SAND2007-3583C

A Partition of Unity FEM for Cohesive Zone Modeling of Fracture

Presented at the USNCTAM

Symposium to Honor Professor Len Herrmann

Jim Cox

Applied Mechanics Development (1526)

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Preview

- Introduction
- PUFEM Displacement Field Enrichment
- Results
  - Aligned mesh -- quantitative
  - Skewed mesh -- discussion of progress
- Observations & Conclusions

Introduction

Objective: A valid means of modeling material localization in finite element analyses.

Goals:

- strong discontinuity idealization
- applicable to cohesive zone modeling
- "continuous discontinuity"
- arbitrary orientation of discontinuity relative to mesh

Approach: Develop a partition of unity FEM (PUFEM) that allows the displacement field to be enriched in the neighborhood of a strong discontinuity.

- can represent a discontinuity without mesh refinement
- can potentially represent the gradients near a surface of localization without mesh refinement

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Background

Initial related studies

- Melenk and Babuska (1996)
  Theory for PUFEM
- Belytschko and Black (1999)
  - developed PUFEM for LEFM
  - used asymptotic displacement fields near a crack tip for enrichment

Origins of this study

ARL (2001)

motivating problem: armor penetration

SNL

motivating problem: HDBT

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Recent Related Studies

PUFEM-Cohesive Zone Studies

- Wells and Sluys (2001)
- Moes and Belytschko (2002)
- Zi and Belytschko (2003) -- tip function addresses tip position but not the field
- de Borst *et al*. (2004) & Mariani *et al*. (2003) tip at element edges

Current work is closer to:

- Strouboulis, Copps, Zhang, and Babuska (2000, 2001, 2003) numerical enrichment functions -- handbook functions

PUFEM Displacement Field Enrichment

- Standard FEM
- PUFEM

Global displacement approximations

$$u(x) = \sum_{i=1}^{N_\Phi} \Phi_i(x) u_i$$

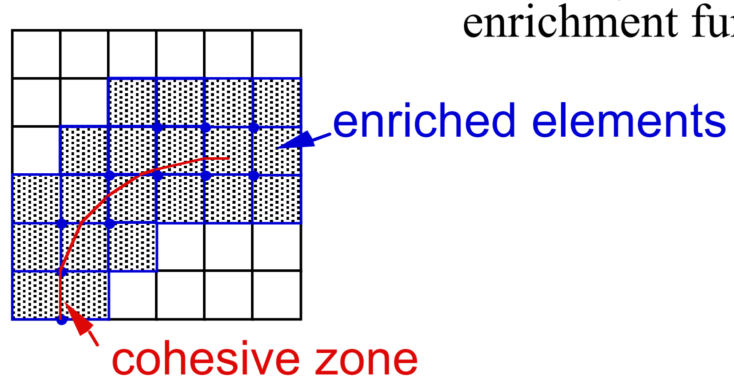
$$u(x) = \sum_{i=1}^{N_\Phi} \Phi_i(x) u_i + \boxed{\sum_{j=1}^{N_\Lambda} \sum_{i=1}^{N_\Phi} \Lambda_j(x) \Phi_i(x) \alpha_{ij}}$$

Element displacement approximations

$$u(x) = \sum_{i=1}^{N_N} N_i(x) u_i$$

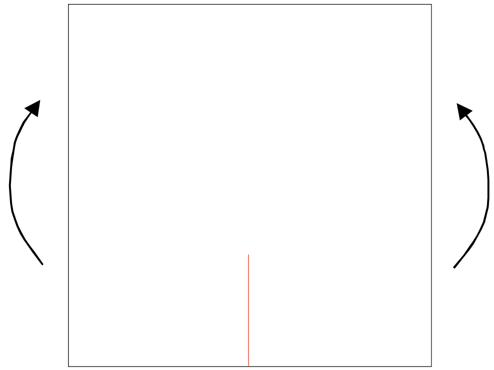
$$u(x) = \sum_{i=1}^{N_N} N_i(x) u_i + \boxed{\sum_{j=1}^{N_\Lambda} \sum_{i=1}^{N_N} \Lambda_j(x) N_i(x) \alpha_{ij}}$$

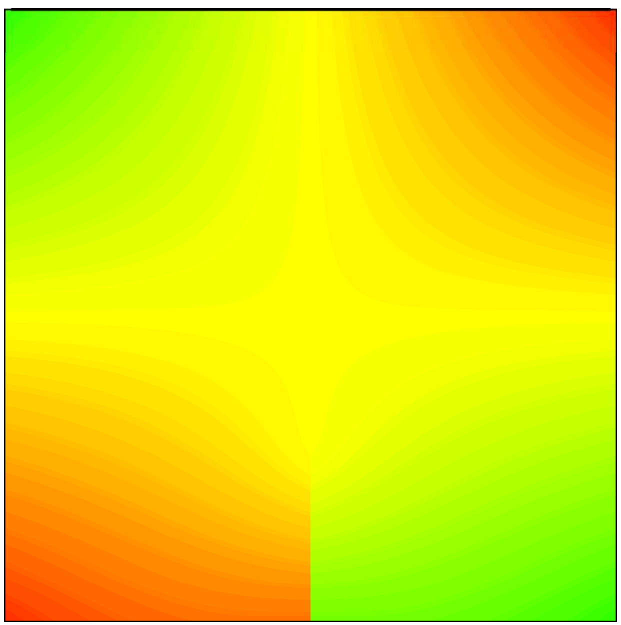
enrichment functions



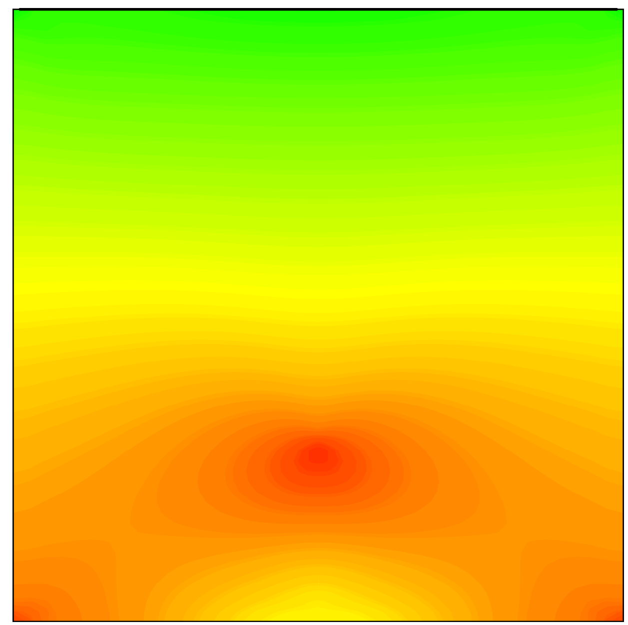
PUFEM Displacement Field Enrichment

Example Problem: concrete 1 m x 1 m in bending





fine-scale FEM solution: u_x



fine-scale FEM solution: σ_{xx}

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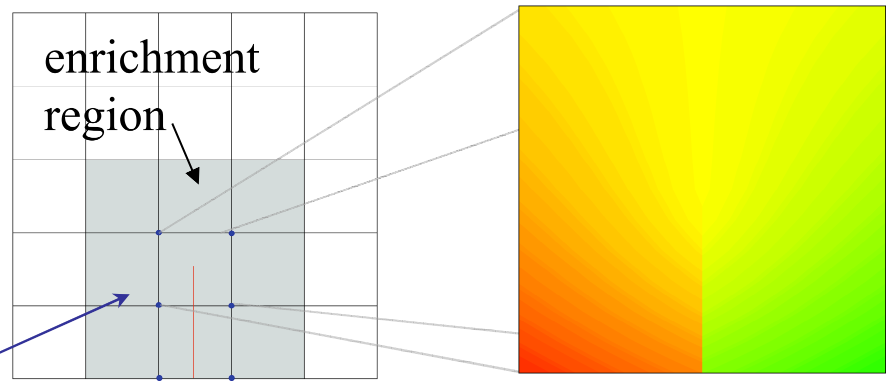
Example response in the "tip-element"

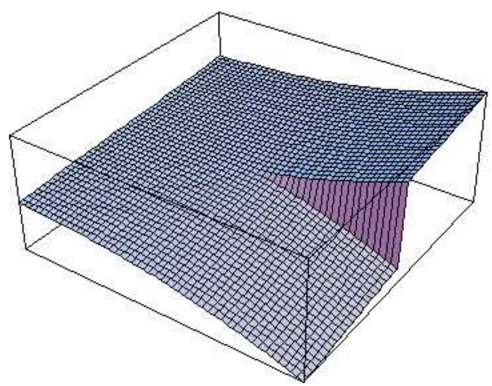
Fine-scale

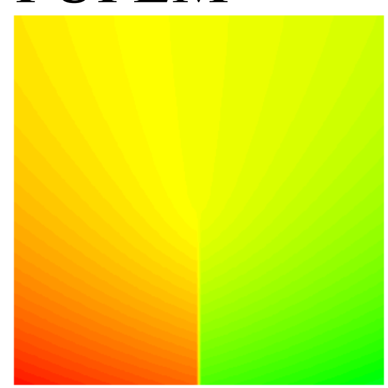


Surface plot view

u_x for tip-element

PUFEM

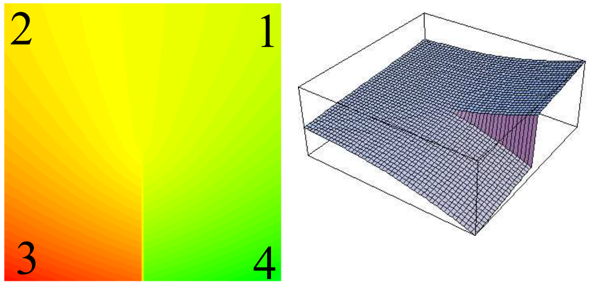




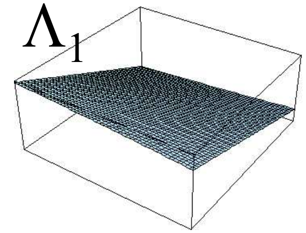
u_x for tip-element

Example enrichment in the “tip element”

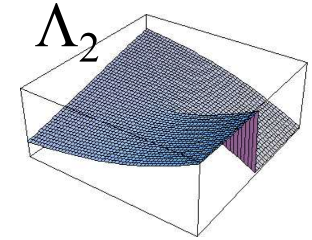
$u(x) =$



Λ_1



Λ_2

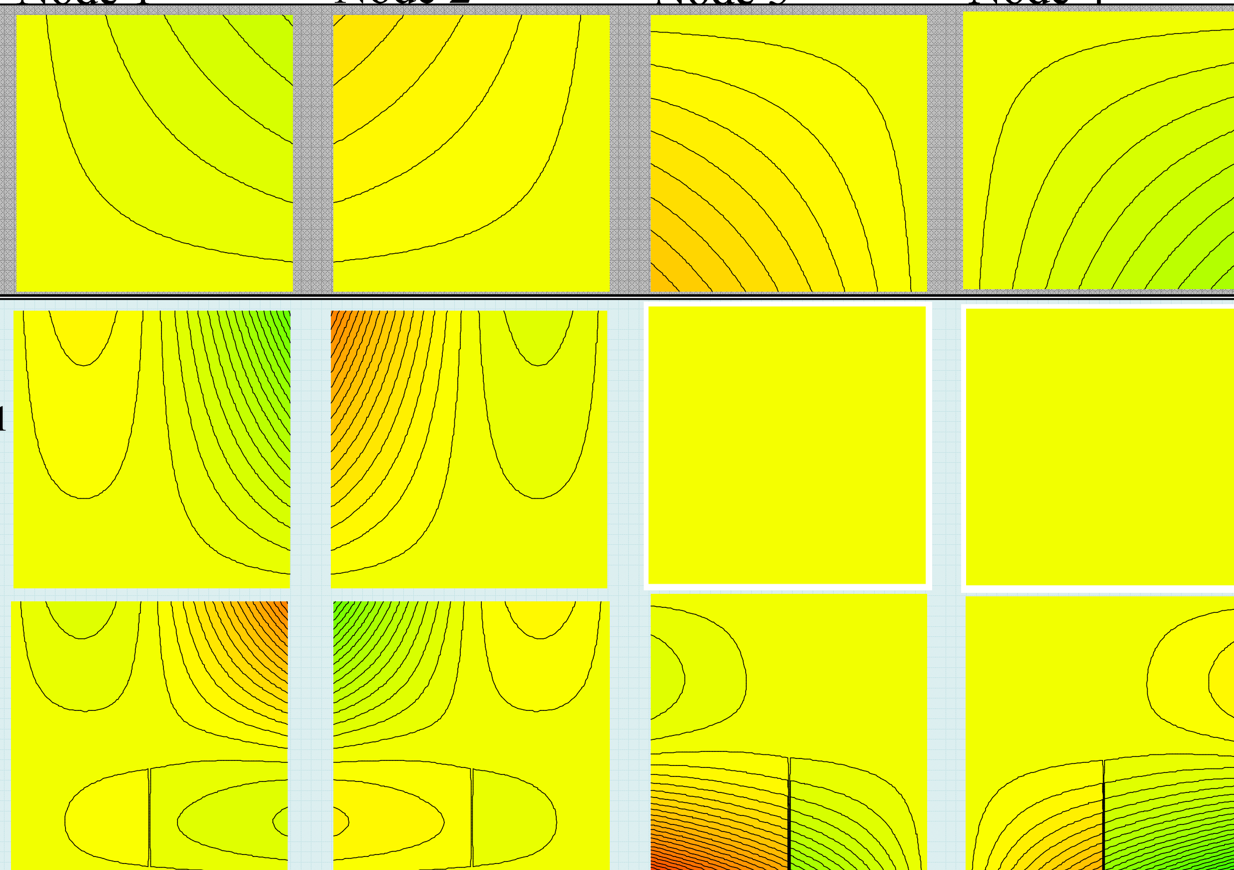


Node 1 | Node 2 | Node 3 | Node 4

$$\sum_{i=1}^{N_N} N_i(x) u_i$$

$$\sum_{i=1}^{N_N} \Lambda_1(x) N_i(x) \alpha_{i1}$$

$$\sum_{i=1}^{N_N} \Lambda_2(x) N_i(x) \alpha_{i2}$$



Enrichment Functions: An Analytical Source

Hong & Kim (2003) used the Muskhelishvili formalism for the inverse problem – assumed linear elastic isotropic material (except for cohesive zone)

Analysis has been used to:

1) verify the proposed solution
2) extend it for field variables required by the PUFEM

Key relationships:

- Displacements

$$u_1 + iu_2 = \frac{1}{2\mu}\left\{\kappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)}\right\}$$

where φ and ψ are analytic functions, and $z = x+iy$.

- Another set of analytic functions simplify $u_{i,j}$ and σ_{ij} expressions

$$\Phi(z) = \varphi'(z) \qquad\qquad \Omega(z) = \left[z\varphi'(z) + \psi(z)\right]'$$

Enrichment Functions: An Analytical Source

- Displacement gradients

$$u_{1,1} + iu_{2,1} = \frac{1}{2\mu}\left[(\bar{z} - z)\overline{\Phi'(z)} + \kappa\Phi(z) - \overline{\Omega(z)}\right]$$

$$u_{2,2} - iu_{1,2} = \frac{1}{2\mu}\left[(z - \bar{z})\overline{\Phi'(z)} + \kappa\Phi(z) + \overline{\Omega(z)} - 2\overline{\Phi(z)}\right]$$

- Stress components

$$\sigma_{11} + i\sigma_{12} = (\bar{z} - z)\overline{\Phi'(z)} + \Phi(z) - \overline{\Omega(z)} + 2\overline{\Phi(z)}$$

$$\sigma_{22} - i\sigma_{12} = (z - \bar{z})\overline{\Phi'(z)} + \Phi(z) + \overline{\Omega(z)}$$

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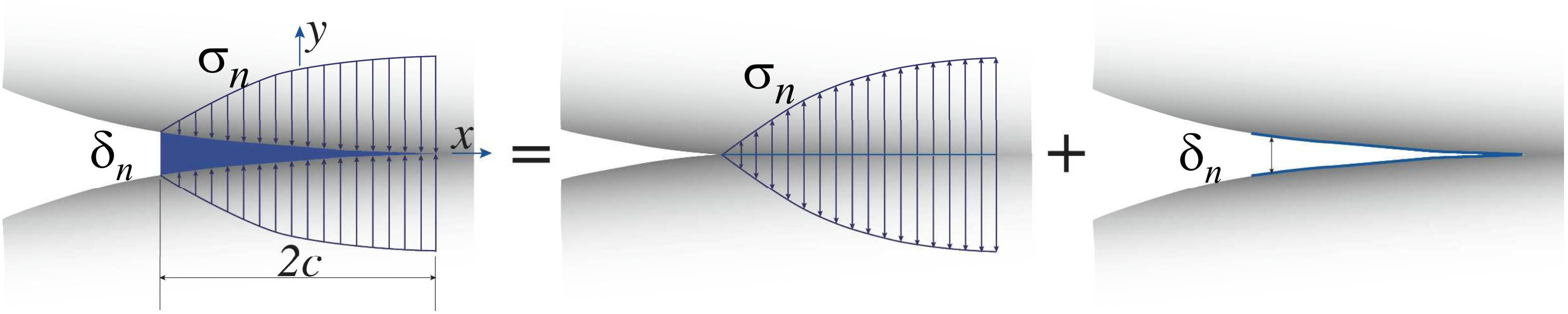
Enrichment Functions: An Analytical Source

- Super-position of two solutions yields a convenient solution form



- Analytic functions

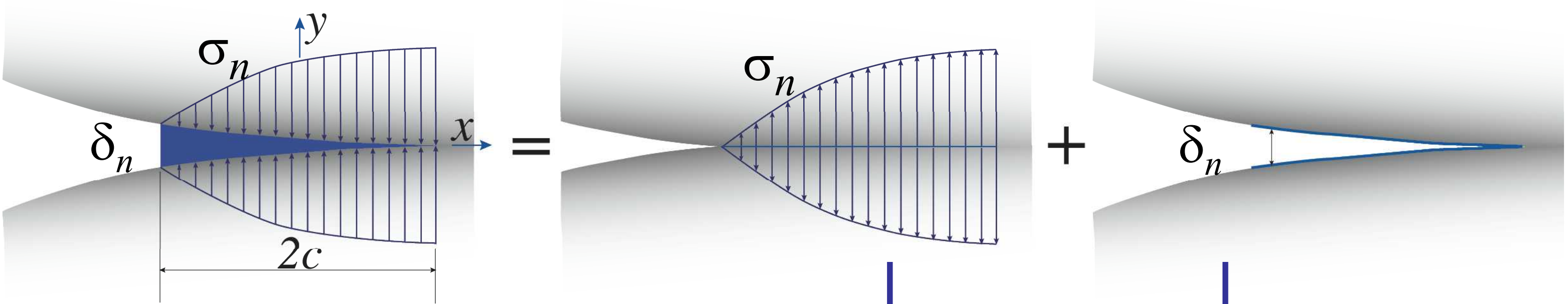
$$\Phi(z) = \frac{1}{2}\left[\sqrt{z+c}F(z) - \sqrt{z-c}G(z) + H(z)\right]$$

$$\overline{\Omega}(z) = \frac{1}{2}\left[\sqrt{z+c}F(z) - \sqrt{z-c}G(z) - H(z)\right]$$

where *F*, *G*, and *H* are *entire* (analytic over the whole domain)

Enrichment Functions: An Analytical Source

- Super-position of two solutions yields a convenient solution form



- Analytic functions F and G

$$F(z/c) = \sum_{n=0}^{N} A_n U_n(z/c) \qquad G(z/c) = \sum_{n=0}^{N} B_n U_n(z/c)$$

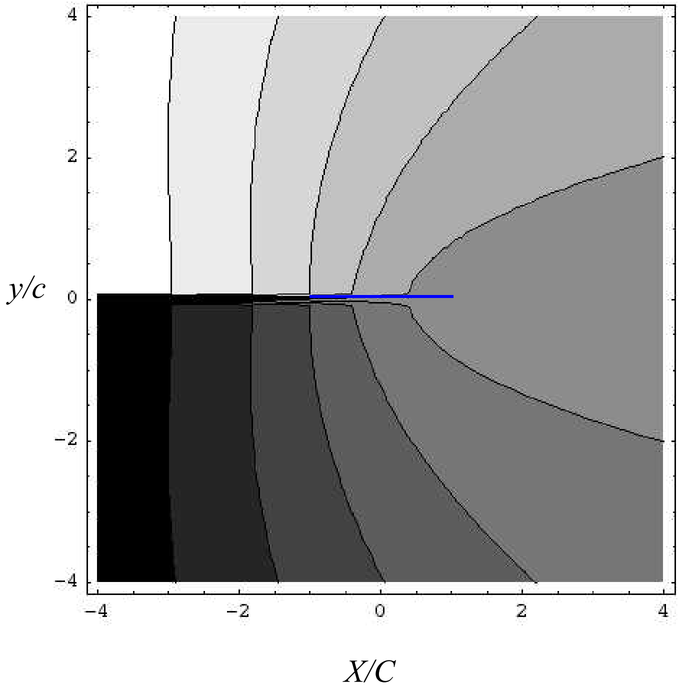
where A_n and B_n are complex coefficients and
U_n are Chebyshev polynomials of the second kind

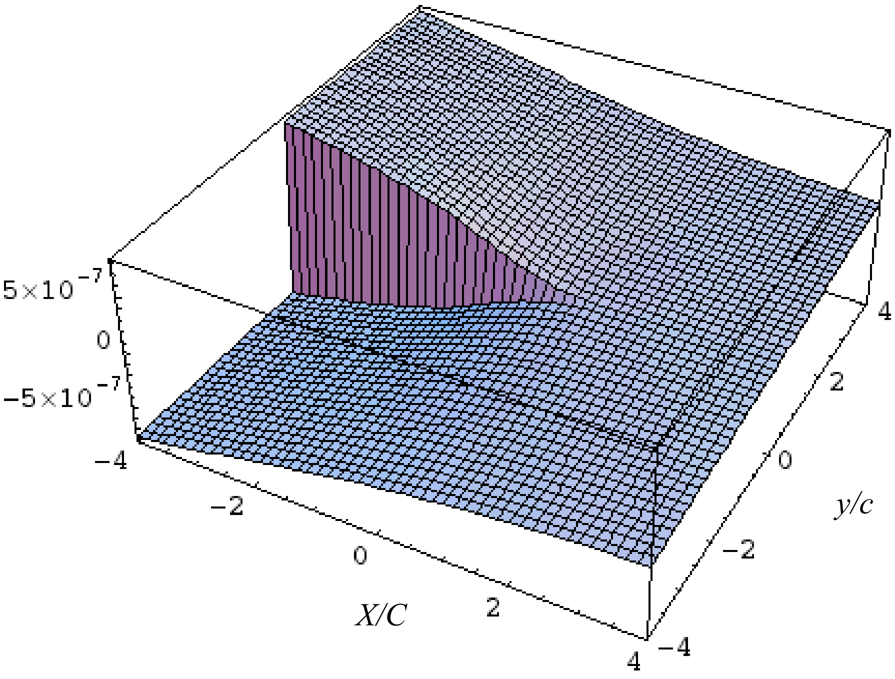
Enrichment Functions: An Analytical Source

- First term considered
  - $F(z)=G(z)=1$
  - $H(z)=0$

Problem for plots
$E=10^7$ psi, $\nu=0.3$

- u_2



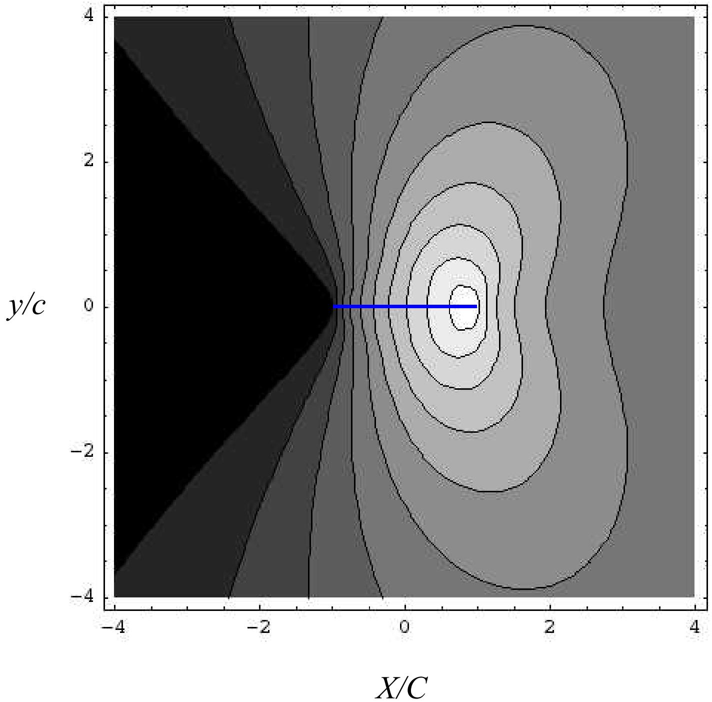


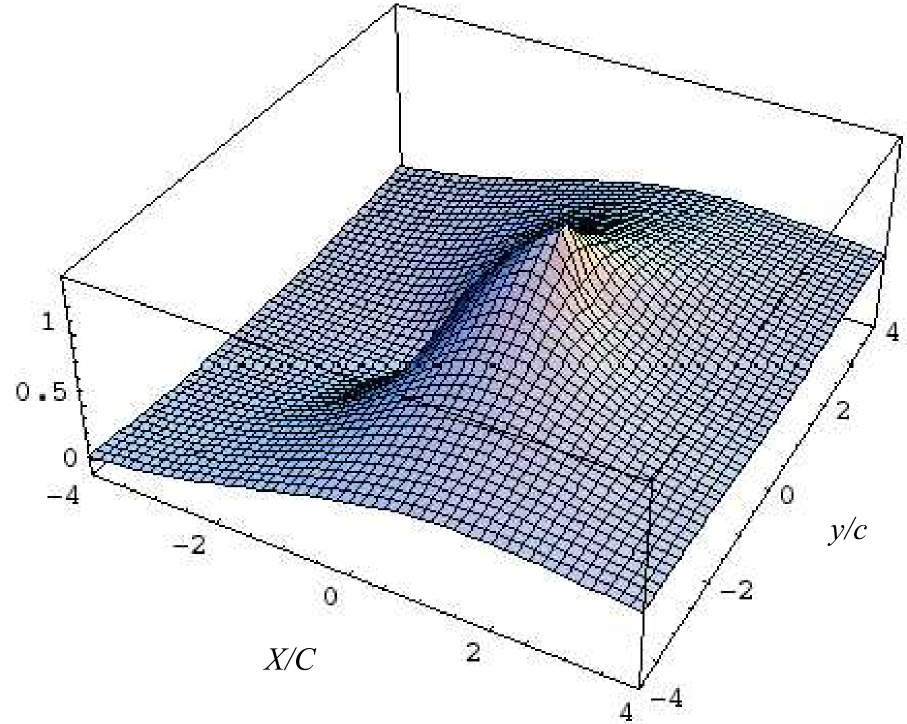
Enrichment Functions: An Analytical Source

- First term considered
  - $F(z)=G(z)=1$
  - $H(z)=0$

Problem for plots
$E=10^7$ psi, $\nu=0.3$

- σ_{22}





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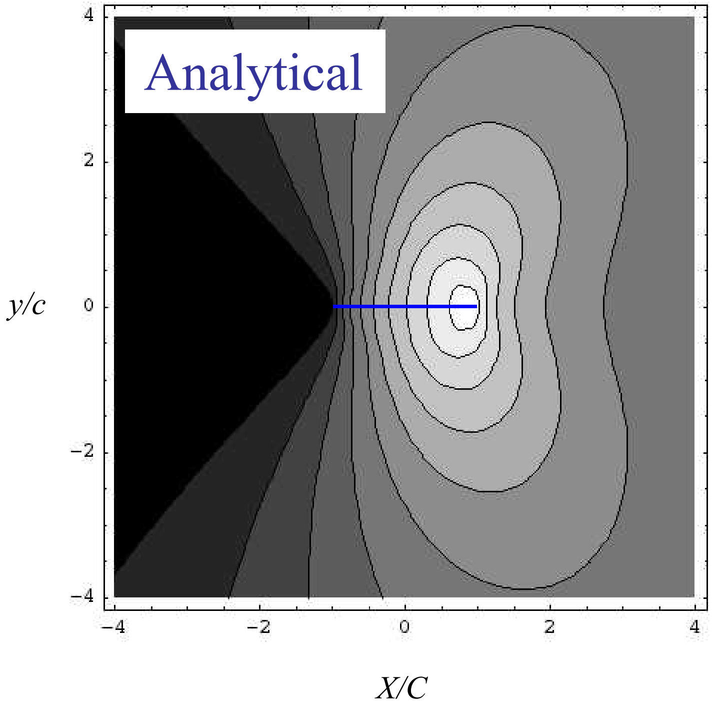
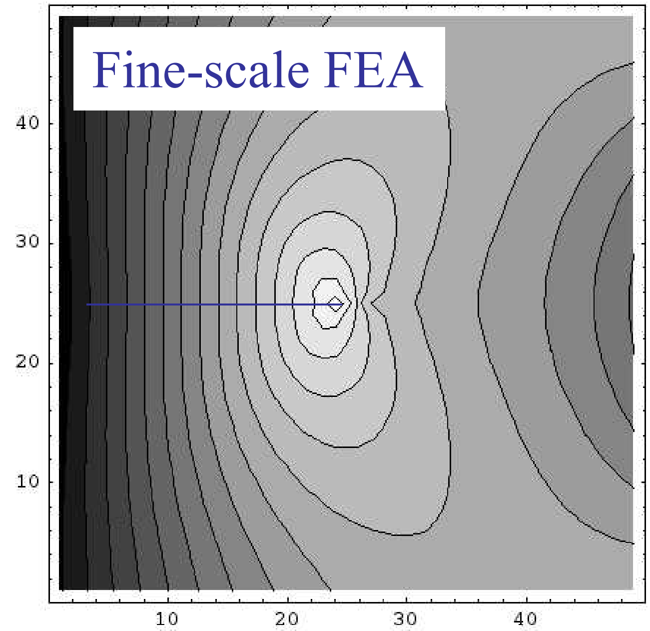
Enrichment Functions: An Analytical Source

- First term considered
  - $F(z)=G(z)=1$
  - $H(z)=0$

Note: problems differ and CZ sizes are not to the same scale.

- Qualitative comparison of σ_{22} with fine-scale FEA





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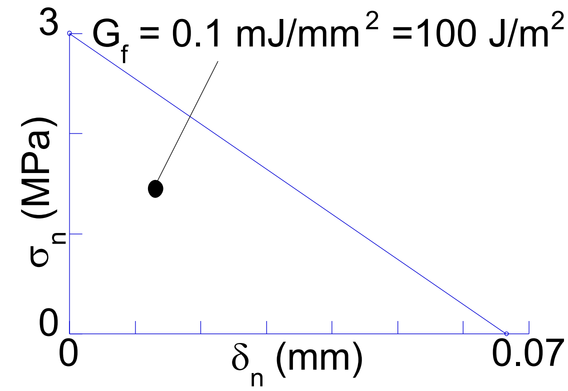
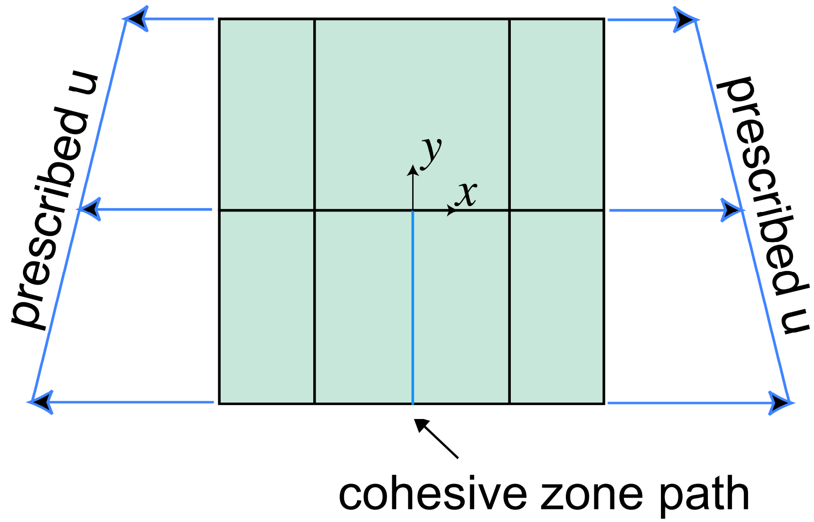
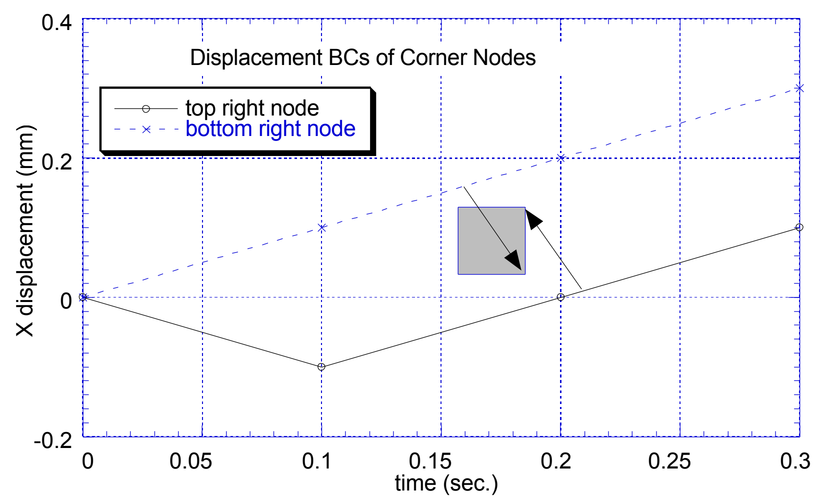
Initial Simple Test Problems

- Concrete test problem
  - relevant to HDBT
  - domain 1 m x 1 m
  - process-zone size ~ O(250 mm)
  - representative concrete tensile properties (except for simplified linear softening)
  - mode I quasistatic crack propagation
  - 2 enrichment functions -> 16-40 dofs



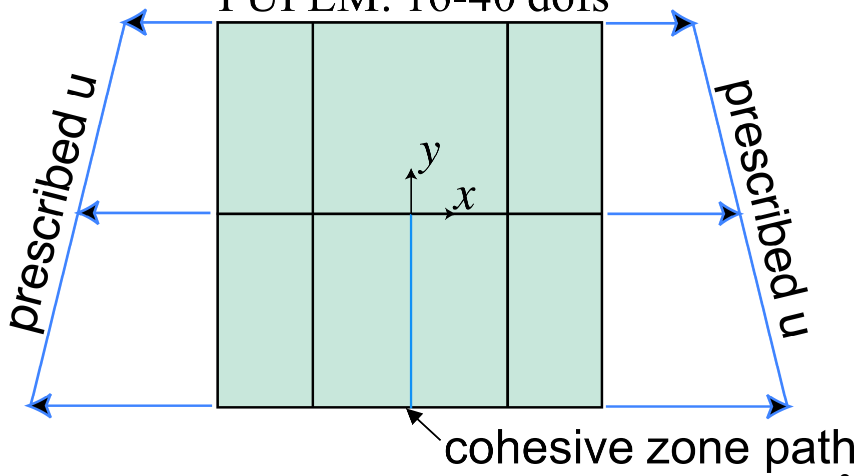
6-Element Test Problem



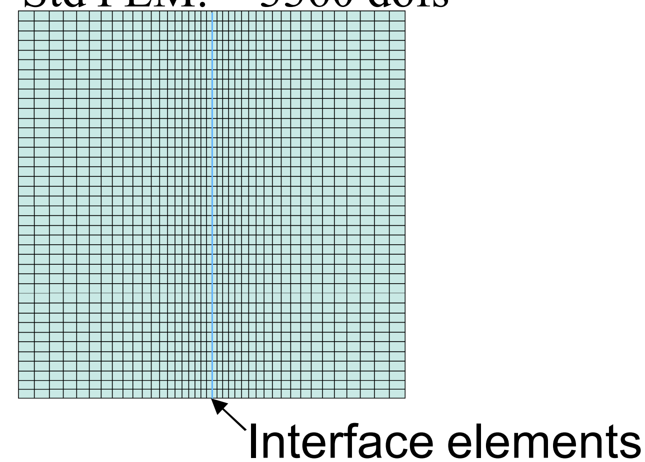


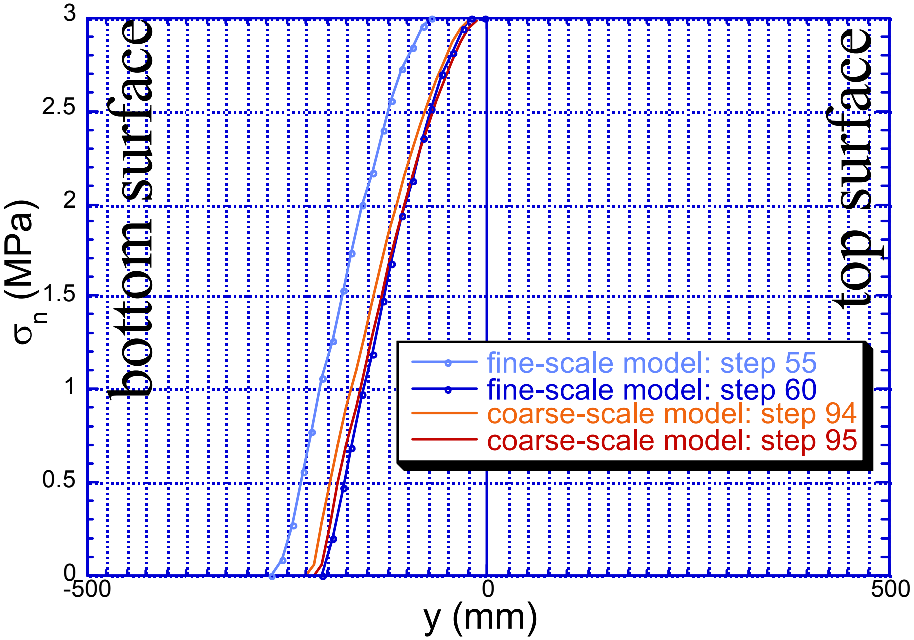
Coarse-scale *vs*. Fine-scale: Qualitative Evaluation

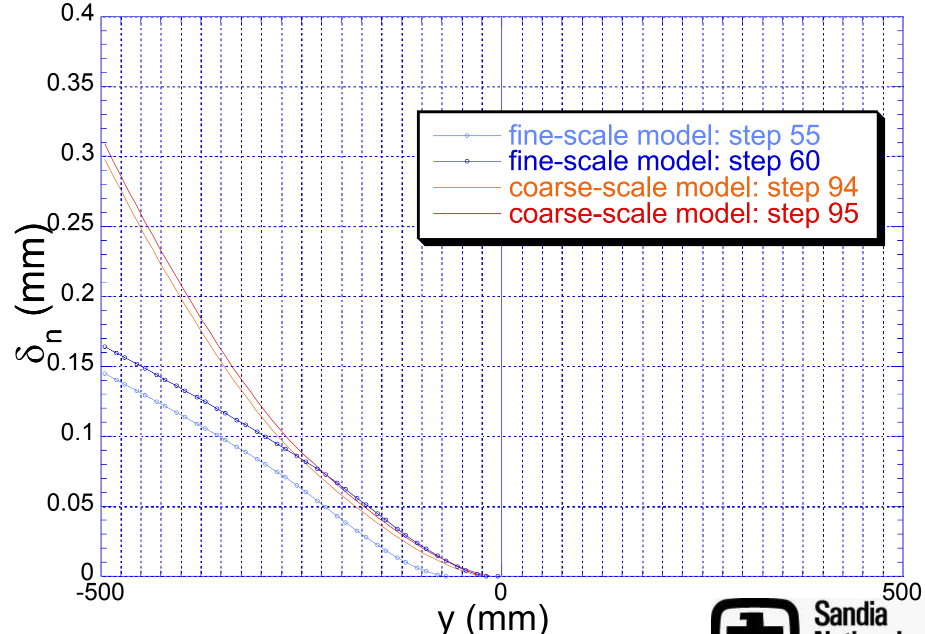
PUFEM: 16-40 dofs



Std FEM: ~ 3360 dofs







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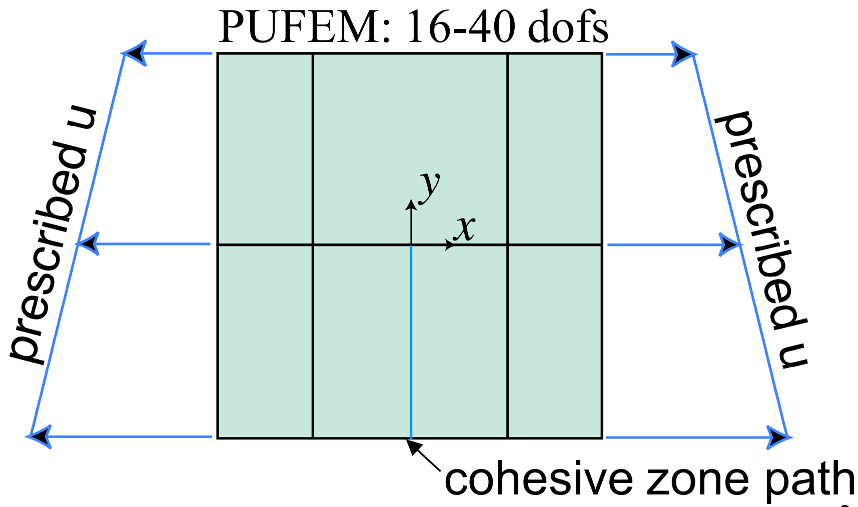
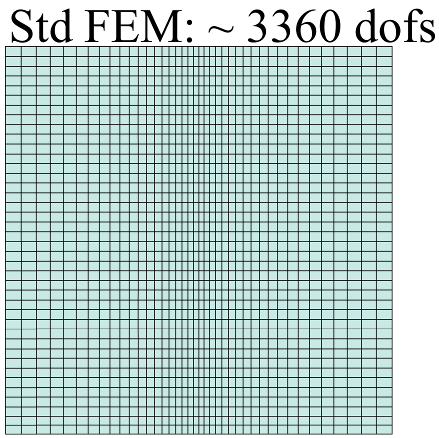
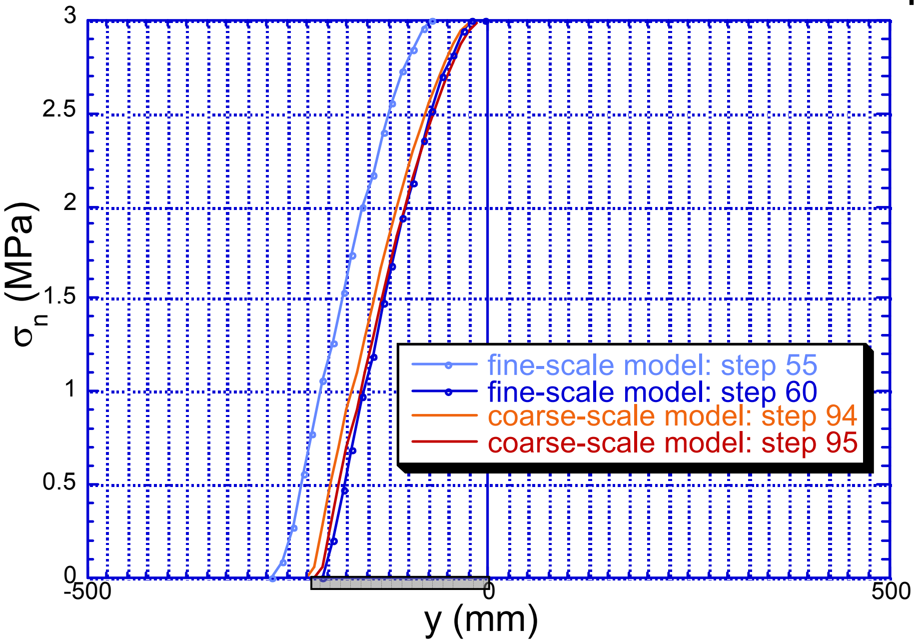
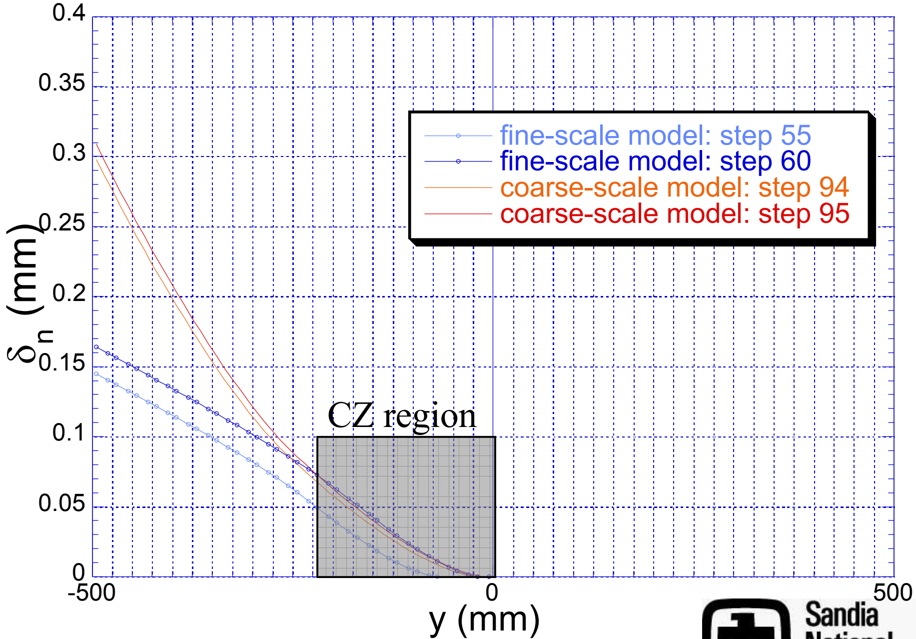
Coarse-scale *vs*. Fine-scale: Qualitative Evaluation









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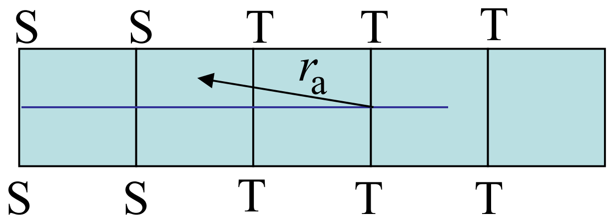
Initial PUFEM Issues

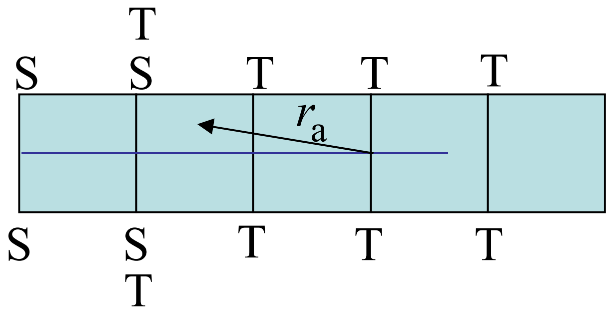
- Crack profiles differed significantly with fine-scale results in the traction free region.
- If several terms are needed to obtain better crack profiles the efficiency will be reduced.
- A length scale exists in the enrichment functions.

Enrichment Modification

- Change to step enrichment
- Analytical radius -- could be applied to the whole plane



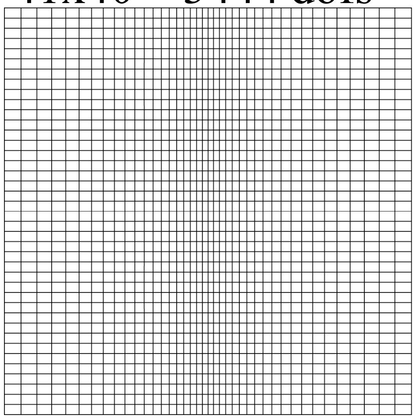
T ~ tip enrichment
S ~ step enrichment



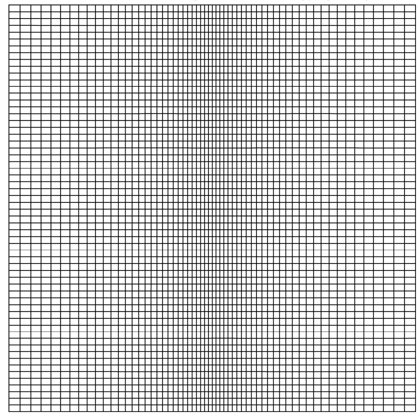
Quantitative Models

- Fine-scale – Standard FEM

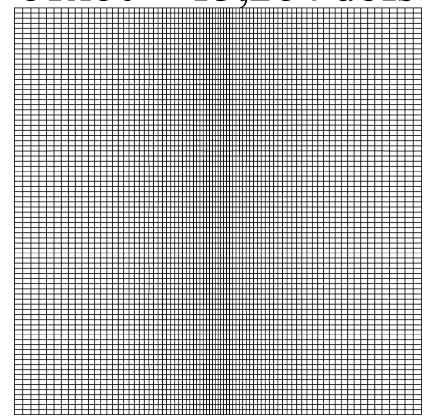
41x40 ~ 3444 dofs



61x60 ~ 7564 dofs

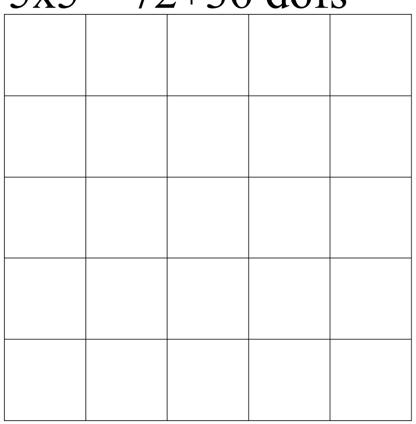


81x80 ~ 13,284 dofs

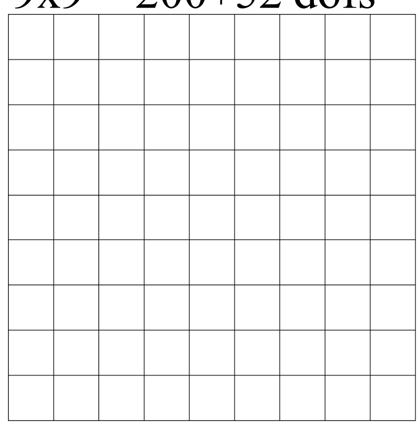


- Coarse-scale – PUFEM

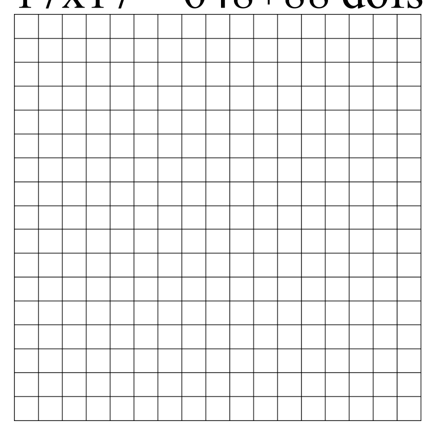
5x5 ~ 72+36 dofs



9x9 ~ 200+52 dofs



17x17 ~ 648+88 dofs



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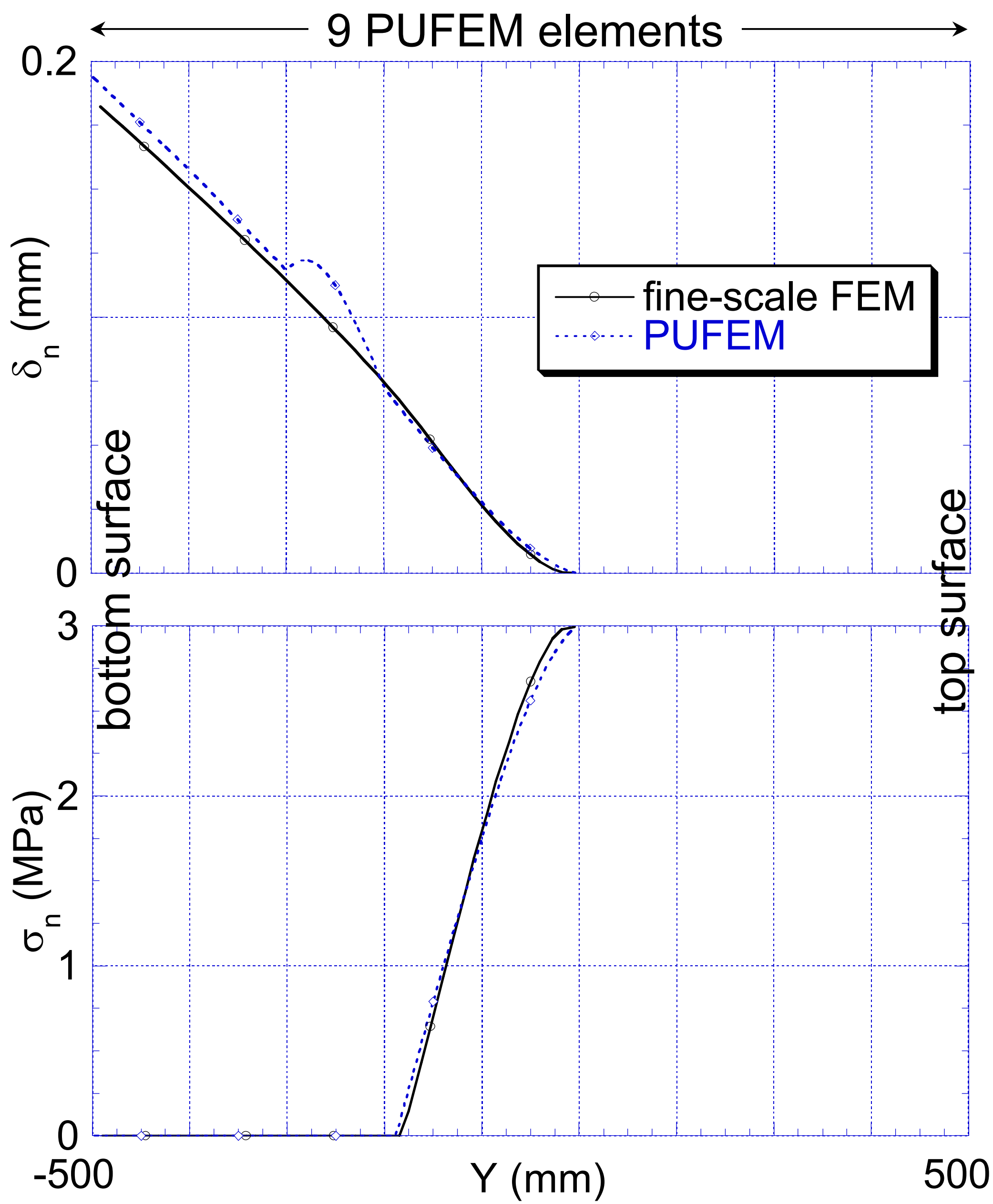
Fine *vs*. Coarse -- Cohesive Zone Response



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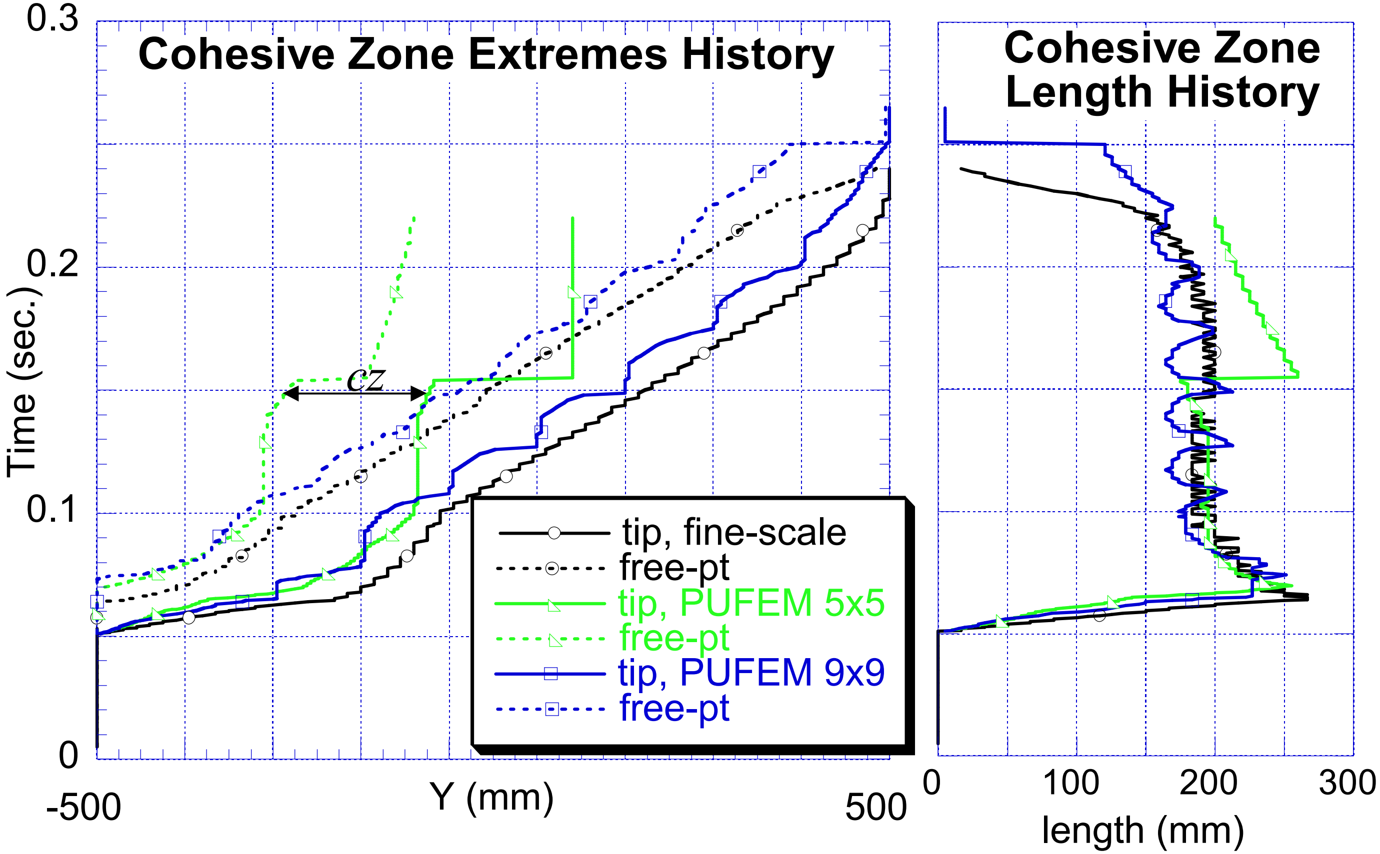
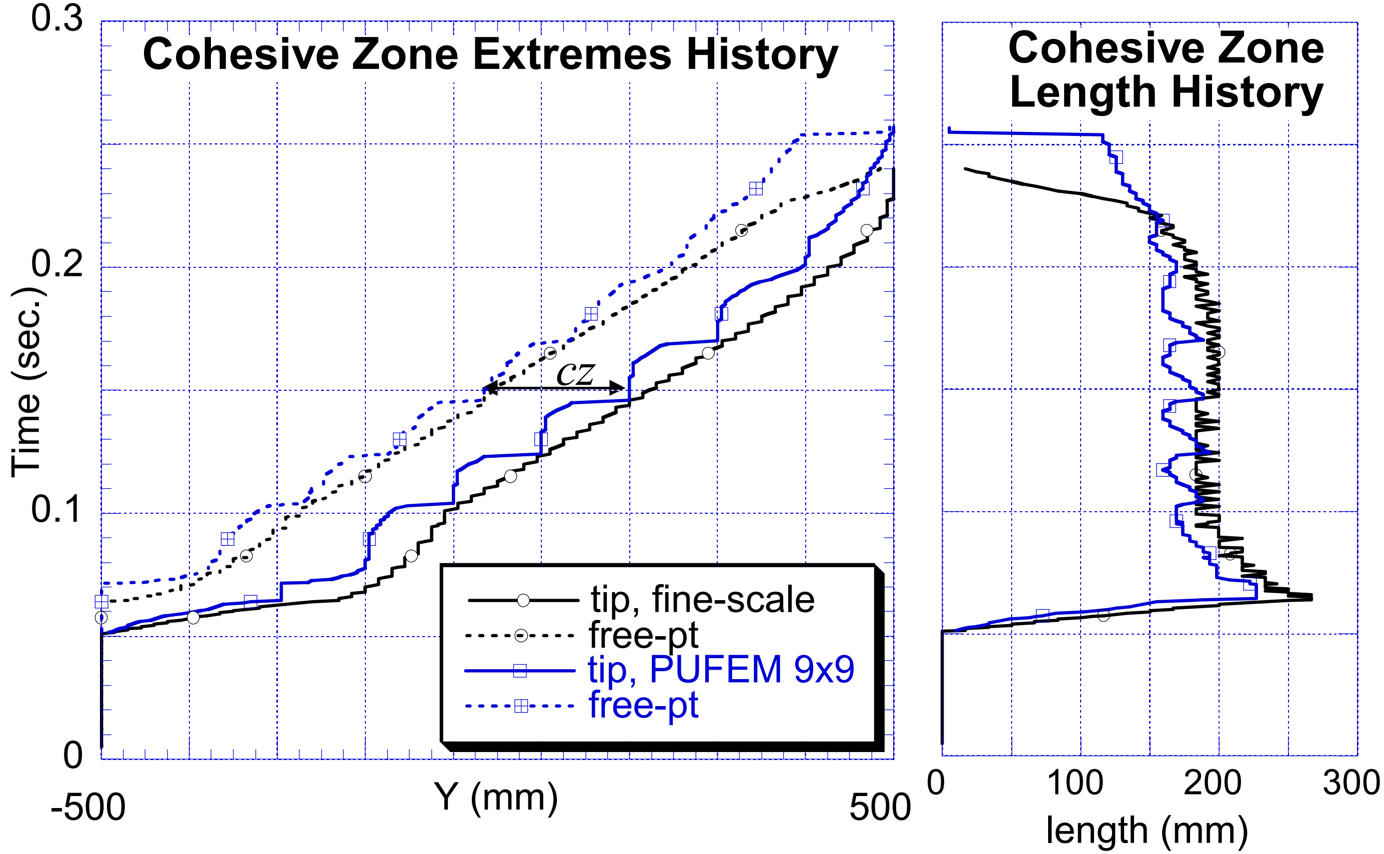
Extremes & Length History



2 coarse meshes

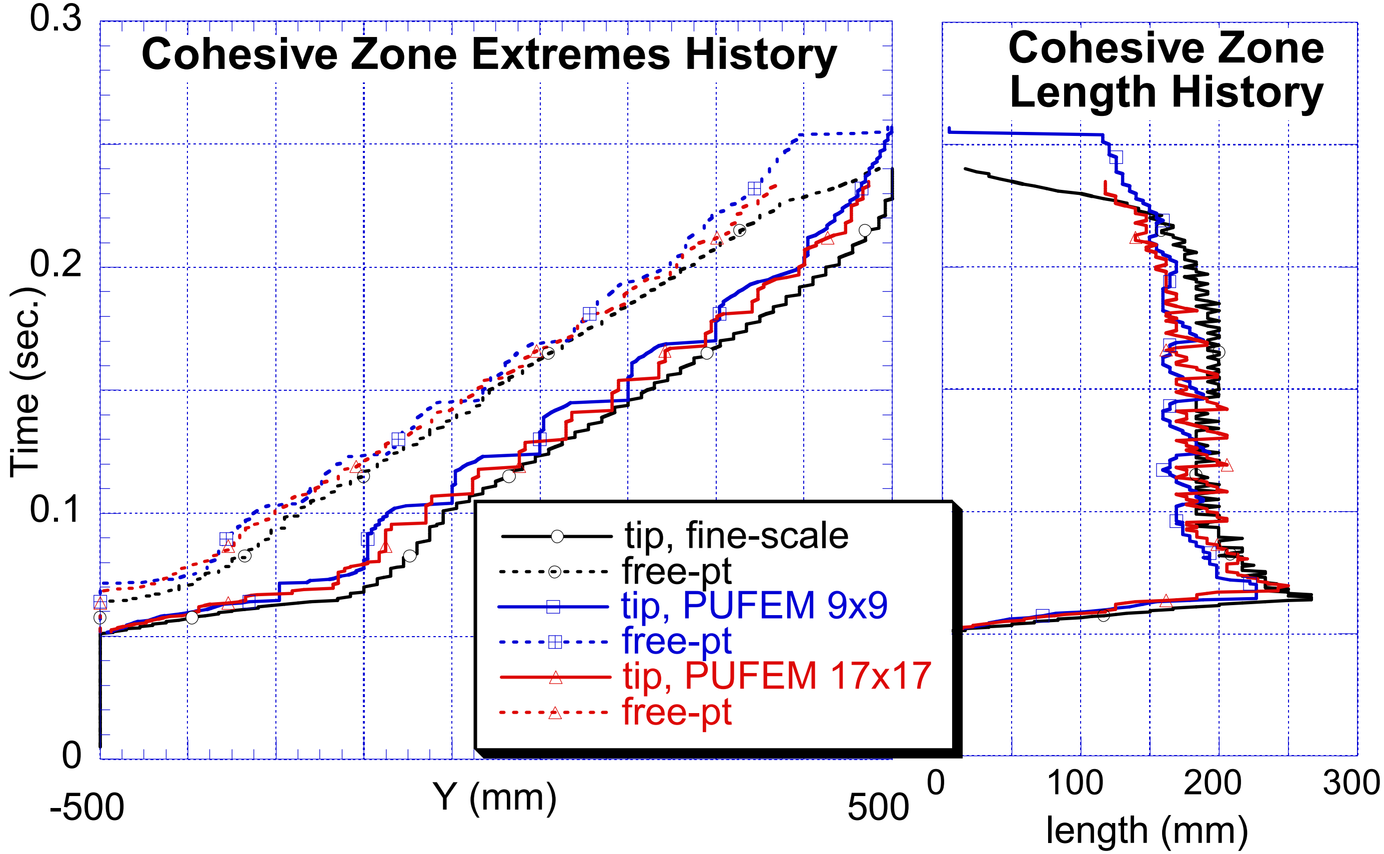
c = 125 mm

Extremes & Length History



decreasing c to 85 mm

Extremes & Length History



refining the PUFEM mesh

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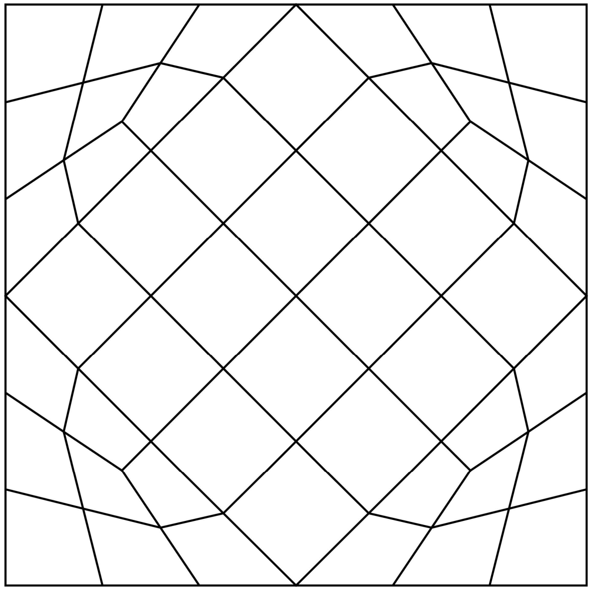
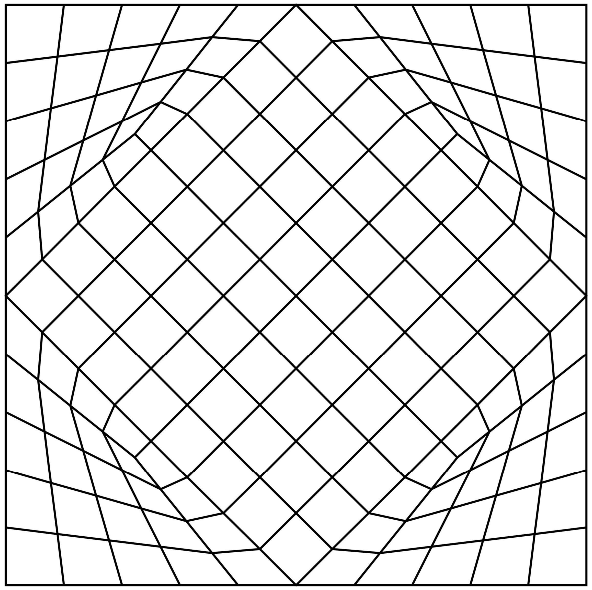
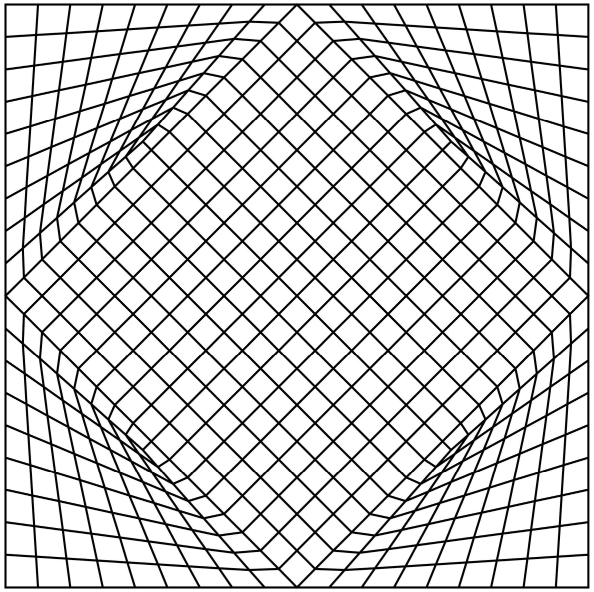
Skewed Mesh Tests



4x4 @ 45°



8x8 @ 45°



16x16 @ 45°

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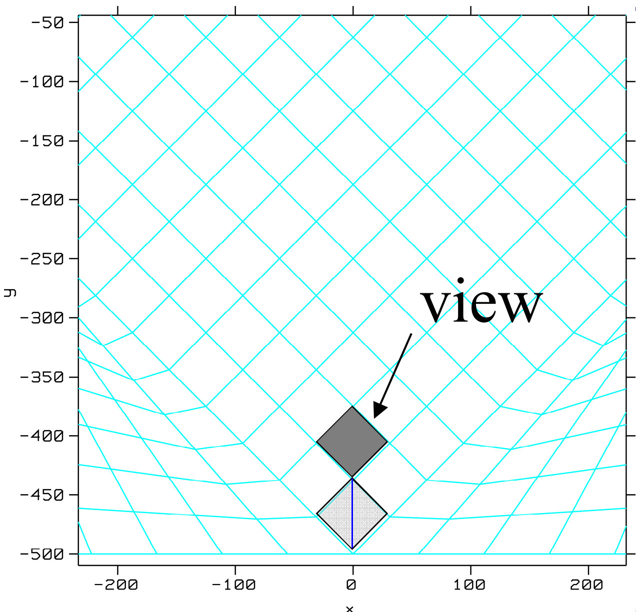
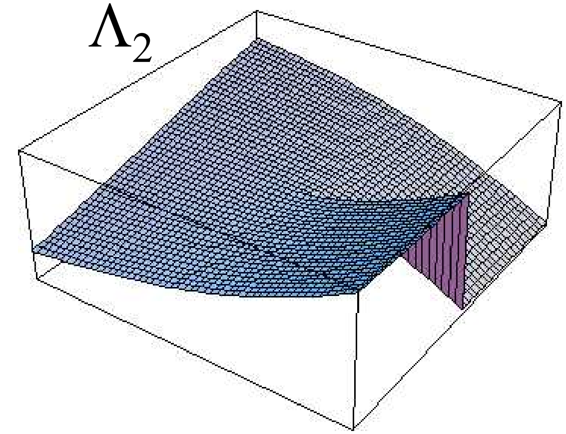
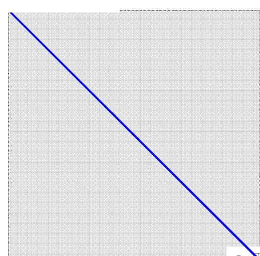
Skewed-mesh: Results and Enrichment



Using a single enrichment Function (G=1)

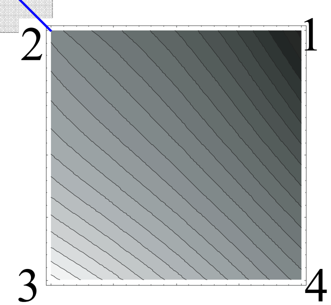
Λ_2

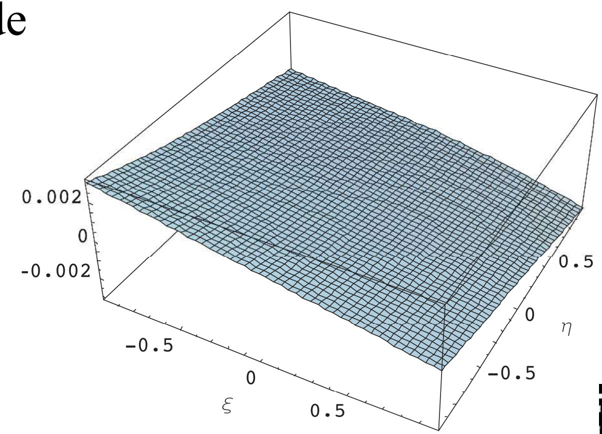




Node 2 is the only enriched node

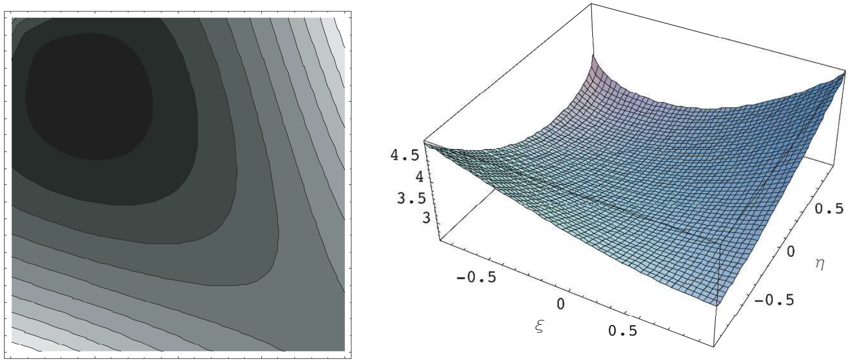
$$u_1(x) =$$



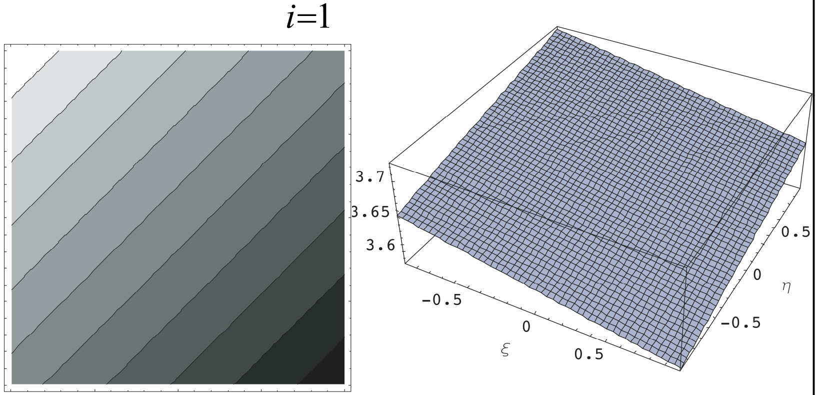


Skewed-mesh: Enrichment

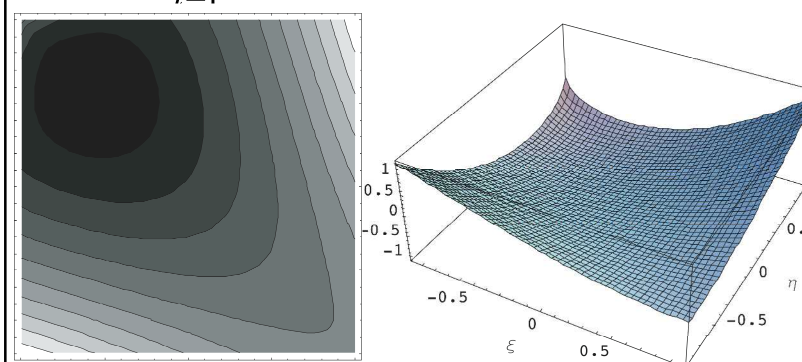
$\sigma_{11}(x) =$



$\sigma_{11}(x) = \sum_{i=1}^{N_N} N_i(x) u_i + \sum_{i=1}^{N_N} \Lambda_2(x) N_i(x) \alpha_{i2}$



"within 2% of being flat"



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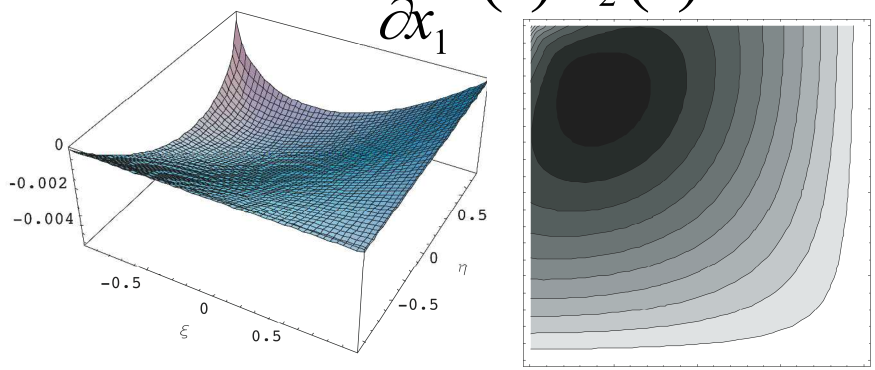
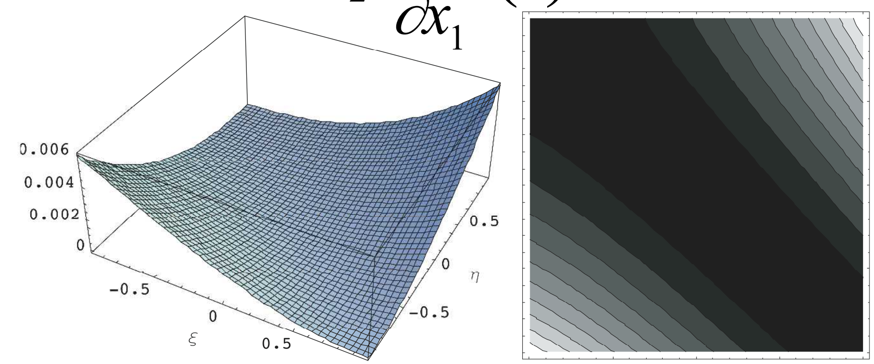
Skewed-mesh: Enrichment

Enrichment contribution to $\varepsilon_{11}(\mathrm{x}) = \dfrac{\partial}{\partial x}\left[\sum_{i=1}^{N_N} \Lambda_2(\mathrm{x}) \mathrm{N}_i(\mathrm{x}) \alpha_{i2}\right]$

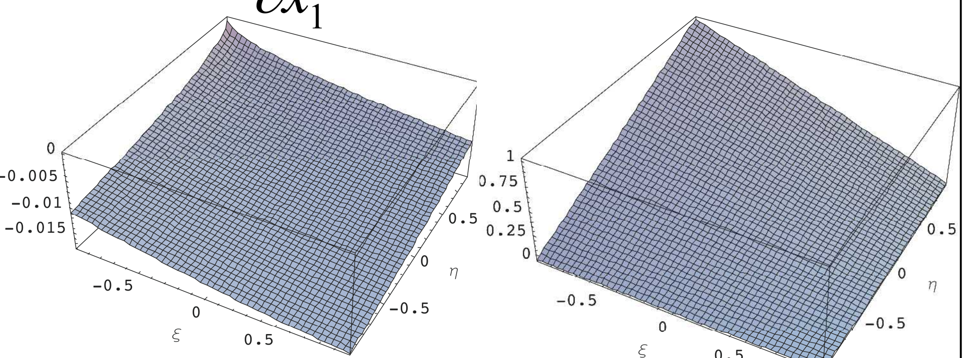
$\alpha_{22} < 0$ for crack opening

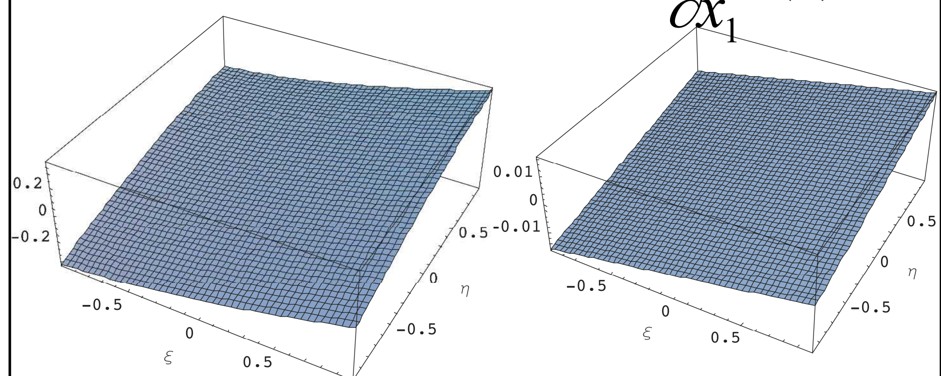
$$-\frac{\partial \Lambda_2}{\partial x_1}(\mathrm{x}) N_2(\mathrm{x}) \quad + \quad -\Lambda_2 \frac{\partial N_2}{\partial x_1}(\mathrm{x})$$





$$-\frac{\partial \Lambda_2}{\partial x_1}(\mathrm{x}) \times N_2(\mathrm{x}) \quad + \quad -\Lambda_2 \times \frac{\partial N_2}{\partial x_1}(\mathrm{x})$$

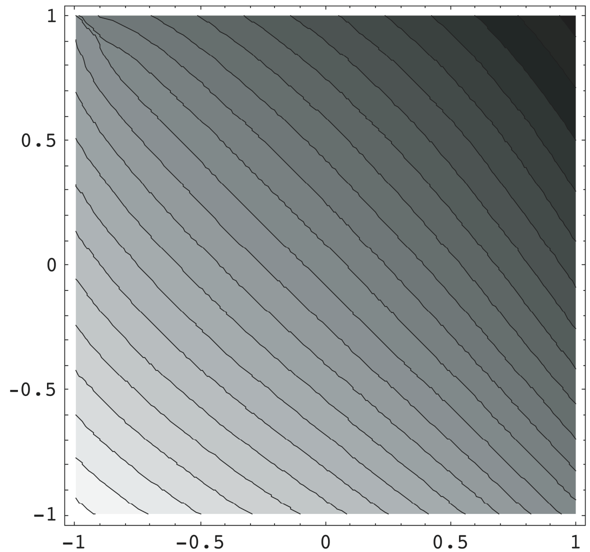


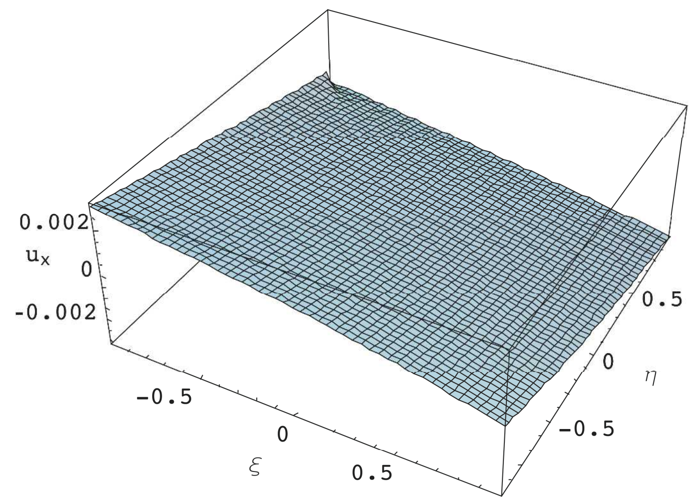


Skewed mesh: Results

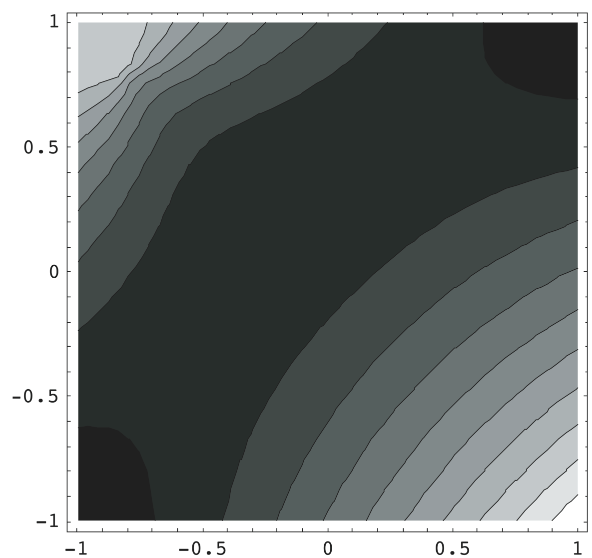
Results for the tip 1/10 of the way through the second element

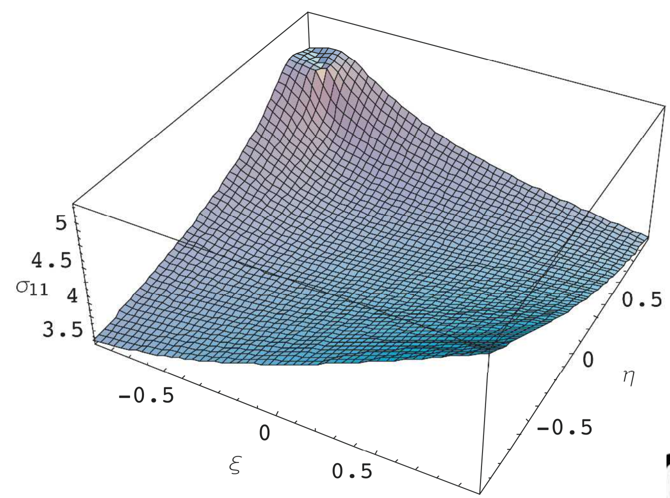
u_1





σ_{11}

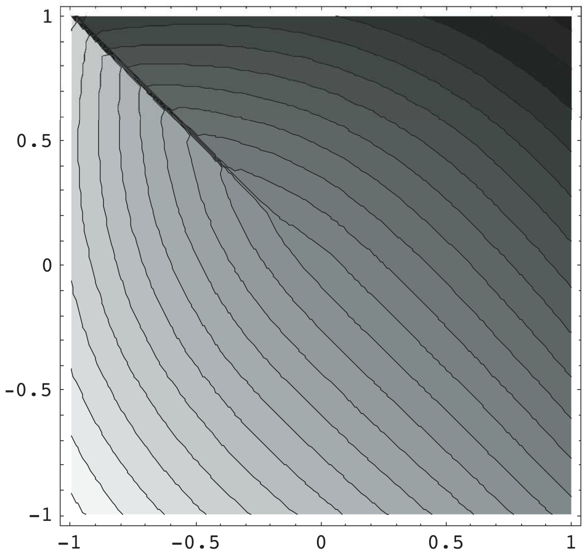


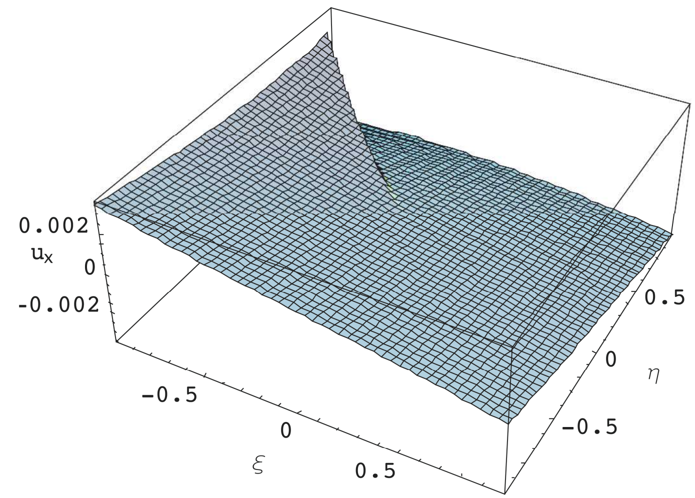


Skewed mesh: Additional Results

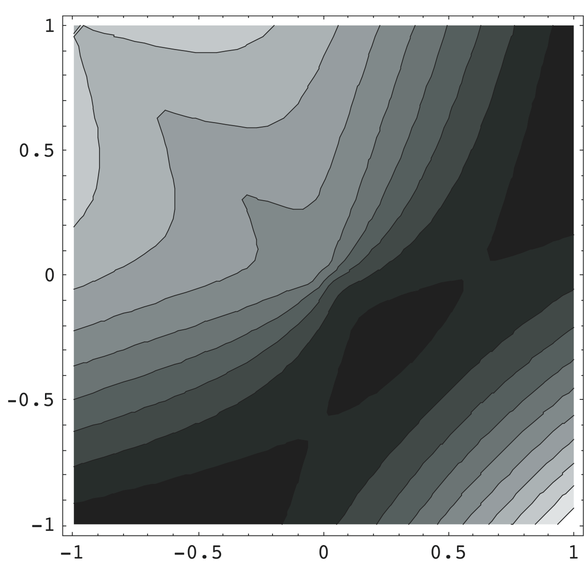
Results for the tip midway through the second element

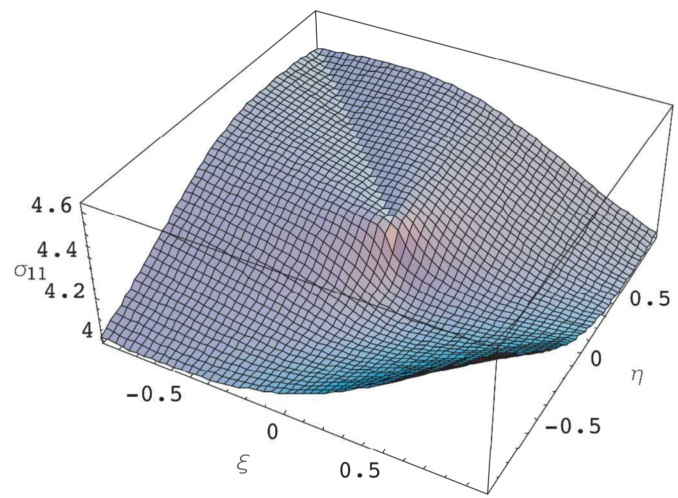
u_1





σ_{11}





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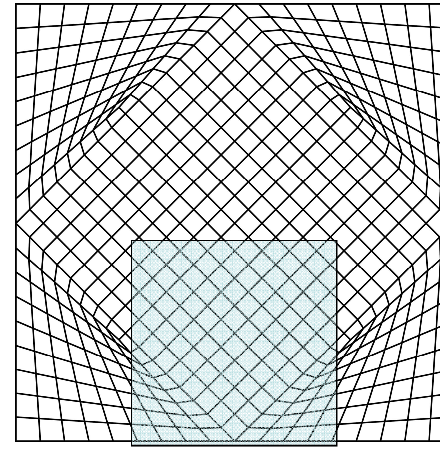
Neighborhood Enrichment

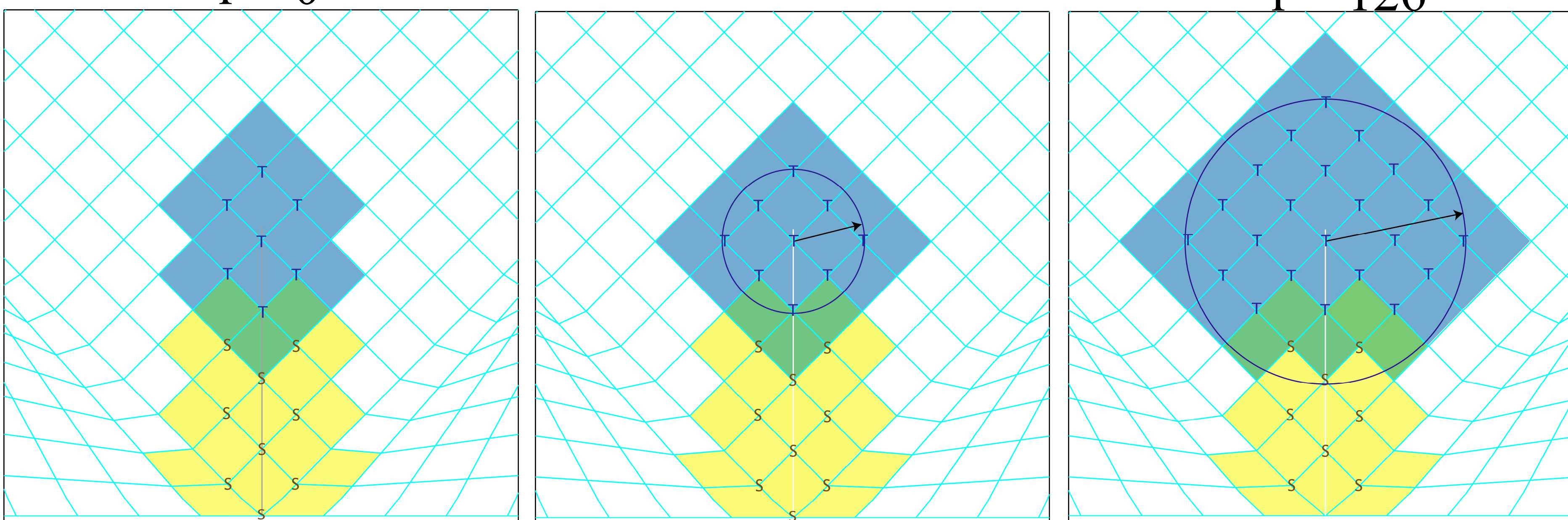
Aka the Mr. Roger's modification



Enriches additional nodes within a user-defined neighborhood of the tip.

Done each time the tip enters a new element.

r = 0   r = 63   r = 126



Observations & Conclusions

- Modification of the enrichment to step enrichment in the wake of the cz can significantly improve the prediction of the crack profile.
- Initial results are not very sensitive to *c*, but adjustment of *c* for the tip-functions may be necessary for some classes of problems.
- PUFEM is exhibiting convergence (with mesh refinement)
- Product form of enrichment has negative effects with a "coarse" skewed-mesh.
- PUFEM for cohesive zone modeling of localization has potential and merits further investigation.
- No free-lunch -- algorithm complexity is high.

Acknowledgements

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