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A Comparison of Mortar and Tied Contact Approaches for Acoustic and Structural Acoustic Meshes

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Motivation

- Acoustic and structural meshes typically generated independently
- Acoustic and structural meshes almost always have different mesh density requirements
- Mesh tying methods have been researched extensively in solid mechanics – but not in acoustics or structural acoustics
- Fully coupled simulations are needed
  - Coupled modes, coupled frequency response

Mesh typing methods are needed for nonconforming wet interface

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Acoustic-Structure Interaction

Equations of motion of solid

$$\rho u_{tt} - \nabla \bullet \sigma = f(x,t) \qquad \Omega_e x[0,T]$$

Acoustic wave equation for fluid

$$\frac{1}{c^2}\phi_{tt} - \Delta\phi = 0 \qquad \Omega_f x[0,T]$$

Boundary conditions

$$\frac{\partial \phi}{\partial n} = \dot{u}_n$$ Continuity of Displacement

$$\sigma_n = -\hat{n}\dot{\psi}$$ Continuity of Stress

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Weak Formulation for Time Domain

Find $\left(u,\phi\right)$ $\quad \left[0,T\right] \to H_1(\Omega_f)x\left(H_1(\Omega_e)\right)^3$

$$\rho(u_{tt},v)_{\Omega_e} - (\sigma,\nabla v)_{\Omega_e} + (\rho\dot{\phi},v)_{\partial\Omega} = (f,v)_{\Omega_e} \qquad \forall v \in \left(H^1(\Omega_f)\right)^3$$

$$\frac{1}{c^2}(\phi_{tt},\psi)_{\Omega_f} + (\nabla\phi,\nabla\psi)_{\Omega_f} + \frac{1}{c^2}(2\nabla\phi\bullet\nabla\dot{\phi},\psi) + \frac{1}{c^2}(\frac{B/A}{c^2}\phi\dot{\phi},\psi) +$$

$$b(\nabla\dot{\phi},\nabla\psi) - (\dot{u}_n,\psi)_{\partial\Omega} = 0 \qquad \forall\psi \in H^1(\Omega_f)g$$

Discretized form:

$$\begin{bmatrix} M_s & 0 \\ 0 & -M_a \end{bmatrix}\begin{bmatrix} \ddot{u} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} C_s & L^T \\ L & -C_a \end{bmatrix}\begin{bmatrix} \dot{u} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & -K_a \end{bmatrix}\begin{bmatrix} u \\ \phi \end{bmatrix} = \begin{bmatrix} f_s \\ f_a \end{bmatrix}$$

Coupling Terms

- **The surface integral is the key to the coupling methods**

$$L_{ij} = \int_{\Gamma} N_{M_i} N_{S_j} \, d\Gamma$$

N_{M_i} Surface shape function on master side

N_{S_j} Surface shape function on slave side

- For conforming meshes, this is the classical structural-acoustic coupling matrix since N_M and N_S are the same functions and are on the same surface

- For mortar methods, N_M and N_S are different functions and are defined on different surfaces, but the integral is still the same

  - Since N_M and N_S are defined on different surfaces, a surface projection method is needed – we use approach of Laursen et al.

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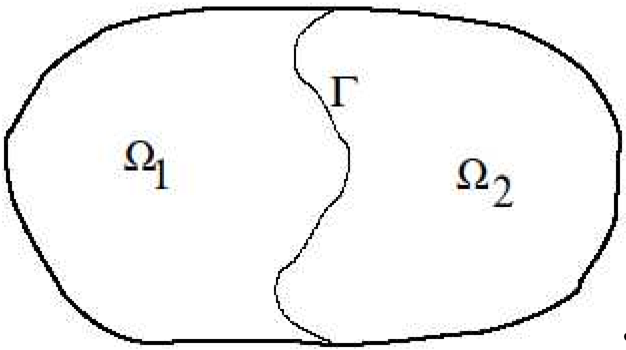
Mesh Tying Methods for Acoustics



Weak formulations

Ω_1

$$\int_{\Omega_1} \left[\frac{1}{c^2} \ddot{\psi}\, \phi + \nabla \psi \bullet \nabla \phi \right] d\Omega_1 = 0$$

Ω_2

$$\int_{\Omega_2} \left[\frac{1}{c^2} \ddot{\psi}\, \phi + \nabla \psi \bullet \nabla \phi \right] d\Omega_2 = 0$$

Constraint equations on interface

- Classical MPC equations $\psi_S = \sum c_i \psi_M$
- Mortar method $\int_{\Gamma} (\psi_1 - \psi_2)\eta \, d\Gamma = 0$

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Discretization of Boundary Constraint

Boundary Constraint Equation:

$$\int_{\Gamma} (\psi_1 - \psi_2)\eta \, d\Gamma = 0$$

Discretization:

$$\psi_M = \sum N_{M_i}(x)\psi_{M\,i} \qquad \psi_S = \sum N_{S_i}(x)\psi_{S_i}$$

$$\eta = N_{S_j}$$

$$\int_{\Gamma} (\psi_1 - \psi_2)\eta \, d\Gamma = \sum_i \int_{\Gamma} N_{M_i} N_{S_j} \, d\Gamma - \sum_i \int_{\Gamma} N_{S_i} N_{S_j} \, d\Gamma$$

Mortar method for acoustics involves same surface integrals as for conforming structural acoustics

Mesh Tying Methods for Structural Acoustics

1. **Conforming finite element approach**
   - **Requires matching meshes**
2. **Classical multipoint constraint equations with ghost nodes**
3. **Mortar method with ghost nodes**
4. **Classical mortar method**
   - **Flemisch et al, 2006**

In all cases we need to evaluate integrals of the type: $\int_{\Gamma} N_M N_S d\Gamma$

N_M Surface shape function on master side

N_S Surface shape function on slave side

Mesh Tying Methods for Structural Acoustics

Ghost nodes approach:

- Add “ghost” acoustic degrees of freedom to solid nodes on wet interface
- Use conforming coupling operators to couple solid nodes on wet interface to appended acoustic dof
- Couple acoustic dof on both sides of wet interface with mortar or standard MPC equations

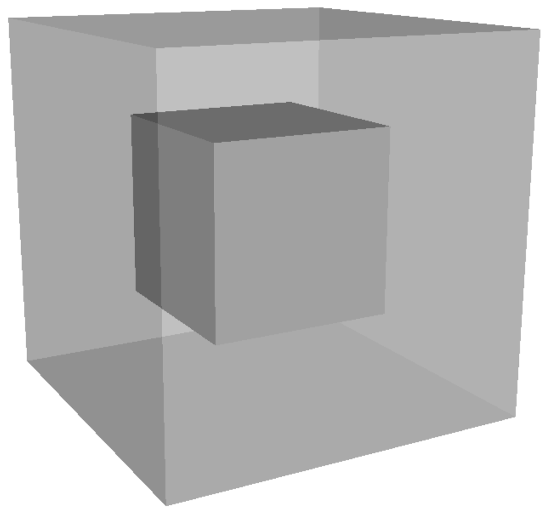
For conforming meshes, this method reduces to a conforming structural acoustics

Same constraint equations for acoustic-acoustic coupling and structural-acoustic coupling

Mesh Tying for Structural Acoustics – Ghost Node Approach

(solid dof + ghost acoustic dof)

Cube-In-Cube Structural Acoustic Example



- Aluminum tank filled with water
- time-dependent pressure load (sinusoid) applied to end of tank
- far-end of tank fixed to rigid wall

We compare the results using 3 methods:

- **Conforming meshes**
- **Nonconforming meshes with ghost nodes and classical MPCs**
- **Nonconforming meshes with ghost nodes and mortar constraints**

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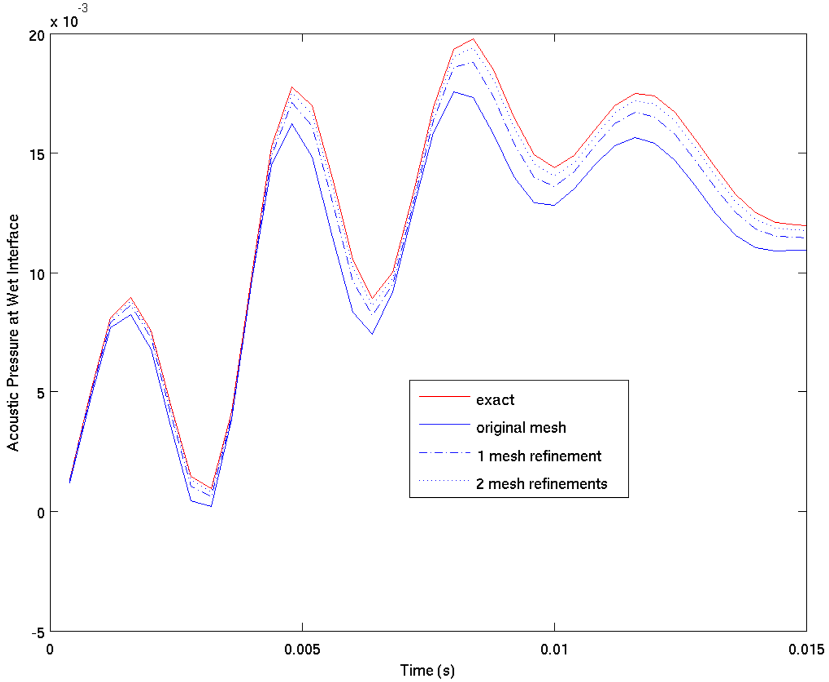
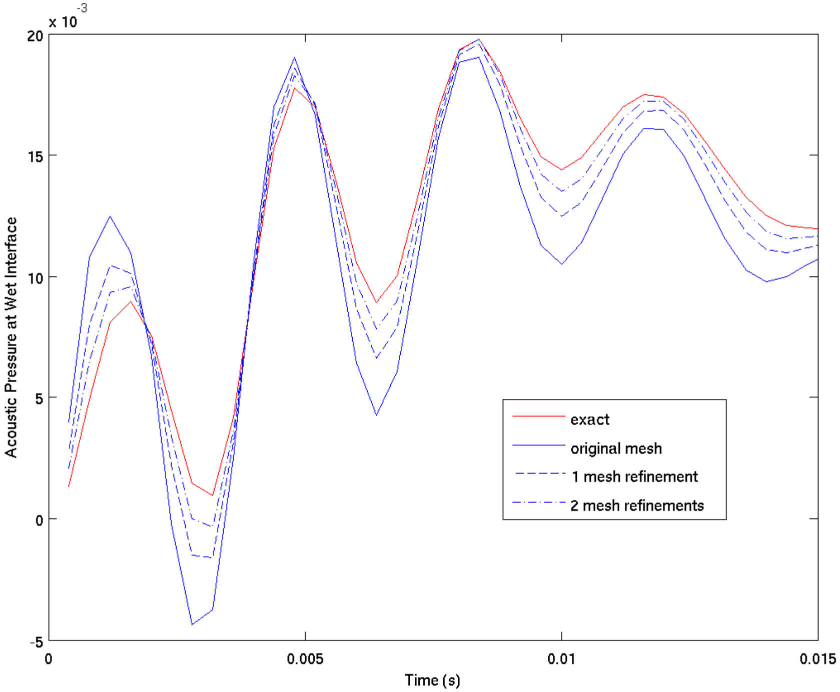
Convergence Results for Cube-in-Cube



Conforming meshes



Ghost nodes approach with classical MPC's

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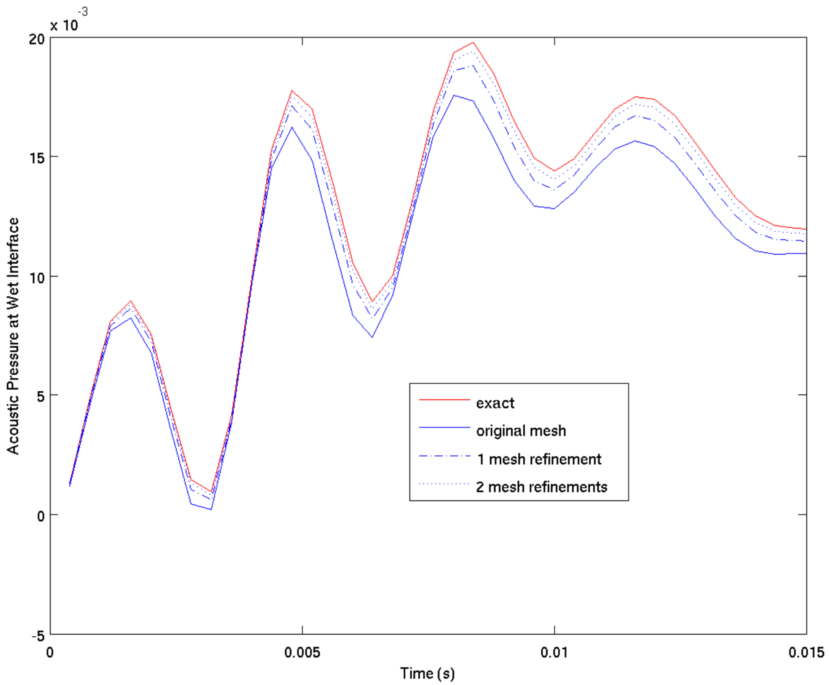
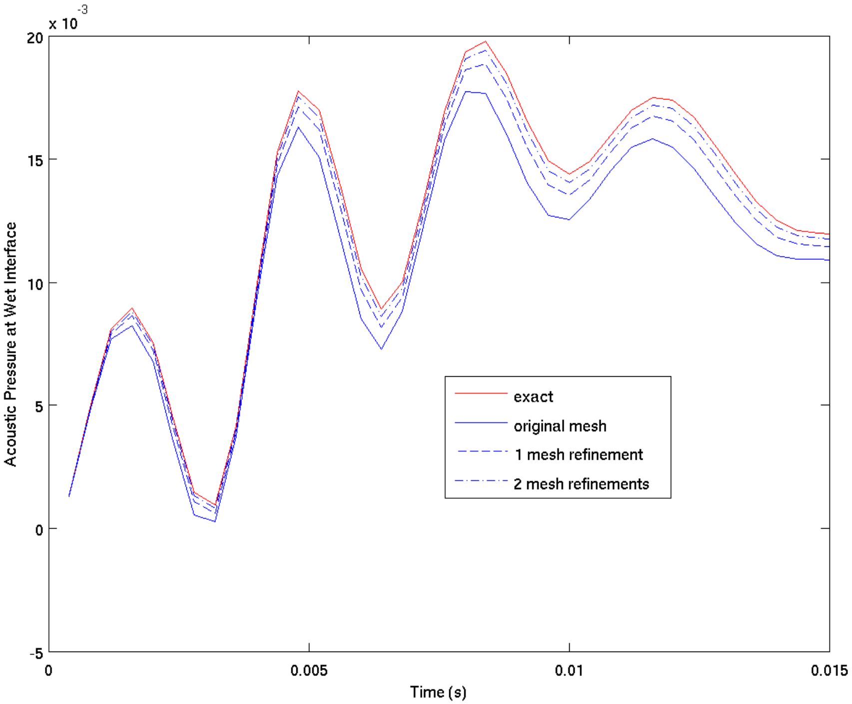
Convergence Results for Cube-in-Cube



Conforming meshes



Ghost nodes approach with mortar method

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Convergence Results for Water-Castor Oil System

- **Two-fluid tank filled with water and castor oil**
- **Assumed that no mixing occurs**

We compare the results using 3 methods:

- **Conforming meshes**
- **Nonconforming meshes with ghost nodes and classical MPCs**
- **Nonconforming meshes with ghost nodes and mortar constraints**

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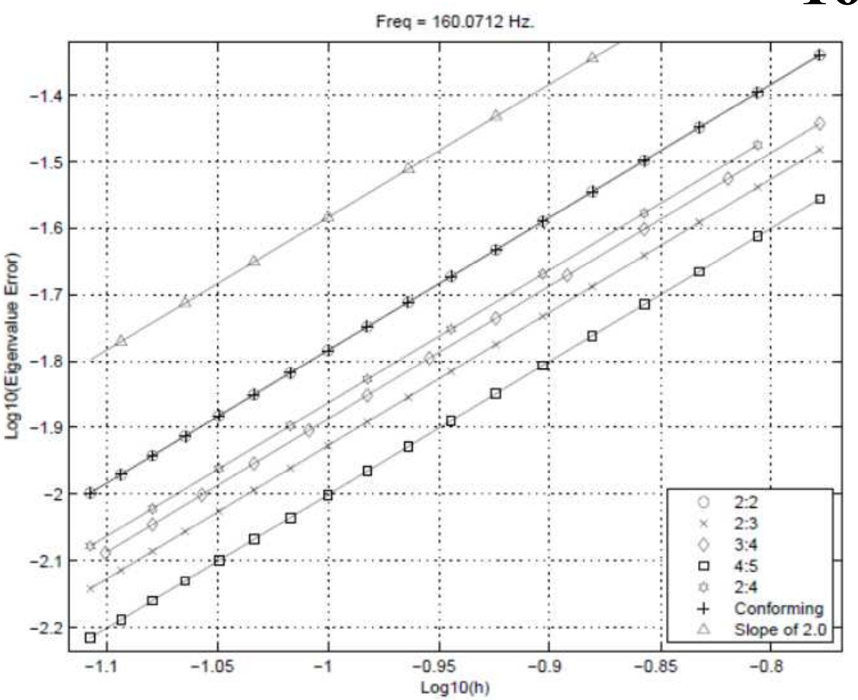
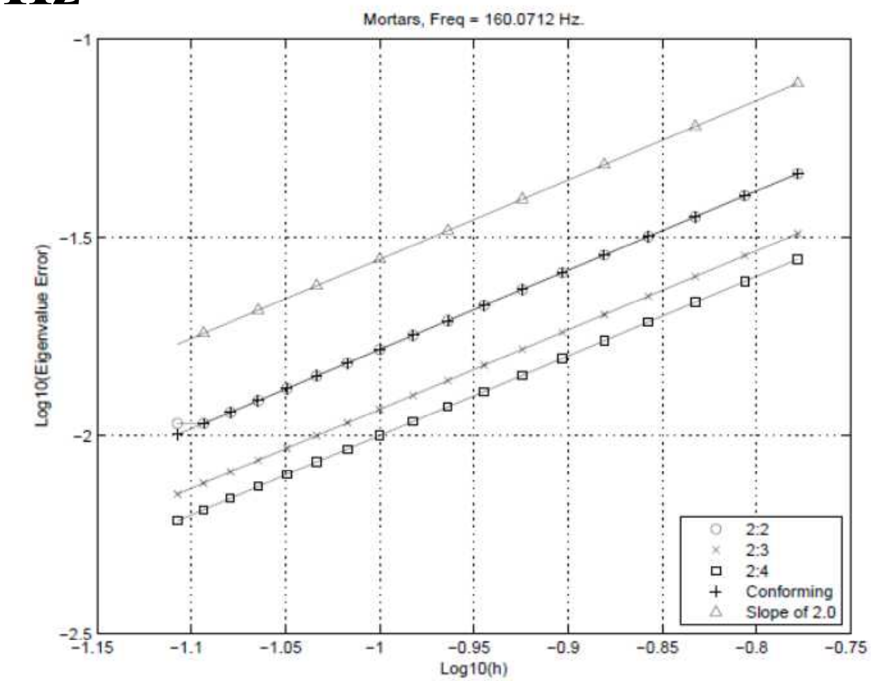
Convergence Results Two-Fluid System

160 Hz



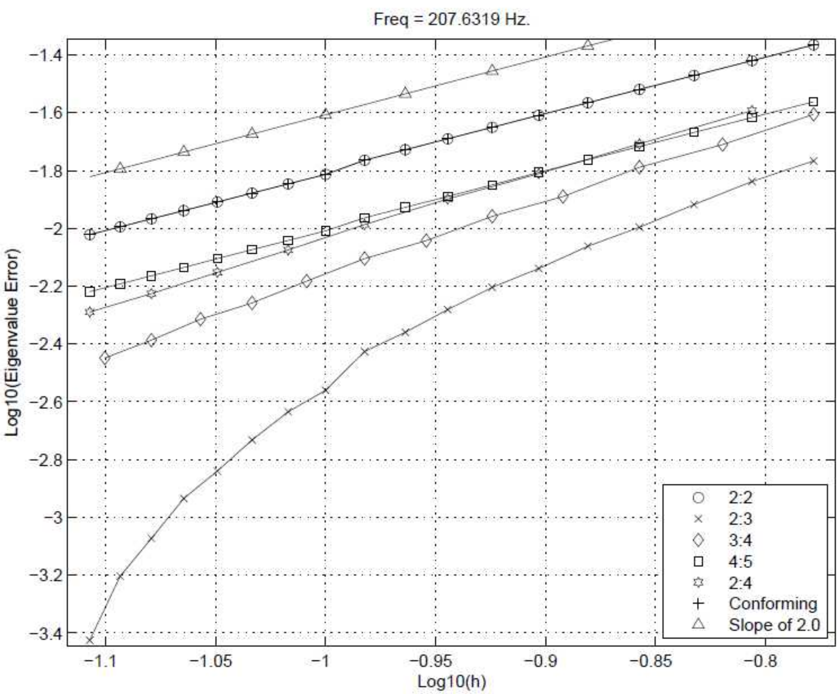
Classical MPCs



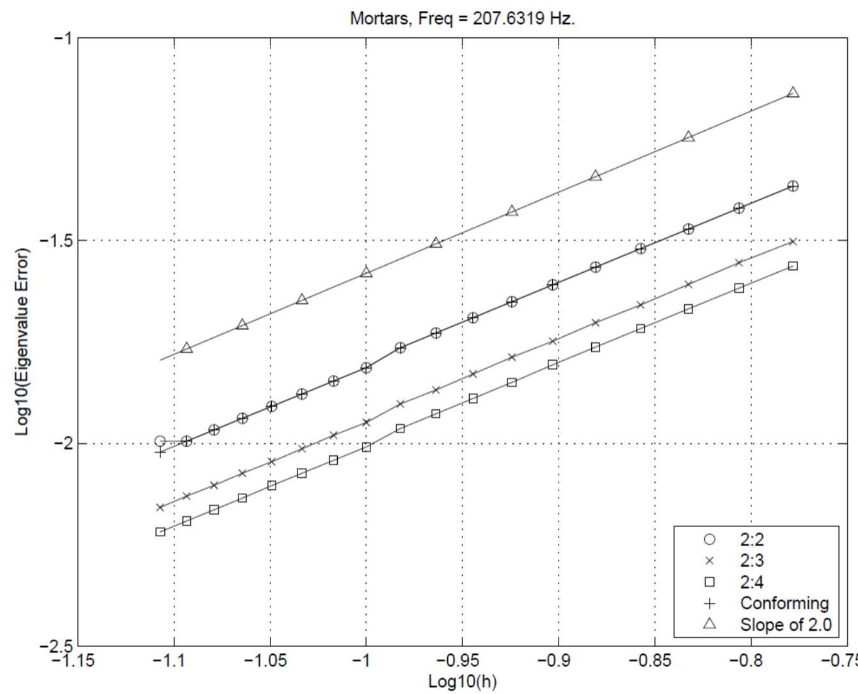
Mortar method

Convergence Results Two-Fluid System

207 Hz



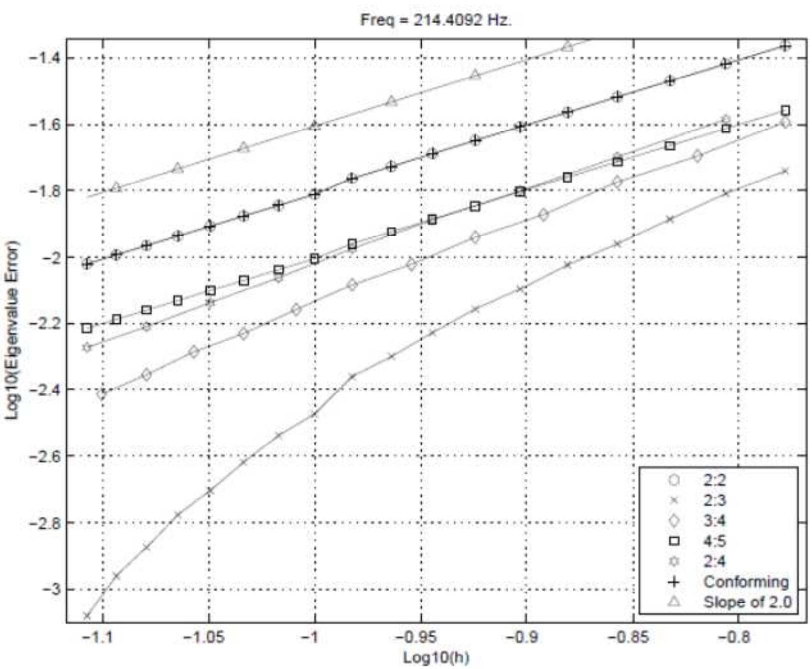
Classical MPCs



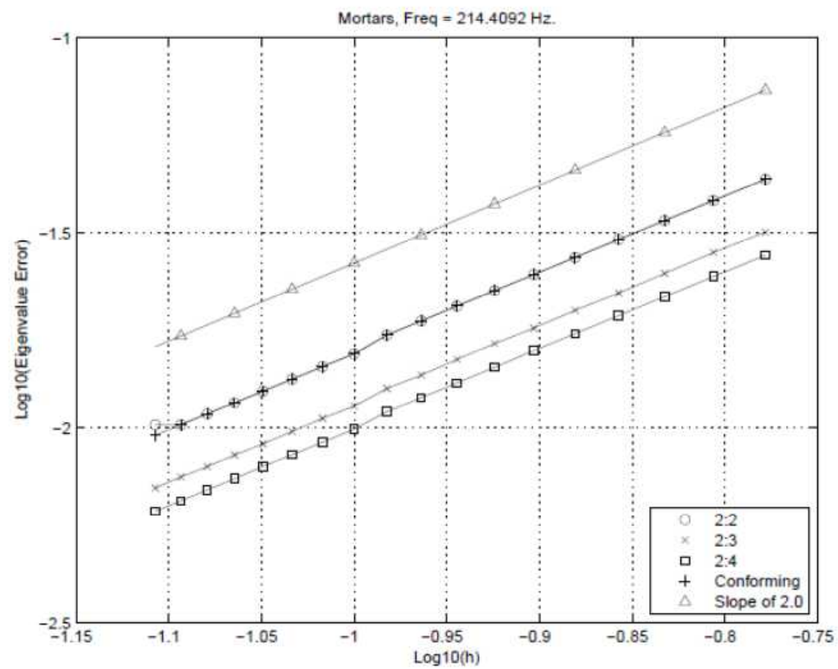
Mortar method

Convergence Results for Two-Fluid System

214 Hz



Classical MPCs



Mortar method

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Observations and Conclusions

- Both Classical MPCs and mortar method demonstrate convergence for both acoustic and structural acoustic mesh tying
- Mortar method appears to converge faster on cube-in-cube transient structural acoustic problem
- For two-fluid system, mortar method appears to give convergence rates that are consistent with theory
- For two-fluid system, classical MPCs show consistent convergent rates for some modes, and inconsistent rates for other modes