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Viscoplasticity Using Peridynamics

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by

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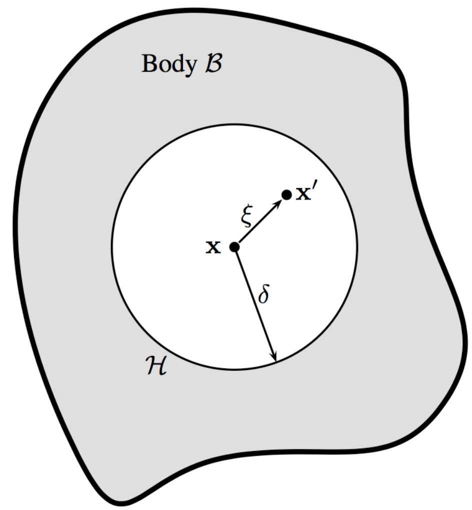
Outline

- Bond-Based Peridynamics
- State-Based Peridynamics
- Kinematics
- Constitutive Model
- Comparison to Experimental Data
- Future Work

Bond-Based Peridynamics

Peridynamic “Bond-based” Model (2000)

$$\rho\ddot{\mathbf{u}}[\mathbf{x},t] = \int_{\mathcal{H}} \mathbf{f}(\mathbf{u}[\mathbf{x}',t] - \mathbf{u}[\mathbf{x},t], \mathbf{x}' - \mathbf{x})dV_{\mathbf{x}'} + \mathbf{b}[\mathbf{x},t]$$



Shortcomings:

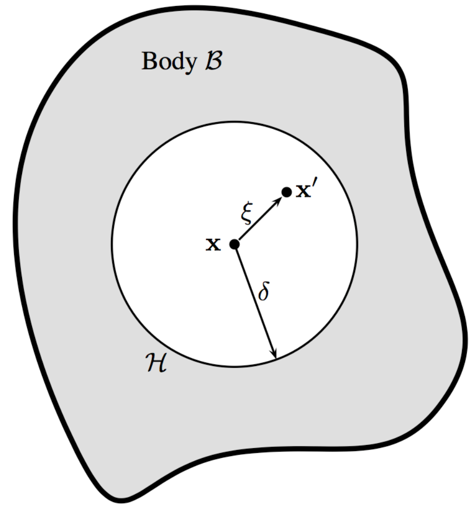
- Must recast constitutive model in terms of pair-wise force functional (as opposed to stress-strain)
- Poisson ratio 1/4 (not entirely true)
- Inelastic behavior a result of volumetric strain (not always physical, think J_2 metal)

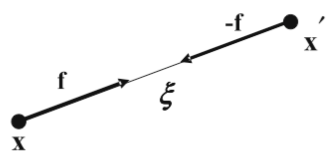
Ref: SA Silling. Reformulation of Elasticity Theory for Discontinuities and Long-Range Rorces. *Journal of the Mechanics and Physics of Solids*, 48(1):175–209, 2000.

State-Based Peridynamics

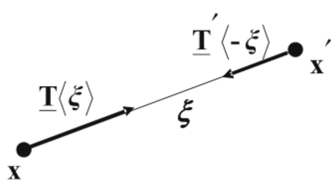
Peridynamic “State-based” Model (2007)

$$\rho\ddot{\mathbf{u}}[\mathbf{x},t] = \int_{\mathcal{H}} \{\underline{\mathbf{T}}[\mathbf{x},t]\langle\mathbf{x}'-\mathbf{x}\rangle - \underline{\mathbf{T}}[\mathbf{x}',t]\langle\mathbf{x}-\mathbf{x}'\rangle\} dV_{\mathbf{x}'} + \mathbf{b}[\mathbf{x},t]$$

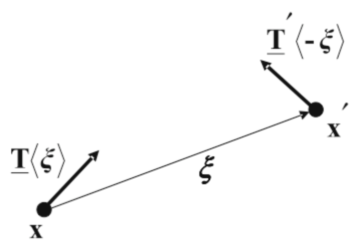




Bond based



Ordinary state based



Non-ordinary state based

Ref: S.A. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari. Peridynamic States and Constitutive Modeling. *Journal of Elasticity*, 88:151-184 (2007).

Constitutive Models

Provide Force-Vector State Field in Terms of Deformation Vector State

$$\underline{\mathbf{T}} = \hat{\underline{\mathbf{T}}}(\underline{\mathbf{Y}}, \Lambda)$$

Where, $\underline{\mathbf{Y}} = (\mathbf{x}' + \mathbf{u}(\mathbf{x}', t)) - (\mathbf{x} + \mathbf{u}(\mathbf{x}, t))$

Examples:

Linear Peridynamic Fluid

$$\underline{t} = \frac{3k\,\theta}{m} \underline{\omega}\, \underline{x}$$

Linear Peridynamic Solid

$$\underline{t} = \frac{3k\,\theta}{m} \underline{\omega}\, \underline{x} + \frac{15\mu}{m} \underline{\omega}\, \underline{e}^d$$

Ref: S.A. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari. Peridynamic States and Constitutive Modeling. *Journal of Elasticity*, 88:151-184 (2007).

Given $\underline{\mathbf{Y}}$ can we define a deformation gradient, $\mathbf{F}$?

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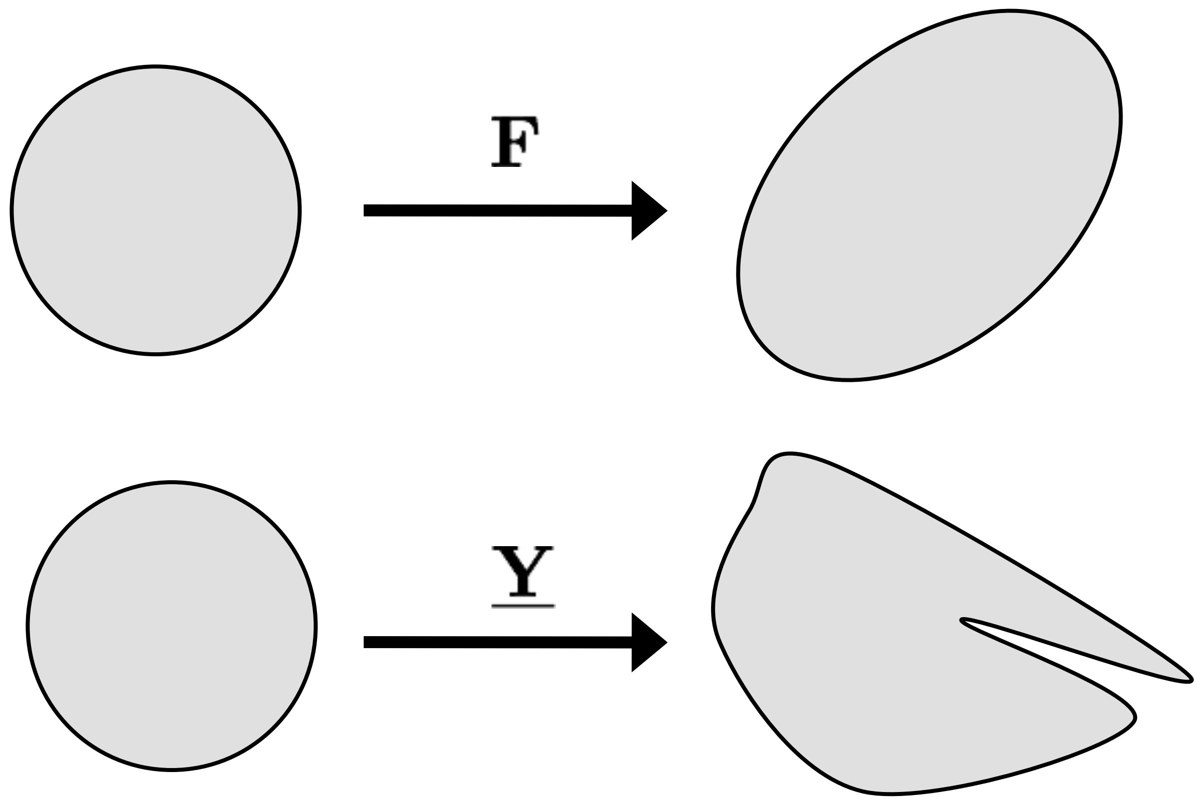
Def. Gradient vs. Def. State



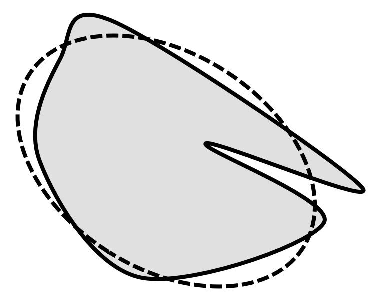
Kinematics

Shape Tensor

$$\mathbf{K}[\mathbf{x}, t] = \int_{\mathcal{H}} \omega\left(|\boldsymbol{\xi}|\right)\left(\boldsymbol{\xi} \otimes \boldsymbol{\xi}\right) \mathrm{d}V_{\boldsymbol{\xi}}$$

Deformation Gradient Tensor

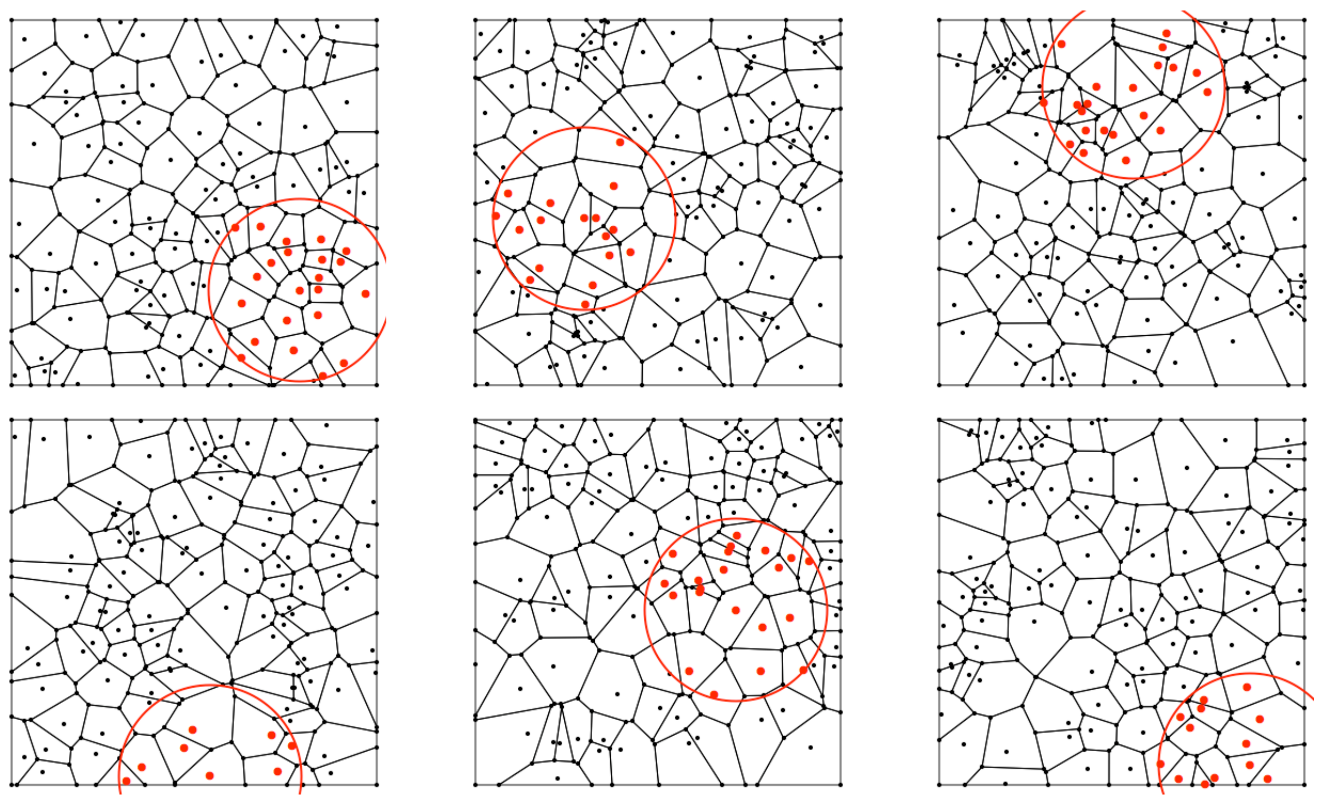
$$\mathbf{F}[\mathbf{x}, t] = \left[\int_{\mathcal{H}} \omega\left(|\boldsymbol{\xi}|\right)\left(\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle \otimes \boldsymbol{\xi}\right) \mathrm{d}V_{\boldsymbol{\xi}}\right] \mathbf{K}^{-1}$$



Ref: S.A. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari. Peridynamic States and Constitutive Modeling. *Journal of Elasticity*, 88:151-184 (2007).

How well does this work?

Test



Assume, $\underline{\mathbf{Y}} = \mathbf{F}_{\text{in}}\, \boldsymbol{\xi}$ Calculate $\mathbf{F}_{\text{out}} = \left[\sum_{j=1}^{k} \omega\left(|\boldsymbol{\xi}|\right) \left(\mathbf{F}_{\text{in}}\, \boldsymbol{\xi} \otimes \boldsymbol{\xi}\right) V_j \right] \mathbf{K}^{-1}$

For all tests cases: $\mathbf{F}_{\text{out}} = \mathbf{F}_{\text{in}}$

Proof

Let $\underline{\mathbf{Y}}_i = F_{ip}\,\xi_p$

$$
\begin{aligned}
F_{ij} &= \left[\int_{\mathcal{H}} \omega(\xi)\, F_{ip}\,\xi_p\,\xi_k \mathrm{d}V_{\boldsymbol{\xi}}\right] K_{kj}^{-1} \\
&= F_{ip}\left[\int_{\mathcal{H}} \omega(\xi)\,\xi_p\,\xi_k \mathrm{d}V_{\boldsymbol{\xi}}\right] K_{kj}^{-1} \\
&= F_{ip}\,K_{pk}\,K_{kj}^{-1} \\
&= F_{ip}\,\delta_{pj} \\
&= F_{ij}
\end{aligned}
$$

Ref: S.A. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari. Peridynamic States and Constitutive Modeling. *Journal of Elasticity*, 88:151-184 (2007).

Conversion of Classical Models to State Based Material Models

Therefore, $\underline{\mathbf{T}} = \hat{\underline{\mathbf{T}}}(\mathbf{F})$

Given the Cauchy stress $\boldsymbol{\mathcal{T}}$ we can find the first Piola-Kirchoff stress

$$\boldsymbol{\sigma} = \det(\mathbf{F})\, \boldsymbol{\mathcal{T}}\, \mathbf{F}^{-T}$$

And the Force-vector State

$$\underline{\mathbf{T}}\langle\boldsymbol{\xi}\rangle = \omega(|\boldsymbol{\xi}|)\, \boldsymbol{\sigma}\, \mathbf{K}^{-1}\, \boldsymbol{\xi}$$

Ref: S.A. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari. Peridynamic States and Constitutive Modeling. *Journal of Elasticity*, 88:151-184 (2007).

Viscoplasticity

Yield Surface

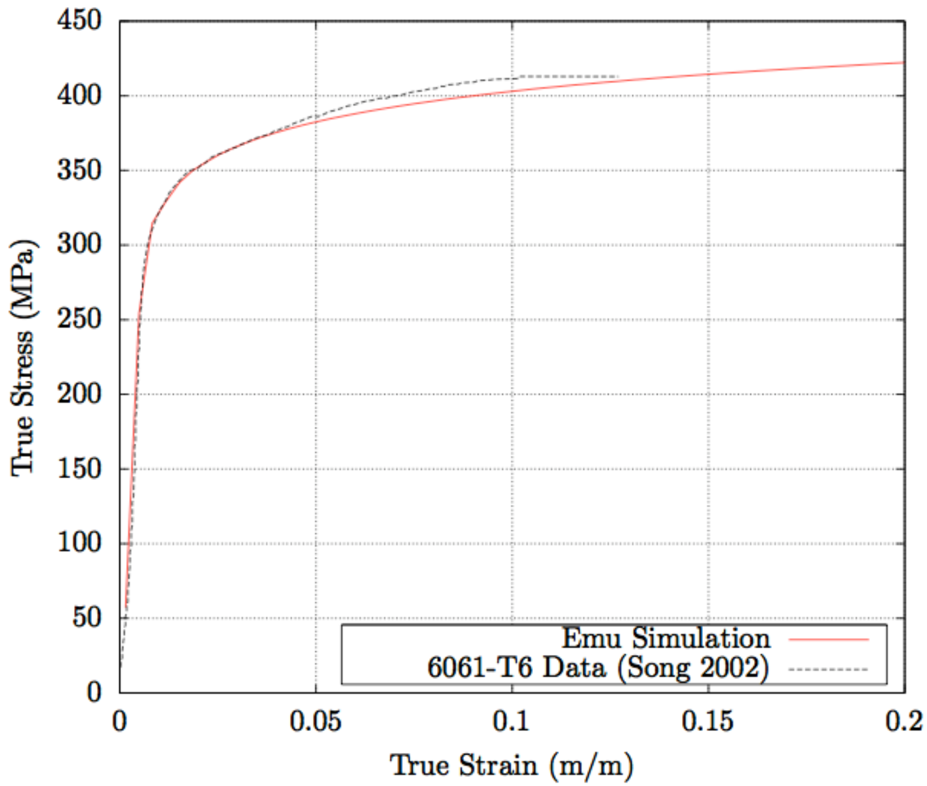
$$\sigma = g\left(1 + \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}^p_0}\right)^{1/m}$$

Where, $g = Y\left(1 + \frac{\varepsilon^p}{\varepsilon^p_0}\right)^{1/n}$ and $\dot{\varepsilon}^p = \sqrt{\frac{2}{3}}\,|\dot{\mathbf{e}}^p|$

Associated Flow Rule

$$\dot{\mathbf{e}}^p = \dot{\lambda}\,\mathbf{Q}$$

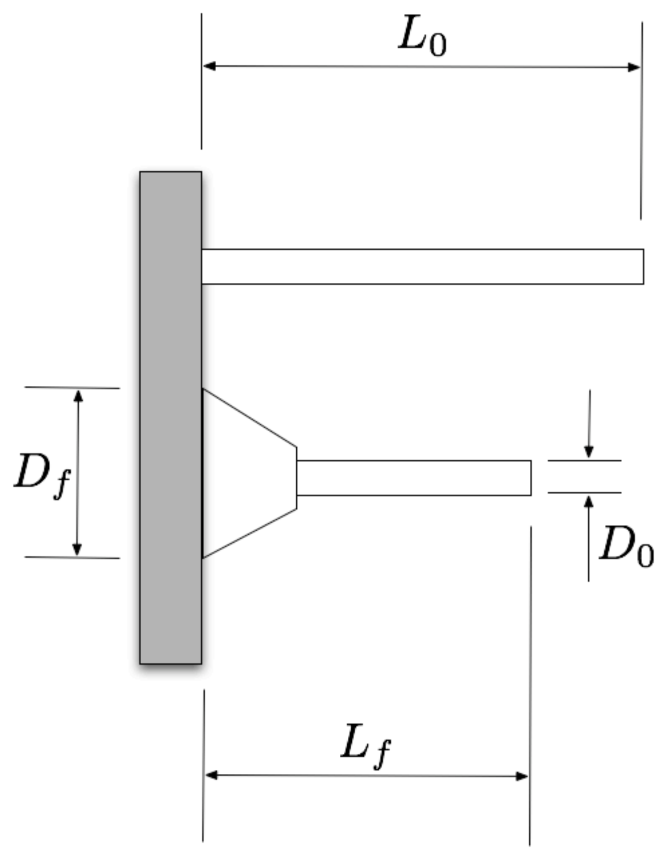
Uniaxial Tension



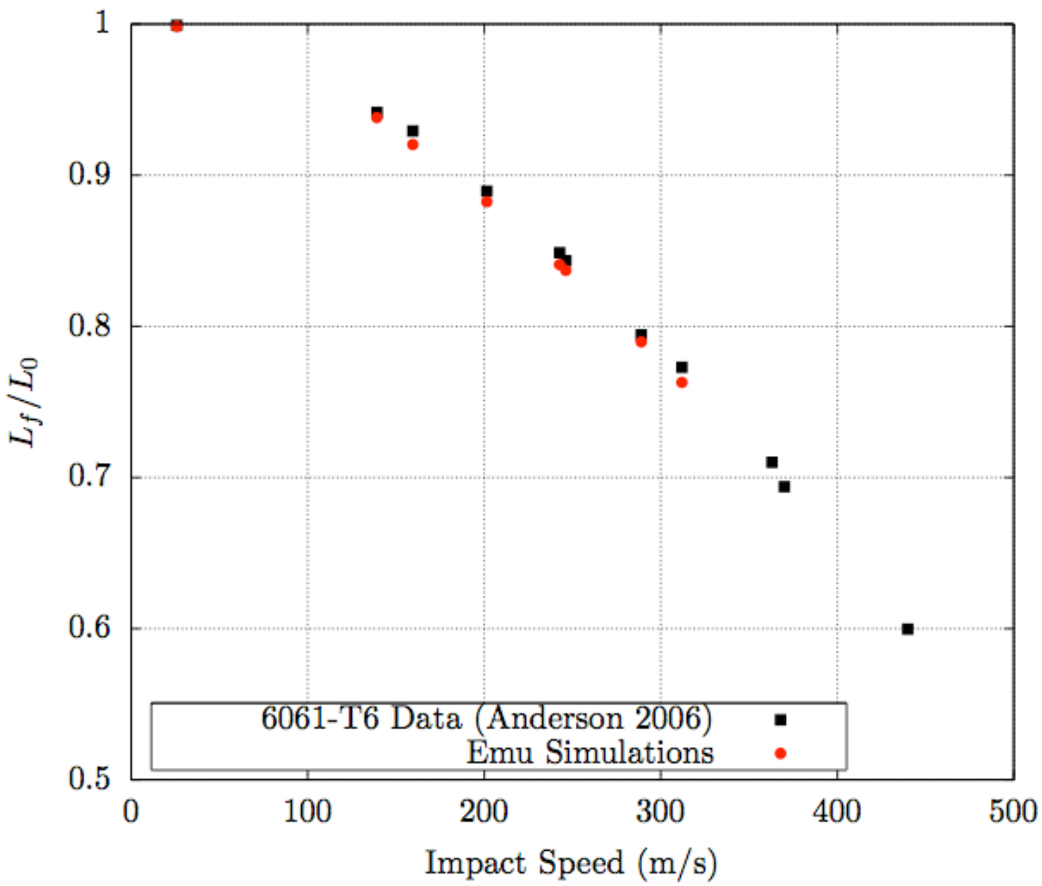
Data was taken from SHPB tests of 6061-T6 Aluminum at a nominal engineering strain rate of 8300/s

Data ref: B. Song, W.W. Chen, and D.J. Frew. Split-Hopkinson Bar Testing of an Aluminum with Pulse Shaping. In Proceedings of the *JSME/ASME International Conference on Materials and Processing*, Honolulu, HI, October 2002. ASME.

Taylor Impact Test

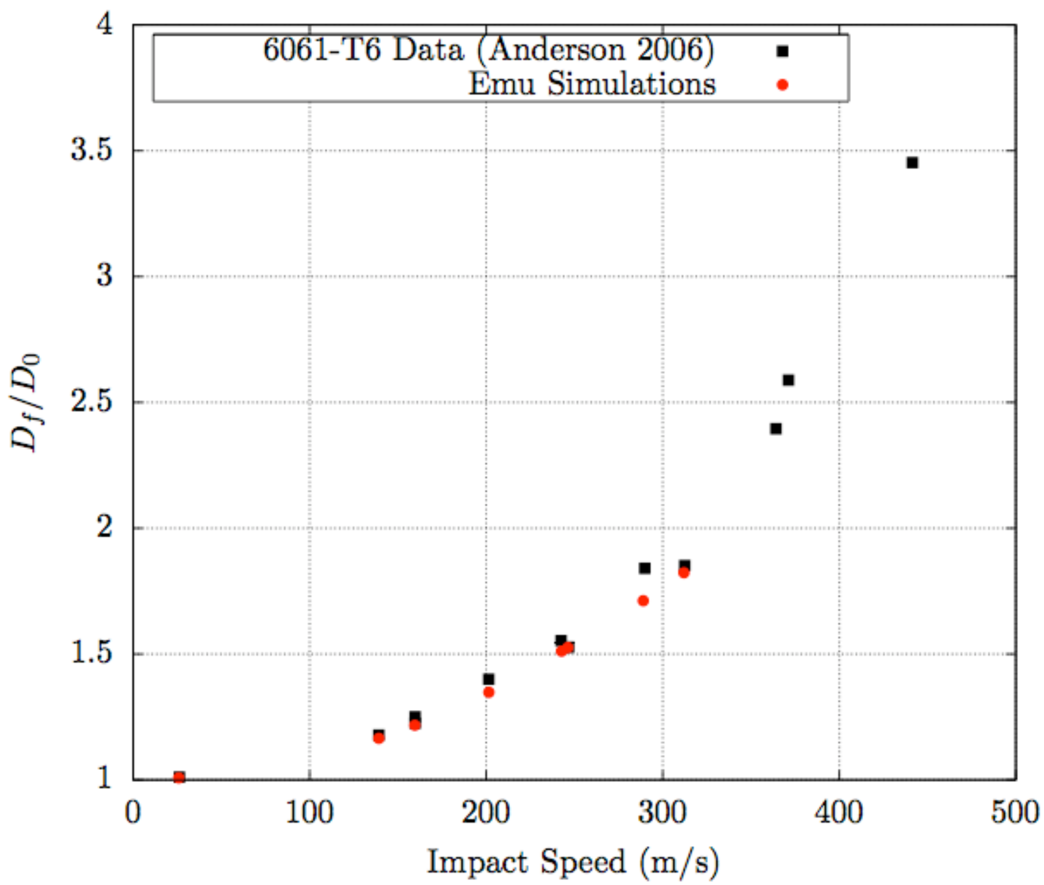


Taylor Impact - Length



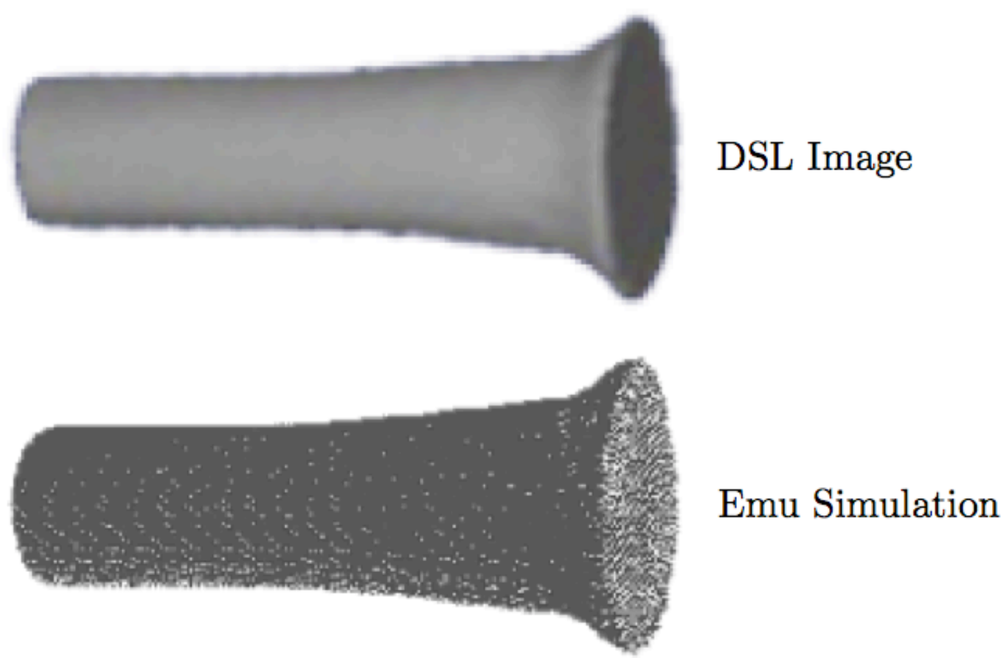
Data ref: C.E. Anderson Jr, A.E. Nicholls, I.S. Chocron, and R.A. Ryckman. Taylor Anvil Impact. In *AIP Conference Proceedings*, volume 845, page 1367. AIP, 2006.

Taylor Impact - Diameter



Data ref: C.E. Anderson Jr, A.E. Nicholls, I.S. Chocron, and R.A. Ryckman. Taylor Anvil Impact. In *AIP Conference Proceedings*, volume 845, page 1367. AIP, 2006.

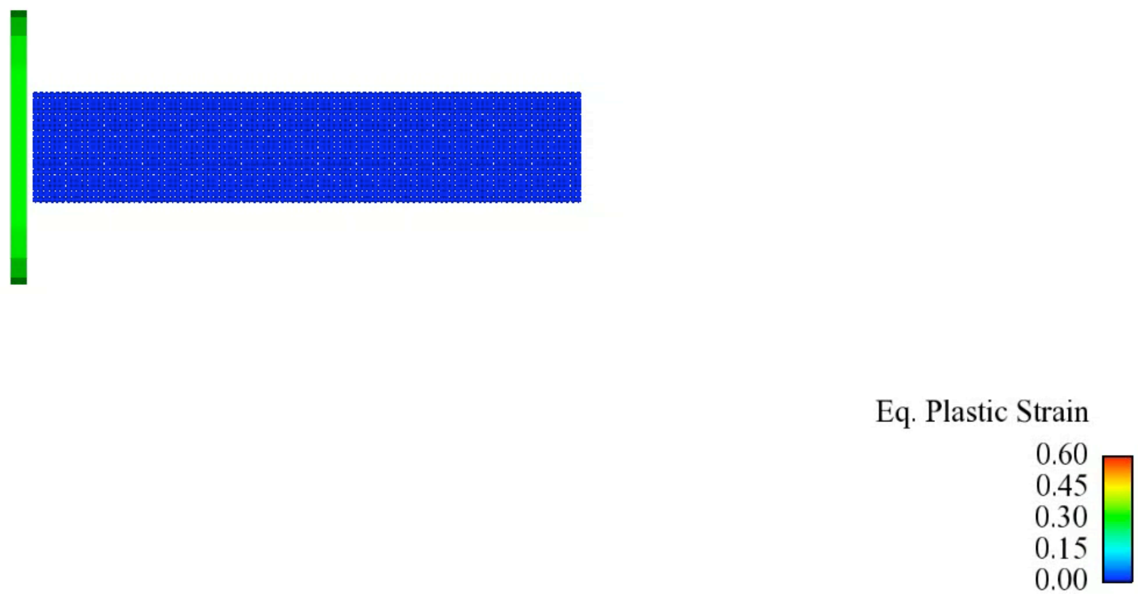
Taylor Impact - DSL Image



Comparison at 289 m/s impact speed (6061-T6 Aluminum)

Data ref: C.E. Anderson Jr, A.E. Nicholls, I.S. Chocron, and R.A. Ryckman. Taylor Anvil Impact. In *AIP Conference Proceedings*, volume 845, page 1367. AIP, 2006.

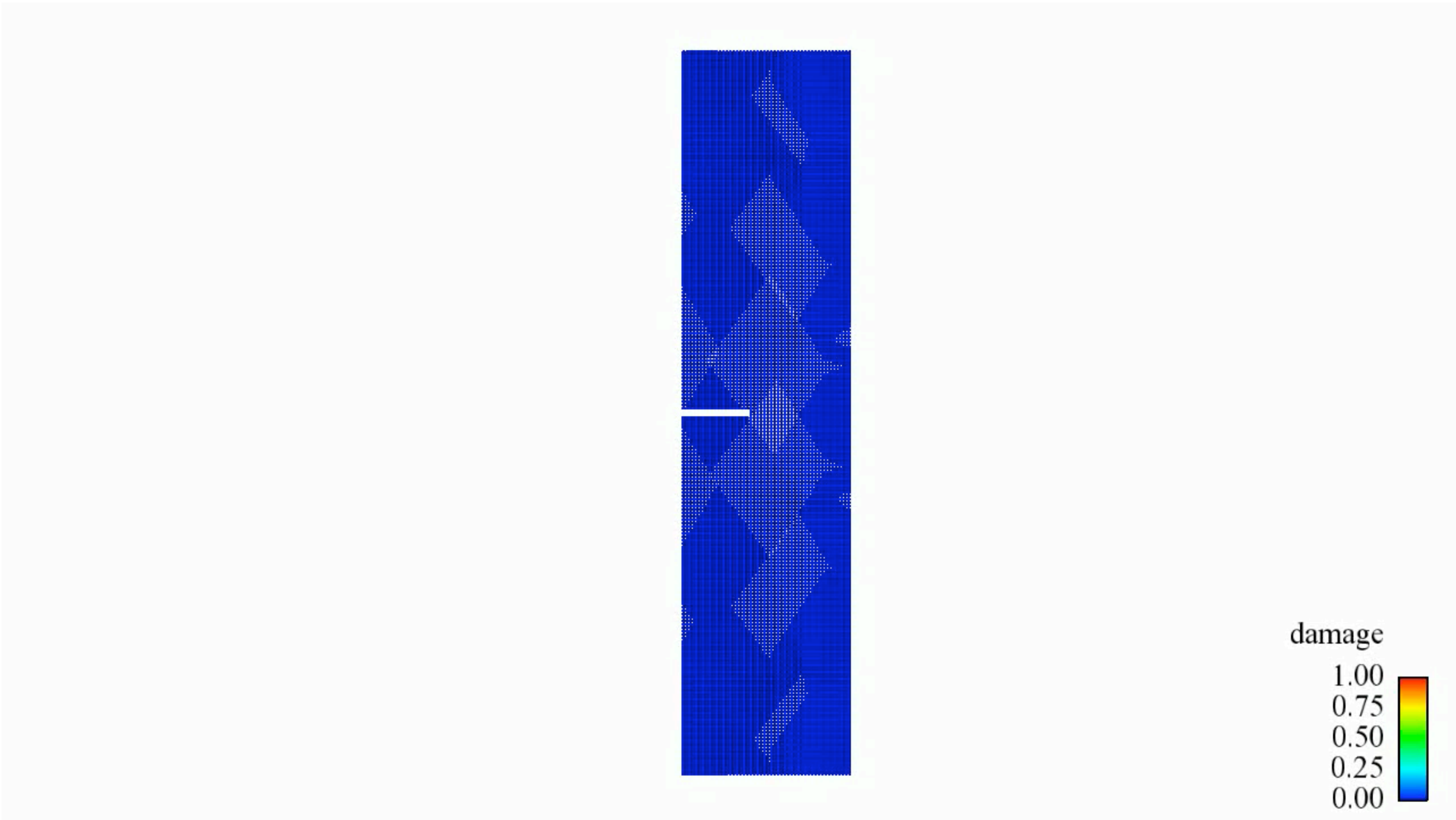
Taylor Impact Simulation



Significance

- Not “curve-fitting” bond models
- Use any constitutive model

Can still do material failure…



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