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Verification of a Conformal Decomposition Finite Element Method for Fluid Dynamics

David R. Noble and Jeremy Lechman

Sandia National Laboratories

Albuquerque, New Mexico

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Motivation: Thermal/Fluid Problems Requiring Enriched Methods

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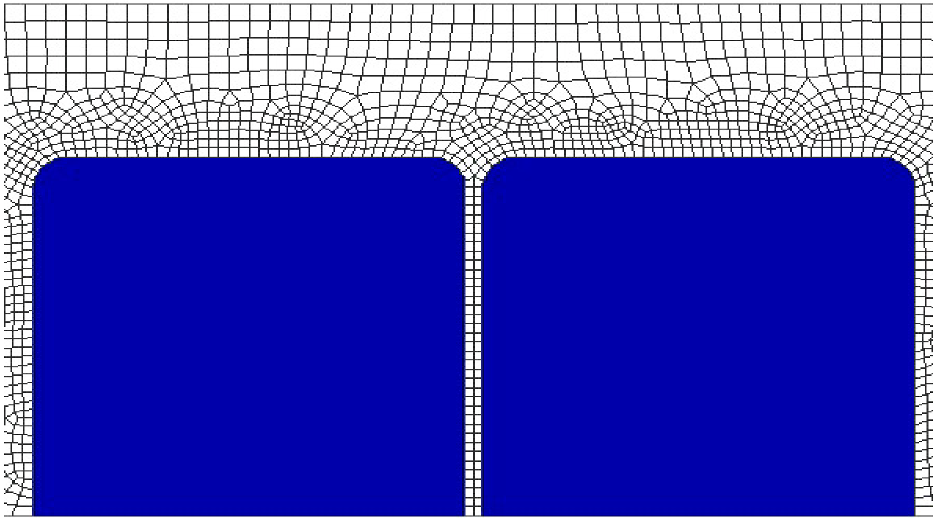
- Laser welding
- Foam decomposition
- Aluminum melting/relocation
- Fuel spills
- Ablation

Common Features

- Coupled thermal and momentum transport
- Significant topological changes
- Strong interfacial physics
- Both strong and weak interfacial discontinuities



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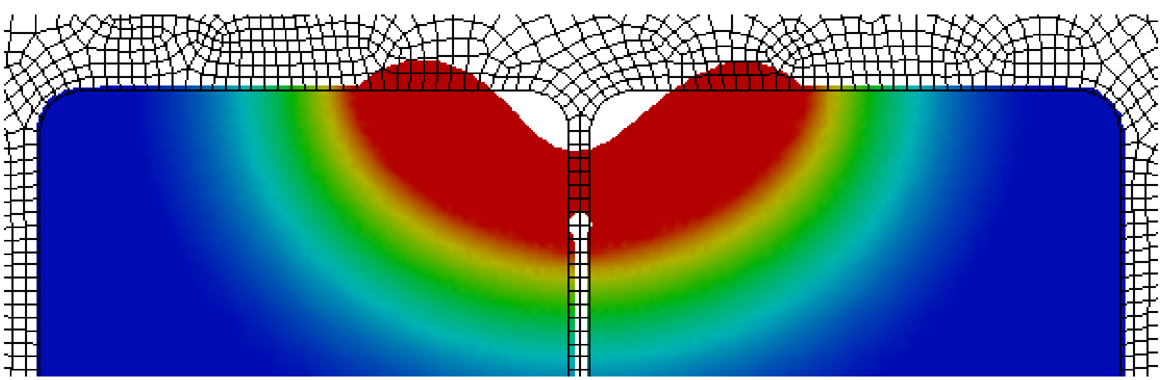
Prototypical Thermal/Fluids Problem: Laser Welding

Material joining by intense localized heating

- Extremely complex interfacial physics
  - Radially distributed laser heating
  - Vapor recoil pressure
  - Vaporization heat loss (latent heat)
  - Radiation and convection heat loss
  - Critical role of surface tension
- Resulting interfacial discontinuities
  - Discontinuous pressure (due to surface tension)
  - Discontinuous velocity gradient (due to discontinuous viscosity)
  - Temperature only solved in fluid and solid (not in gas)



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Finite Element Methods for Interfaces in Fluid/Thermal Applications Tested at Sandia

Boundary Fitted Meshes

- Supports wide variety of interfacial conditions accurately
- Requires boundary fitted mesh generation
- Not feasible for arbitrary topological evolution (ALE)
  - Mesh quality degrades with evolution, phase breakup and merging are precluded.

Diffuse Interface Level Set Methods

- Avoids boundary fitted mesh generation
- Use phase-averaged properties across interface
  - Smears physics across interface

eXtended Finite Element Methods (XFEM)

- Dolbow et al. (2000), Belytchko et al. (2001)
- Successfully applied to numerous problems ranging from crack propagation to phase change to multiphase flow
- Supports weak conditions accurately, mixed and Dirichlet conditions are actively researched (Dolbow et al.)
- Avoids boundary fitted mesh generation
- Supports general topological evolution (subject to resolution requirements)
- Requires modified matrix structure and element assembly including interpolation and integration

Conformal Decomposition Finite Element Method (CDFEM)

- Enrichment by adding nodes along interfaces

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Beyond XFEM: Conformal Decomposition Finite Element Methods (CDFEM)

Simple Concept

- Decompose non-conformal elements into conformal ones
- Obtain solution on conformal elements

Related Work

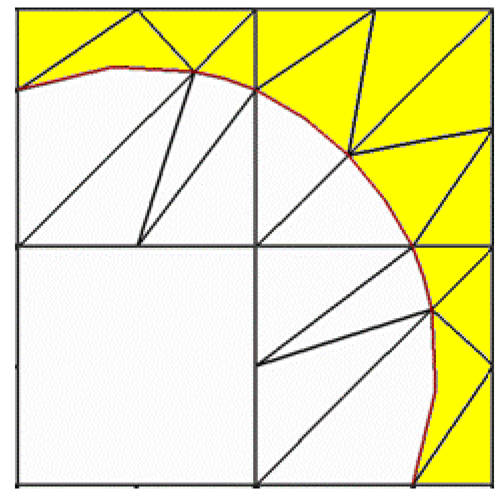
- Li et al. (2003) FEMCGAN: FEM on Cartesian Grid with Added Nodes
  - Focus on Cartesian Grid. Considered undesirable because it lost original matrix structure.

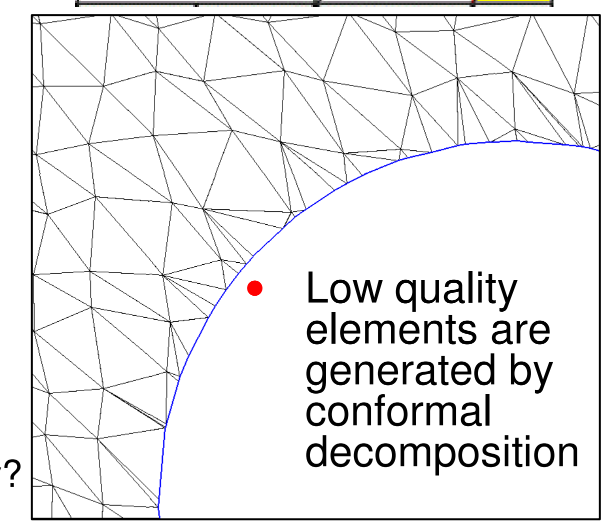
Properties

- Supports wide variety of interfacial conditions accurately (identical to boundary fitted mesh)
- Avoids boundary fitted mesh generation
- Supports general topological evolution (subject to resolution requirements)
- Requires modified matrix structure (additional elements)
  - Similar to finite element adaptivity
- Uses standard finite element assembly including data structures, interpolation, and quadrature

Questions

- Accuracy? Linear system conditioning? Conformal elements can have vanishing size and quality.
- Relationship to XFEM?
- Three dimensional implementation for arbitrary interface geometry?





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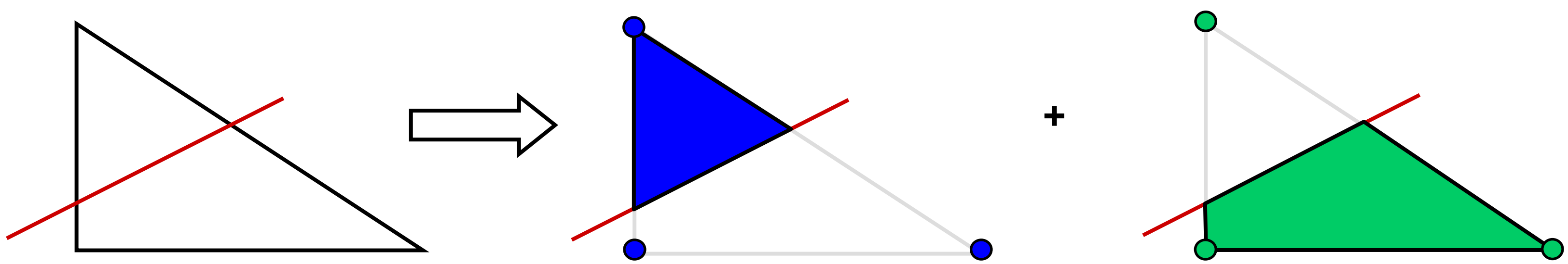
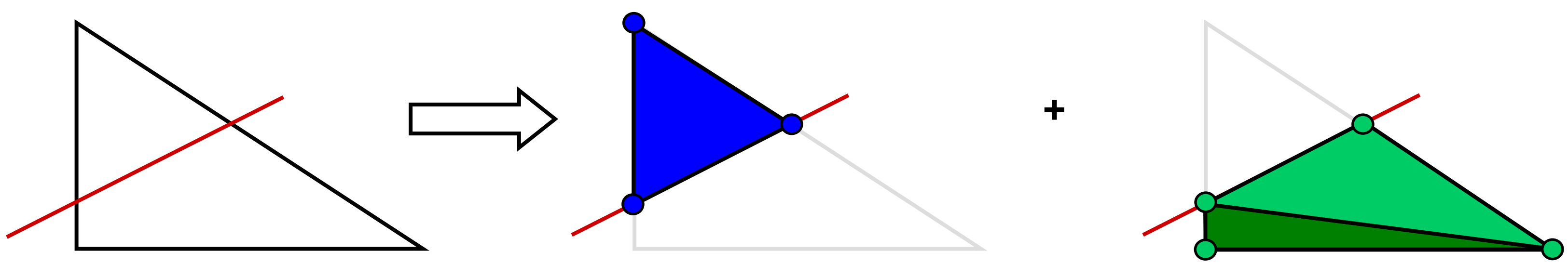
XFEM – CDFEM Comparison

XFEM Approximation with Heaviside Enrichment



CDFEM Approximation



- Identical IFF interfacial nodes in CDFEM are constrained to match XFEM values at nodal locations
- CDFEM space contains Heaviside XFEM space

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XFEM – CDFEM Comparison, cont'd

Approximation

- CDFEM space contains Heaviside XFEM space
  - Accuracy of CDFEM no less than XFEM? Li et al. (2003)
  - CDFEM can recover XFEM solution by constraining interfacial nodes
    - Separate linear algebra step outside of element assembly routines
- CDFEM space captures essential feature of "ridge" enrichment space

Boundary Conditions

- CDFEM readily handles interfacial Dirichlet conditions
  - Simply apply Dirichlet conditions to interfacial nodes
- Gives another view of difficulty with Dirichlet conditions in XFEM
  - CDFEM recovers XFEM when interfacial nodes constrained to XFEM space
  - CDFEM provides optimal solution for Dirichlet problem when interfacial nodes are given by Dirichlet conditions
  - Attempting to satisfy both sets of constraints simultaneously over-constrains the problem

Implementation

- Conformal decomposition can be performed external to all assembly routines

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XFEM - CDFEM Requirements Comparison

	XFEM	CDFEM
Volume Assembly	Conformal subelement integration, specialized element loops to use modified integration rules	Standard Volume Integration
Element Generation	None. Subelements conform to interface but not one another	Elements must conform to one another as well as interface
Surface Flux Assembly	Specialized volume element loops with specialized quadrature	Standard Surface Integration
Phase Specific DOFs and Equations	Different variables present at different nodes of the same block	Block has homogenous dofs/equations
Dynamic DOFS and Equations	Require reinitializing linear system	Require reinitializing linear system
Various BC types on Interface	Dirichlet BCs are research area	Standard Techniques available
Avoiding sliver elements	Automatic	Requires constraining added nodes to XFEM space

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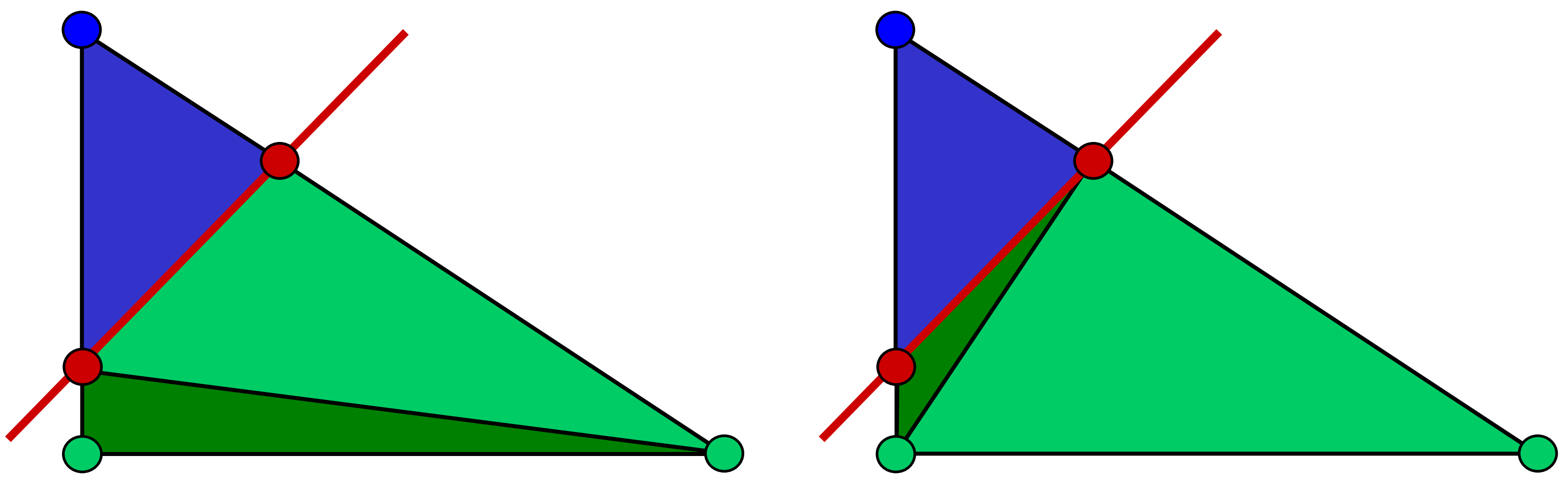
CDFEM – Level Set Implementation in Two Dimensions

Conformal Decomposition Algorithm in Two Dimensions

- Isosurface of piecewise linear level set field on triangles generates C^0 line segments
- Parent non-conformal triangular elements decomposed into conformal triangular elements
- Must choose how to decompose quadrilateral into triangles
  - Babuška and Aziz: Large angles more detrimental to accuracy than small angles
  - Diagonal chosen to cut largest angle



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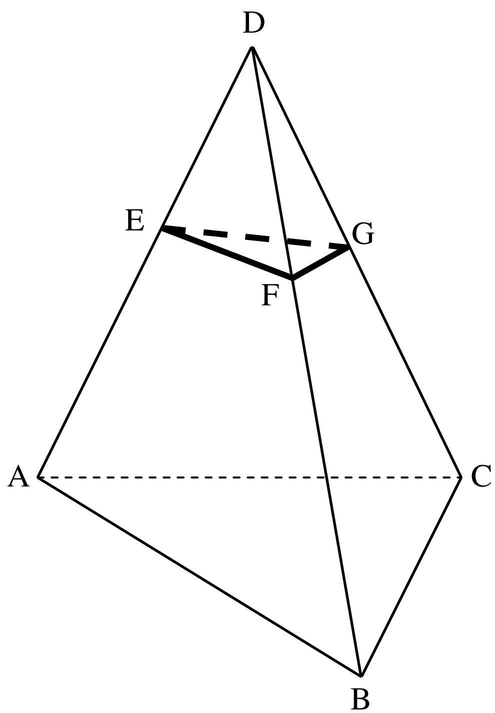
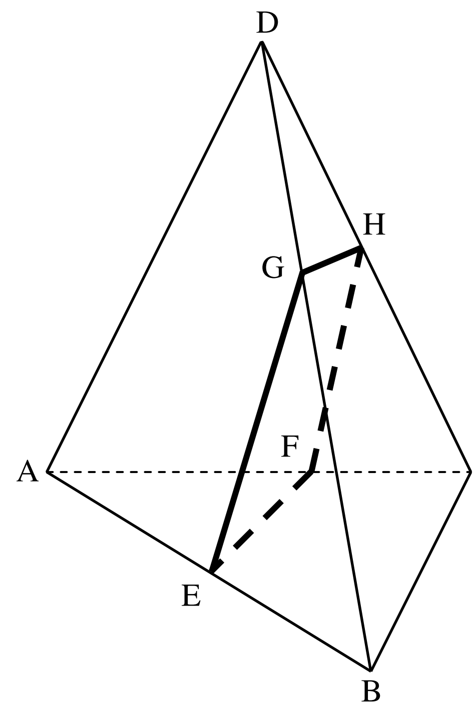
CDFEM – Level Set Implementation in Three Dimensions

Conformal Decomposition Algorithm in Three Dimensions

- Isosurface of piecewise linear level set field on tetrahedra generates C^0 planar polygons
- Parent non-conformal tetrahedral elements decomposed into conformal tetrahedral elements – Intermediate wedges generated
  - wedge + tetrahedra
  - wedge + wedge





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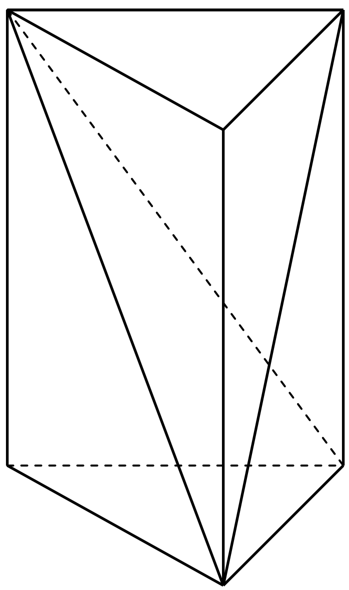
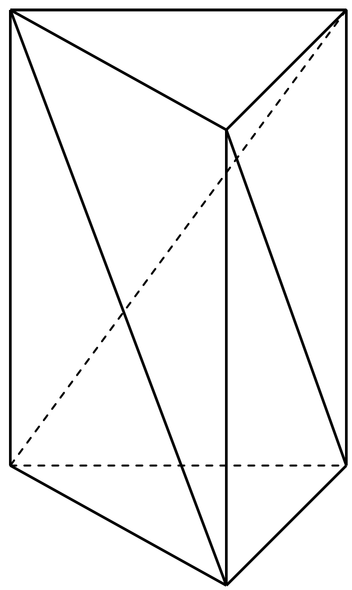
CDFEM – Level Set Implementation in Three Dimensions – cont'd

- Decompose faces of wedges into triangles and then generate tetrahedra
  - Desired strategy is again to choose the diagonals to cut largest angles
  - Non-tetrahedralizable wedge called Schonhardt's polyhedron may be generated
  - Current strategy depends on face
    - Interfacial faces – cut largest angle, Non-interfacial faces – select node with largest level set magnitude (prefers edges that are not aligned with interface)



Wedge amenable to generation of tetrahedra



Schonhardt's Polyhedron – Non-tetrahedralizable without Steiner points

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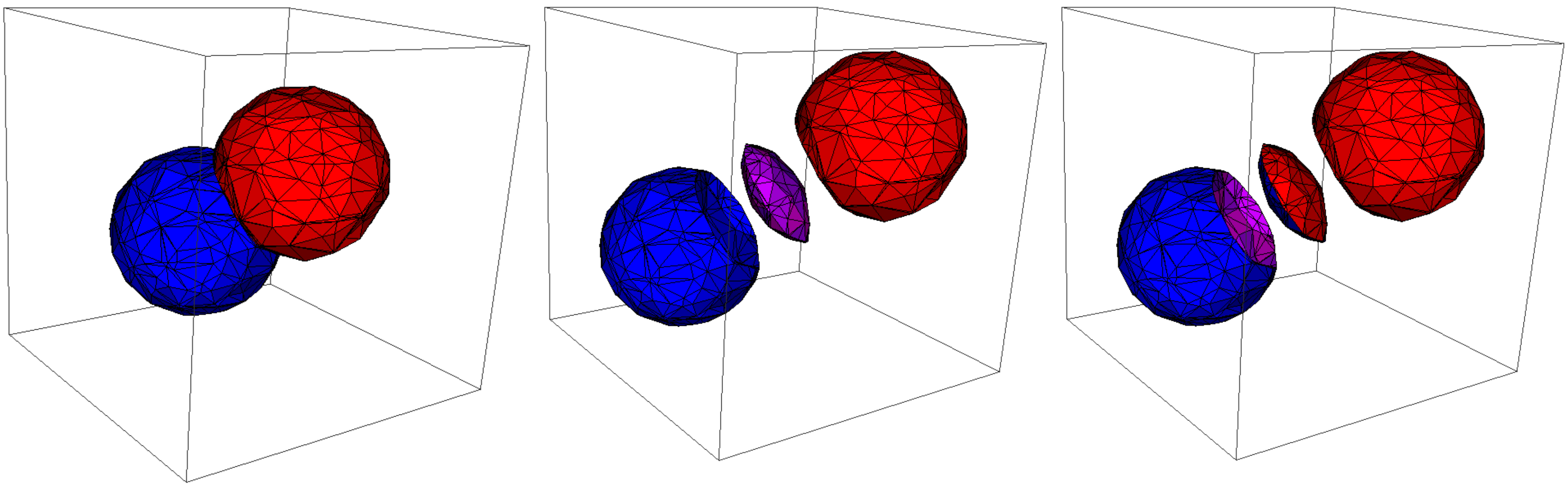
Multiphase CDFEM Implementation



CDFEM – Level Set Implementation in Production FEM Code

- Arbitrary number of phases defined by multiple level set fields
- Dynamic decomposition of blocks and sidesets
- Creation of sidesets on interfaces for bc application
- Phase specific material properties, equations, source terms, etc.
- No changes needed to physics code

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CDFEM Verification

- Two-Dimensional Potential Flow About a Cylinder (static)
  - Analytical solution provides quantitative measure of accuracy
    - Accuracy of velocity potential and its gradient computed in volume and on interface
  - Allows experiments with various boundary conditions
  - Used as test of effect on matrix conditioning
- Three-Dimensional Potential Flow About a Sphere (static)
  - Analytical solution provides quantitative measure of accuracy
    - Accuracy of velocity potential and its gradient computed in volume and on interface
  - Allows experiments with various boundary conditions
- Two-Dimensional Viscous, Incompressible Couette Flow (static)
  - Analytical solution provides quantitative measure of accuracy
  - Test of conformal decomposition for viscous, incompressible flow
- Three-Dimensional Viscous Flow about a Periodic Array of Spheres (static)
  - Comparison with Boundary Element results
  - Examines behavior of decomposition up to sphere overlap
- Advection of Weak Discontinuity (dynamic)
  - Shows ability to capture discontinuities
  - Analytical solution provides quantitative measure of accuracy
- Solidification of 1-D Bar (dynamic)
  - Shows ability to capture discontinuities
  - Analytical solution provides quantitative measure of accuracy

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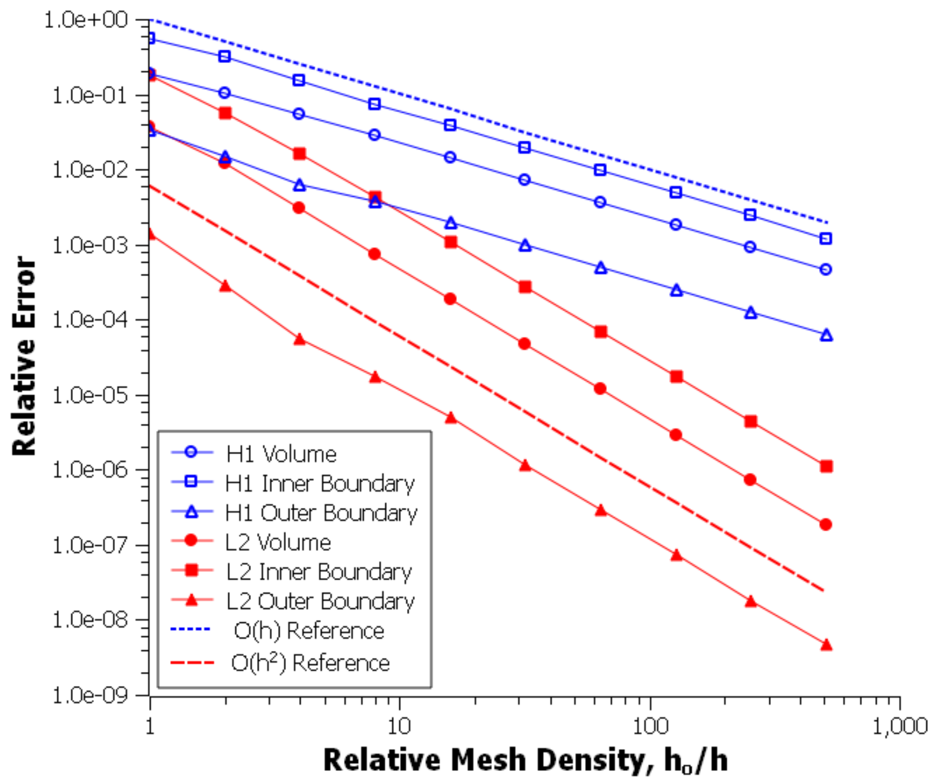
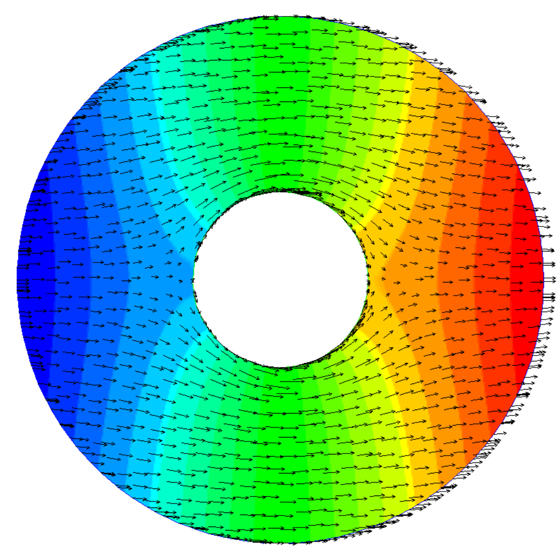
CDFEM Simulation of Steady, Potential Flow about a Circular Cylinder

- Embedded curved boundaries
- Dirichlet BC on outer surface, Natural BC on inner surface
- Optimal convergence rates for solution and gradient both on volume and boundaries





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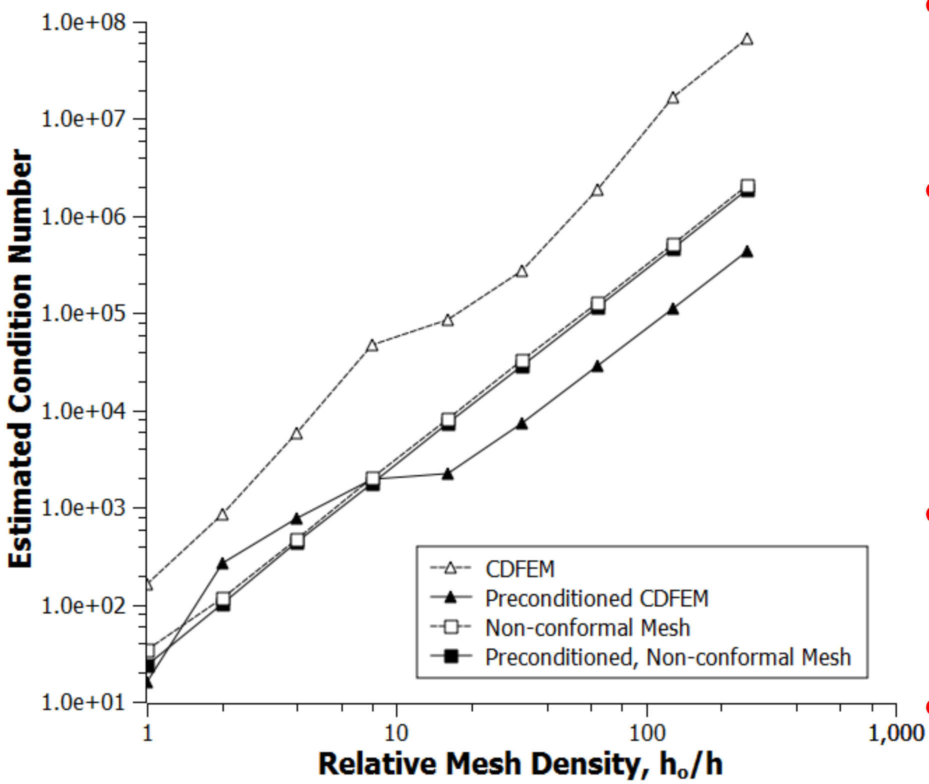
Linear System Conditioning for CDFEM Simulation of Steady, Potential Flow about a Circular Cylinder

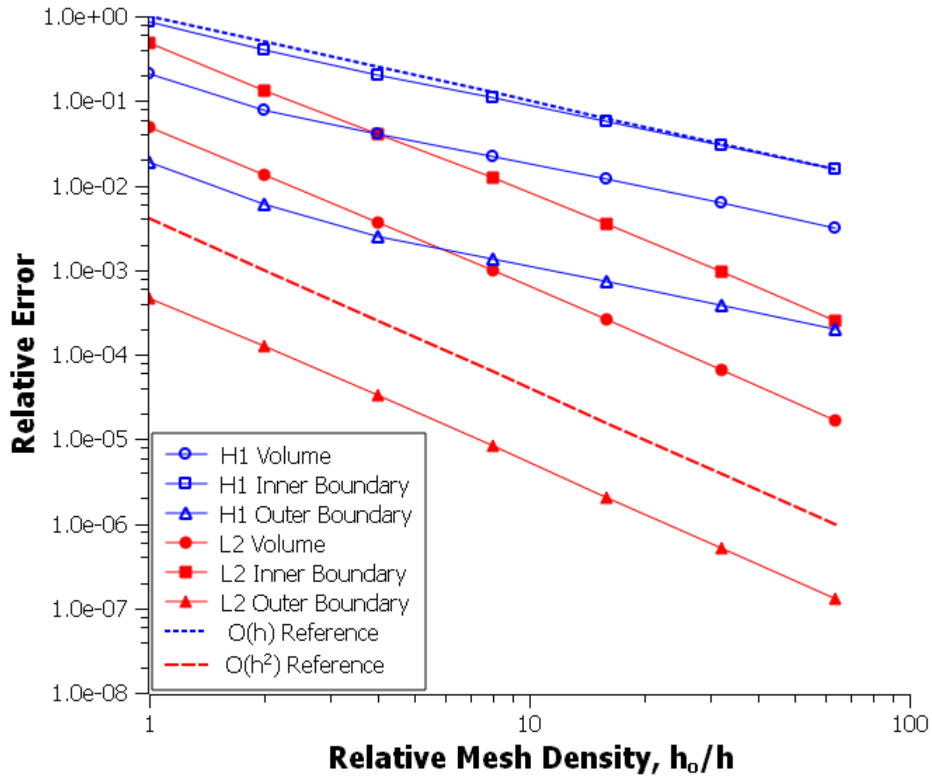


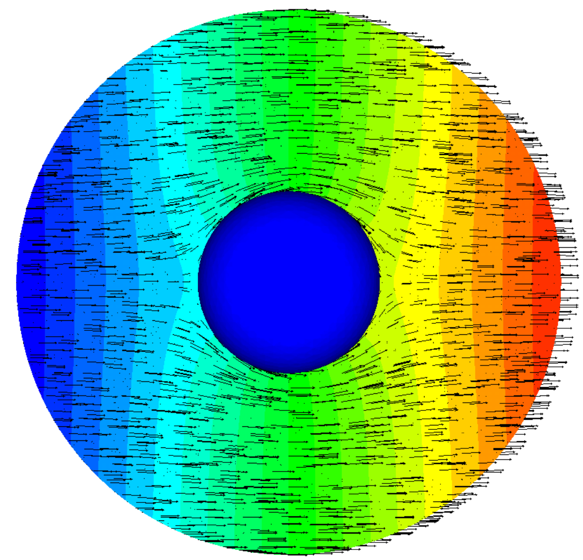
- Expectation
  - Nearly degenerate elements expected to degrade conditioning of the matrix resulting from finite element assembly
- Evaluation
  - TRILINOS package used to estimate extreme eigenvalues
  - Condition estimates generated with and without Jacobi preconditioning
  - Compared to simple conduction system using un-decomposed mesh
- Results
  - Preconditioned system exhibits expected $O(h^{-2})$ scaling

- Poor conditioning from CDFEM easily removed by standard preconditioning
  - Consistent with findings of Graham and McLean (2006) for anisotropic refinement

CDFEM Simulation of Steady, Potential Flow about a Sphere

- Embedded curved boundaries
- Dirichlet BC on outer surface, Natural BC on inner surface
- Optimal convergence rates for solution and gradient both on volume and boundaries





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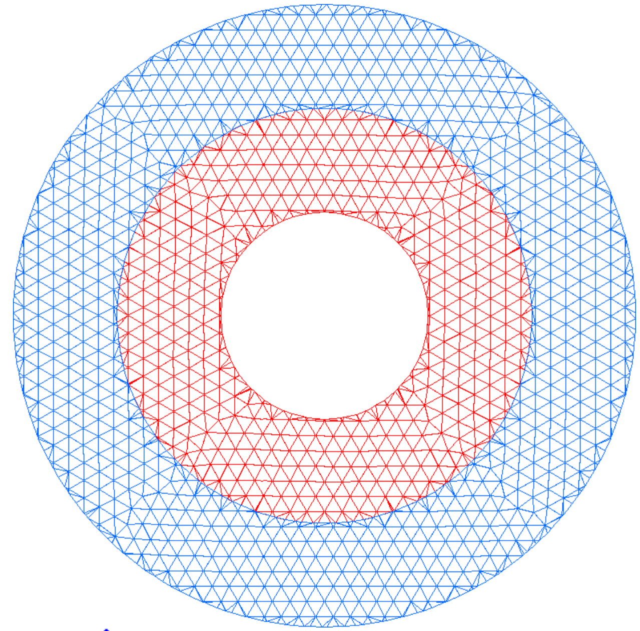
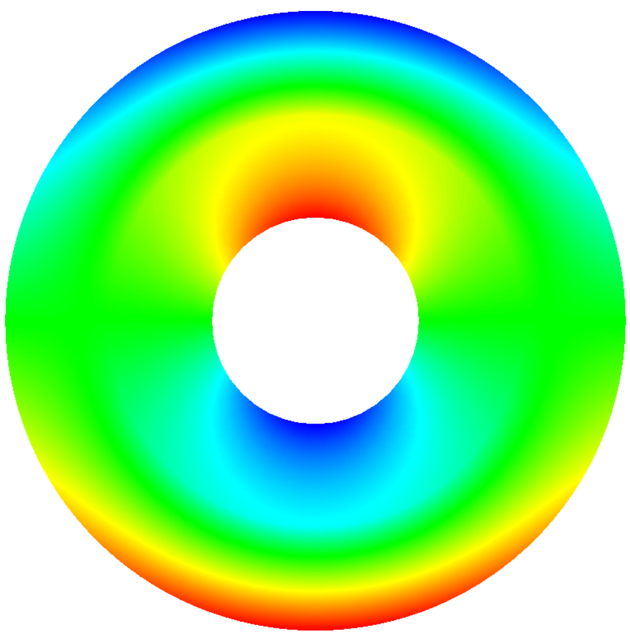
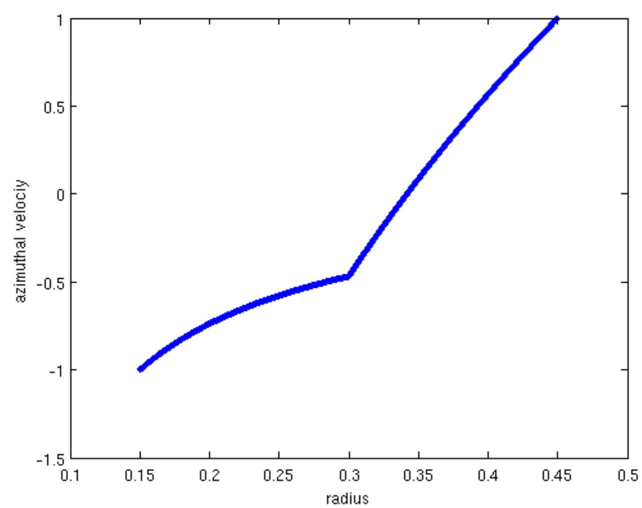
CDFEM Simulation of Steady, Fluid-Fluid Interface Problem: Couette Flow

- Two-Phase Flow between concentric cylinders
  - Counter-rotating cylinders
  - 4:1 viscosity ratio
  - No surface tension
- Dirichlet conditions on inner and outer surfaces, weak discontinuity along interface
- Cut regular, unstructured mesh along outer, inner, and interfacial radii







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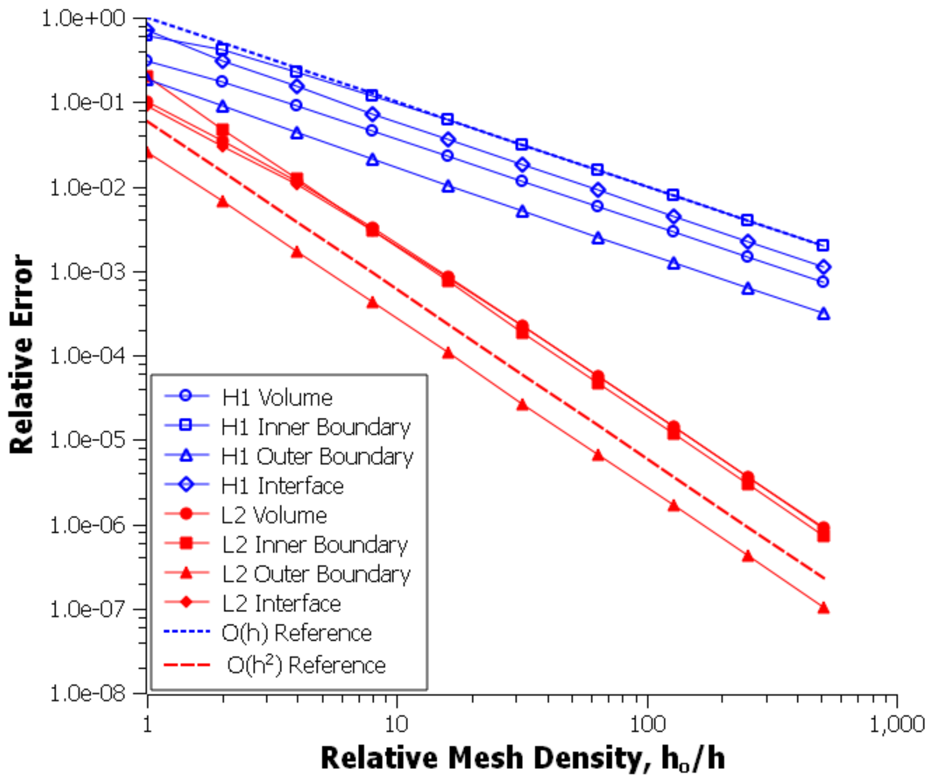
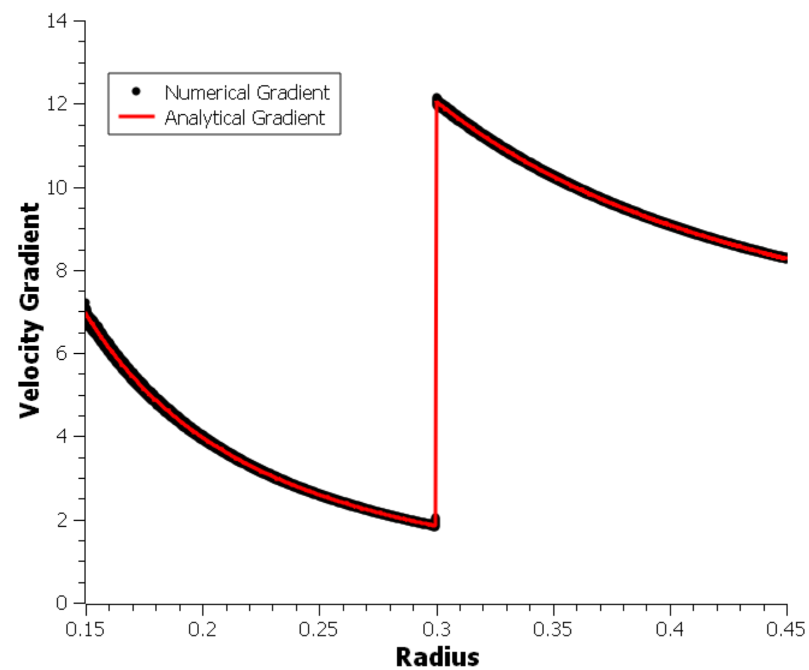
CDFEM Simulation of Steady, Fluid-Fluid Interface Problem: Couette Flow

- Embedded curved boundaries
- Dirichlet BC on inner and outer surface
- Weak discontinuity in velocity captured sharply and accurately
- Optimal convergence rates for solution and gradient both on volume and boundaries





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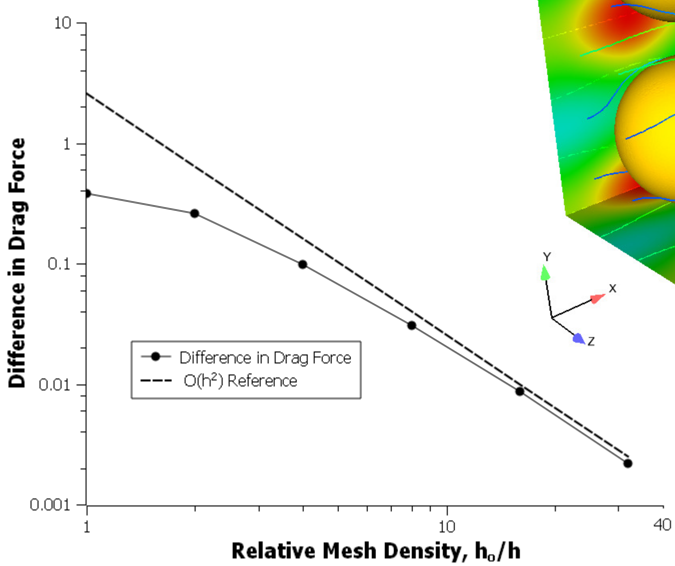
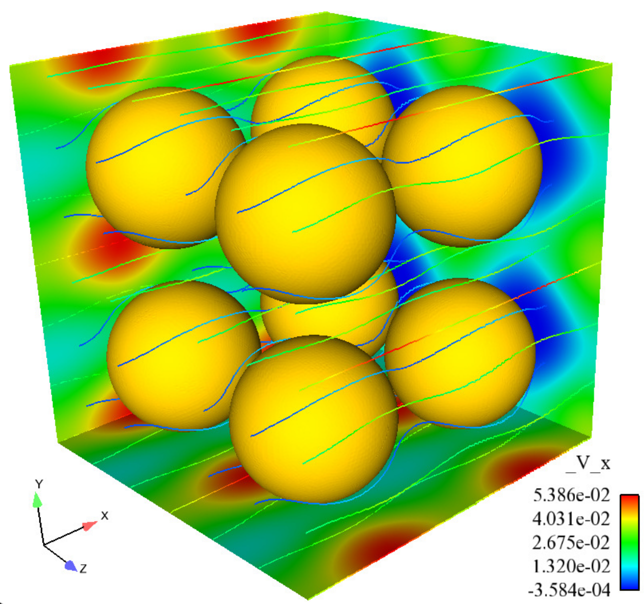
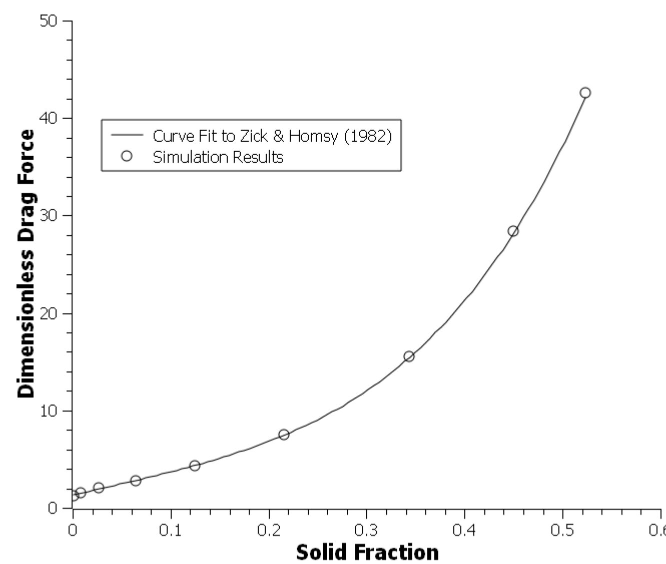
CDFEM Simulation of Steady, Viscous Flow about a Periodic Array of Spheres

- Embedded curved boundaries
- Dirichlet BC on sphere surface
- Accurate results right up to close packing limit
- Sum of nodal residuals provides accurate/convergent measure of drag force







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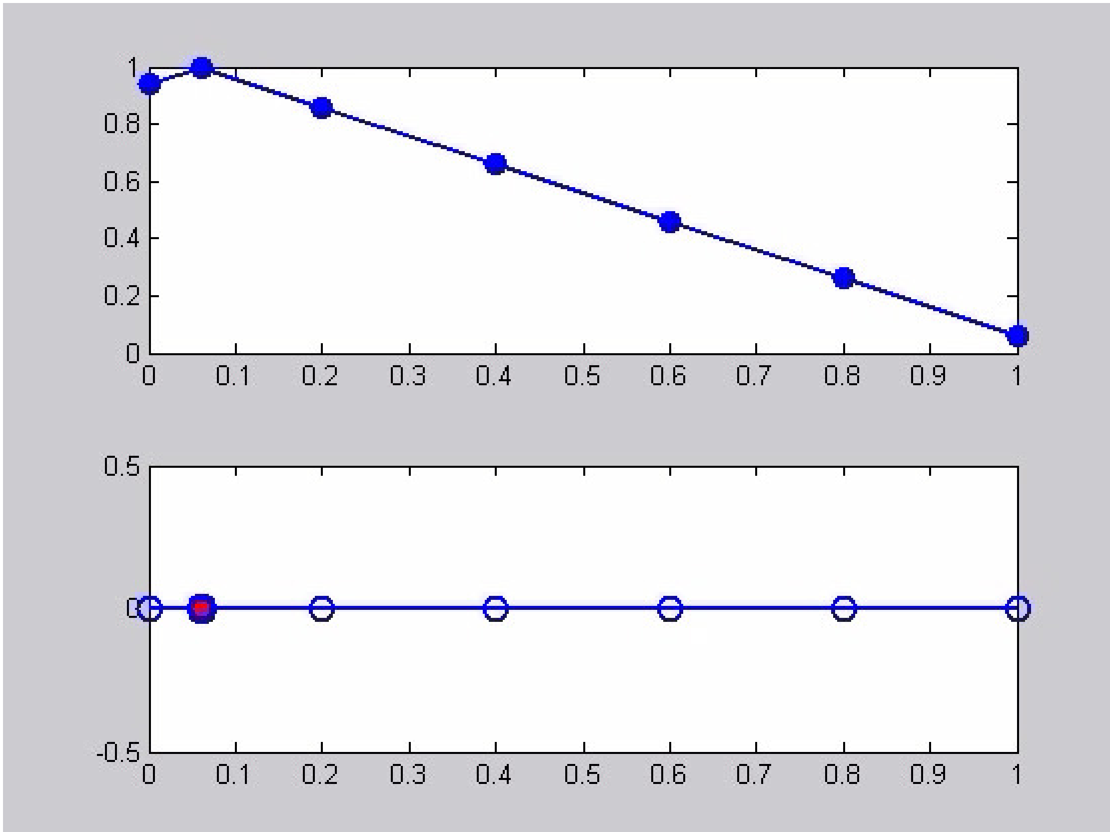
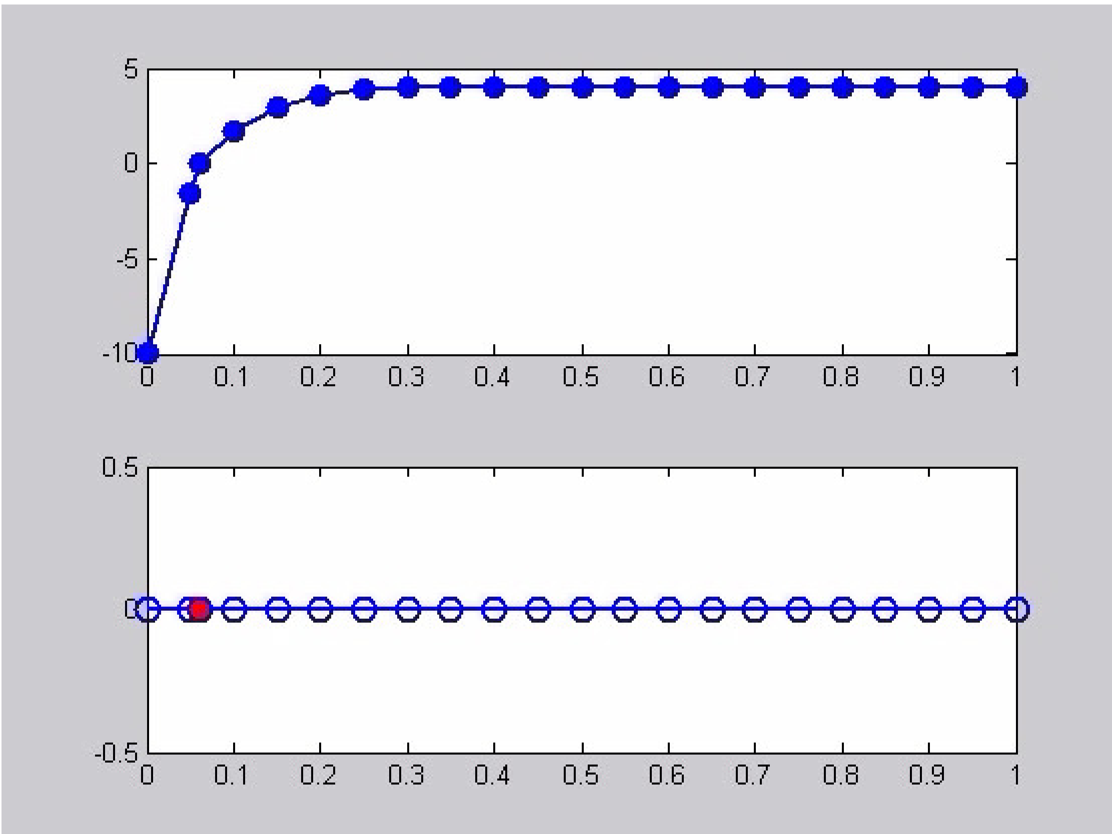
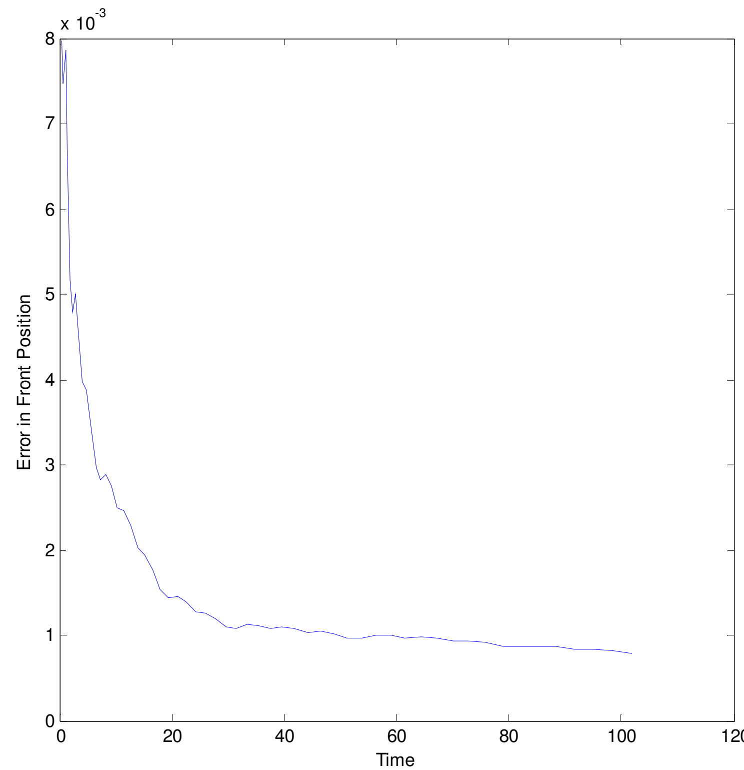
Dynamic CDFEM: 1-D Advection of a Piecewise Linear Field



- Exact preservation of linear field
- Does not pollute Max-Min

Dynamic CDFEM: 1-D Phase Change





- Great agreement with exact solution

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Summary and Conclusions

CDFEM Foundation

- Simple method for handling arbitrary interfacial discontinuities
  - Transparent to underlying finite element assembly
- Recovers Heaviside XFEM when added nodes are constrained to lie in XFEM space
  - Theoretically expected to provide accurate solutions

CDFEM Implementation

- Readily employed for arbitrary number of phases in two and three dimensions

CDFEM Validation for Fluid Dynamics

- Optimal convergence rates both on the volume and on the curved interface for diffusion and flow problems
- Linear system conditioning degrades from nearly degenerate elements but is restored using standard preconditioning

Future Work

- Fully transient multiphase flows in multiple dimensions

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