SAND2009-4406C

Numerical Approaches for the Quadratic Eigenvalue Problem on Large Structural Acoustic Systems

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Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company,
for the United States Department of Energy's National Nuclear Security Administration
under contract DE-AC04-94AL85000.

outline

- Problem Definitions
- Solution Approaches
  - modal projection (with variants)
  - Iterative solution of full system
  - others
- Numerical Tradeoffs
  - simple piston
  - cylindrical cavity
- Extensions
- Summary

problem definition

The quadratic eigenvalue problem is important for many systems including tightly coupled structural acoustics.

$$\left(\begin{bmatrix} k_s & 0 \\ 0 & k_a \end{bmatrix} + \lambda \begin{bmatrix} c_s & L \\ L^T & c_a \end{bmatrix} + \lambda^2 \begin{bmatrix} m_s & 0 \\ 0 & m_a \end{bmatrix}\right)\begin{bmatrix} u \\ \psi \end{bmatrix} = 0 \qquad (1)$$

where subscripts "s" and "a" represent the structural and acoustic regions respectively and L is a coupling matrix. Here "u" represents the structural dofs and ψ is the acoustic response. Damping matrices, c_s and c_a, are associated with energy loss such as radiation.

QEVP linearization

Equation (1) is linearized for solution as a generalized, linear eigen problem of order 2*N*. We use one of these two linearizations:

$$\left(\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} - \lambda \begin{bmatrix} 0 & M \\ -M & -C \end{bmatrix}\right)\begin{bmatrix} \dot{\varphi} \\ \varphi \end{bmatrix} = 0$$

or

$$\left(\begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} - \lambda \begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}\right)\begin{bmatrix} \dot{\varphi} \\ \varphi \end{bmatrix} = 0$$

The first provides a simple LHS with solves of standard, uncoupled matrices. The second is more forgiving of singular K. Note: K here includes both the structural and acoustic components.

Solution Approaches

- Linearization of Solution
  - Modal Projection
  - Iterative Solution on full system
- Recasting as generalized eigenvalue problem
  - limited to zero energy loss

Projection Approach

In projection approaches, a standard modal analysis is performed omitting the damping/coupling matrix. Eigenvectors from that problem are used to project the full problem to a small, dense system. Left and right eigenmodes of that system are projected back to physical space.

solve,

$$\left(K - \lambda' M\right)\Phi = 0$$

truncate modes and generate:

$$\overline{K} = \Phi^T K \Phi$$
$$\overline{M} = \Phi^T M \Phi$$
$$\overline{C} = \Phi^T C \Phi$$

These small matrices are linearized and a dense system solve is performed (as with dggev).

Projection Solution

$$\left(\begin{bmatrix} 0 & \bar{I} \\ -\bar{K} & -\bar{C} \end{bmatrix} - \lambda \begin{bmatrix} \bar{I} & 0 \\ 0 & \bar{M} \end{bmatrix}\right)\begin{bmatrix} \dot{\bar{\varphi}} \\ \bar{\varphi} \end{bmatrix} = 0$$

The system is not symmetric, so the left eigenvalues must also be determined. Eigenvalues are unchanged, but vectors require projection out to the physical space,

$$\varphi = \Phi\bar{\varphi}$$

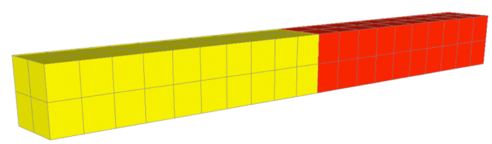
The method is straightforward, and easy to implement. For large systems, the computational complexity is in solution of the uncoupled eigenmodes used as a basis. Modal truncation introduces the most significant issue.

Iterative Solution of Full System

- ARPACK, ANASAZI and other software packages provide solution of nonsymmetric generalized eigenvalue problems, provided that sparse linear solvers are available.
- For large linear systems, Salinas uses parallel domain decomposition solvers such as FETI and GDSW.
- Some linearizations allow us to solve two symmetric systems of order N.
- Advantage: no modal truncation.

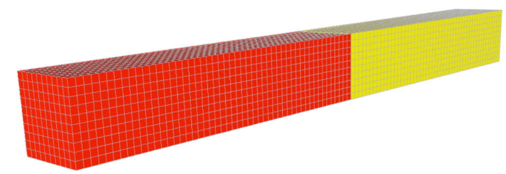
Numerical Comparisions

- Models
  - small piston
  - large piston
  - cavity
- Evaluations
  - modal convergence
  - solution times



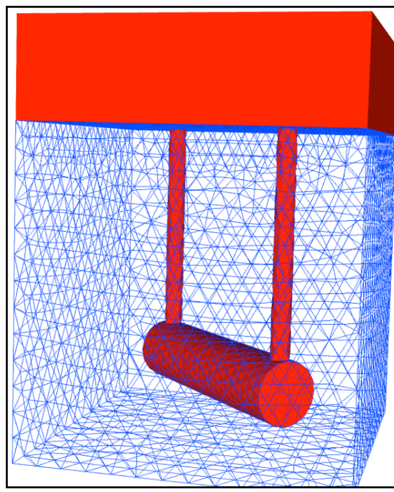
Small Piston

80 Hex elements. 2x2x20



Large Piston

10k Hex elements. 10x10x100



cylindrical cavity

acoustic: 40k

structural: 67k

All tet4. Air/soil

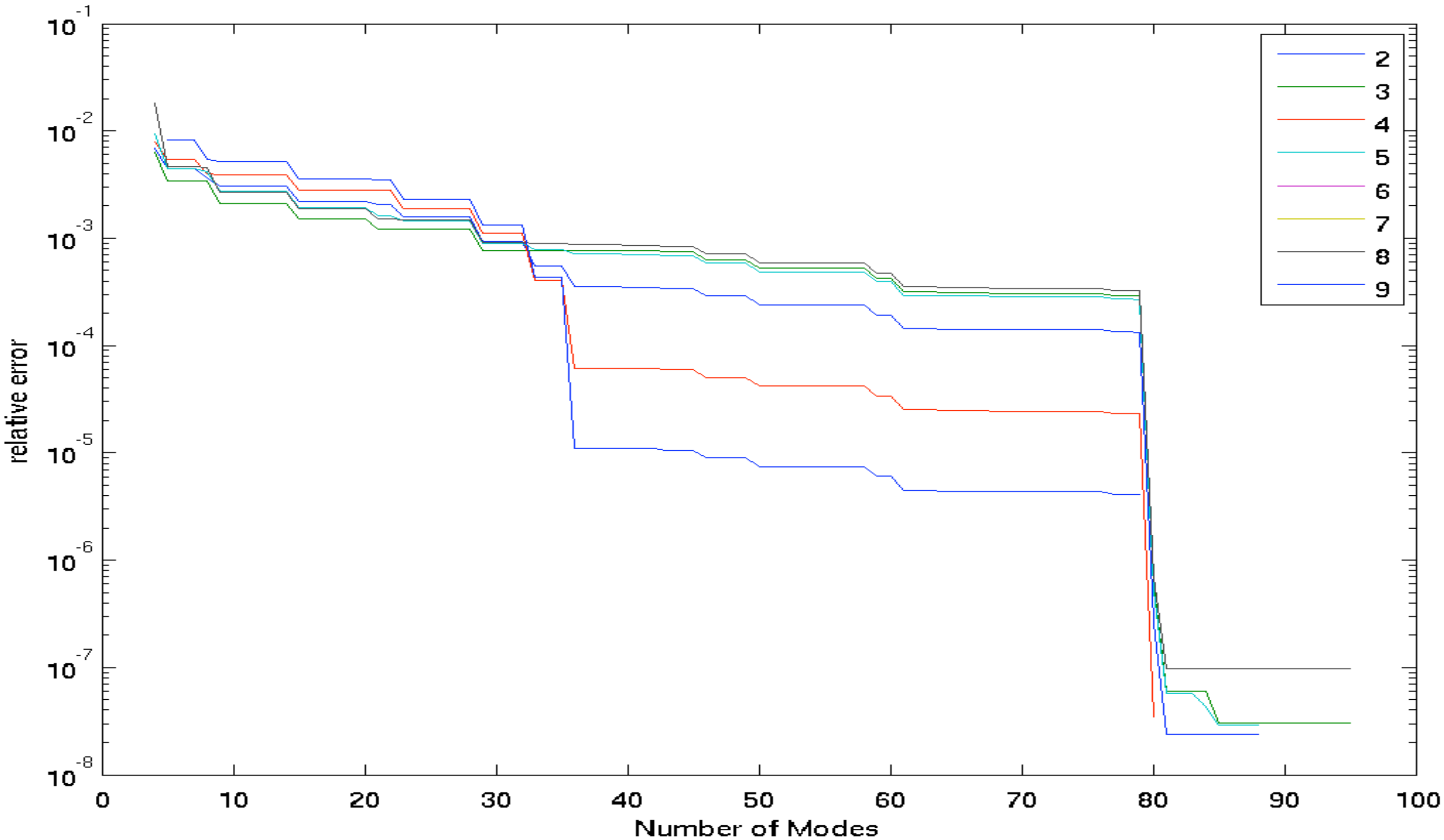
Piston Model

- Structural and acoustic sections of a beam.
- Fictitious materials chosen for high coupling.
- Closed form solutions are available.†

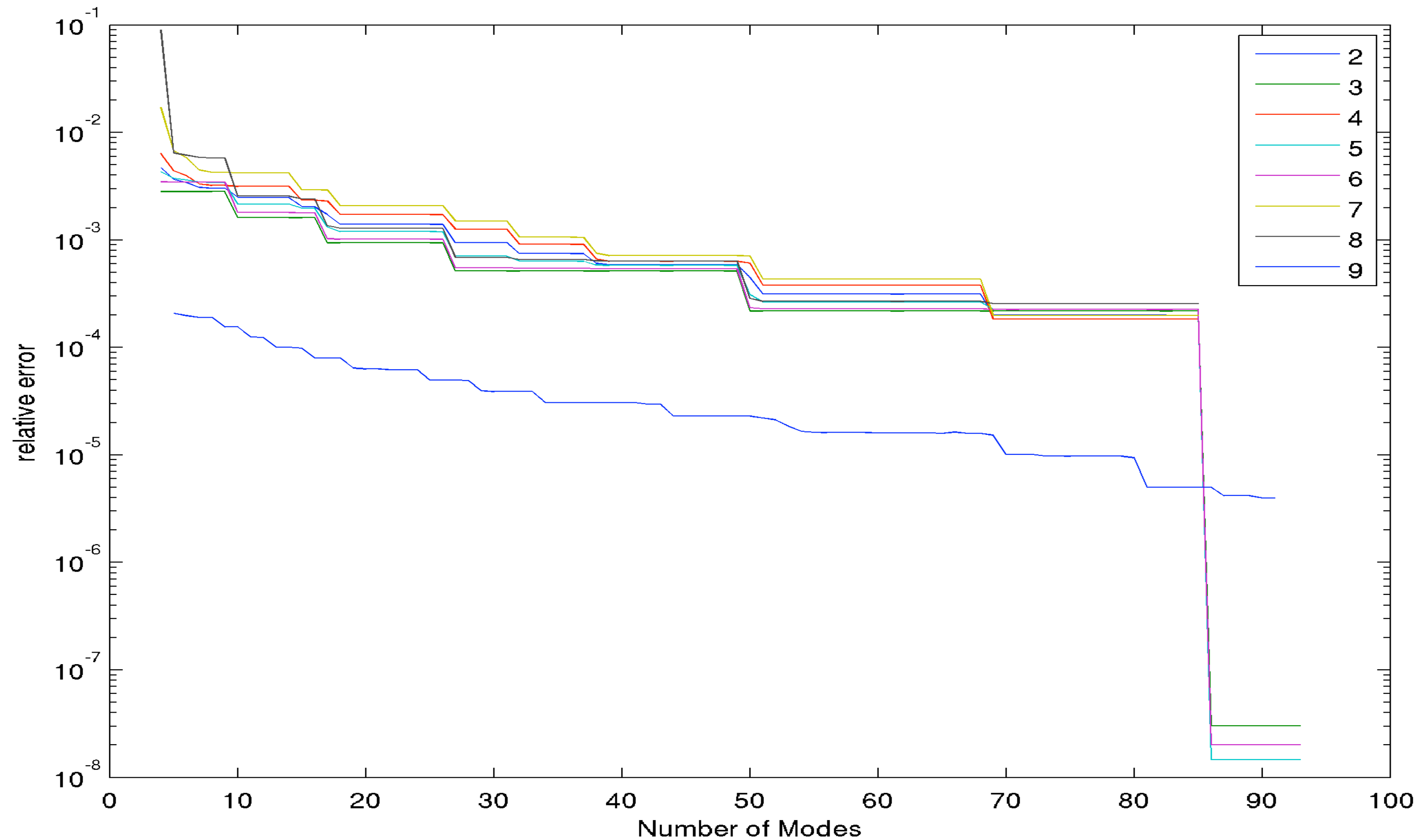
$$Err = \frac{|\lambda - \lambda_{ref}|}{\lambda}$$

Kinsler, Frey and Sanders, “Fundamentals of Acoustics”, 3rd edition, John Wiley & Sons, NY, 1982.

Small Piston modal convergence



Large Piston modal Convergence



Cavity Model
Modal Convergence

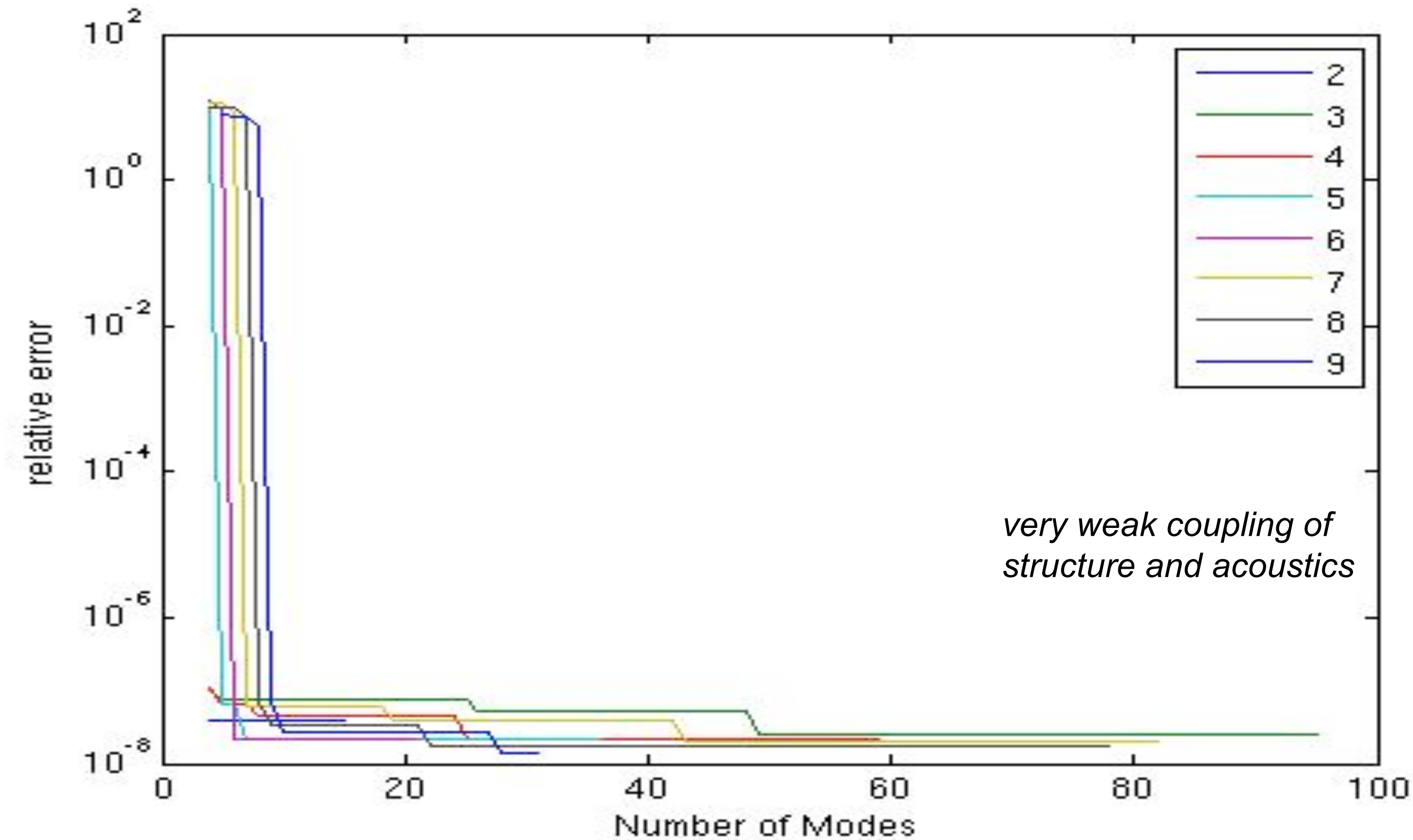


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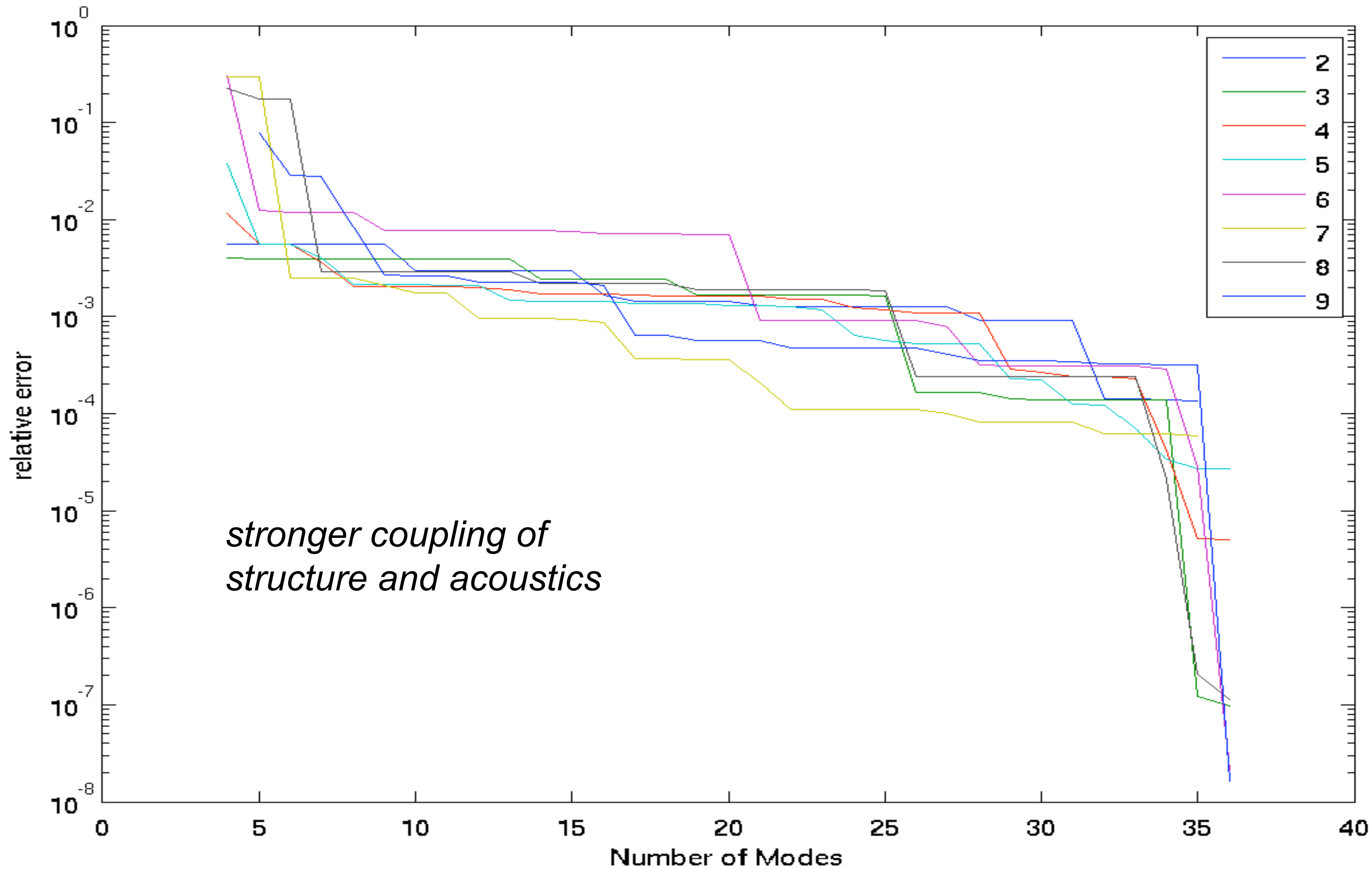
figure to be updated for more modes.

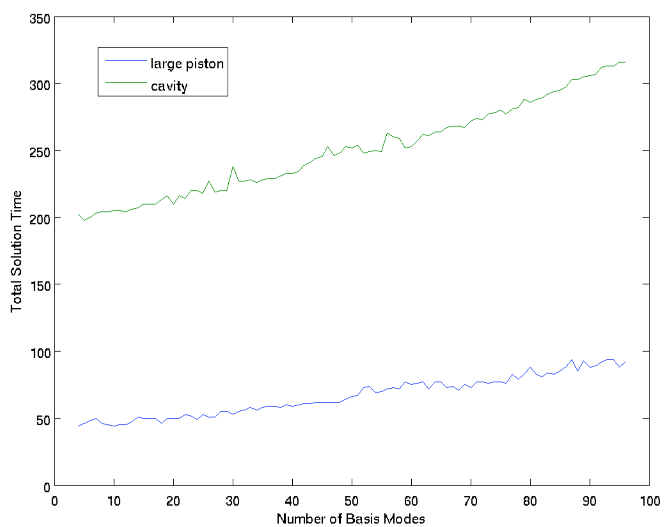
Cavity Model (heavy fluid) Modal Convergence



Full Solution Times

computing lowest 10 system modes

Model	Method	Basis	Time
Small Piston	projection	96 modes	7
	Projection	10 modes	<1
	iteration	full	1
Large Piston	projection	96 modes	92
	Projection	10 modes	44
	iteration	full	36
Cavity	projection	96 modes	316
	Projection	10 modes	205
	iteration	full	268



Modal Convergence

Sensor Model

Garth Reese:

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Model Details:

1.1 M hex8 elements

95% acoustic mesh

high coupling

Conclusions

- A small number of modes is typically sufficient of convergence of the modal projection method. However, this is quite problem dependent.
- Full iteration is not significantly more expensive than projection methods, and avoids the question of modal truncation.
- Both methods depend on scalability of the linear solver.