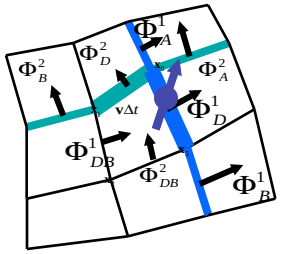
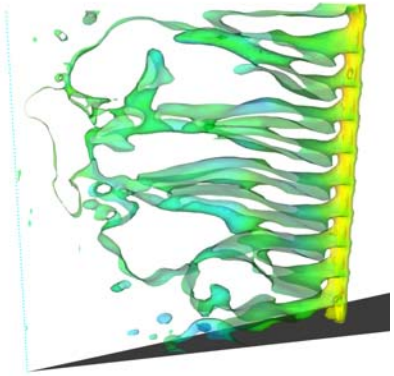
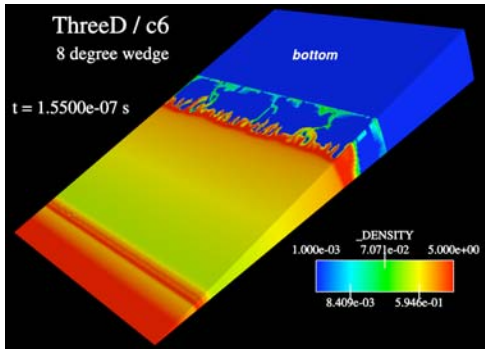
SAND2009-1111C

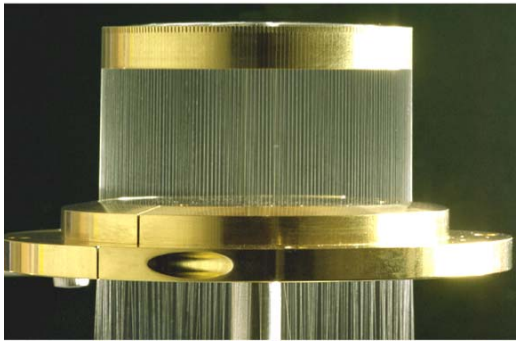
Computational Science and Engineering
March 2-6, 2009
Miami Hilton Downtown
Miami, Florida

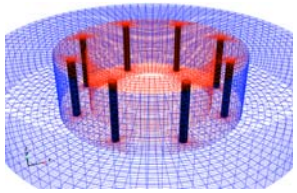
High Resolution Multiphysics Simulations of Z-Pinch with ALEGRA











Bill Rider

ALEGRA Team led by Allen Robinson

Sandia National Laboratories, Albuquerque

SIAM CSE Meeting, March 3 2009, MS49 (2 of 2)

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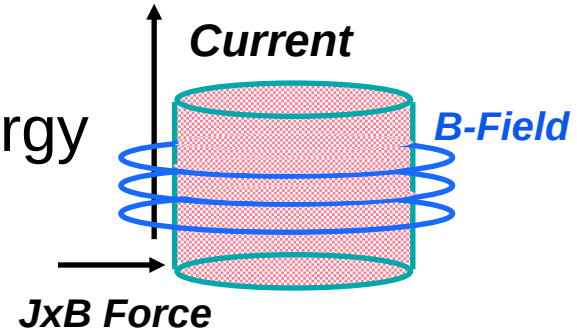
ALEGRA Team

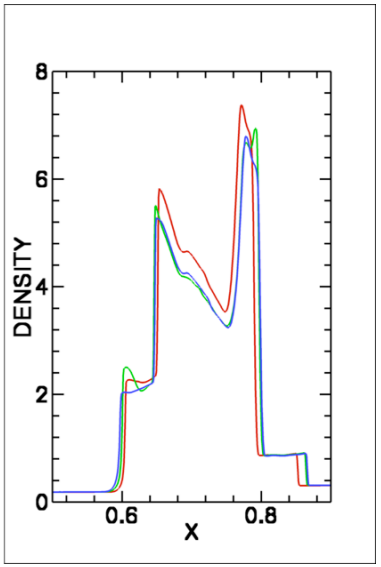
- **Allen C. Robinson (Project Leader)**
- Thomas A. Brunner, Susan Carroll, Richard Drake, Christopher J. Garasi, Thomas Gardiner, Thomas Haill, Heath Hanshaw, David Hensinger, Marlin Kipp, Duane Labreche, Raymond Lemke, Edward Love, Christopher Luchini, Stewart Mosso, John Niederhaus, Curtis C. Ober, Sharon Petney, **William J. Rider**, Guglielmo Scovazzi, Chris Seifert, O. Erik Strack, Randall Summers, Timothy Trucano, V. Greg Weirs, Michael Wong, Thomas Voth

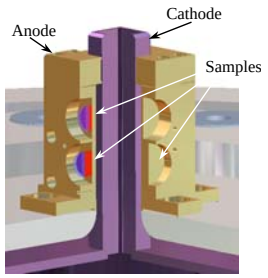
Outline of the Talk

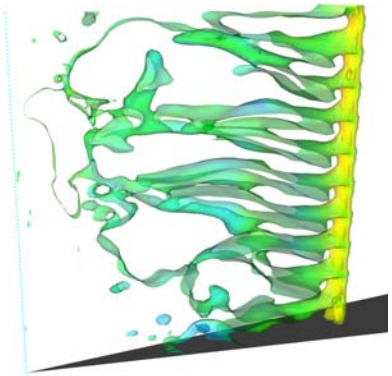
- **Governing Equations**
- **Solution Procedure**
- **Multimaterial dynamics**
  - Adaptive Remap step
  - Mixed cell treatment
  - Conservation of energy
- **Compatible MHD**
- **Applications**
  - Code Verification
  - Z-pinch implosions

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{u}) + \mathbf{u}(\nabla \cdot \mathbf{B}) = 0$$









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Governing Equations

- **Mass**

$$\frac{\partial f_k \rho_k}{\partial t} = -\nabla \cdot \left(f_k \rho_k \left(\mathbf{u} - \mathbf{u}_g \right) \right)$$

- **Momentum**

$$\frac{\partial \rho \mathbf{u}}{\partial t} = -\nabla \cdot \left(\rho \left(\mathbf{u} - \mathbf{u}_g \right) \mathbf{u} - \mathbf{T} - \mathbf{T}^M + p_r \mathbf{I} \right) + \mathbf{b}$$

- **Energy**

$$\frac{\partial \rho \left(e + e_r + 1/2 \mathbf{u}^T \mathbf{u} + 1/2 \mathbf{B}^T \mathbf{B} \right)}{\partial t} =$$

$$- \quad \nabla \cdot \left[\rho \left(\mathbf{u} - \mathbf{u}_g \right) \left(e + e_r + 1/2 \mathbf{u}^T \mathbf{u} + 1/2 \mathbf{B}^T \mathbf{B} \right) \right]$$

$$- \quad \nabla \cdot \left[\mathbf{u} \left(\mathbf{T} + p_r \right) + \left(\mathbf{u} \mathbf{B} \right) \mathbf{B} - \mathbf{q} \right] + \mathbf{u}^T \mathbf{b} + S_e$$

- **Magnetics - Faraday's law**

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \left(\mathbf{B} \times \left(\mathbf{u} - \mathbf{u}_g \right) \right) + \left(\mathbf{u} - \mathbf{u}_g \right) \left(\nabla \cdot \mathbf{B} \right) + \nabla \times \mathbf{E}' = 0$$

- **Involution constraint, Ampere's law**

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{H} = \mathbf{J}$$

Governing Equations: Radiation Included

- **Energy Equation**

$$\frac{\partial\left(\rho\epsilon+e_r+\frac{1}{2}\mathbf{u}^T\mathbf{u}+\frac{1}{2\mu_0}\mathbf{B}^T\mathbf{B}\right)}{\partial t} =$$
$$-\nabla\cdot\left(\rho\left(\mathbf{u}-\mathbf{u}_g\right)\left(\rho\epsilon+e_r+\frac{1}{2}\mathbf{u}^T\mathbf{u}+\frac{1}{2\mu_0}\mathbf{B}^T\mathbf{B}\right)\right)$$
$$+\nabla\cdot\left(\mathbf{u}\left(\mathbf{T}+\mathbf{T}^M-p_r\mathbf{I}\right)-\mathbf{q}\right)+\mathbf{J}\cdot\mathbf{E}'$$

- **Ion-Electron Temperature**

$$\rho\frac{de_e}{dt}=\mathbf{T}_e:\nabla\mathbf{u}-\nabla\cdot\mathbf{q}_e+\mathbf{J}\cdot\mathbf{E}'+\rho C_{Ve}\frac{\theta_i-\theta_e}{\tau_{ei}}-\int_0^\infty\left(\kappa\left(4\pi B_\nu-cE_\nu\right)\right)d\nu,$$
$$\rho\frac{de_i}{dt}=\mathbf{T}_i:\nabla\mathbf{u}-\nabla\cdot\mathbf{q}_i+\rho C_{Ve}\frac{\theta_e-\theta_i}{\tau_{ei}}$$

- **Rad.** $\frac{1}{c}\frac{\partial I}{\partial t}+\mathbf{\Omega}\cdot\nabla we=-\sigma_t we+\frac{\sigma_s}{4\pi}\int_{4\pi}we\,\mathbf{\Omega}'+\sigma_a\frac{cB(T_m)}{4\pi}+S_I$

Lagrangian Equations - Step 1
(Conduction step done next)

- **Lagrangian frame in integral form**

$$\frac{d}{dt}\int_{\Omega_t} \rho \, dv = 0 \quad \Longrightarrow \quad \dot{\mathbf{x}} = \mathbf{u}$$

$$\frac{d}{dt}\int_{\Omega_t} \rho \dot{\mathbf{u}} \, dv = \int_{\Omega_t} \nabla \cdot (\mathbf{T} + \mathbf{T}^M) \, dv$$

$$\frac{d}{dt}\int_{\Omega_t} \rho e \, dv = \int_{\Omega_t} \mathbf{T} \cdot \nabla \mathbf{u} \, dv$$

$$\frac{d}{dt}\int_{\Gamma_t} \mathbf{B} \cdot \mathbf{n} \, dA = 0$$

Remesh (step 2) - Remap (step 3)

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}_g - \mathbf{u}$$

Remesh is the process of selecting the target mesh for the remap (u_g).

$$\frac{\partial f_k}{\partial t} = -\left(\mathbf{u} - \mathbf{u}_g\right) \nabla f_k$$

$$\frac{\partial_r f_k \rho_k}{\partial t} = -\nabla \cdot \left(f_k \rho_k \left(\mathbf{u} - \mathbf{u}_g\right)\right)$$

$$\frac{\partial_\rho \mathbf{u}}{\partial t} = -\nabla \cdot \left(\rho \mathbf{u} \left(\mathbf{u} - \mathbf{u}_g\right)\right)$$

$$\frac{\partial_\rho e}{\partial t} = -\nabla \cdot \left(\rho e \left(\mathbf{u} - \mathbf{u}_g\right)\right)$$

$$\frac{\partial_\rho K}{\partial t} = -\nabla \cdot \left(\rho K \left(\mathbf{u} - \mathbf{u}_g\right)\right)$$

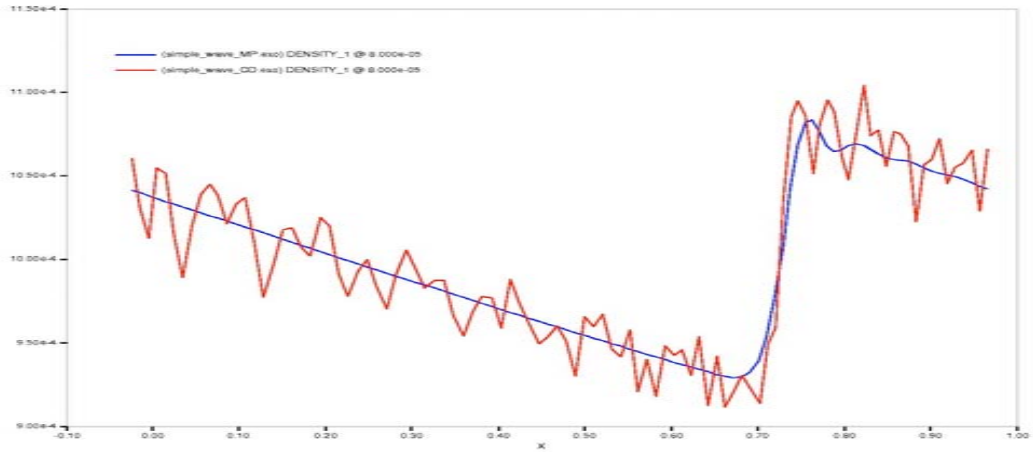
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left(\mathbf{B} \times \left(\mathbf{u} - \mathbf{u}_g\right)\right) - \left(\mathbf{u} - \mathbf{u}_g\right)\left(\nabla \cdot \mathbf{B}\right)$$

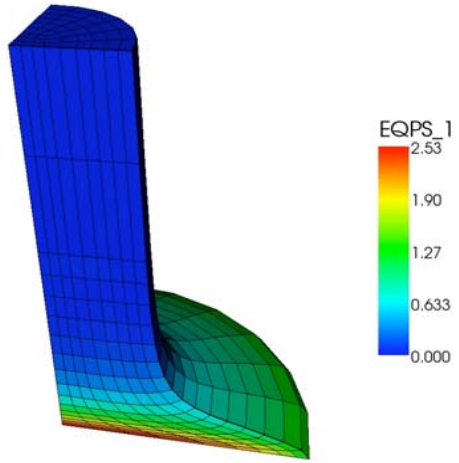
If $u_g = 0$ the method is pure Eulerian,

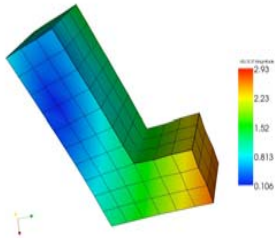
if $u_g = u$ the method is pure Lagrangian.

Multi-material Lagrangian

- **The Lagrangian hydro algorithm in ALEGRA is based on the method from the SNL code PRONTO.**
  - It uses the Von Neumann-Richtmyer time-space staggering, but the energy equation is first-order in time, but conservative.





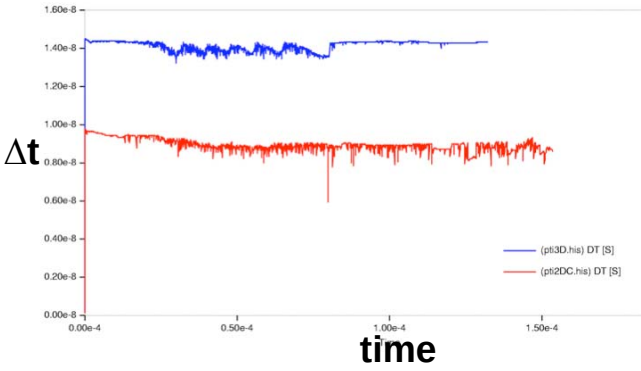


  - Implementing and bringing on line a second-order method.

Numerical analysis of basic Lagrangian methodology

- **Stability analysis using several classical techniques**
  - A more precise definition of stability for ALE hydro and Lagrangian step
  - **Produced a new time step control for ALEGRA (allows a large safety factor)**
  - Lagrangian analysis included impact of Q on the stability

$$\frac{\ell}{|u|+c} \to \min\left(\frac{\ell}{|u|}, \frac{\ell}{c}\right)$$



- **Artificial viscosity analysis**
  - Based an assessment of a simple flyer problem (Mie-Gruneisen) and unexpected results.
  - Defined an analysis of analytical Q's for a Mie-Gruneisen EOS .
  - **Redefined the quadratic Q** (much larger than typical and material dependent), linear Q is unchanged.

$$p = p_0 + \rho_0 U_s u_p + \rho\Gamma(e - e_1)$$

$$\rho_0 U_s \left(\tfrac{1}{\rho_1} - \tfrac{1}{\rho_0}\right) = u_p$$

$$p_1 = p_0 + \rho_0 u_p U_s$$

$$e_1 - e_0 = -\tfrac{1}{2}(p_0 + p_1)\left(\tfrac{1}{\rho_1} - \tfrac{1}{\rho_0}\right)$$

$$C_2 = s + \frac{\Gamma}{2}$$

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Multi-material Treatment

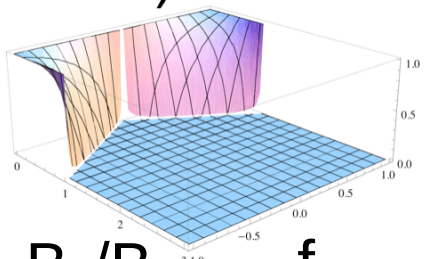
- **The traditional treatment is "constant volume," which means that volume fractions are constant through the Lagrangian step.**
  - This is clearly wrong because different materials will evolve differently based on their relative stiffness.
- **ALE will produce multimaterial Lagrangian treatments (mixed cells cannot be avoided).**
  - The remap process will create mixed material elements out of thermodynamic equilibrium
  - The elements *should be* brought into equilibrium before moving on to the remap requiring an adjustment of the volume fractions

$$\Delta f_k = f_k\left(\frac{p_k-\bar{p}}{B_k}\right)+f_k\frac{\left(B_k-\bar{B}\right)}{B_k}\frac{\Delta V}{V} \quad \begin{cases} f_k^1 = f_k^0 :+\xi\,\Delta f_k ; \rho_k \coloneqq \dfrac{f_k^0\rho_k}{f_k^1} \\ e_k^1 = e_k^0 - \dfrac{\xi\bar{p}\,\Delta f_k}{m_k} \end{cases}$$

Modifications (x) are required for a robust & stable algorithm

- **Based on averaging,** $\bar{p} = \bar{B}\sum_k \frac{f_k p_k}{B_k}$ $\qquad \bar{B} = \left(\sum_k \frac{f_k}{B_k}\right)^{-1}$
- **Large changes in material volume are dangerous and must be limited, however**
  - For small volume fractions and dissimilar materials the algorithm can be unstable if Δf_k is too large,
  - but not allowing enough of a change keeps materials in unphysical states (doesn't allow adjustment)
  - Stability is assured through

$$\xi = \min_k \left(\alpha, \frac{f_k}{|\Delta f_k|}, \frac{m_k e_k}{|\bar{p}\,\Delta f_k|}, \frac{\delta f_{\max}}{|\Delta f_k|} \right)$$



B_1/B_2 $\qquad$ f_1

  - We also damp stiff materials more strongly

$$\alpha = \alpha_0 \phi(f) / \gamma_k^2 \text{ where } \gamma_k = B_k / p_k; \alpha_0 = 0.05 - 0.25; \phi(f) = \tanh(30 f_k)$$

* Similar approaches by LeBlanc, Tipton, Miller & Puckett,…

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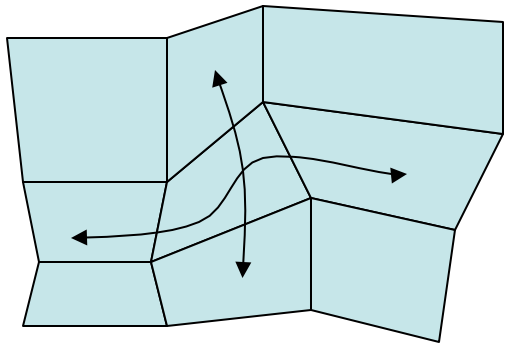
The remap algorithm is extremely important to ALEGRA's performance

- **Eulerian style remesh/remap usage is very common with ALEGRA users.**
- **Remap Overview Summary**
  - Interface reconstruction options.
  - Remap algorithms:
    - Hydro
    - MHD
    - Solid kinematics
  - **DeBar modifications to kinetic and magnetic energy to support shocks.**

ALEGRA Hydro Remap Summary

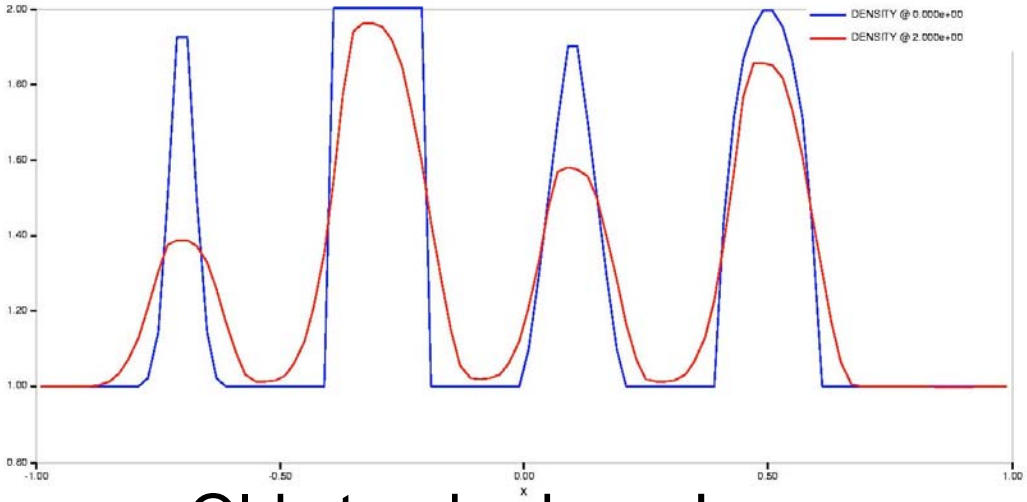
- **Element based reconstruction operators utilize one-dimensional mesh topology associated stencils (quads and hexes). Volume or mass coordinates are used.**
- **Momentum remapped using half-interval shift.**



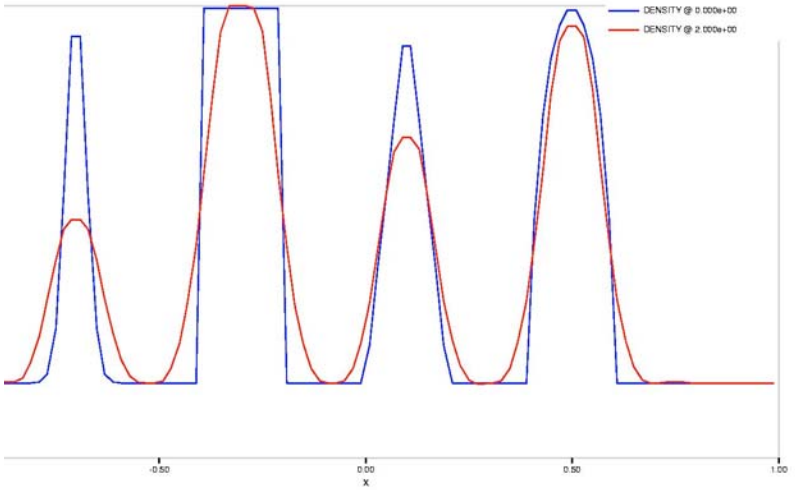
- ***It is desirable to replace this split remap methodology.***
  - *An unsplit remap algorithm for hydro is in progress.*

Hydro Remap Improvements

- **Remap methods have been improved with an improved van Leer method on variable meshes and a third-order scheme.**



Old standard van Leer

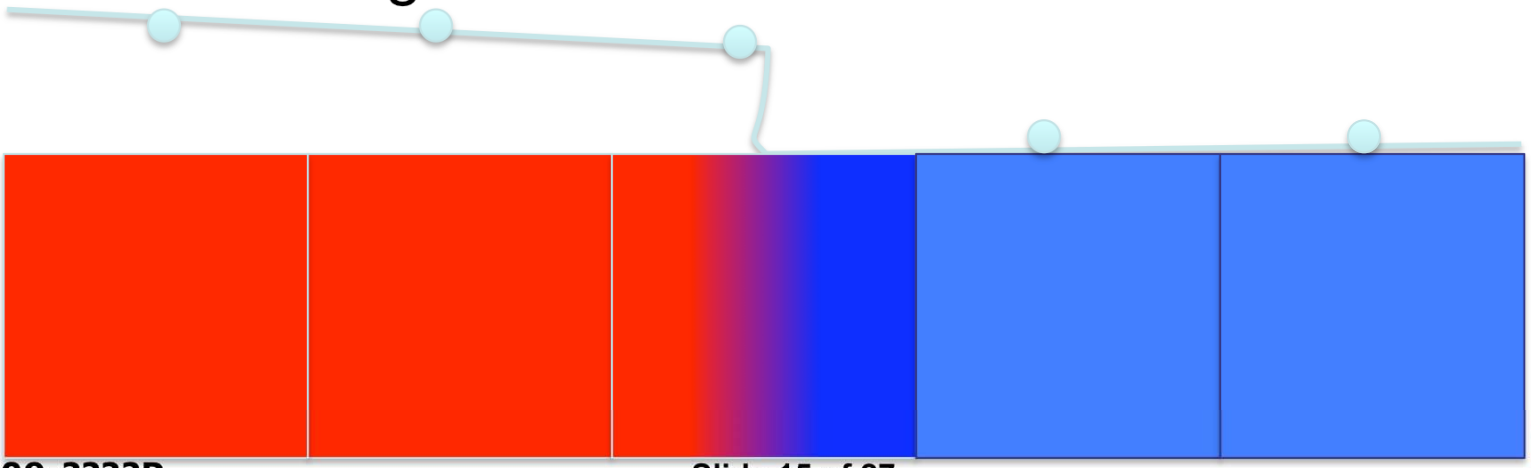


3rd Order (modified PPM)

- **These new methods are now ALEGRA's default method along with an adaptive mixed cell methodology.**

The mixed element advection: the fundamental idea

- **A mixed cell precludes the computation of accurate approximations, and high-order approximations do produce anomalous side-effects at interfaces.**
- **We choose "safer" methods at interfaces (i.e., minmod).**
  - Minmod advection picks the smallest local derivative for its numerical stencil (most dissipative second-order monotone method).
  - This has the impact of biasing the differencing into single material regions.



Details of the new adaptive interface aware remap in ALEGRA

- **Third-order remap based on three element parabolic conservative interpolation.**
  - For robustness, the edge values are third-order, but bounded by neighboring data,

$$\phi_{j+1/2} = \tfrac{1}{6}\left(2\phi_{j+1} + 5\phi_j - \phi_{j-1}\right) \rightarrow \tfrac{1}{2}\left(\phi_{j+1} + \phi_j\right) - \tfrac{1}{6}\left(\Delta_{j+1/2}\,\phi - \Delta_{j-1/2}\,\phi\right)$$

$$\Delta_{j+1/2}\,\phi = \min\text{mod}\left\lfloor \phi_{j+1} - \phi_j, 4\left(\phi_j - \phi_{j-1}\right)\right\rfloor \quad \Delta_{j-1/2}\,\phi = \min\text{mod}\left\lfloor \phi_j - \phi_{j-1}, 4\left(\phi_{j+1} - \phi_j\right)\right\rfloor$$

- **Mixed cell remap is now lower order**
  - Many codes use donor cell in mixed cells (LLNL codes)*
  - Instead Alegra uses the minmod scheme (the most dissipative second order "TVD" method)

$$\phi_{j+1/2} = \phi_j + \tfrac{1}{2}\min\text{mod}\left\lfloor \phi_{j+1} - \phi_j, \phi_j - \phi_{j-1}\right\rfloor$$

  - **Effectively uses one-sided differencing in mixed cells, only differencing into the pure material region (closer values). Huge impact on calculations!**

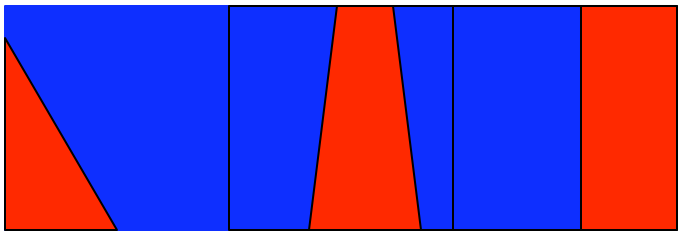
*we have found that donor cell is too diffusive, negatively impacting validation results.

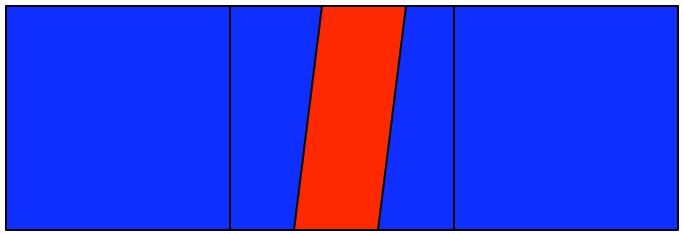
More details of the adaptive remap based on mixed cells

- **For the elements (cells) adjacent to an interface, the bounding step in the parabolic reconstruction is modified to be more dissipative,**

$$\Delta_{j+1/2}\,\phi = \min \mathrm{mod}\left\lfloor \phi_{j+1} - \phi_j, 2\left(\phi_j - \phi_{j-1}\right)\right\rfloor \quad \Delta_{j-1/2}\,\phi = \min \mathrm{mod}\left\lfloor \phi_j - \phi_{j-1}, 2\left(\phi_{j+1} - \phi_j\right)\right\rfloor$$

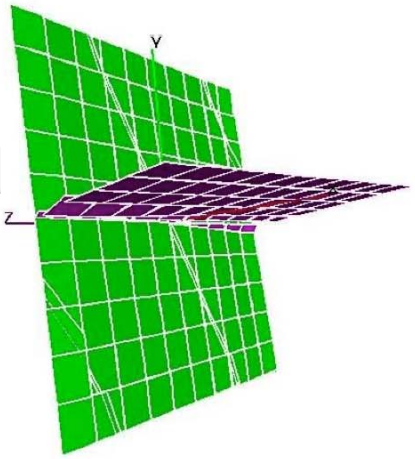
- **Donor cell approximations are used in two cases (could use minmod for the left case):**

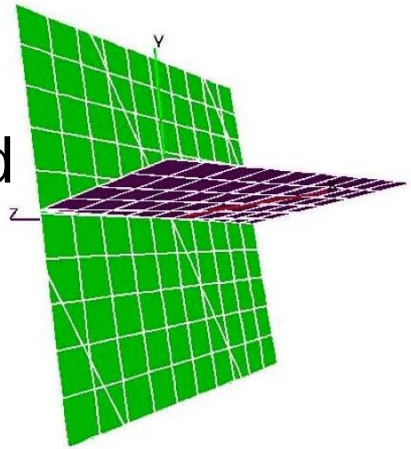




Multimaterial Remap*

- **The remap uses high resolution finite volume differencing for continuous fields**
- **Interface reconstruction with linearity preserving and automatic interface ordering (PIR) is used for discontinuous field (material interfaces).**

unsmoothed 

smoothed 

***Work by Mosso**

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“DeBar fix” for energy conservation

- **We have a modern implementation of DeBar’s kinetic energy treatment.**

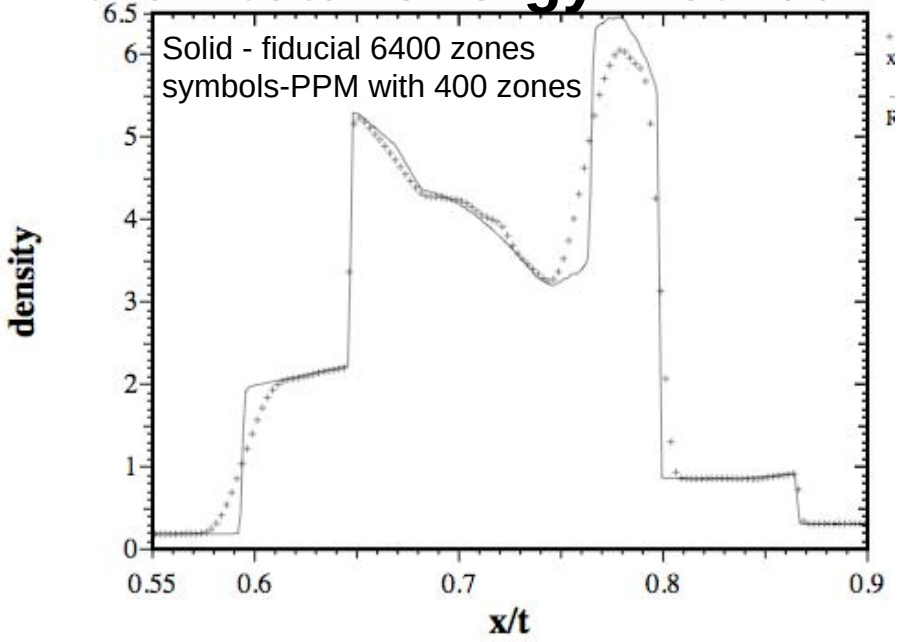
$$KE = \frac{1}{nodes} \sum_{nodes} \frac{1}{2}\mathbf{u}^2$$

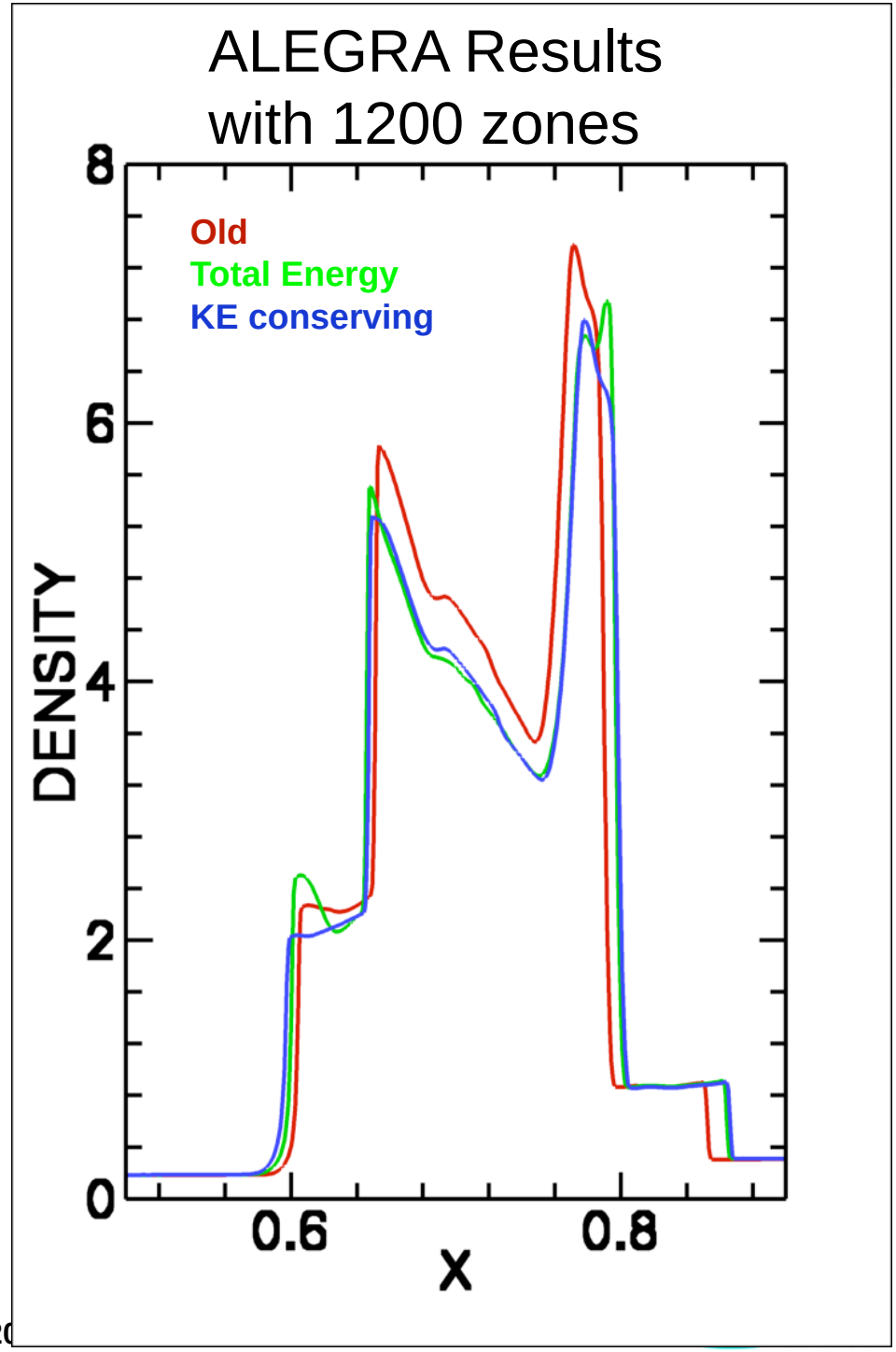
$$\rho e = \rho e + (KE_{remapped} - \frac{1}{nodes} \sum_{nodes} \frac{1}{2}\mathbf{u}^2{}_{remapped})$$

- **It corrects for the process of remap on the kinetic energy and allows full conservation of energy.**
  - We have a switch (Q/p >0.001) to turn the fix off away from shocks.
  - We can also take care to limit the amount of cooling of a material due to the fix which is used in the Z-pinch implosions, this is NOT the default setting.
  - Robust/automatic controls are desirable.

Results: W-C Blast Wave

This demonstrates the ability of the KE conservation to produce correct results and is more flexible and robust than total energy method.





Stable Time-Step Estimation

- **Physics-based time step:**

$$\Delta t_1 := \frac{h}{c + 2c_2 h |\nabla \cdot \mathbf{v}|}$$

- **Time step based on Fourier analysis of time integrator (impact of Q's dissipation):**

$$\Delta t_2 := \frac{h}{c\left(\sqrt{1+\xi_{\max}^2} + \xi_{\max}\right)}$$

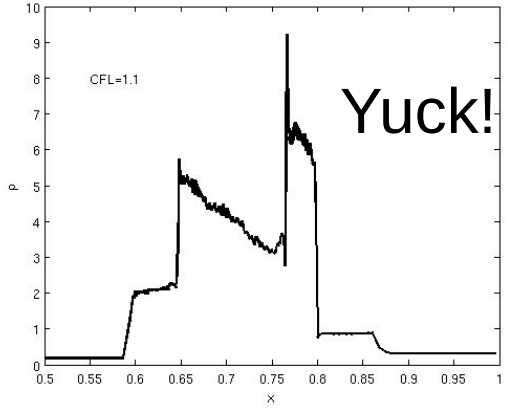
$$\xi_{\max} = c_1 + c_2 h c^{-1} |\nabla \cdot \mathbf{v}|$$

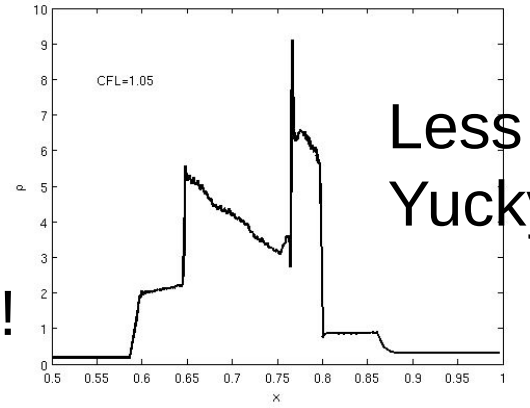
- **Final time step is the most restrictive of the two:**

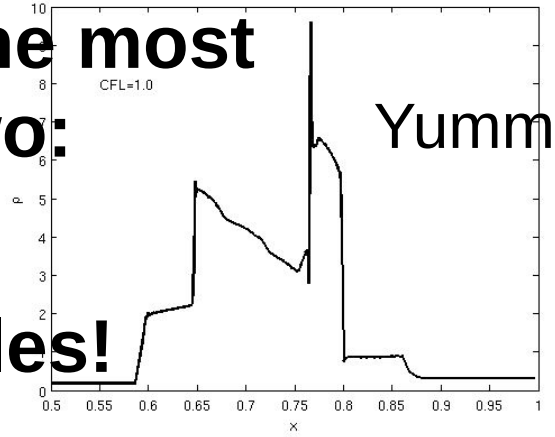
$$\Delta t = CFL \cdot \min(\Delta t_1, \Delta t_2)$$

- **Fourier analysis rules!**

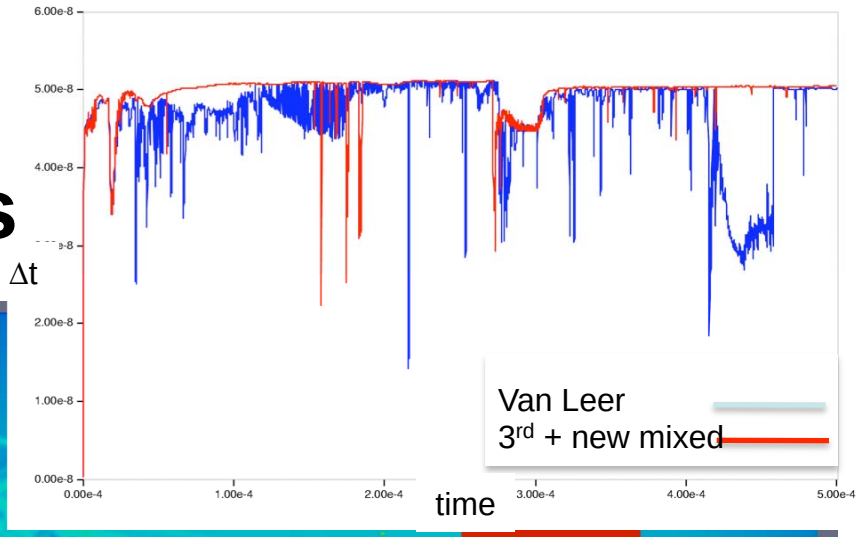
IBW Problem

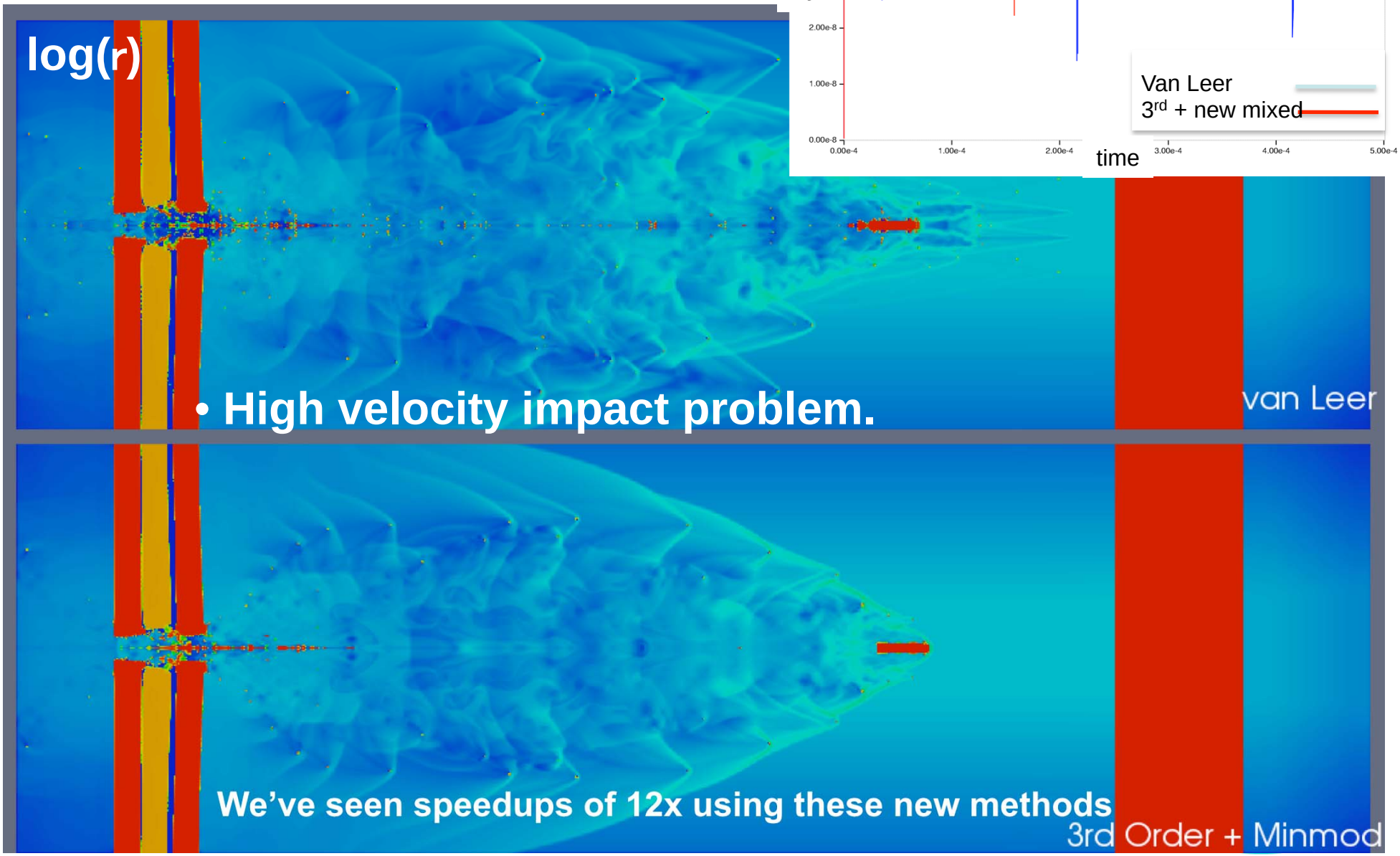






Impact on ALEGRA Results





Magneto-hydrodynamic treatment is important for many applications.*

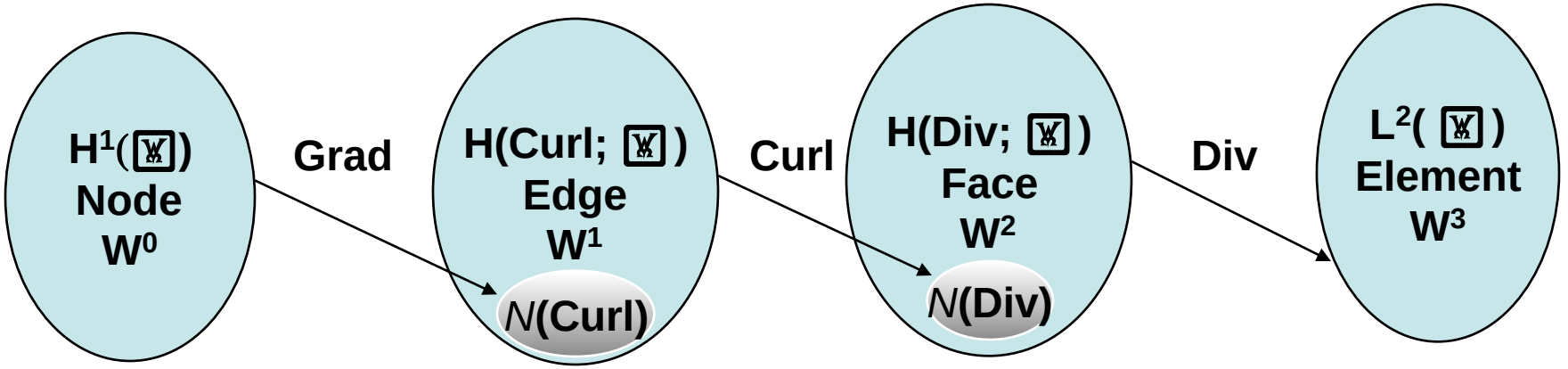
- Uses a compatible formulation that preserves the divergence free magnetic field automatically.
- A software package, *Intrepid*, makes the implementation relatively seamless with ALEGRA (and other codes)
- These properties are maintained through the remap as well.
- ALEGRA includes magnetic diffusion (using multilevel solves via Trilinos) as well as ideal MHD

* work by Bochev and Robinson

Magnetic Flux Density Remap

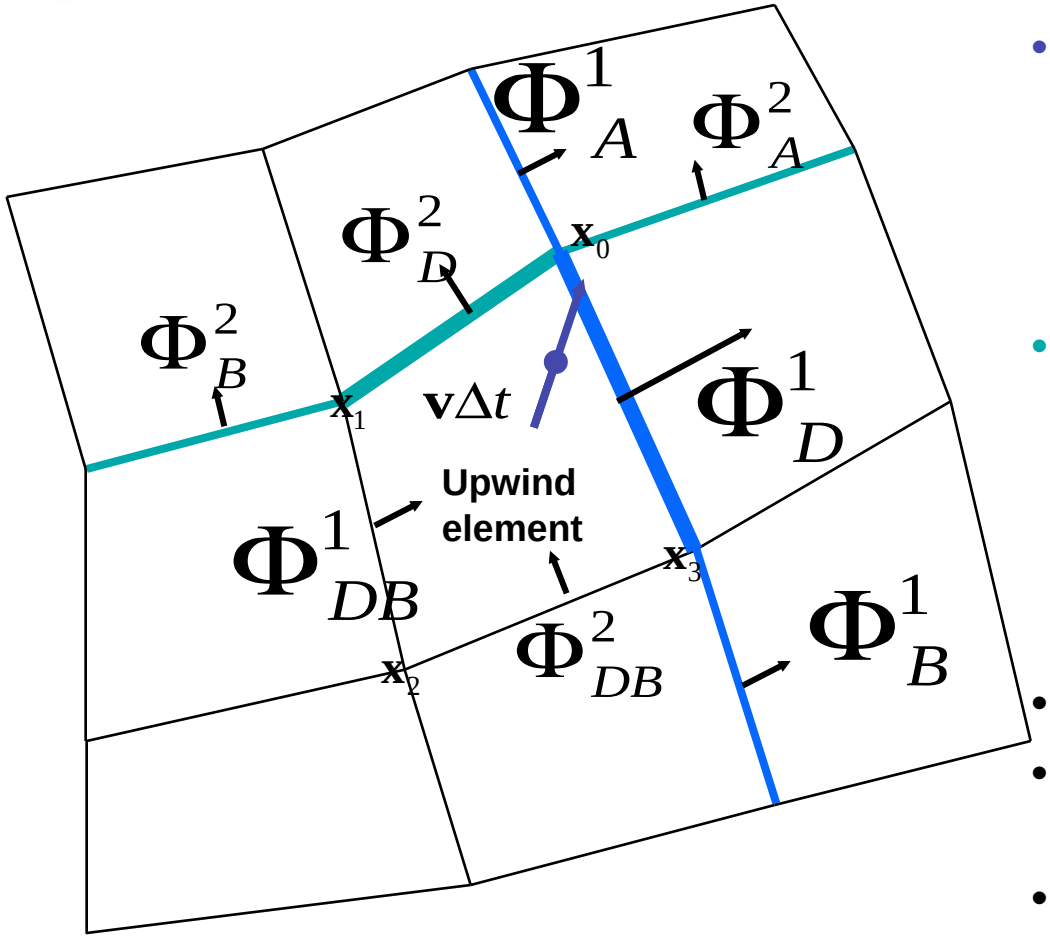
- **The Lagrangian step maintains the discrete divergence free property via flux density updates given only in term of curls of edge centered variables.**
  - The remap should not destroy this property.
- **Constrained transport (CT) is the name for a basic approach for updating the fluxes to preserve the divergence free property.**
  - CT is fundamentally unsplit.

Mimetic discretizations (deRham complex) are natural for MHD



- Magnetic flux and vector potential circulation are invariants in ideal MHD and thus natural degrees of freedom.
- Circulation is the degree of freedom for the edge element.
- Flux in the degree of freedom for the face element.
- Discrete node, edge, face element representations matching these properties are possible using mimetic FE to solve the magnetic diffusion equation in an operator split context.
- In 3D we thus ensure that fluxes exactly satisfy the divergence free property for the magnetic flux density.

CT on unstructured quad and hex grids (CCT)



- **Define the low order or donor method by integrating the total flux through the upwind characteristic of the total face element representation of the flux density.**
- **High order method constructs a modification to the flux so that it varies across the element face. Compute flux density gradients in the tangential direction using the blue and the green faces.**
- **All contributions are combined.**
- **Electric field updates are located on edges.**
- **Take curl to get updated fluxes.**
- ***Requires tracking flux and circulation sign conventions*.**

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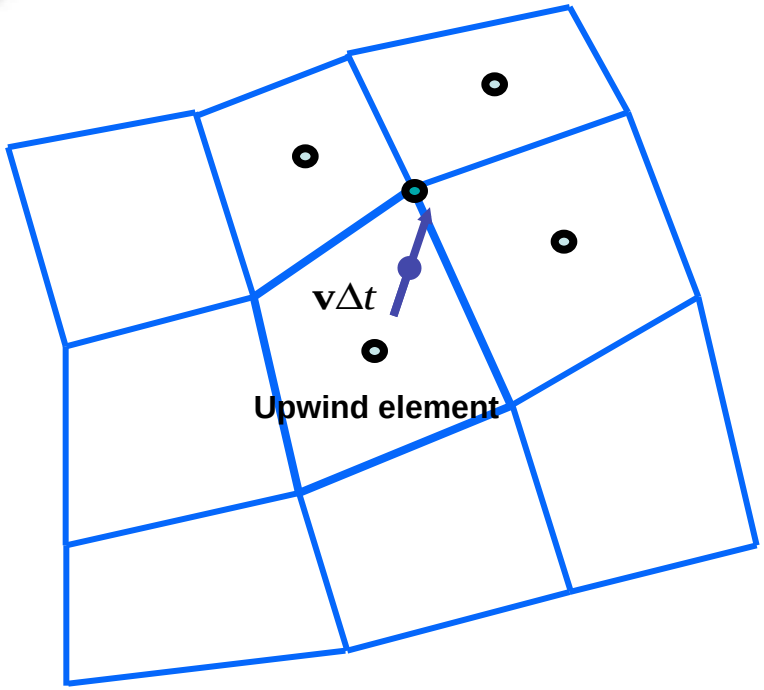
Improved CCT Algorithm



- Compute B at nodes from the face element representation at element centers. This must be second order accurate. Patch recovery (PR) suggested. Other means are possible.
- Compute trial cross face element flux coefficients on each face using these nodal B.
- Limit on each face to obtain cross face flux coefficients which contribute zero total flux.
- Compute the edge flux contributions in the upwind element by a midpoint integration rule at the center of the edge centered motion vector.
- 3D coding recently been implemented and is at the verification stage.

NOTE: All CT algorithms will not conserve magnetic energy. This has consequences for shocks.

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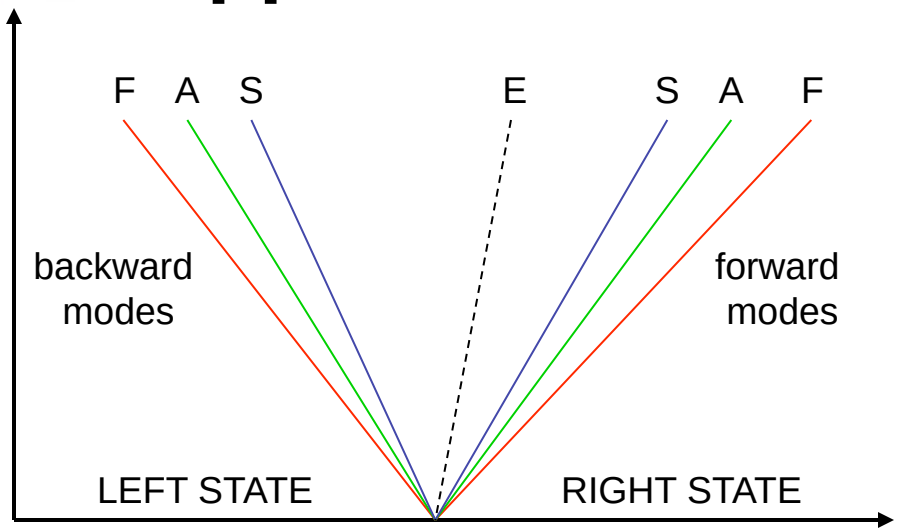
Applications: Ideal MHD shock tube

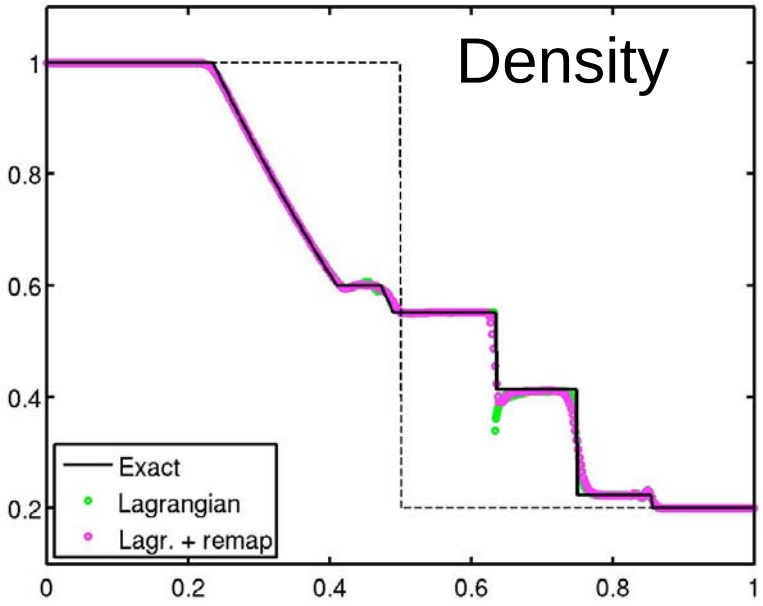


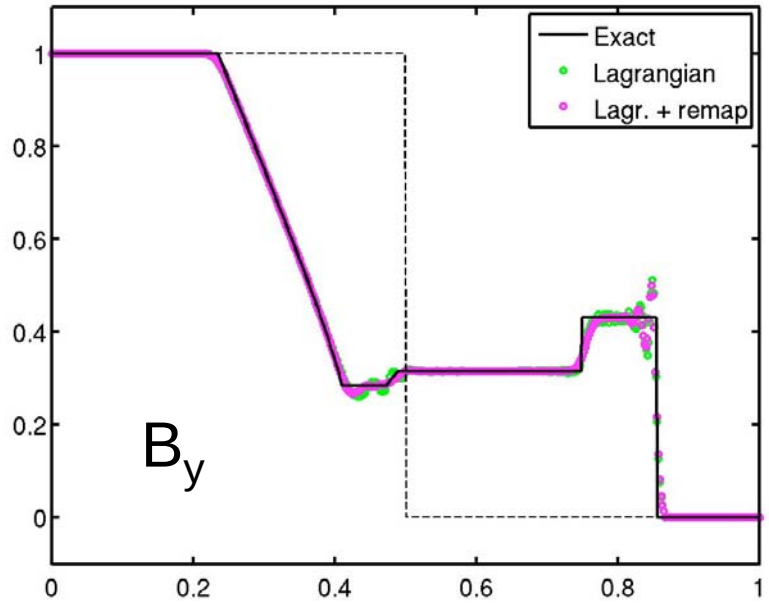
F → $v_x \pm c_f$ “fast”

A → $v_x \pm c_a$ “Alfven”

S → $v_x \pm c_s$ “slow”

E → v_x “entropy”





*** work by Niederhaus and Robinson**

3D DeBar for magnetic energy

- **Implemented in 3D so far.**

$$ME = \int \mathbf{B} \bullet \hat{\mathbf{A}} / (2\mu) dv$$

$$\rho e = \rho e + ME_{remapped} - \int (\mathbf{B}_{remapped} \bullet \mathbf{B}_{remapped}) / (2\mu) dv$$

- **Implementation uses the same switches/limiters as the KE DeBar**
- **Optional control for KE and/or ME debar**

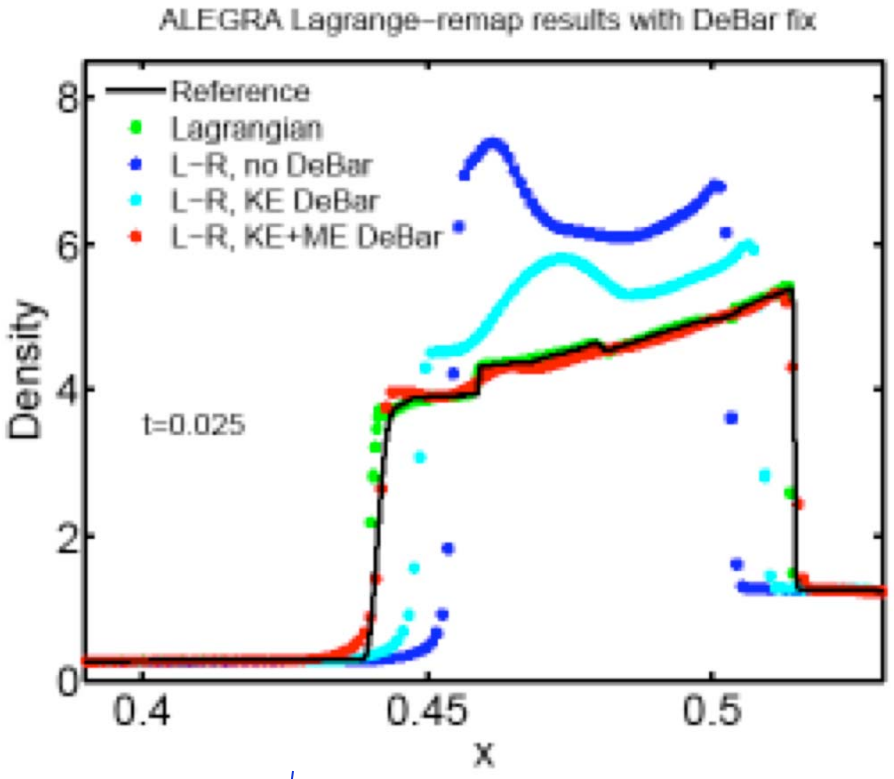
Magnetized Woodward-Colella problem

Uniform **B** field added:

$$\mathbf{B} = 15\hat{x} + 15\hat{y}$$

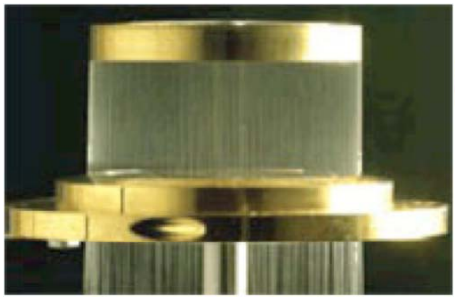
- Magnetic/kinetic/internal energy densities then have nearly same magnitude

- Failure to conserve magnetic energy exposed in MHD shocks:
  - -Shock speeds
  - -Post-shock states

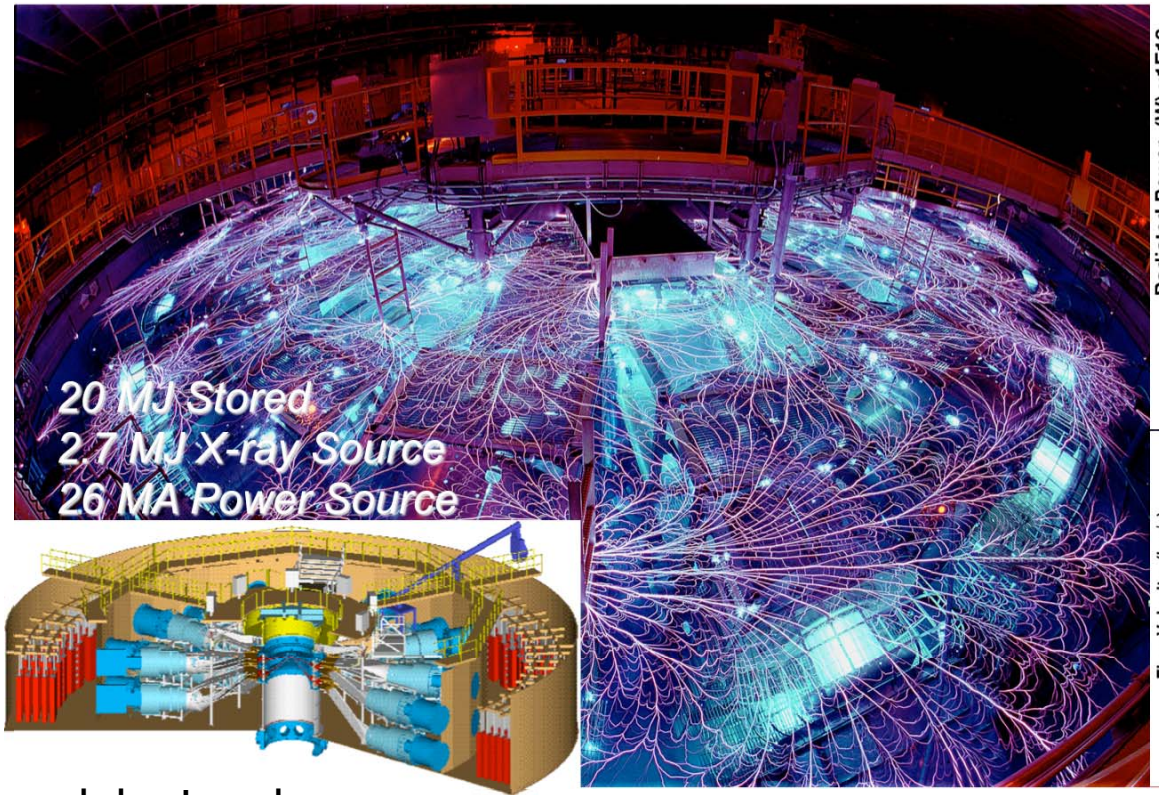


- Solution is improved with DeBar correction for KE only
- Solution acceptable only with full DeBar correction (KE + ME)
  - -Correct speeds, states and rate of convergence

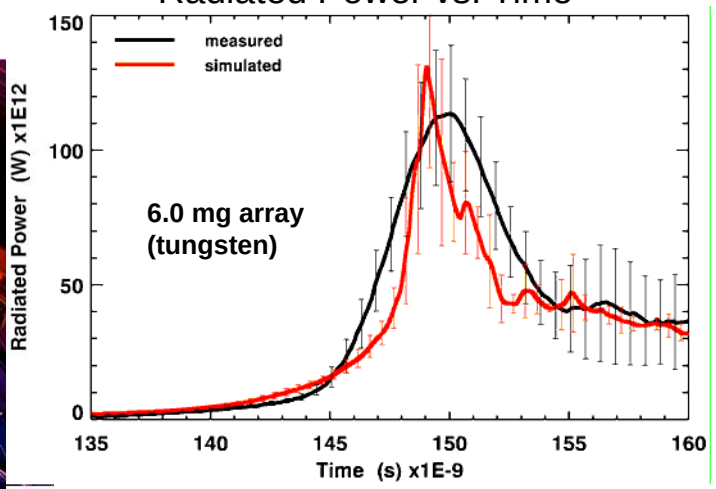
Applications: Z-Pinch*

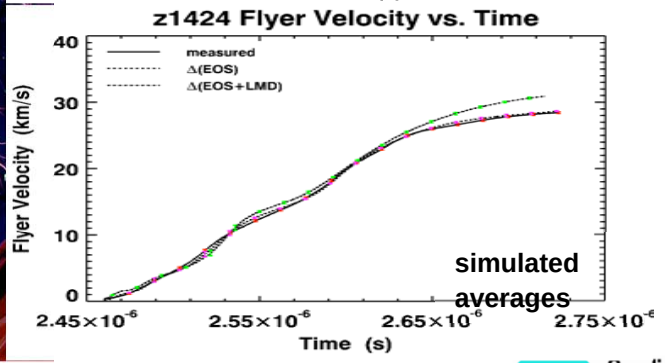


- **ALEGRA has provided predictive simulations of magnetic flyers and wire array implosions conducted on SNL's Z-machine.**



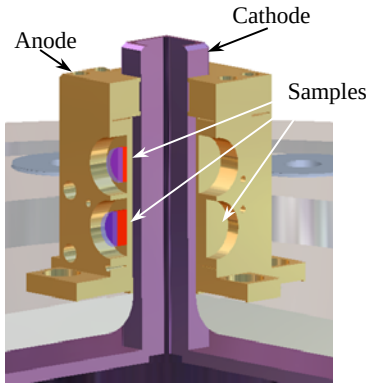
Radiated Power vs. Time

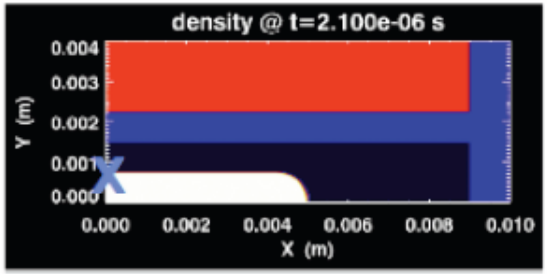


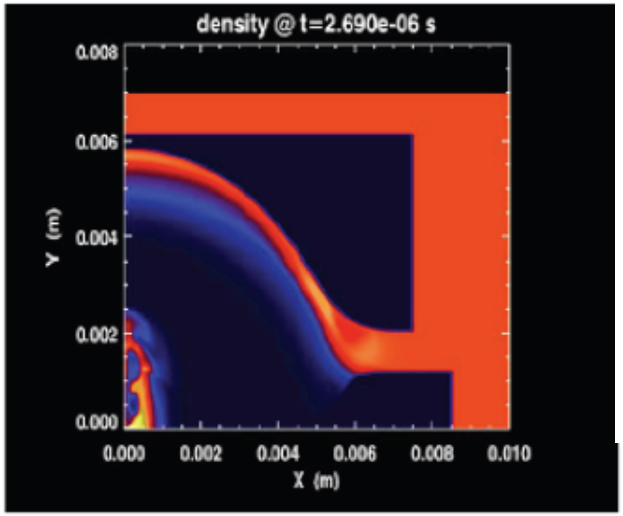


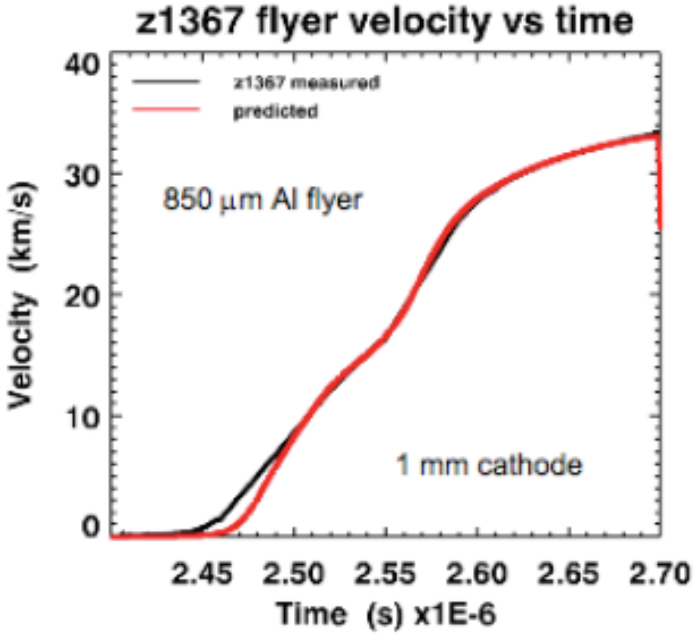
* work by Lemke, …

Flyer Application



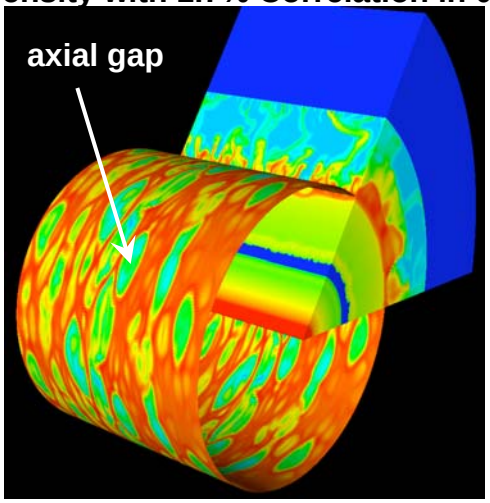


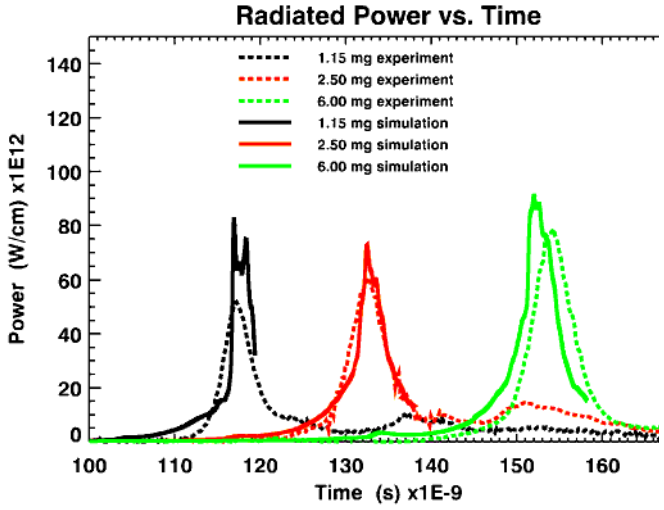


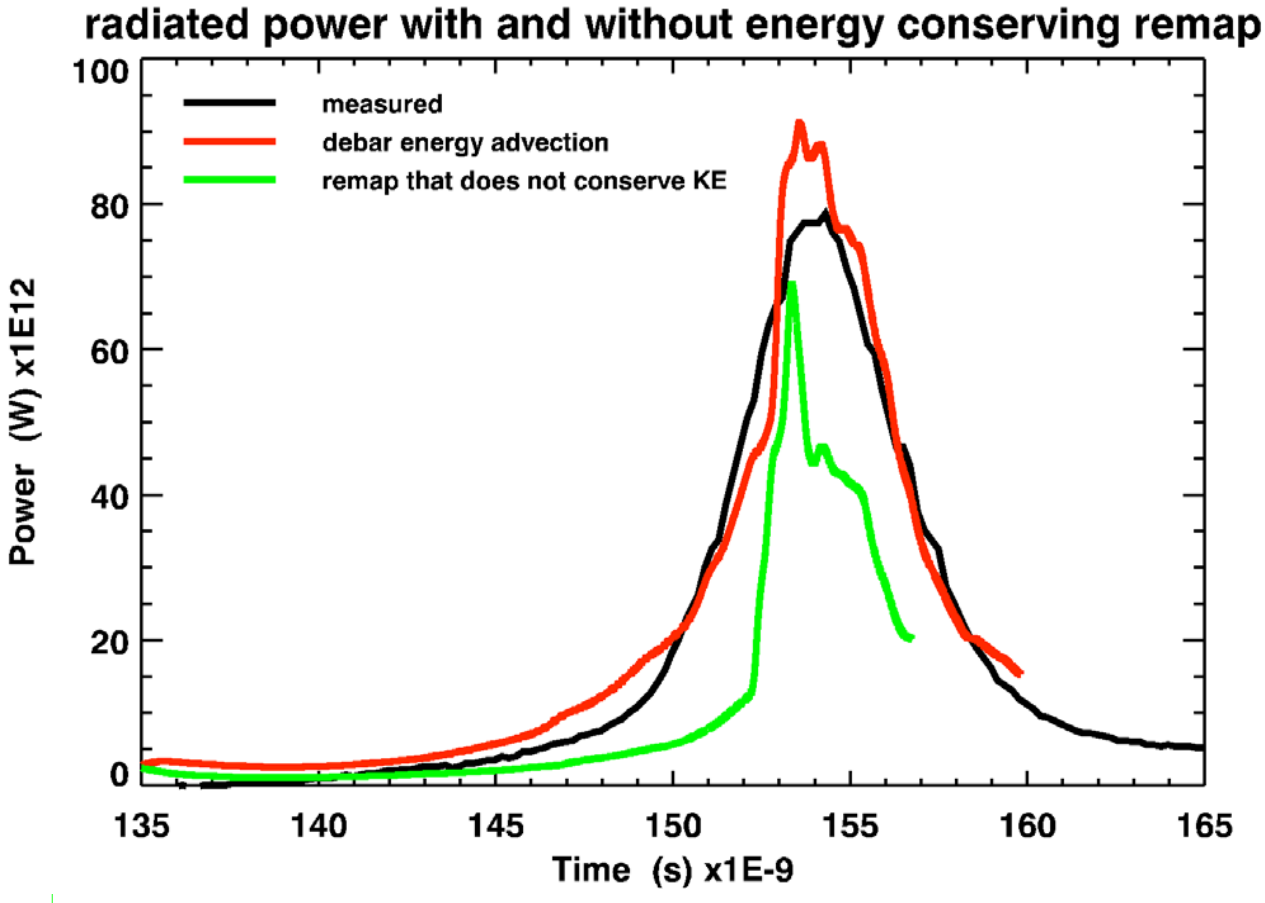


Wire Array Implosions require 3-D and (almost) all the physics!

Density with 1.7% Correlation in θ



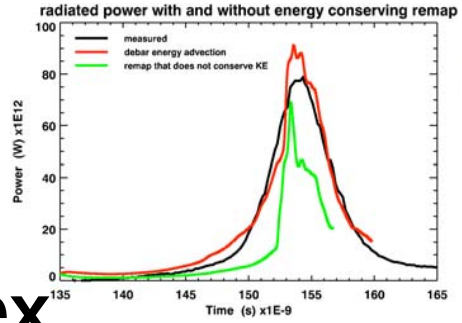


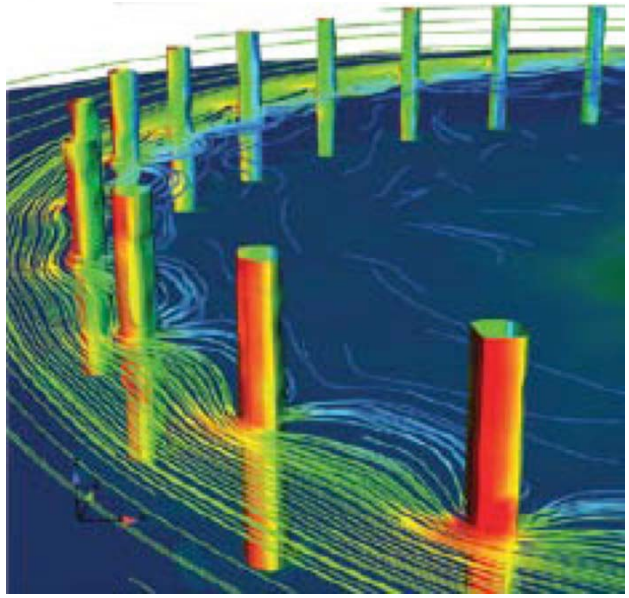


Simulates a wide range of parameters

Not possible w/o correct energy conservation!

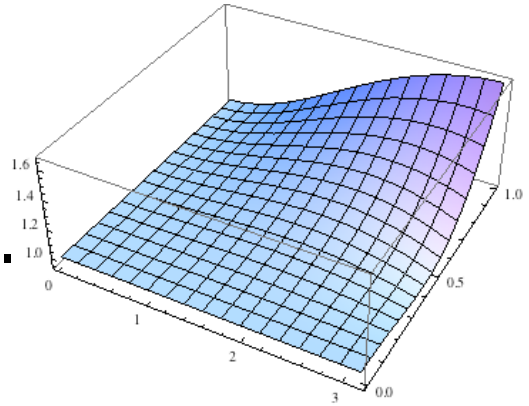
Conclusions





- **ALEGRA is a complex multiphysics code developed at SNL principally under the ASC program**
- **It has many advanced numerical methods that have been improved greatly of late.**

- **MHD is an important aspect of many applications for ALEGRA**
- **Applications demonstrated are Z-pinch dynamics and complex material modeling.**



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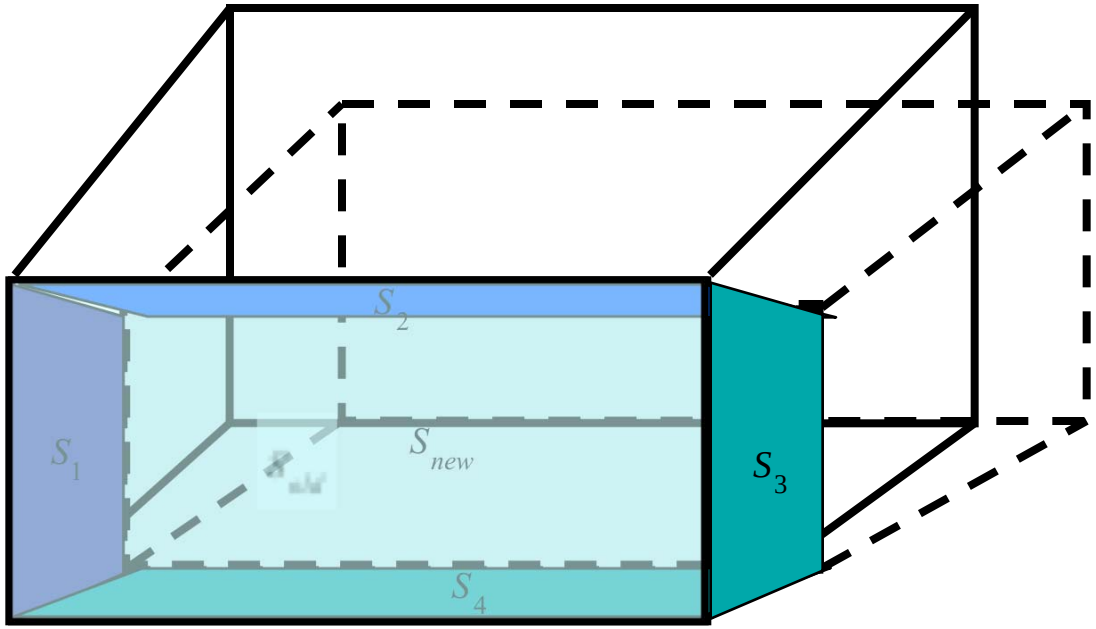
Flux remap step

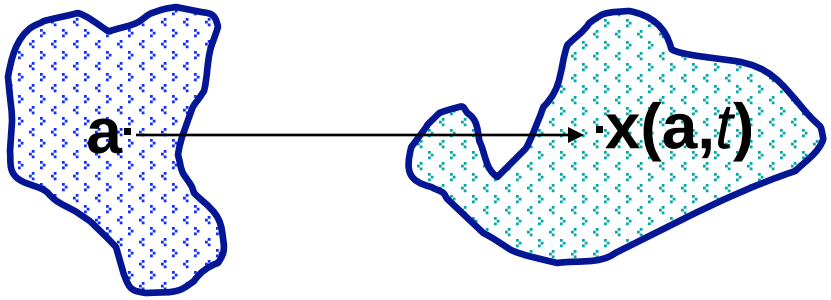
$$\int_S \mathbf{B} \bullet d\mathbf{a} = 0$$

$$\int_{S_{old}} \mathbf{B} \bullet d\mathbf{a} + \int_{S_{new}} \mathbf{B} \bullet d\mathbf{a} + \sum_{i=1}^{4} \int_{S_i} \mathbf{B} \bullet (\mathbf{v}_g \Delta t \times \mathbf{dl}) = 0$$



$$\int_{S_{old}} \mathbf{B} \bullet d\mathbf{a} + \int_{S_{new}} \mathbf{B} \bullet d\mathbf{a} + \sum_{i=1}^{4} \int_{S_i} \mathbf{dl} \bullet (\mathbf{B} \times \mathbf{v}_g \Delta t) = 0$$

Basic Solid Kinematics



(Reference) Material Coordinates

$\mathbf{x}(\mathbf{a},t)$

(Current) Spatial Coordinates

Deformation gradient and inverse:

$$\mathbf{F} = \partial \mathbf{x} / \partial \mathbf{a}$$

$$\mathbf{G} = \mathbf{F}^{-1} = \partial \mathbf{a} / \partial \mathbf{x}$$

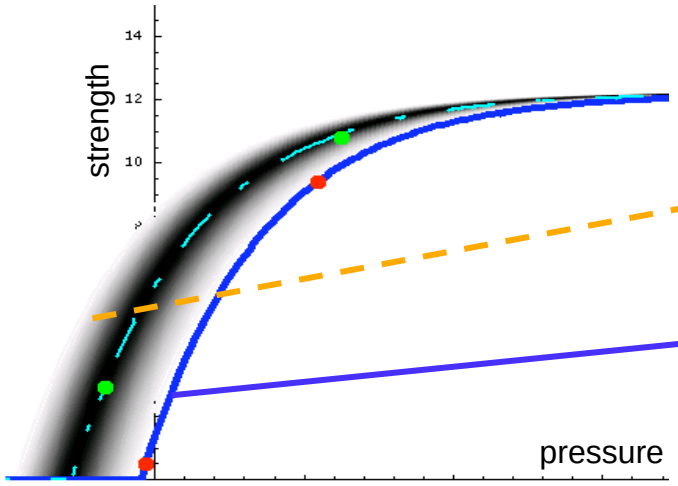
Polar Decomposition: $\mathbf{F} = \mathbf{VR}$

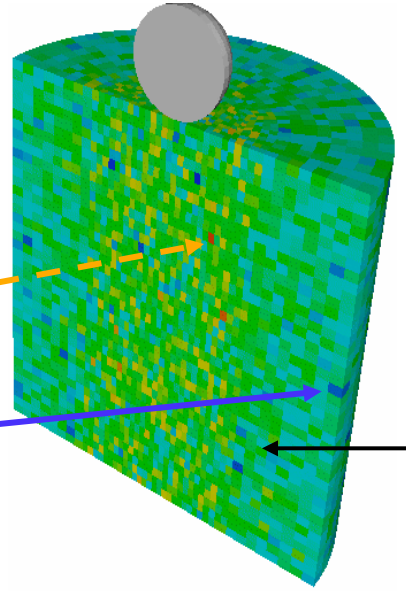
Symmetric Positive Definite (Stretch) Tensor

Proper Orthogonal (Rotation) Tensor

Material Heterogeneity is Integral to Dynamic Failure

Spatially Variable Strength Profile for Ceramics

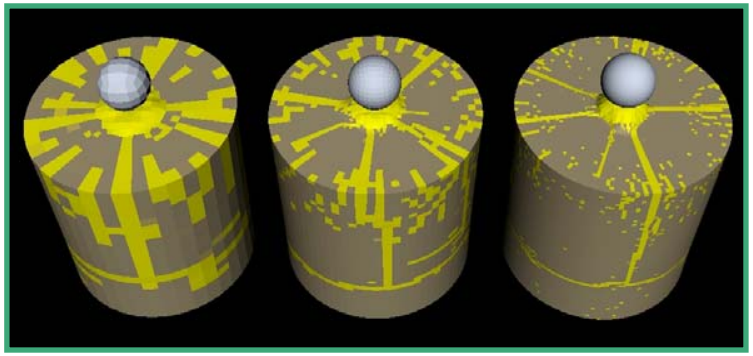




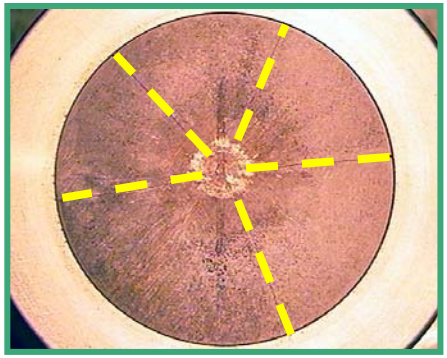
Initial state: small elements are stronger on average, but also more variable

Weibull distribution of strength*:

$$\sigma = \bar{\sigma}\left[\frac{\bar{V}\ln R}{V\ln(1/2)}\right]^{1/m}$$



Reduced Mesh Dependence: Same Model with Uncertainty, Size, and Rate Effects



Comparison to Experiment

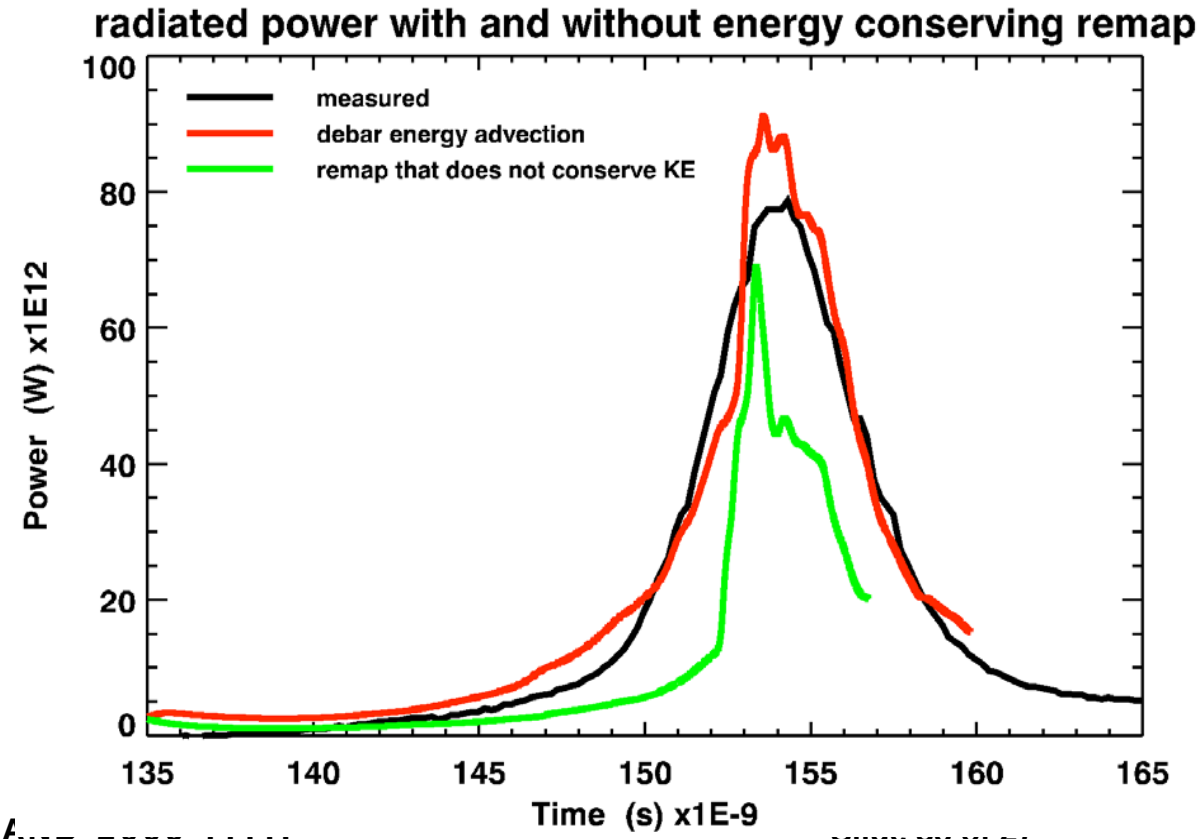
Similar crack morphology for different mesh sizes

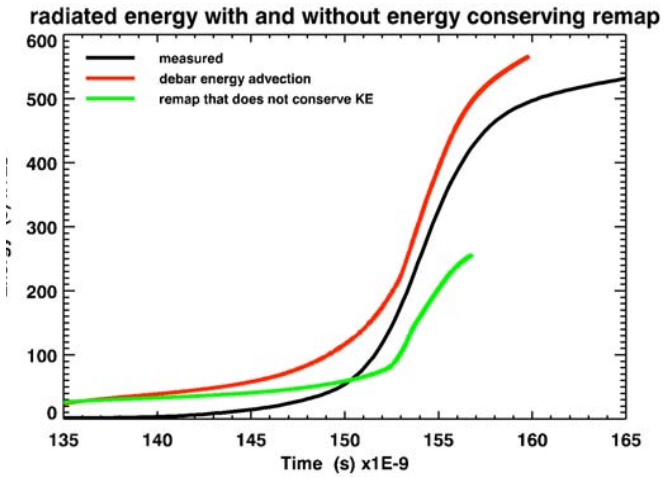
Formal validation and uncertainty quantification will help identify remaining issues

*Brannon, RM, Wells, JW and Strack, OE 'Validating Theories for Brittle Damage', Metallurgical and Materials Transactions A, 38A, p. 2861-8, 2007

Algorithm Impact: 3D Z-Pinch Implosion

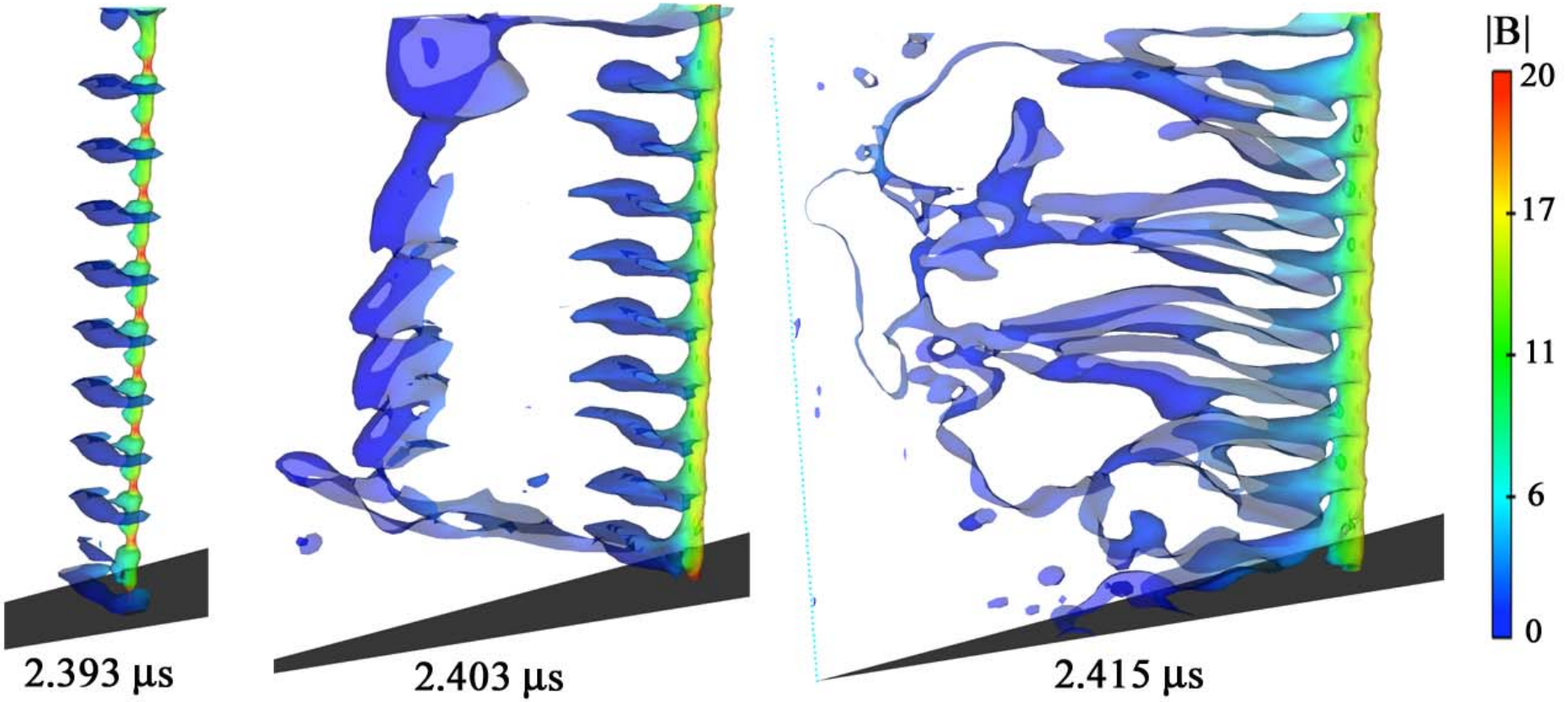
- **This shows the impact of using KE DeBar remap. The radiated power is the key metric. Results are courtesy Ray Lemke.**





Sinusoidal Core Perturbation

Volume fraction isosurface with magnetic field strength



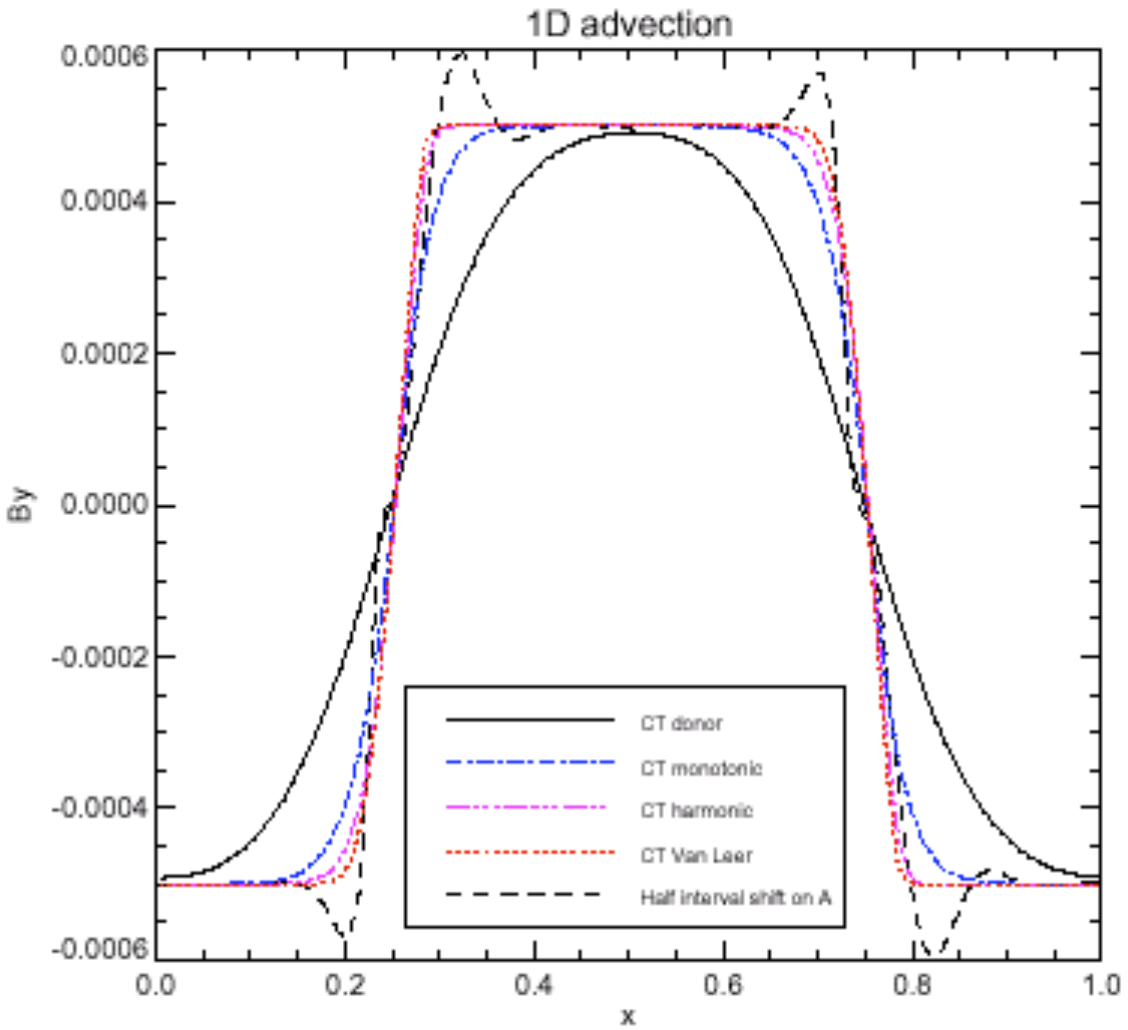
- Is this a variation of the m=0 instability for wire arrays?
- Are the local dynamics governed by the the strength of the local field versus the global?

A quote or two to start us off

"For the numerical analyst there are two kinds of truth; the truth you can prove, and the truth you see when you compute." – Ami Harten

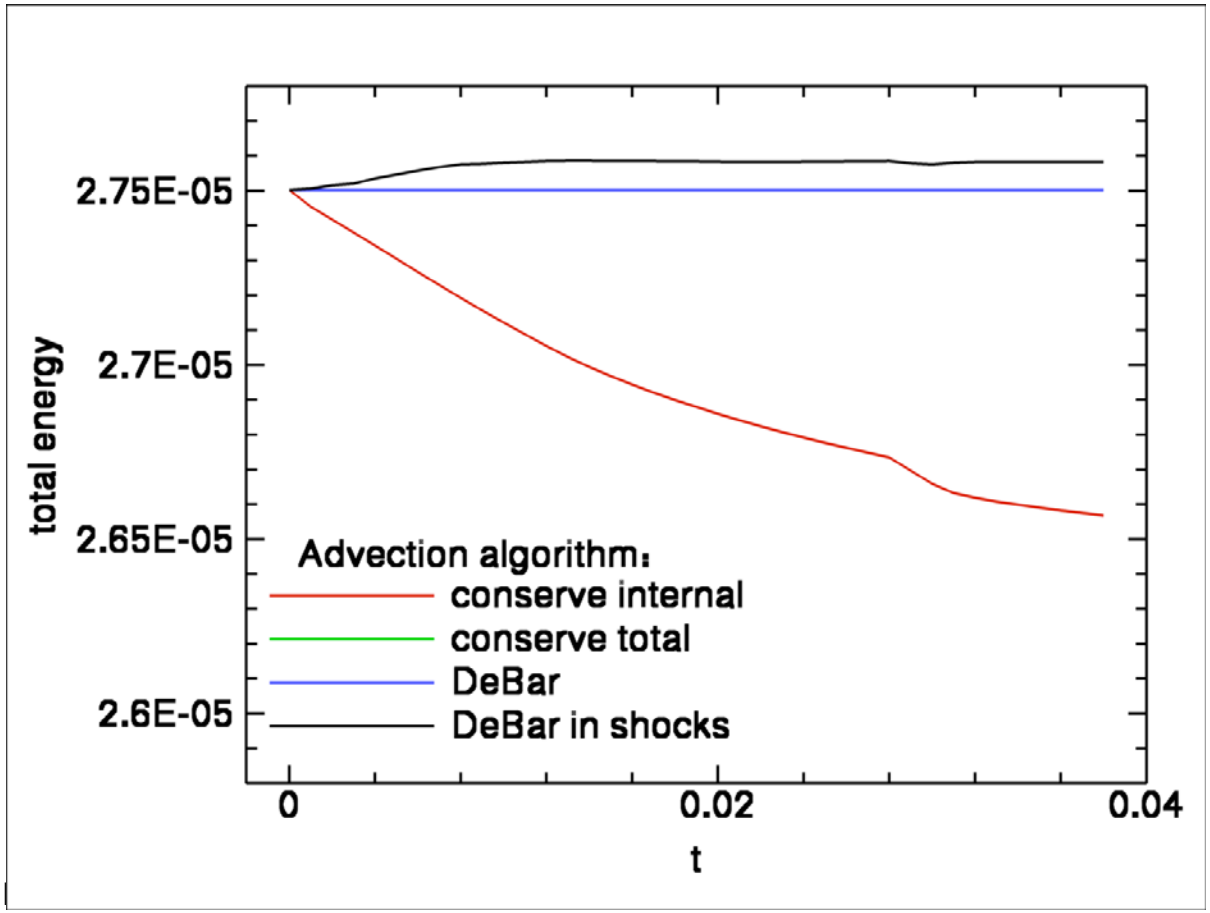
"When C++ is your hammer, every problem looks like your thumb." -– *Anonymous*

CT 1D advection



Results with W-C Blast Wave Problem (continued)

Total energy behavior - internal vs total energy advection vs DeBar & DeBar at shocks (with Q/p>0.001)



The green curve is hidden by the blue curve.

Definitions

- **Lagrangian:**
  - Mesh moves with material points.
  - Mesh-quality may deteriorate
- **REMESH**
  - Mesh-quality is adjusted to improve solution-quality or robustness.
  - **Eulerian** sets new mesh to original location
- **REMAP**
  - algorithm transfers dependent variables to the new mesh.

Mesquite Remesh: Underlying Methodology

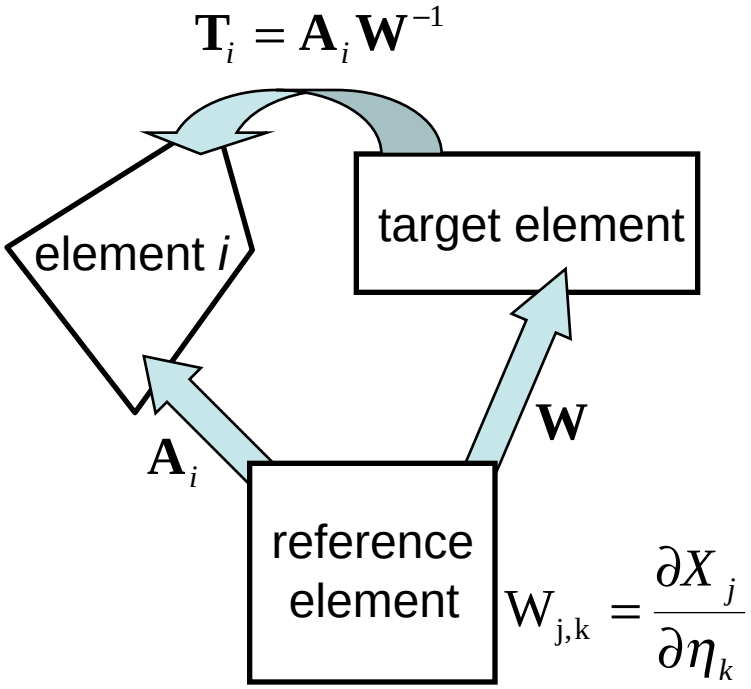
- Define an element quality metric -

$$\mu_i = \frac{\| \mathbf{T}_i \|_F^2}{\det \mathbf{T}_i^a}$$

- Write objective function -

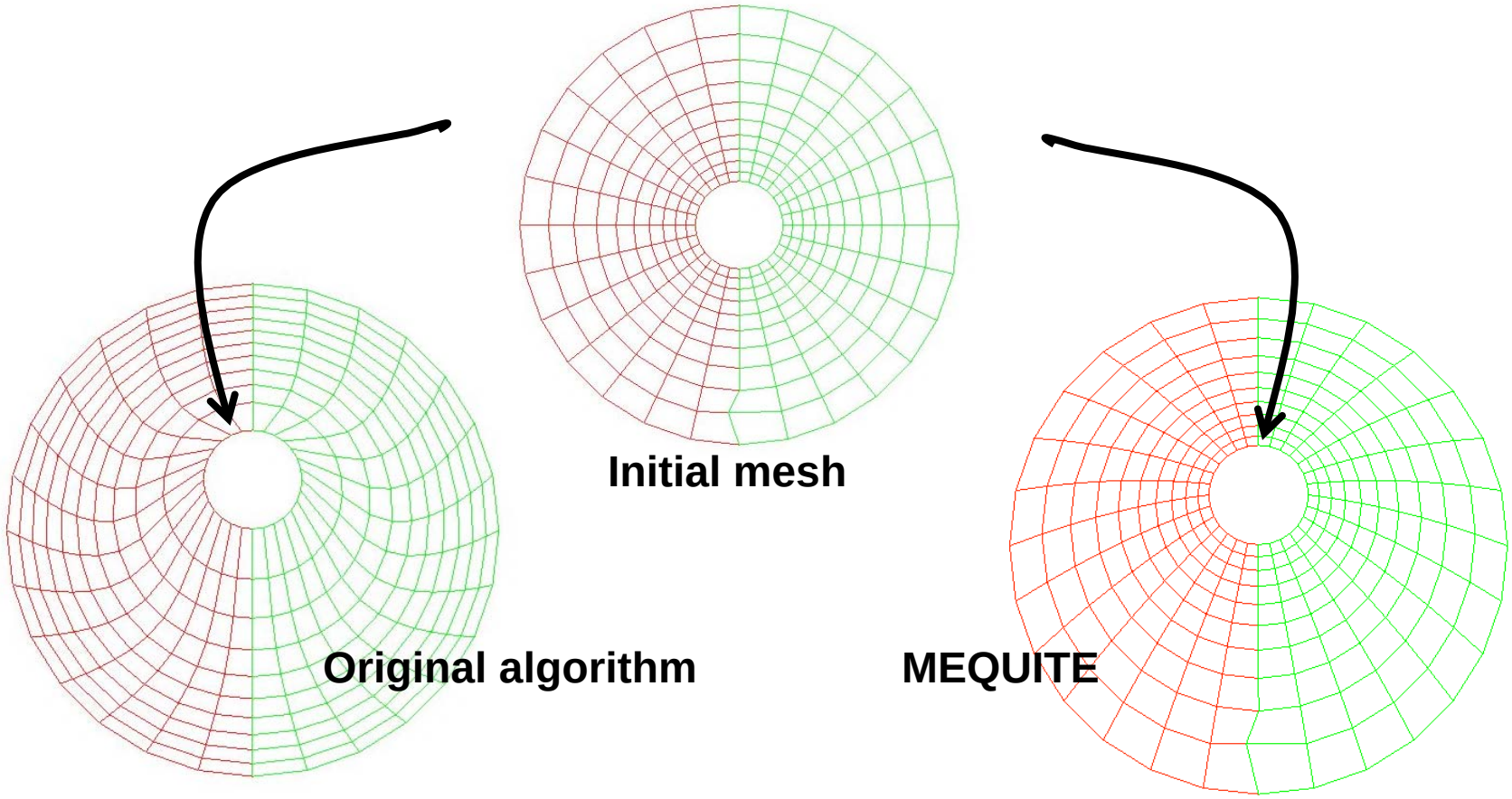
$$F_{\mu,\hat{\Omega}} = \sum_{i \in \hat{\Omega}} \mu_i^p$$

- Minimize *F* w.r.t. vertex locations.



A significant issue for our applications is the appropriate selection of a target element (i.e. **W**) and quality metric.

A Comparison of Equipotential and MESQUITE Meshes



MESQUITE implementation preserves mesh grading without additional controls.

Improved Time Integration Algorithms

- ***Central-difference*** **method is** ***unconditionally unstable*** **(w/o Q).**
- **Midpoint predictor-corrector is stable.**
- **Predictor-corrector is exactly energy conservative.**
- **Predictor-corrector is 2nd-order accurate.**
- **Tumbling block is kickin, SLIRP is kickin too**
- **Periodic breaking wave problem confirms theoretical results.**
- **This slide isn't fluffy enough.**