Optimal Allocation of Resources Under Uncertainty Following an Anthrax Attack

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Paul T. Boggs

Department 8961
Sandia National Laboratories
Livermore, CA 94551, USA

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LDRD
LABORATORY DIRECTED RESEARCH & DEVELOPMENT

Collaborators

- Jaideep Ray (PI)
- Youssef Marzouk (MIT)
- David Gay (1411)
- Nicole Lemaster

Outline

Outline

Anthrax! Panic!

- First indication of attack is an unusual number of people showing up at the hospital very sick
- It will take some time to diagnose anthrax
- Much is not known
  - Where the attack took place
  - When the attack happened
  - The size of the attack
  - How many people were exposed
  - Whether more than one city was attacked

The emergency manager must make decisions about how to allocate resources in the face of huge uncertainty

Anthrax Treatment

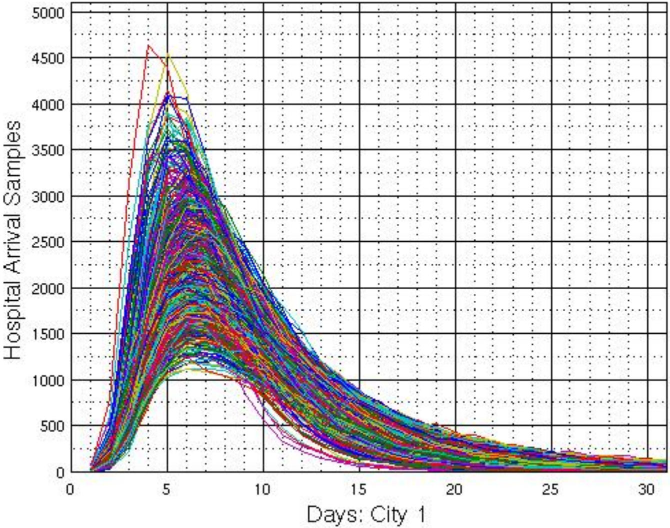
- Everyone gets antibiotics (85% will die)
- Treatment is having lungs drained and being put on ventilator (45% will die)
- Resources are doctors and ventilators

Outline

Assessing the Uncertainty

- We developed a technique to compute a joint probability density function (PDF) that captures the uncertainty
  - Based on the number of people who show symptoms on day j
  - This only becomes known as people arrive and state when they got sick
  - We developed a way to predict total number of victims who became sick on day j
  - We also use other assumptions from the medical literature
- We can sample the PDF to obtain scenarios of the number of people, requiring treatment, who arrive on subsequent days

Assessing the Uncertainty: Early in Attack



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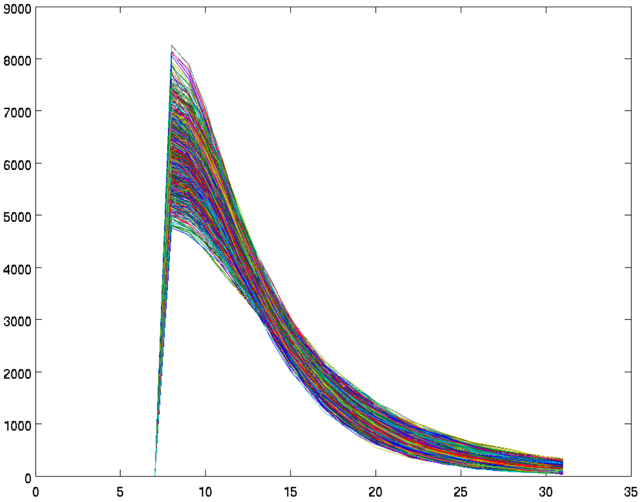
Assessing the Uncertainty; Later in Attack



Optimal Allocation Under Uncertainty

- Early in the incident, the PDF will have very wide variance indicating much uncertainty
- The PDF will narrow as more data are collected
- The optimization problem is to allocate resources in such a way that one minimizes some measure of the expected total number of deaths
- Given the uncertainty, make decisions that you will not regret later
- Optimization should permit conservative allocation in the case of subsequent attacks

An Optimization Model - Least Regret

On each day we can compute a new PDF

- Sample the PDF to obtain set of scenarios
- For scenario k compute an optimal schedule of allocations of resources, r_k
  (r_{kj} is the allocation of on day j for scenario k)
- Let D_k^* be the optimal number of deaths obtainable in scenario k
- Let $D_k(r)$ be the number of deaths in scenario k with allocation r

An Optimization Model (cont'd)

- Next, compute an allocation schedule, r^* such that $D_k(r^*)$ doesn't differ too much from D_k^* for all k
- Ship the allocation, r_1^*, for today

Repeat the whole procedure on each day with resources reduced by r_1^*

Optimal Resource Allocation for Each Scenario

- For each scenario, the optimal allocation is the solution to a classical resource allocation problem
- Find r_k that solves

$$\min_{r} D_k(r)$$

Optimal Resource Allocation for Each Scenario

Constraints

- For each day j, $r_{kj} \geq 0$
- $\sum_{j=1}^{T} r_{kj} \leq R$ (total available resources)
- $r_{kj} \leq N_k^j$
  where N_k^j is the number of people who arrive on day j in scenario k
- $D_{k,j} = r_{kj} * t_j + (N_k^j - r_{kj}) * u_j$
  t_j is the percent of people, treated on day j, who die
  u_j is the percent of people, untreated on day j, who die
- Assume that people who arrive later in attack are more treatable, i.e., $t_j > t_{j+1}$

This is a linear programming problem and is easy to solve

Least-Regret Formulation

- Create least-regret optimization problem - "Don't to too badly in any scenario" i.e., don't deviate too much from the optimal number of deaths: Find r^* satisfying

$$\min_r .5 * \sum_{k=1}^{K} (D_k(r) - D_k^*)^2$$

- Constraints are different, in particular

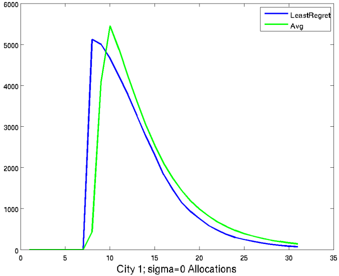
$$D_k^j \geq \begin{cases} t_j * N_{k,j} & \text{if } r_j > N_{k,j} \\ t_j * r_j + (N_{k,j} - r_j) * u_j & \text{else} \end{cases}$$

This problem is a large, convex quadratic programming problem, but for one city can be solved in a few minutes

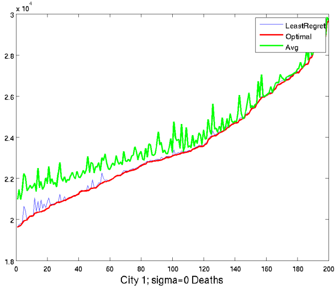
Stochastic Optimization Formulation

- Let a be the allocation schedule we seek
- We have $0 \leq a_j \leq \sigma R$ for each day j

Results - One City



(a) Allocations



(b) Deaths

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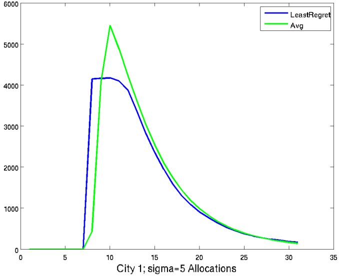
Conserving Resources

- Add constraint

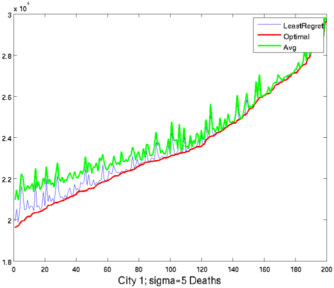
$$\min_{r} .5 * \sum_{k=1}^{K} (D_k(r) - D_k^*)^2 + .5\sigma \sum_{i=1}^{T} r_j^2$$

- $\sigma \geq 0$ is a penalty parameter
- As σ is increased, the allocation schedule "flattens"

Results: One City with Penalty

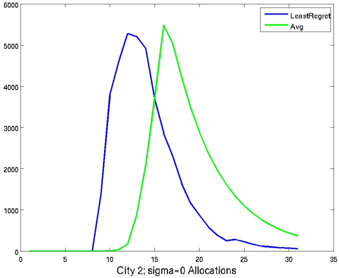


(c) Allocations

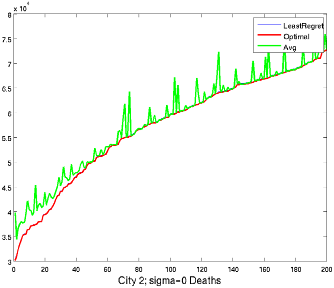


(d) Deaths

Results: Second City with No Penalty



(e) Allocations



(f) Deaths

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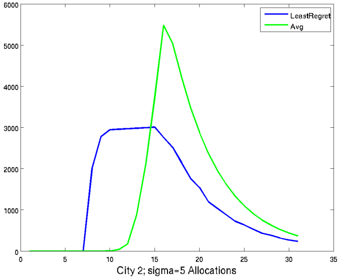
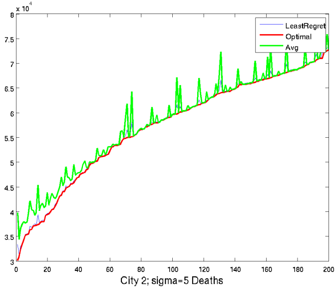
Results: Second City with Penalty



(g) Allocations



(h) Deaths

Two Cities Attacked

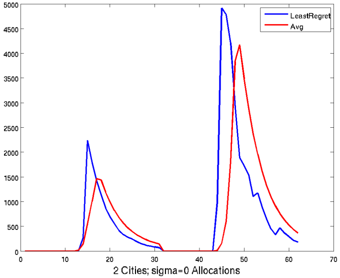
- A life is a life
- Must impose "social" constraint
  Each city must suffer proportionally

$$\frac{D_k^1}{TotArriv_1^k} * (1 - \pi) \leq \frac{D_k^2}{TotArriv_2^k} \leq \frac{D_k^1}{TotArriv_1^k} * (1 + \pi)$$

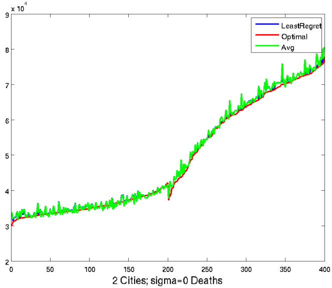
  for $0 \leq \pi \leq 1$
- We call this the "equilibration-of-pain" constraint

This constraint is only applied to resource allocations, not for the least regret

Results: Two Cities with No Penalty



(i) Allocations



(j) Deaths

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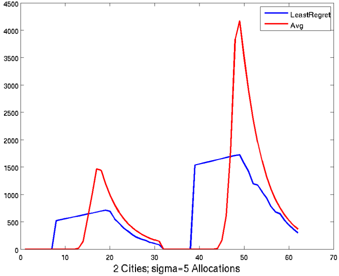
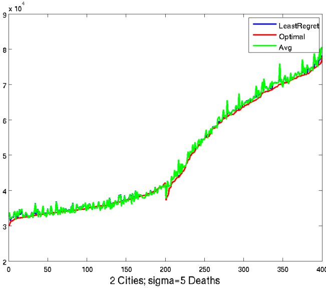
Results: Two Cities with Penalty



(k) Allocations



(l) Deaths

Summary: We have created a useful tool for emergency managers

- Developed novel approach for addressing uncertainty in an anthrax attack where we can
  - Generate PDF to capture the uncertainty based on real data and established assumptions
  - Sample the PDF to produce representative scenarios
  - Use a 2-stage, "Least-Regret" optimization strategy to determine allocation schedule
  - Provides an allocation schedule over entire attack period
  - Update easily as new data become available

Summary: We have created a useful tool for emergency managers

- Can accommodate many constraints
  - General resource allocation conditions, e.g., multiple supply points, shipping delays, etc.
  - Social constraints, e.g., equilibration-of-pain
  - Penalty constraints to flatten resource allocation and hedge against future attack
- This approach is made possible by increase in computational resources **and** improvements in optimization algorithms

Outline

Recent Enhancements

- Developed a 2-stage stochastic optimization with recourse model
- Created several objective function formulations
- Modeled the ability to reuse resources as patients die
- Are testing these approaches

Future Research

- Extension to multi-stage stochastic optimization with recourse
- General resource allocation model with appropriate constraints and classes of constraints
- Other epidemics, e.g., contagious spread where spread is coupled to allocation of resources

“And miles to go before I sleep.”

Thanks!