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More Connections Are Not Always Better

Braess-like Paradoxes on a Bipartite Transaction Network

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Sandia National Laboratories, NM

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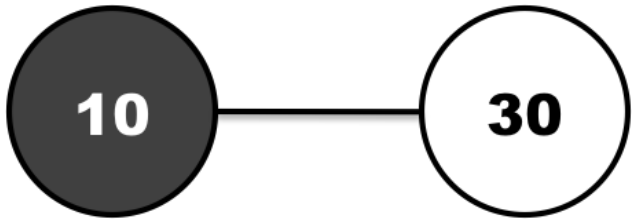
Naive Exchange Model

Budget-Constrained Bilateral Exchange of One Good with Unit Price[1]

- Bilateral exchange must occur if feasible



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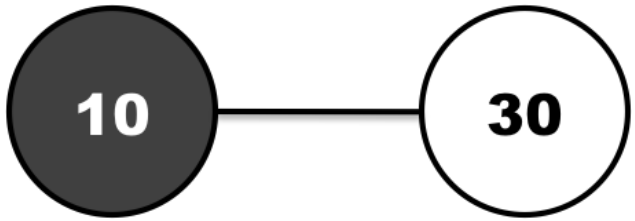

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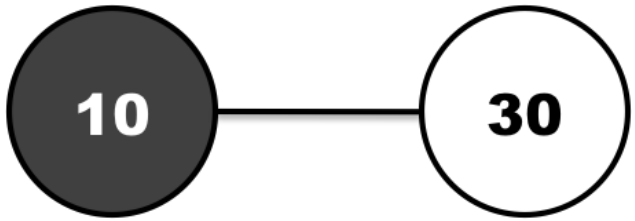

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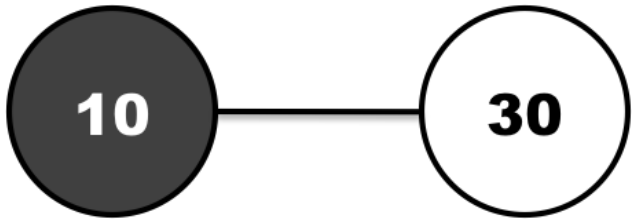

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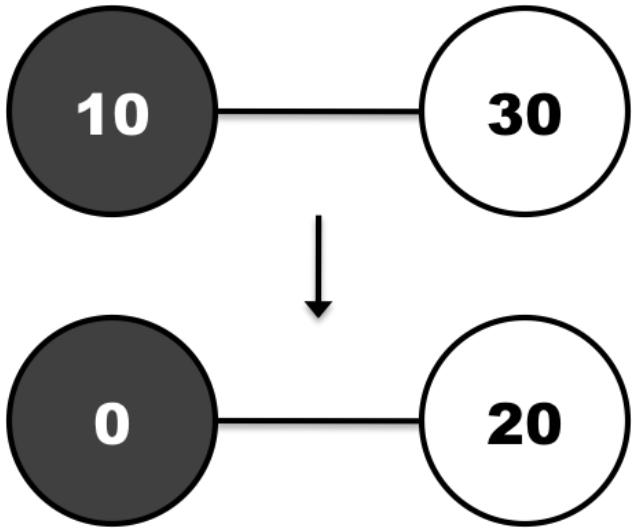

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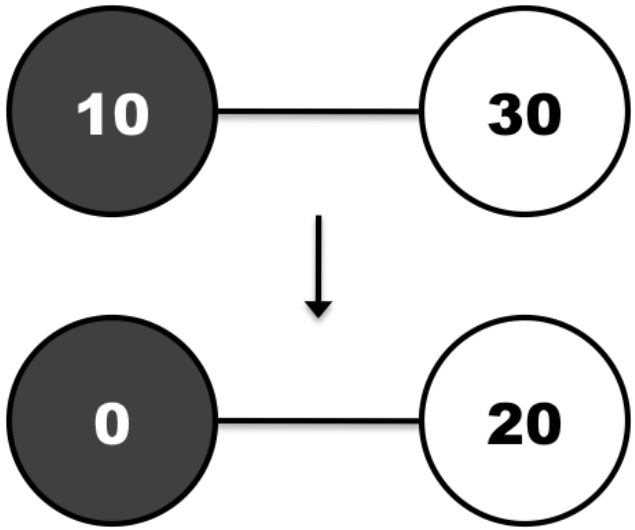

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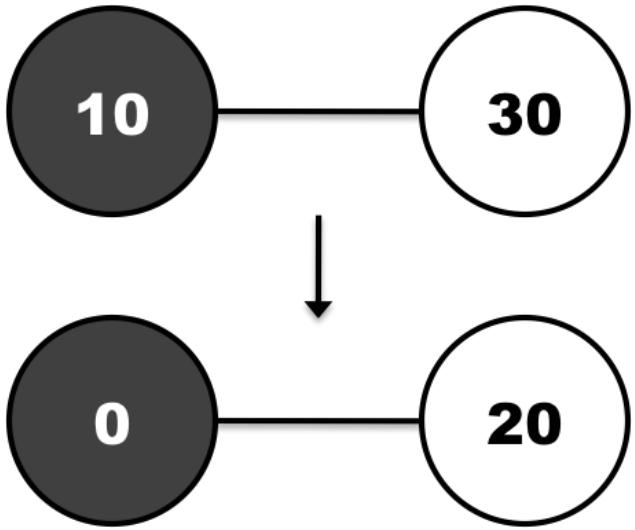
  - No further exchange



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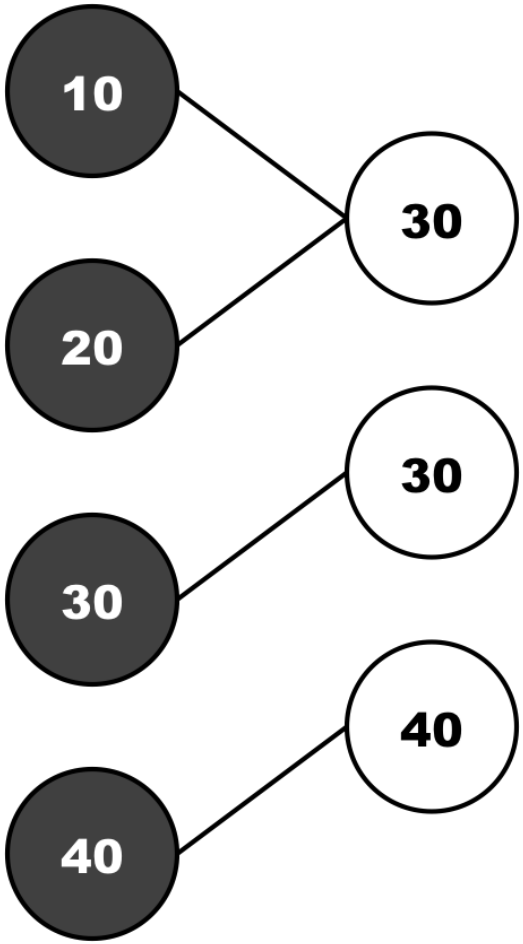
Statement of the Problem for Asynchronous Exchange on a Bipartite Graph



- Example of transactions on a multicomponent bipartite graph (buyers: gray, sellers: white)

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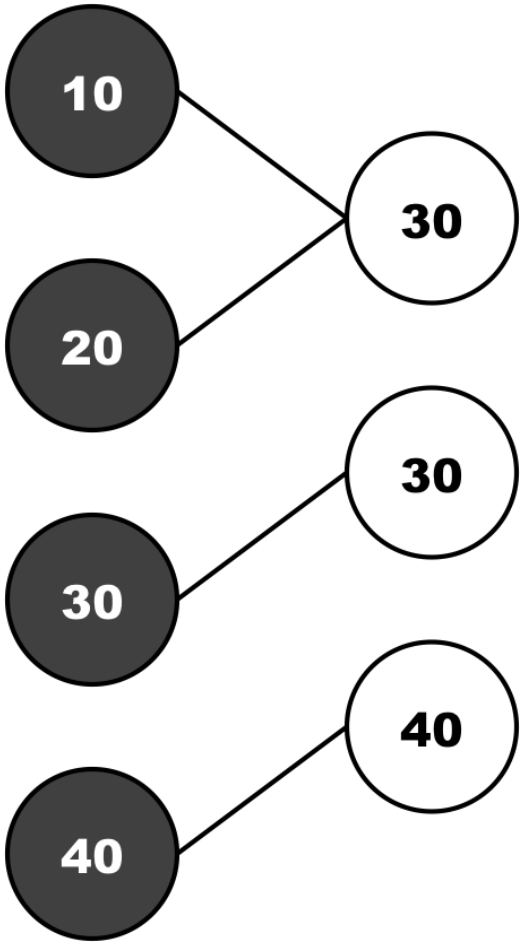

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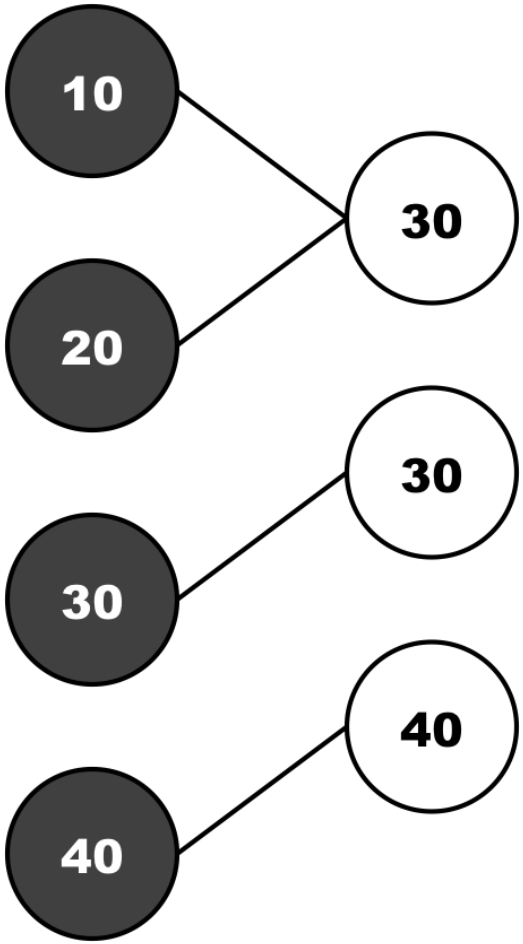


- Example of transactions on a multicomponent bipartite graph (buyers: gray, sellers: white)
- In this example, all demands can be met regardless of the order in which transactions occur
- What is necessary and sufficient for that to be true?

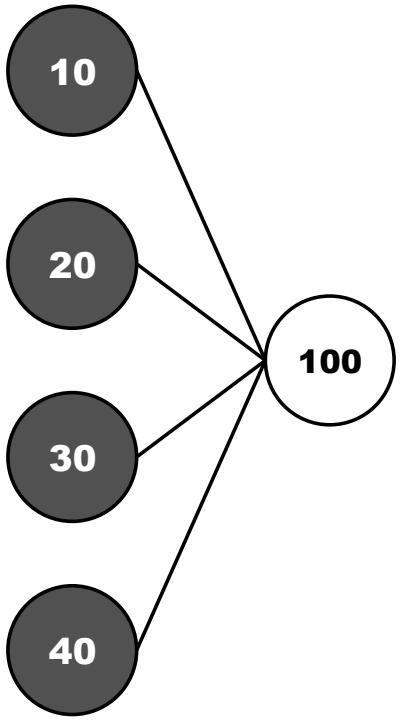
Definition

A trading session on the graph consists of one of the (up to $L!$) possible sequences of all possible trades on the graph of L links.

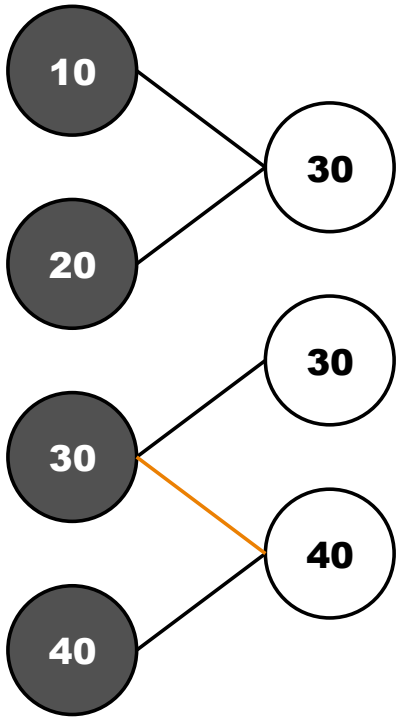
Theorem

Given that supply equals demand, the demands are reduced to zero at the end of every trading session iff each component (for which, within that component, supply equals demand) is complete bipartite.

How to Connect For Success?
Example: Star graph (local monopoly) always works



How to Connect For Success?
Example: Starvation on an incomplete component



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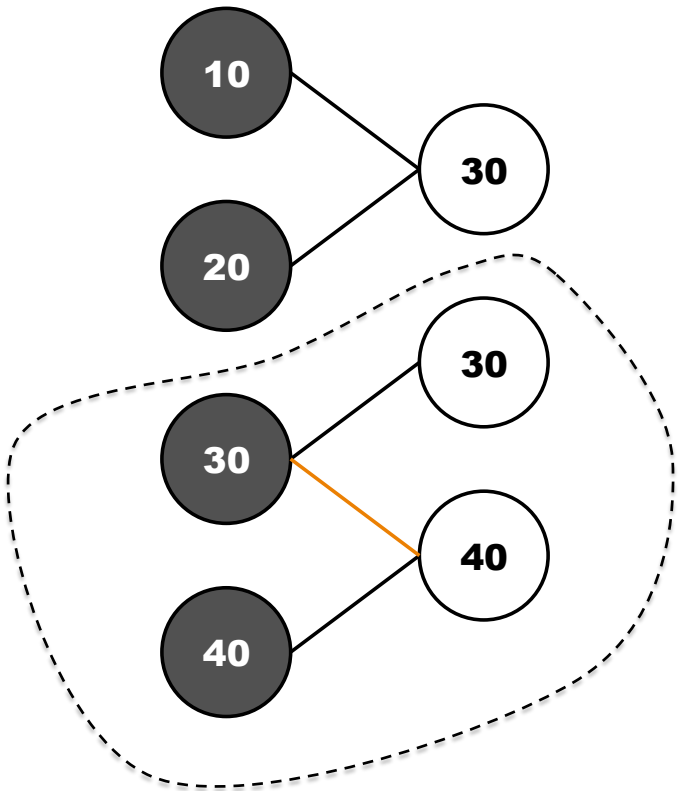

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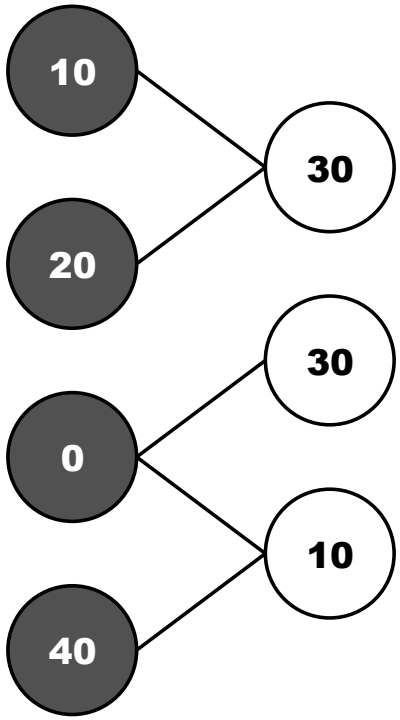

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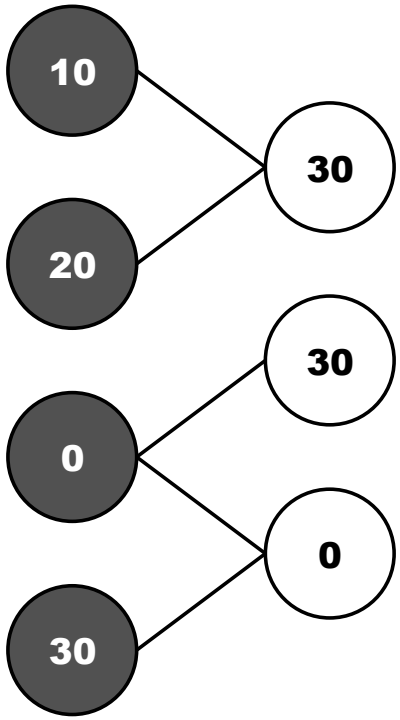

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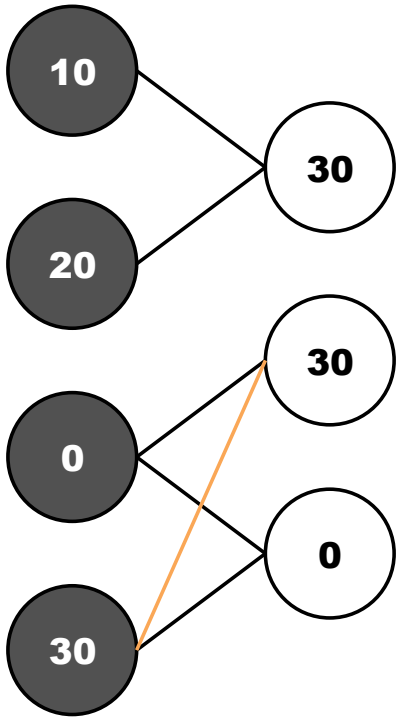
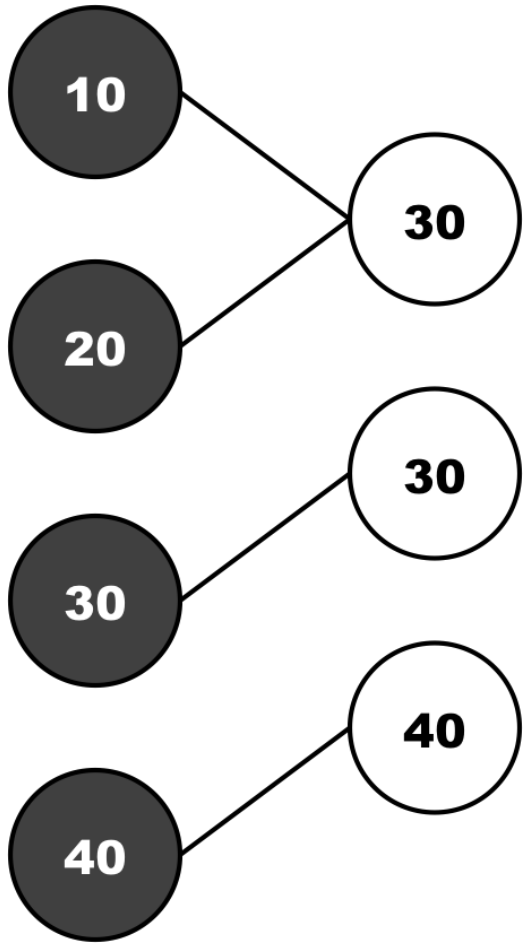


How to Connect For Success?
Enumeration

- Begin with the minimally connected example (four links) and add all possible links one at a time



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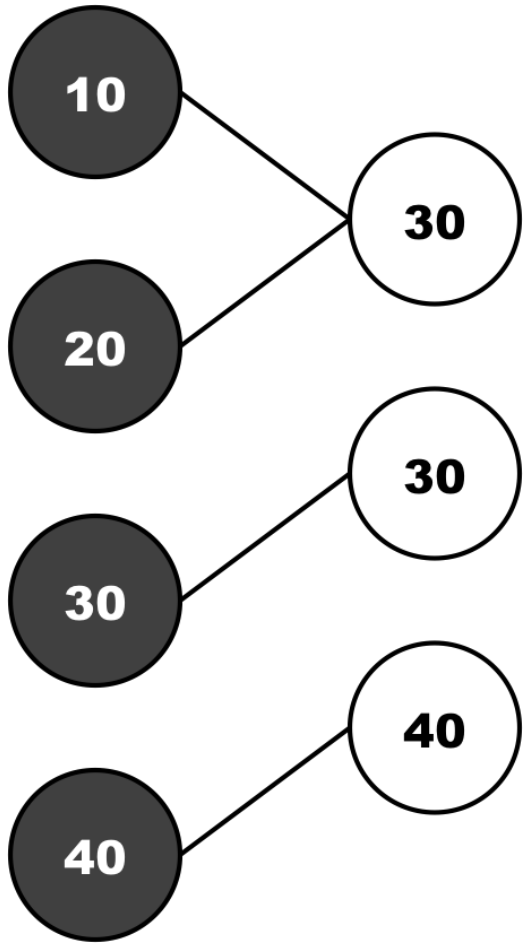

How to Connect For Success?
Enumeration

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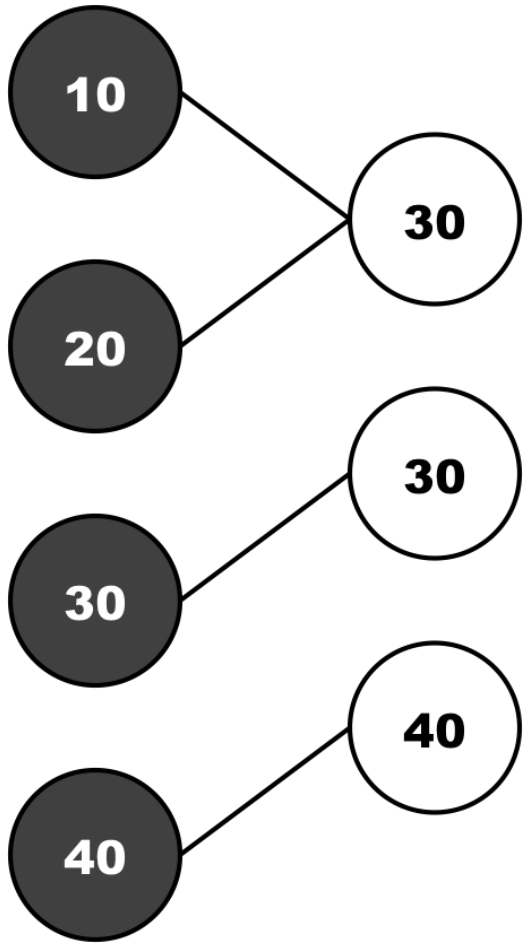
- Record the fraction of trading sessions that do NOT meet demand ("infeasible")
- Also record the maximum demand left unmet after each trading session



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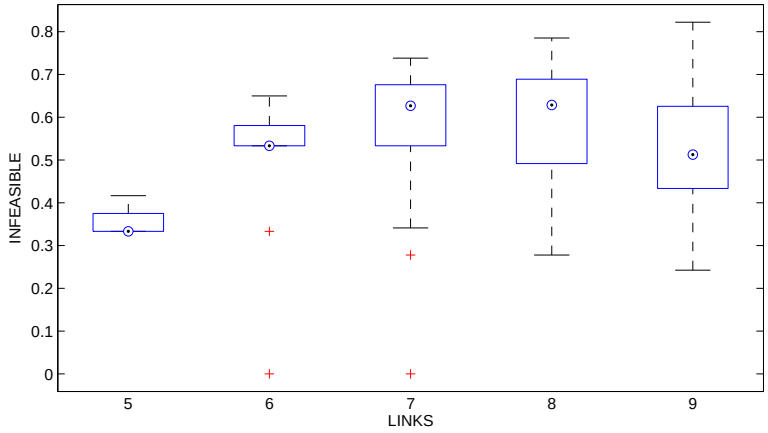
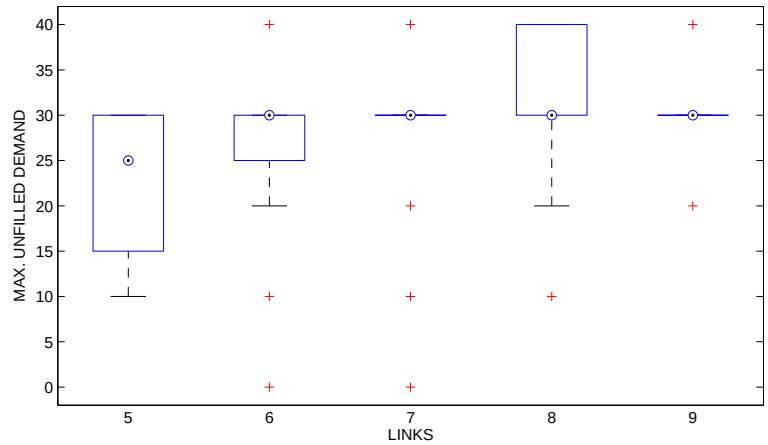
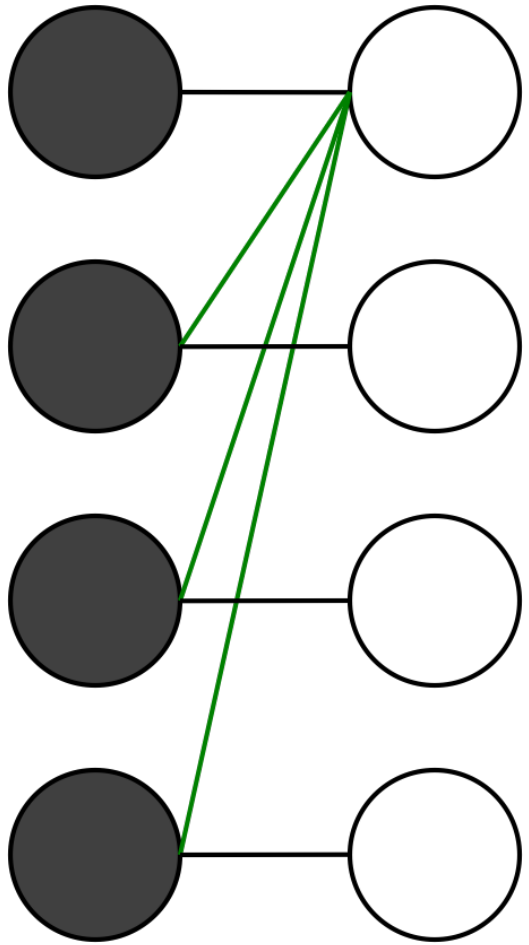




How to Connect For Success?
What Reserves Would Be Needed to Meet Demand?

Lower Bound

The reserves that would be required by sellers in order to meet demand in the worst case has a lower bound that is proportional to the number of buyers.



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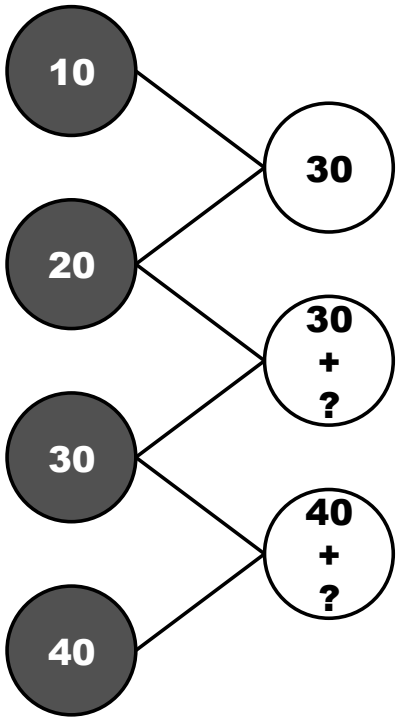
Tomorrow's "Smart Grid"

- "Smart Grid" has the potential to turn 5×10^7 households into market points (from 200 now)[a]



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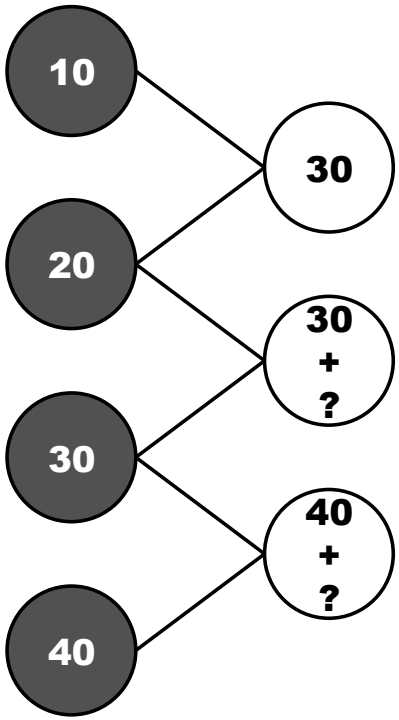

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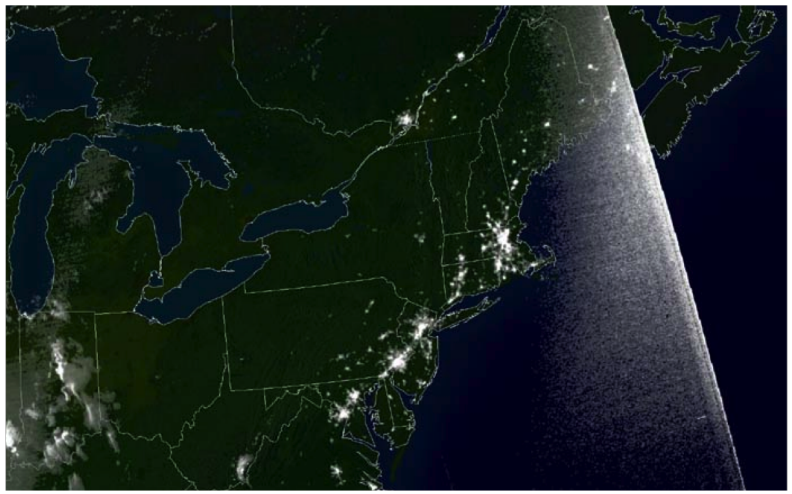

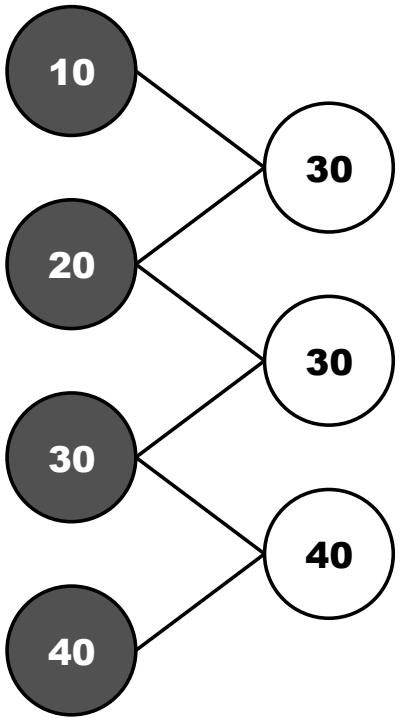

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  - Otherwise maintaining complete components may be challenging
- Enumeration and Lower Bounds show that reserve requirements (or unmet demand) could be large if these conditions were not satisfied