



VisWeek 09  
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SAND2009-6641C

# ParaView

## *Statistics*

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Sandia is a multiprogram laboratory operated by Sandia Corporation,  
a Lockheed Martin Company, for the United States Department of Energy's  
National Nuclear Security Administration under Contract DE-AC04-94-AL85000



# Outline

- Statistics in General
- Statistics in VTK
- Statistics in ParaView
- Algorithm Details



# VTK Filters



# Tasks

- **Learn** from input data. Also called **Train** in the machine learning/classification community.
- **Derive** further (related and/or more user-accessible) information from minimal statistics.
- **Appraise** the model; detect
  - problems with assumptions (independence, goodness of fit); and
  - stability problems (numerical & sensitivity).
- **Assess** some data using what was learned.





# Design Pattern

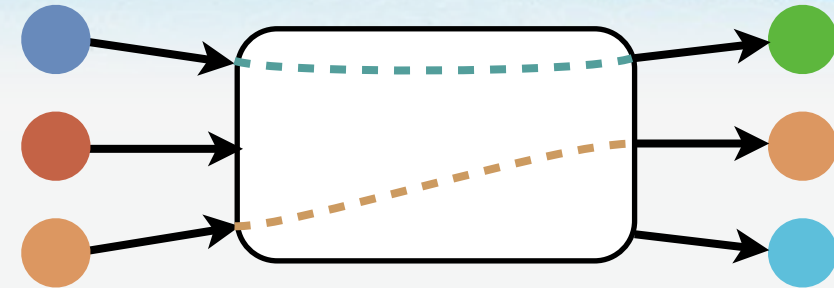
With distributed data, most statistics algorithms look like trendy applications of

- **Learn** – Map-Reduce
- **Derive** – Embarrassingly Parallel Reduce
- **Appraise** – Map-Reduce
- **Assess** – Embarrassingly Parallel Map



# VTK Statistics

- Filters have **inputs** for
  - Data to learn or assess
  - Model parameters (e.g., k-means start points)
  - Pre-existing model for assessment
- Filters have **outputs** for
  - Possibly-assessed data
  - Model output
  - Assessment summary information

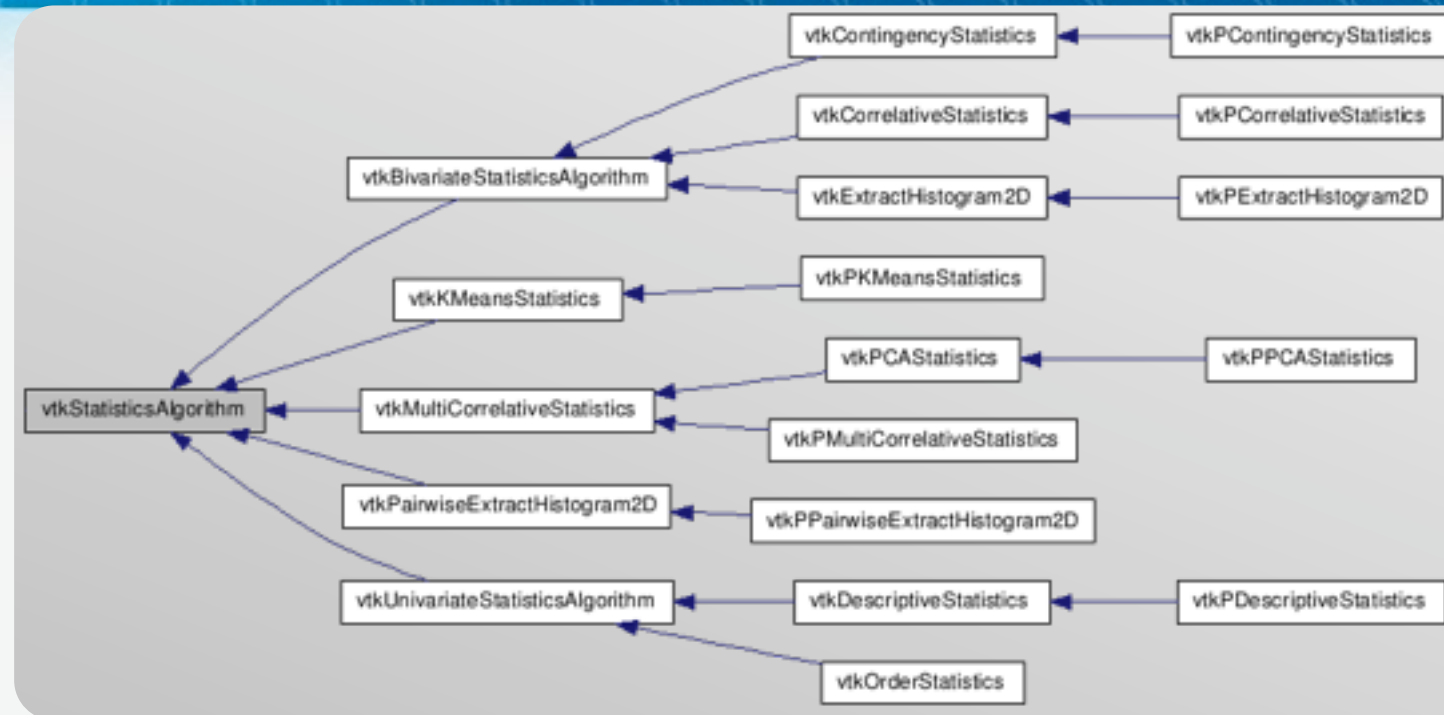




# VTK Statistics

- Filters include
  - Contingency tables
  - Descriptive statistics
  - *k*-means clustering
  - Order statistics (quantiles)
  - Principal component analysis
  - Bivariate histogram (for parallel coords)
- Currently no filters implement Appraise but all implement Learn, Derive, & Assess.

★ Filters in blue have parallel implementations.



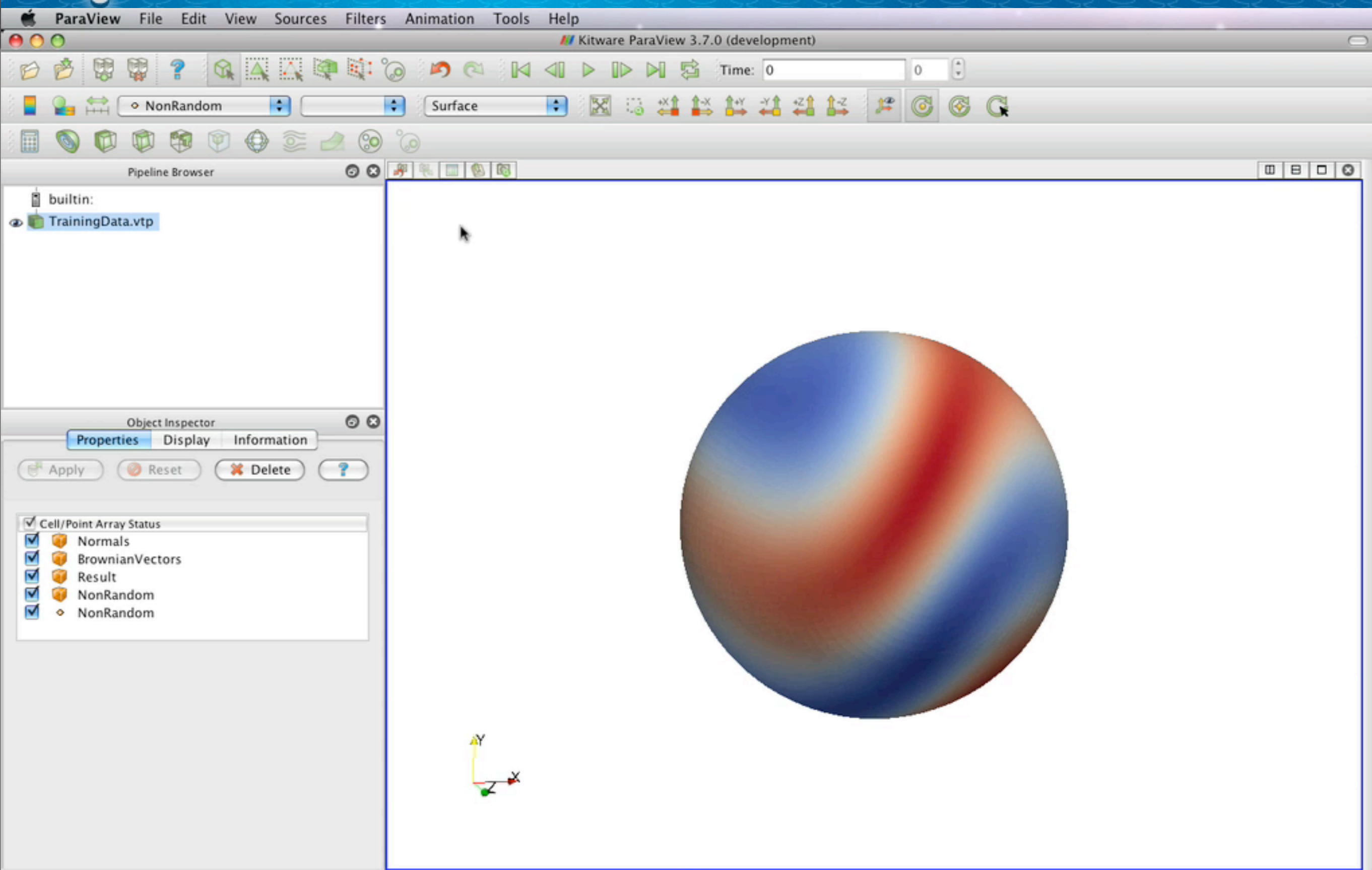


# ParaView Interface



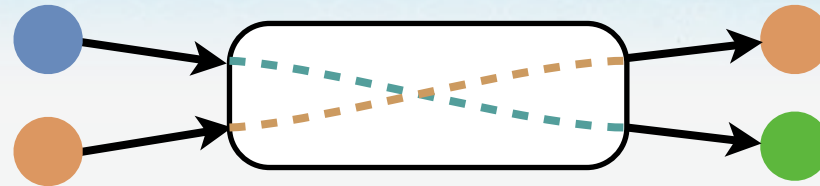


# ParaView Statistics





# ParaView Statistics



- Filters have **inputs** for
  - Data to learn or assess
  - Pre-existing model for assessment
- Filters have **outputs** for
  - Model output
  - Possibly-assessed data
- Notice reversed output order (for ease of use)!





# Statistics Caveats

- ⚠ In data-parallel mode, **point** arrays will have **distorted** statistics: shared points are counted once per process instead of just once.
- ⚠ Distortion may be introduced by your mesh (spatially varying sampling frequency).
- ⚠ Tasks that perform random sampling will choose a **different** random sample each time the filter is re-executed.





# Algorithm Details





# Details: Contingency

## **Learn + Derive**

- Counts number of occurrences of all combinations of values
- Marginalizes with respect to each array component
- Computes information entropies



# Details: Contingency

## Assess

- Assigns probability from contingency table to each observation.
- Computes Pointwise Mutual Information (PMI) of each observation.
- Note that when you Learn from a different dataset or a subset of the data, any values not encountered during Learn will be assessed with 0 probability. This can make the output look noisy.



# Details: Descriptive

## **Learn**

- Computes the min, max, mean, and M2–M4 centered sums.

## **Derive**

- Adds columns for standard deviation, variance, and estimators for skewness and kurtosis.

## **Assess**

- Tags each observation with signed (or unsigned) number of deviations from the mean.



# Details: $k$ -means

## Learn

- Iteratively updates  $k$  cluster centers  $x_i$  until maximum count or relative tolerance met.
- Initial  $x_i$  are taken from a uniform random distribution over each array's bounds **or** a third input table for model parameters.

## Derive

- Compares total error of each  $(k, x_i)$  set to determine lowest-error fit. (Not useful in ParaView: only a single value of  $k$  is allowed.)





# Details: $k$ -means

## **Derive**, *cont.*

- Use in VTK allows comparisons between multiple  $k$  values and initial cluster centers.

## **Assess**

- Tags each observation with 2 values:
  - Integer ID of nearest cluster center
  - Distance to cluster center (Euclidean)



# Details: Multicorrelative

## Learn

- Computes means of arrays and covariances of array pairs

## Derive

- Computes Cholesky decomposition of the covariance matrix (used in **Assess**).

## Assess

- Uses the inverse of the covariance matrix to tag each observation with its Mahalanobis distance.



# Details: Multicorrelative

- Output table is densely packed with multiple matrices and vectors.
- Covariance matrix is symmetric; only the top half is stored.
- Cholesky decomposition is lower-triangular.
- Overall:  $N+1 \times N+1$  table for  $N$  arrays.

	Column	Mean	BrownianVectors_0	BrownianVectors_1	BrownianVectors_2	Result
0	BrownianVectors_0	0.0130061	0.0903729	-0.00155543	0.00117395	0.000430427
1	BrownianVectors_1	0.0202801	0.300621	0.0863474	0.00163257	-0.00264618
2	BrownianVectors_2	-0.00266763	-0.00517405	0.293804	0.0905124	-0.0040427
3	Result	0.00479249	0.00390508	0.00562544	0.300775	0.0898239
4	Cholesky	1587	0.00143179	-0.00898141	-0.0132915	0.299273





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**Mean**

**Covariance**

**Cholesky decomposition**

**#Vals**





# Details: PCA

## **Learn**

- Identical to multicorrelative statistics

## **Derive**

- Optionally normalizes covariance matrix, then computes SVD to get eigenanalysis.

## **Assess**

- Projects each observation into the new basis, which may be truncated to a fixed dimension or a fixed “energy.”



# Details: PCA

- Output table is densely packed with multiple matrices and vectors.
- Multicorrelative output is identical but without the final  $N+1$  rows.

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4	Cholesky	1587	0.00143179	-0.00898141	-0.0132915	0.299273
5	PCA 0	1.06379	-0.0652366	0.490468	0.582203	-0.645156
6	PCA 1	1.01444	0.826499	-0.411326	0.380697	-0.052727
7	PCA 2	0.970223	-0.518189	-0.76089	0.262885	-0.288821
8	PCA 3	0.951554	-0.210058	0.106293	0.668581	0.705391
9	PCA Cov	0	0.0903729	0.0863474	0.0905124	0.0898239



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8	PCA 3	0.951554	-0.210058	0.106293	0.668581	0.705391
9	PCA Cov	Unused	0.0903729	0.00390508	0.00562544	0.0898239

**Covariance**

**Cholesky decomposition**

**Eigenvalues**

**Eigenvectors (row vectors)**

**Eigenvector normalization**



