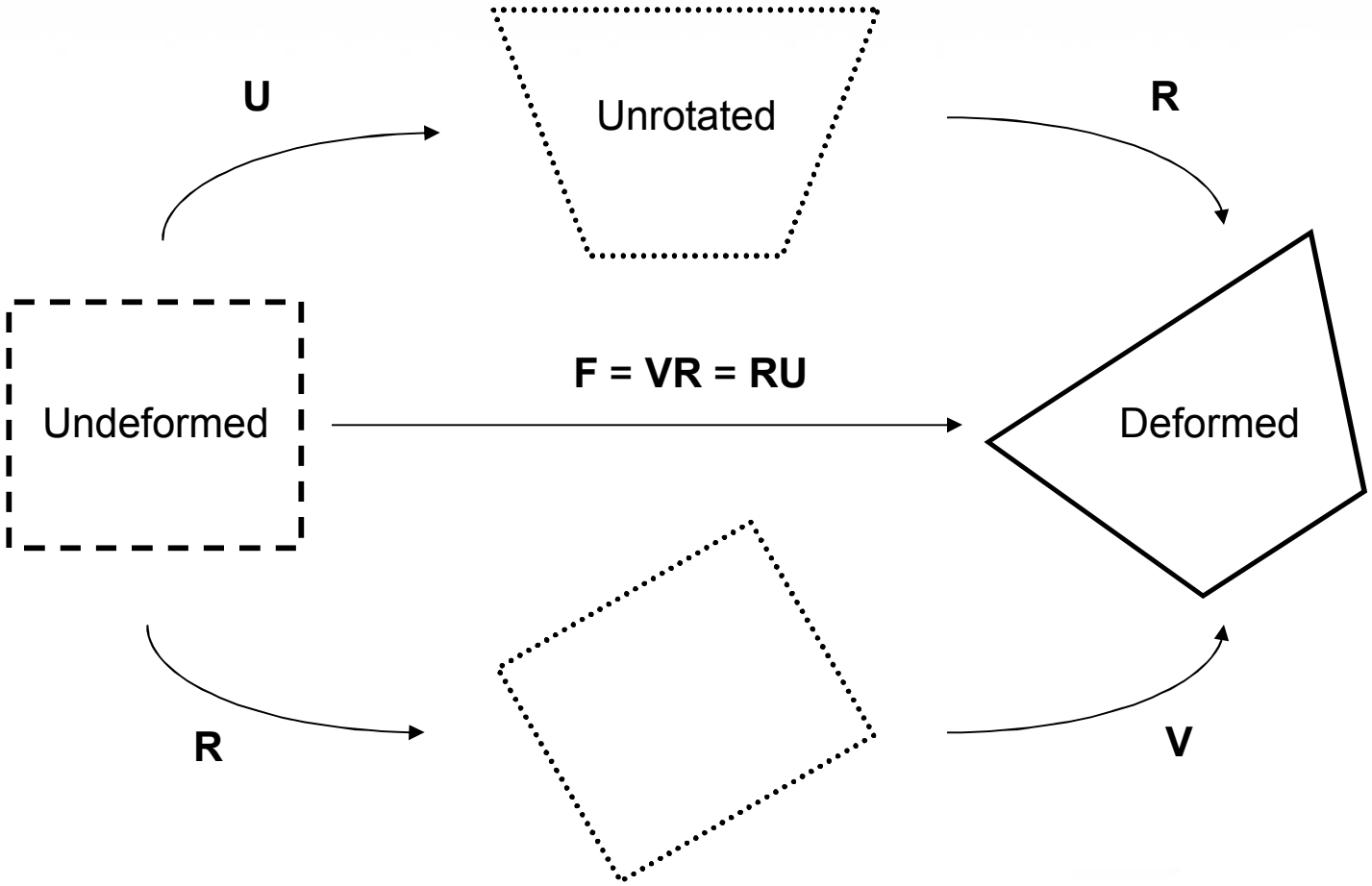


Total Hourglass Control: Theory

- Incremental formulation (ignoring viscosity)
  - Hourglass forces computed in the unrotated configuration by addition of previous step hourglass forces and an increment based on the velocity field

$$\dot{q}_{i\alpha} = \frac{1}{V} v_{iI} \gamma_{\alpha I}$$

$$\gamma_{\alpha I} = \frac{V}{\delta}\left(\Gamma_{\alpha I} - \frac{1}{V} x_{jJ} \Gamma_{\alpha J} B_{jI} \right)$$

$$\Delta Q_{i\alpha} = \varepsilon \Delta t \left(2G^{\tan} \delta_{ij} \delta_{\alpha\beta} \right) \dot{q}_{j\beta}$$

$$Q_{i\alpha}\big|_{n+1} = R_{ij}\left(Q^{u}_{j\alpha}\big|_{n} + R^{-1}_{jk} \Delta Q_{k\alpha} \right)$$

  - Incremental updates do not guarantee orthogonality with the mean strain
  - Hourglass operator $\gamma_{\alpha I}$ is not invariant under rigid body rotations

Total Hourglass Control: Theory

- Eigenvalue and eigenvector algorithms for polar decomposition
  - Jacobi diagonalization
    - Iterative
    - Robust
    - Widely used and tested
  - Analytic, small perturbation algorithm (Scherzinger, W.M. and Dohrmann, C.R., CMAME, 2008)
    - Analytic solution
    - Robust
      - Uses the Wilkinson shift
      - Accurate for small eigenvalues and very close eigenvalues
    - Less widely used and tested
    - Compares favorably with multiple widely used algorithms

Total Hourglass Control: Theory

- Total formulation (ignoring viscosity)
  - The hourglass forces computed from a total deformation calculated using the total rotation R_{ij}

$$\hat{d}_{ij} = x_{iI} - R_{ij} X_{jI}$$

$$q_{i\alpha} = \frac{1}{V} \hat{d}_{iI} \gamma_{\alpha I} \quad \left(= \frac{1}{V} R_{ij} u_{jI} \gamma_{\alpha I} \right)$$

$$\gamma_{\alpha I} = \frac{V}{\delta} \left(\Gamma_{\alpha I} - \frac{1}{V} x_{jJ} \Gamma_{\alpha J} B_{jI} \right)$$

$$Q_{i\alpha} = \varepsilon \left(2G^{\tan} \delta_{ij} \delta_{\alpha\beta} \right) q_{j\beta}$$

  - This should eliminate the accumulation of error due to incremental updates of hourglass forces

Total Hourglass Control: Analysis

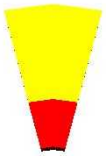
- The total hourglass formulation is appropriate for the Presto/Adagio SO element
  - The SO element employs the total deformation gradient and strongly objective incremental kinematics (Rashid, M.M., IJNME, 1993)
  - The total rotation is calculated in the SO element regardless of the hourglass formulation employed
- The numerical cost of the two hourglass formulations is similar
  - The incremental formulation requires a backward rotation of the hourglass force increment and then a forward rotation of the hourglass forces
  - The total formulation requires a forward rotation of the reference coordinates (or displacement) and an additional application of the hourglass operator if hourglass viscosity is employed
  - For the SO element, the only additional cost for the total formulation is the differencing of current coordinates and rotated reference coordinates (which can be avoided)

Total Hourglass Control: Examples

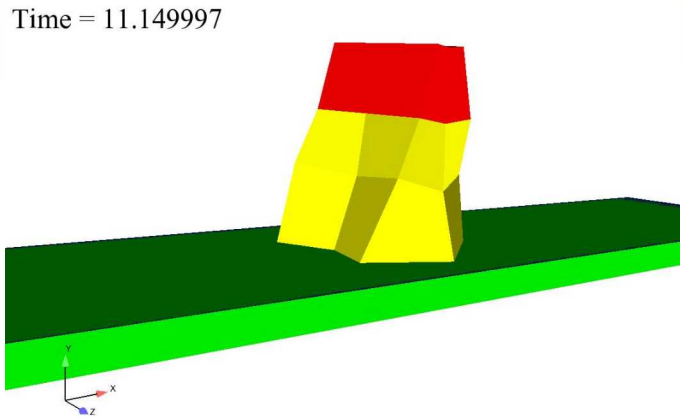
- Both formulations are implemented in Presto/Adagio
  - The incremental formulation is the default for the SO element and is the only available formulation for the other solid elements
  - The total formulation is an option for the SO element activated by a line command
- Two example problems
  - Rotating wedge with contact
    - Extremely large rotations and large deformation
    - Shows improvement with total hourglass formulation
  - Plate punch
    - Contact between punch and plate with moderate deformation
    - Shows total formulation is sensitive and not currently viable for certain problems

Total Hourglass Control: Examples

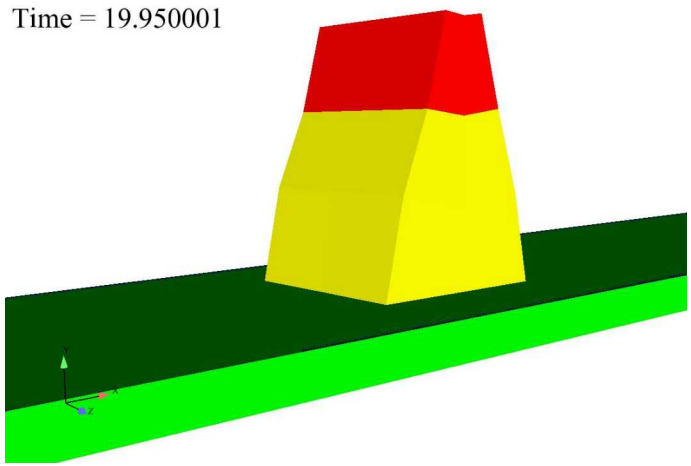
- Rotating wedge with contact



Incremental formulation

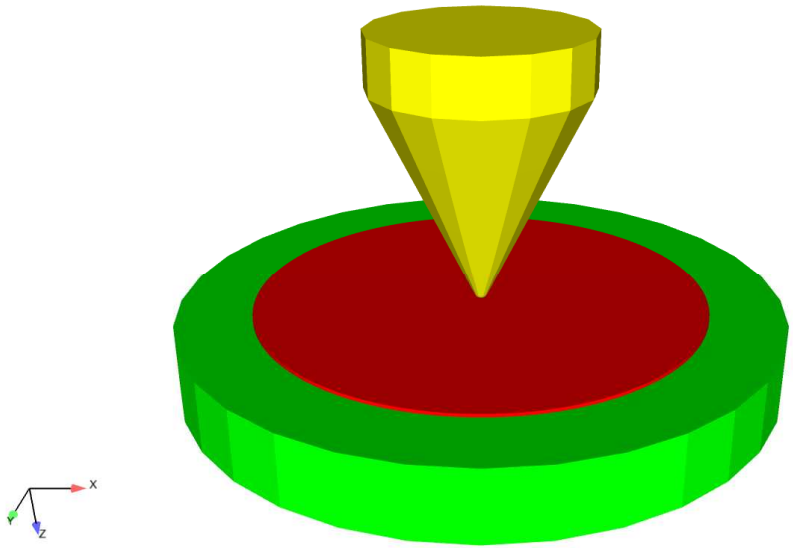


Total formulation

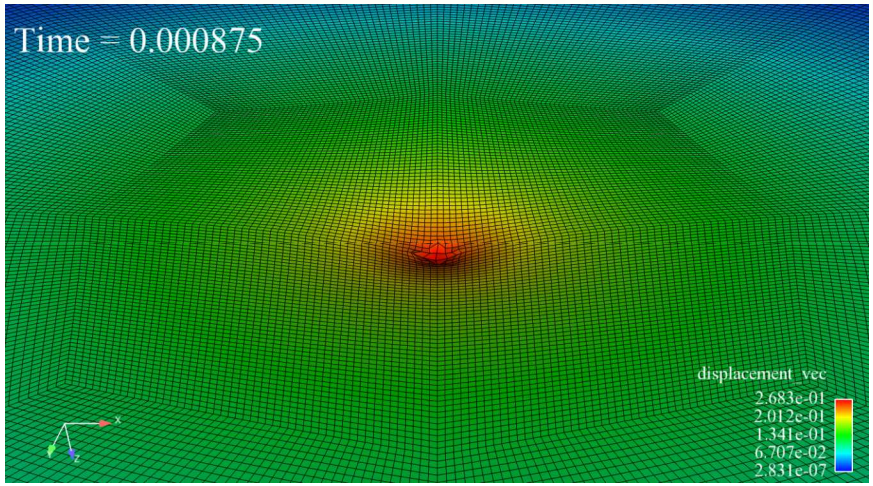


Total Hourglass Control: Examples

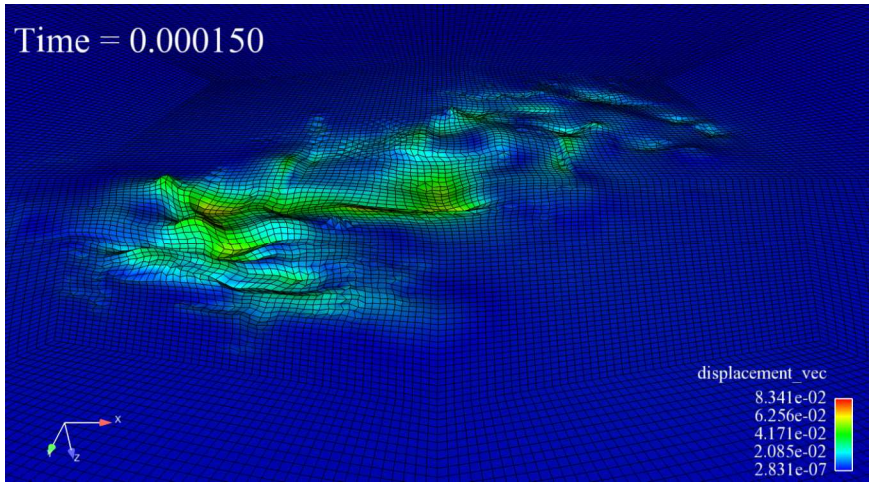
- Plate punch



Incremental formulation



Total formulation



Total Hourglass Control: Examples

- Numerical cost for a particular problem involving large rotations

	Incremental Formulation	Total Formulation	Increase
Total Runtime	514 hrs 12 min	524 hrs 54 min	2.1 %
Internal Force Computation Time	110 hrs 33 min	117hr 40 min	6.4 %

Total Hourglass Control: Conclusions

- Total hourglass formulation often shows improvements
  - Large rotation, large deformation problems
  - Little change in results for most problems
  - Significant improvements for many problems (e.g. rotating wedge)
- Certain problems display unacceptable error
  - Strong hourglass excitement (e.g. through contact)
  - The incremental formulation often shows error as well

Total Hourglass Control: Conclusions

- Investigation of total formulation
  - The uniform gradient hex gradient operator B_{ij} and hourglass deformations $q_{i\alpha}$ are not orthogonal
    - The one-point integration gradient operator is orthogonal to the hourglass deformations but its use did not show improvement for the plate punch problem
    - Alternative methods of calculating the hourglass deformations are currently under consideration
  - The total hourglass formulation is sensitive to the eigenvector decomposition (used to calculate the total rotation R_{ij} and for the incremental kinematics)
    - No clear dependence on various eigenvector decomposition algorithms
    - More investigation is required

Total Hourglass Control: Conclusions

- Future work
  - Combine incremental and total formulations
  - Integrate rotation R_{ij} incrementally
  - Reformulate SO element to calculate in the reference configuration