SAND2012-0283C

Tutorial on Experimental Dynamic Substructuring Using the Transmission Simulator Method

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Experimental Structural Dynamics

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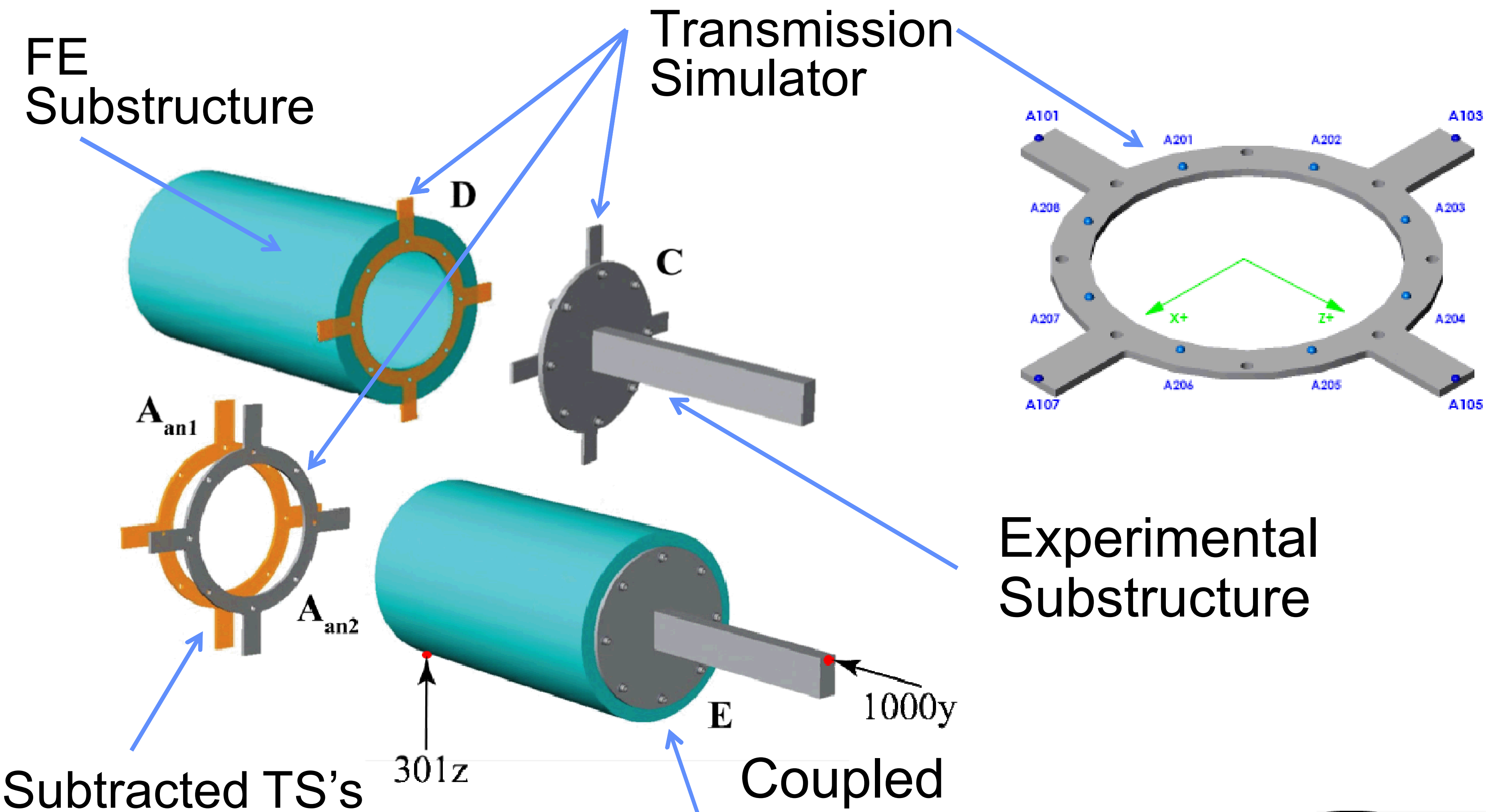
Introduction to the Transmission Simulator Method – Also called Modal Constraint for Fixture and Subsystem (MCFS)

- **Method utilizes an instrumented fixture called the transmission simulator (TS)**
- **TS is connected to the experimental substructure with exactly the same connection geometry in the joint interface, therefore it includes the stiffness and damping of the joint**
- **Analytical model of the TS exists as well**
- **Translational measurement dof at convenient locations on the TS may be chosen – Only requirement is that the kept mode shapes of the measurement dof must be independent**

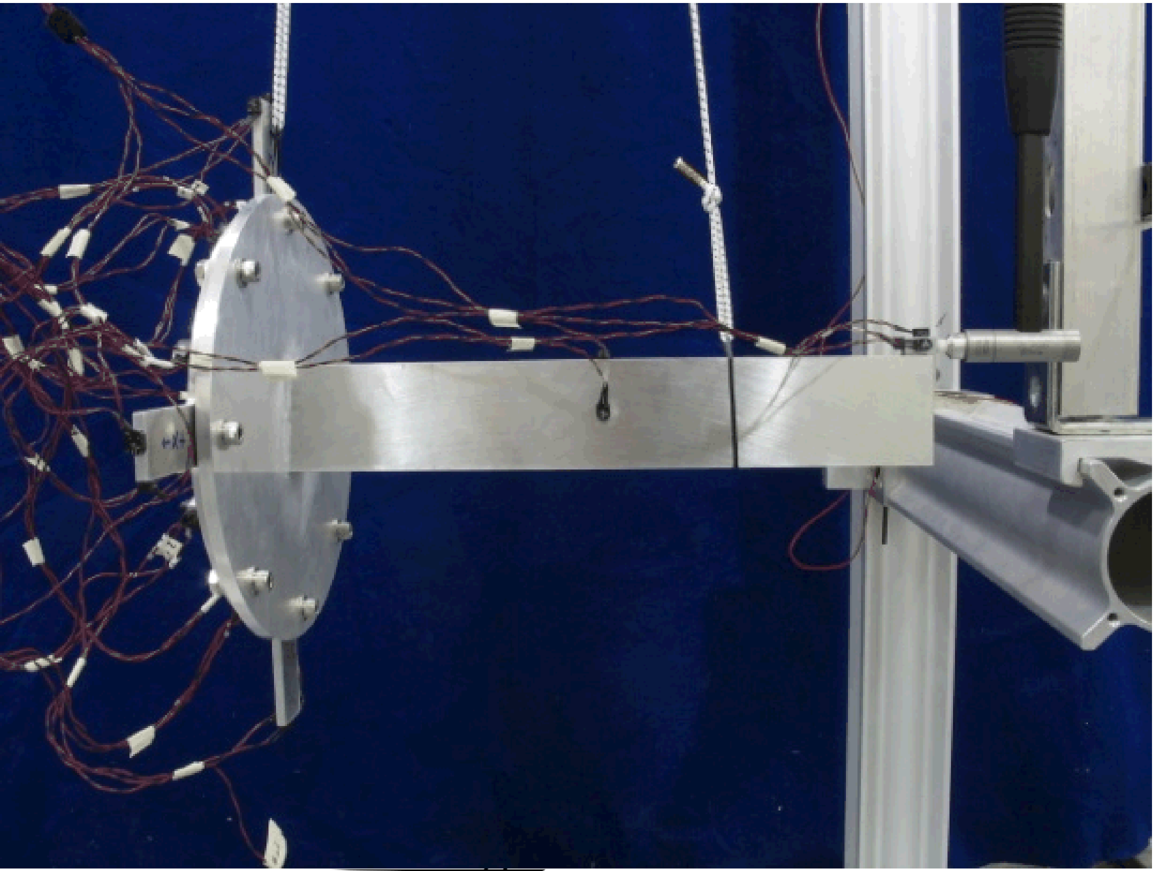
Introduction to the TS Method

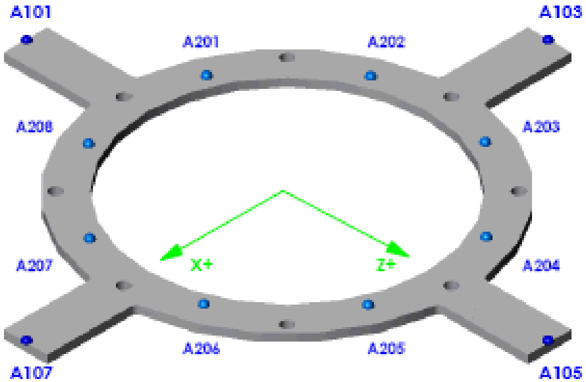
- Connections are made through the modal dof of the free modes of the TS – these inherently contain rotations as well as translations – so rotations are not being neglected, only approximated
- The mass of the TS improves the shape basis vectors – i.e. allows the true connection motion to be approximated with fewer basis vectors than pure free modes with no mass loading

Picture of Concept



Experimental Substructure and Transmission Simulator





Overview CMS Equations

Substructures concatenated ⟶ $$\mathbf{M}\ddot{\bar{\mathbf{u}}} + \mathbf{K}\bar{\mathbf{u}} = \bar{\mathbf{f}} + \bar{\mathbf{g}}$$

Modal Truncation ⇨ $$\bar{\mathbf{u}} \cong \mathbf{R}\bar{\eta}$$

$$\mathbf{M}\mathbf{R}\ddot{\bar{\eta}} + \mathbf{K}\mathbf{R}\bar{\eta} = \bar{\mathbf{f}} + \bar{\mathbf{g}} + \bar{\mathbf{r}}$$

$$\mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\bar{\eta}} + \mathbf{R}^T\mathbf{K}\mathbf{R}\bar{\eta} = \mathbf{R}^T\bar{\mathbf{f}} + \mathbf{R}^T\bar{\mathbf{g}} + \bar{\mathbf{0}}$$

$$\mathbf{M}_m\ddot{\bar{\eta}} + \mathbf{K}_m\bar{\eta} = \bar{\mathbf{f}}_m + \bar{\mathbf{g}}_m$$

Couple mass/stiffness ⇨ $$\bar{\eta} \cong \mathbf{L}\bar{\varepsilon}$$

$$\mathbf{L}^T\mathbf{M}_m\mathbf{L}\ddot{\bar{\varepsilon}} + \mathbf{L}^T\mathbf{K}_m\mathbf{L}\bar{\varepsilon} = \mathbf{L}^T\bar{\mathbf{f}}_m + \bar{\mathbf{0}}$$

$$\widetilde{\mathbf{M}}\ddot{\bar{\varepsilon}} + \widetilde{\mathbf{K}}\bar{\varepsilon} = \tilde{\bar{\mathbf{f}}}$$

Component Mode Synthesis Theory

Generalized framework is from deKlerk, Rixen and Voormeeren for a truncated set of mode shapes – start with one substructure integer #*s*

$$\mathbf{M}^{(s)}\ddot{\bar{\mathbf{u}}}+\mathbf{C}^{(s)}\dot{\bar{\mathbf{u}}}+\mathbf{K}^{(s)}\bar{\mathbf{u}}=\bar{\mathbf{f}}^{(s)}+\bar{\mathbf{g}}^{(s)}$$

Modal Truncation ➡ $\bar{\mathbf{u}}\simeq\mathbf{R}\bar{\eta}$

$$\mathbf{M}^{(s)}\mathbf{R}\ddot{\bar{\eta}}^{(s)}+\mathbf{C}^{(s)}\mathbf{R}\dot{\bar{\eta}}^{(s)}+\mathbf{K}^{(s)}\mathbf{R}\bar{\eta}^{(s)}=\bar{\mathbf{f}}^{(s)}+\bar{\mathbf{g}}^{(s)}+\bar{\mathbf{r}}^{(s)}$$

$$\mathbf{R}^{T}\mathbf{M}^{(s)}\mathbf{R}\ddot{\bar{\eta}}^{(s)}+\mathbf{R}^{T}\mathbf{C}^{(s)}\mathbf{R}\dot{\bar{\eta}}^{(s)}+\mathbf{R}^{T}\mathbf{K}^{(s)}\mathbf{R}\bar{\eta}^{(s)}=\mathbf{R}^{T}\bar{\mathbf{f}}^{(s)}+\mathbf{R}^{T}\bar{\mathbf{g}}^{(s)}+\mathbf{R}^{T}\bar{\mathbf{r}}^{(s)}$$

$$\mathbf{R}^{T}\bar{\mathbf{r}}^{(s)}=\bar{\mathbf{0}}$$

$$\mathbf{R}^{T}\mathbf{M}^{(s)}\mathbf{R}\ddot{\bar{\eta}}^{(s)}+\mathbf{R}^{T}\mathbf{C}^{(s)}\mathbf{R}\dot{\bar{\eta}}^{(s)}+\mathbf{R}^{T}\mathbf{K}^{(s)}\mathbf{R}\bar{\eta}^{(s)}=\mathbf{R}^{T}\bar{\mathbf{f}}^{(s)}+\mathbf{R}^{T}\bar{\mathbf{g}}^{(s)}$$

$$\mathbf{M}_{m}^{(s)}\ddot{\bar{\eta}}^{(s)}+\mathbf{C}_{m}^{(s)}\dot{\bar{\eta}}^{(s)}+\mathbf{K}_{m}^{(s)}\bar{\eta}^{(s)}=\bar{\mathbf{f}}_{m}^{(s)}+\bar{\mathbf{g}}_{m}^{(s)}$$

Concatenate all substructures together

Reduced # rows ➡ $\mathbf{M}_{m}\ddot{\bar{\eta}}+\mathbf{C}_{m}\dot{\bar{\eta}}+\mathbf{K}_{m}\bar{\eta}=\bar{\mathbf{f}}_{m}+\bar{\mathbf{g}}_{m}$

Key to TS method is soft constraint in terms of modal coordinates of TS

$$\mathbf{B}\bar{\mathbf{u}}_c = \bar{\mathbf{0}}$$

$$\mathbf{B}\mathbf{R}_c\bar{\eta} \cong \bar{\mathbf{0}}$$

Fewer rows $\Rightarrow$ $$\Psi_c^{BD+}\mathbf{B}\mathbf{R}_c\bar{\eta} \cong \Psi_c^{BD+}\bar{\mathbf{0}}$$

$$\Psi_c^{BD+} = \begin{bmatrix} \Psi_c^{+} & 0 & 0 \\ 0 & \ldots & 0 \\ 0 & 0 & \Psi_c^{+} \end{bmatrix}$$

Constraint is now formulated in terms of TS modal coordinates

$$\Psi_c^{BD+}\mathbf{B}\mathbf{R}_c\overline{\eta} = \overline{\mathbf{0}}$$

⇕

$$\Psi_c\overline{\gamma} \cong \mathbf{R}_c^{(s)}\overline{\eta}^{(s)} \qquad \overline{\gamma} \cong \Psi_c^{+}\mathbf{R}_c^{(s)}\overline{\eta}^{(s)} \qquad \overline{\gamma} = \textit{Modal coordinates of TS}$$

$$\tilde{\mathbf{B}}\overline{\eta} \cong \overline{0}$$

where

$$\tilde{\mathbf{B}} = \Psi_c^{BD+}\mathbf{B}\mathbf{R}_c$$

Primal method to enforce constraint

$$\tilde{\mathbf{B}}\bar{\eta} \cong \bar{0}$$

$$\bar{\eta} \cong \mathbf{L}\bar{\varepsilon}$$

This connects substructures through stiffness/mass/damping

$$\tilde{\mathbf{B}}\mathbf{L}\bar{\varepsilon} \cong \bar{0}$$

Choose $\mathbf{L}$ in the null space of $\tilde{\mathbf{B}}$

$$\tilde{\mathbf{B}}\tilde{\mathbf{L}} = [zeros]$$

Connected by forces

$$\mathbf{M}_m\ddot{\bar{\eta}} + \mathbf{C}_m\dot{\bar{\eta}} + \mathbf{K}_m\bar{\eta} = \bar{\mathbf{f}}_m + \bar{\mathbf{g}}_m$$

Connected through K,M,C

$$\mathbf{L}^T\mathbf{M}_m\mathbf{L}\ddot{\bar{\varepsilon}} + \mathbf{L}^T\mathbf{C}_m\mathbf{L}\dot{\bar{\varepsilon}} + \mathbf{L}^T\mathbf{K}_m\mathbf{L}\bar{\varepsilon} = \mathbf{L}^T\bar{\mathbf{f}}_m + \mathbf{L}^T\bar{\mathbf{g}}_m$$

$$\tilde{\mathbf{L}}^T\bar{\mathbf{g}}_m = \tilde{\mathbf{L}}^T\mathbf{R}^T\bar{\mathbf{g}} = \bar{0}$$

Final connected eqns of motion

$$\tilde{\mathbf{M}}\ddot{\bar{\varepsilon}} + \tilde{\mathbf{C}}\dot{\bar{\varepsilon}} + \tilde{\mathbf{K}}\bar{\varepsilon} = \tilde{\bar{\mathbf{f}}}$$

where

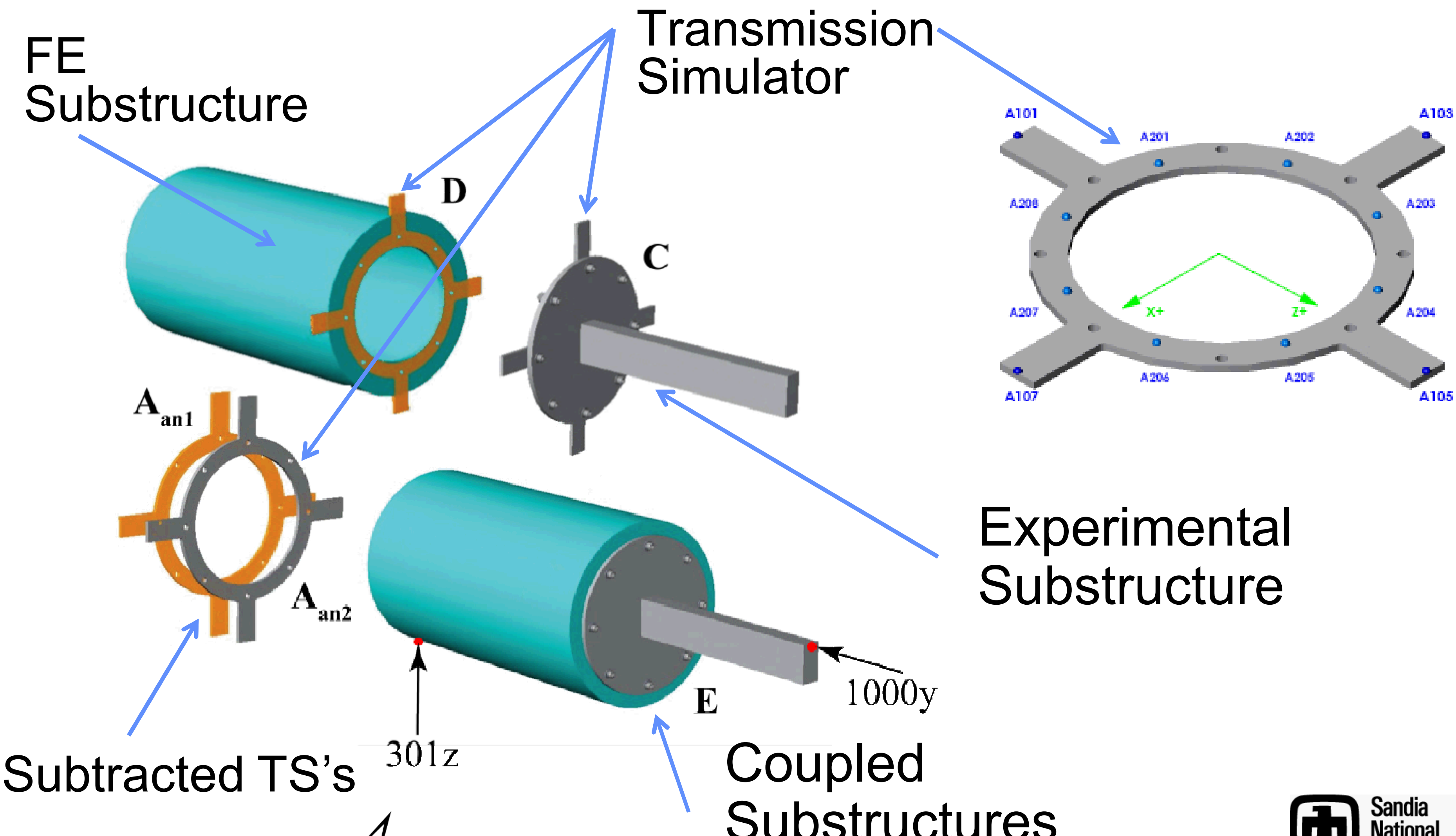
$$\tilde{\mathbf{M}} = \tilde{\mathbf{L}}^T \mathbf{M}_m \tilde{\mathbf{L}}$$

$$\tilde{\mathbf{C}} = \tilde{\mathbf{L}}^T \mathbf{C}_m \tilde{\mathbf{L}}$$

$$\tilde{\mathbf{K}} = \tilde{\mathbf{L}}^T \mathbf{K}_m \tilde{\mathbf{L}}$$

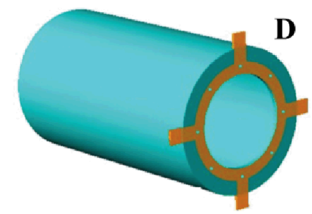
$$\tilde{\bar{\mathbf{f}}} = \tilde{\mathbf{L}}^T \bar{\mathbf{f}}_m$$

Picture of Concept

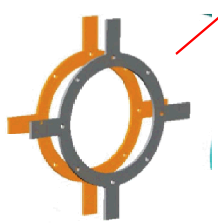


Eigenvalue Example with matrices

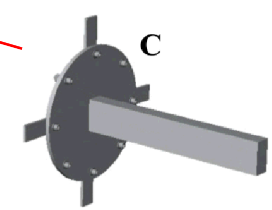
FE model has substructure + TS
Exp model has substructure with TS
Subtract 2 TS substructures
Neglect damping matrices initially



$$\begin{bmatrix}\omega_{FE}^2 & 0 & 0\\ 0 & \omega_{EXP}^2 & 0 \\ 0 & 0 & -2\omega_{TS}^2\end{bmatrix}\begin{Bmatrix}\eta_{FE}\\ \eta_{EXP}\\ \eta_{TS}\end{Bmatrix} - \omega^2 \begin{bmatrix} I_{FE} & 0 & 0\\ 0 & I_{EXP} & 0\\ 0 & 0 & -2I_{TS}\end{bmatrix}\begin{Bmatrix}\ddot{\eta}_{FE}\\ \ddot{\eta}_{EXP}\\ \ddot{\eta}_{TS}\end{Bmatrix} = \begin{Bmatrix}0\\ 0\\ 0\end{Bmatrix}$$



$$\begin{Bmatrix}y_{FE}\\ y_{EXP}\\ y_{TS}\end{Bmatrix} = \begin{bmatrix} R_{FE} & 0 & 0\\ 0 & R_{EXP} & 0\\ 0 & 0 & R_{TS}\end{bmatrix}\begin{Bmatrix}\eta_{FE}\\ \eta_{EXP}\\ \eta_{TS}\end{Bmatrix}$$



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Constraints

$\mathbf{y}_{FEmeas} = \mathbf{y}_{EXPmeas}$ and $\mathbf{y}_{EXPmeas} = \mathbf{y}_{TSmeas}$

Consider only first set of constraint equations

$$\mathbf{R}_{TS}^{+}\mathbf{R}_{FEmeas}\overline{\boldsymbol{\eta}}_{FE} = \mathbf{R}_{TS}^{+}\mathbf{R}_{EXPmeas}\overline{\boldsymbol{\eta}}_{EXP}$$

$$\mathbf{R}_{TS}^{+}\mathbf{R}_{FEmeas}\overline{\boldsymbol{\eta}}_{FE} - \mathbf{R}_{TS}^{+}\mathbf{R}_{EXPmeas}\overline{\boldsymbol{\eta}}_{EXP} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{R}_{TS}^{+} & 0 \\ 0 & \mathbf{R}_{TS}^{+} \end{bmatrix} \begin{bmatrix} \mathbf{I} & -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{FEmeas} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{EXPmeas} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}_{TS} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\eta}_{FE} \\ \boldsymbol{\eta}_{EXP} \\ \boldsymbol{\eta}_{TS} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\widetilde{\mathbf{B}} = \begin{bmatrix} \mathbf{R}_{TS}^{+} & 0 \\ 0 & \mathbf{R}_{TS}^{+} \end{bmatrix} \begin{bmatrix} \mathbf{I} & -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{FEmeas} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{EXPmeas} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}_{TS} \end{bmatrix}$$

Connecting the substructures through M K

$$\overline{\boldsymbol{\eta}} = \widetilde{\mathbf{L}}\overline{\boldsymbol{\varepsilon}}$$

$$\widetilde{\mathbf{B}}\widetilde{\mathbf{L}}\overline{\boldsymbol{\varepsilon}} = \overline{\mathbf{0}}$$

$$\mathbf{L}^T \begin{bmatrix} \boldsymbol{\omega}_{\mathbf{FE}}^2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\omega}_{\mathbf{EXP}}^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -2\boldsymbol{\omega}_{\mathbf{TS}}^2 \end{bmatrix} \mathbf{L}\overline{\boldsymbol{\varepsilon}} - \omega^2 \mathbf{L}^T \begin{bmatrix} \mathbf{I}_{\mathbf{FE}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{\mathbf{EXP}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -2\mathbf{I}_{\mathbf{TS}} \end{bmatrix} \mathbf{L}\ddot{\overline{\boldsymbol{\varepsilon}}} = \overline{0}$$

Solving above eigenvalue problem gives new frequencies and another set of eigenvectors (name them $\boldsymbol{\Phi}$) and new generalized coordinates $\overline{\boldsymbol{\beta}}$

$$\overline{\mathbf{y}} = \mathbf{R}\mathbf{L}\boldsymbol{\Phi}\overline{\boldsymbol{\beta}}$$

Damping Approximation

The damping matrix is

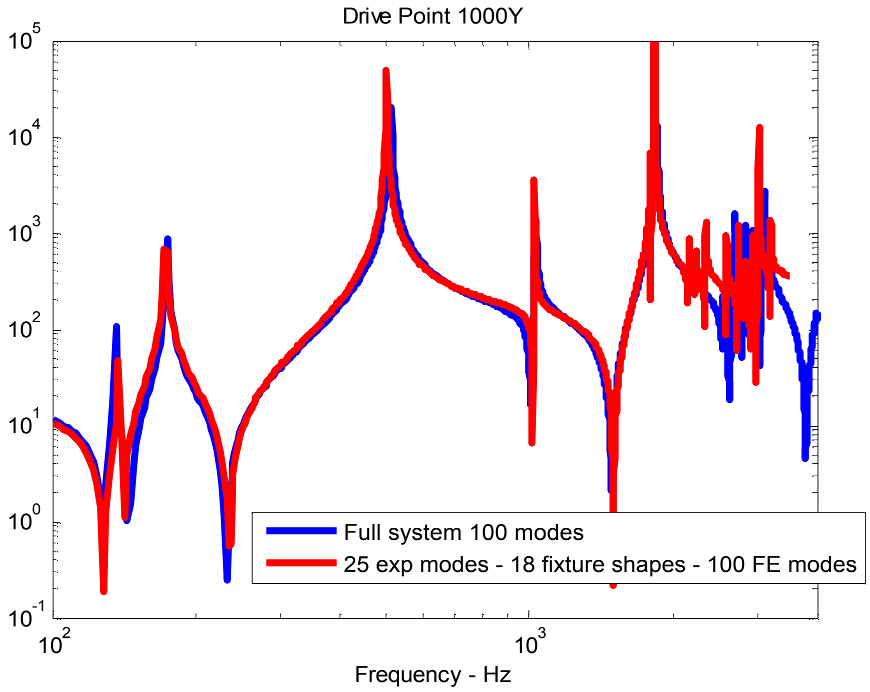
$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}\begin{bmatrix} 2\varsigma\omega_{FE} & 0 & 0 \\ 0 & 2\varsigma\omega_{EXP} & 0 \\ 0 & 0 & -4\varsigma\omega_{TS} \end{bmatrix}\mathbf{L}\mathbf{\Phi}$$

From which we usually just take diagonal values and set them equal to

$$2\varsigma_{new}\omega_{new}$$

Example from cylinder/plate/beam

A driving point axial response on the cylinder plate and beam yields



Frequency Based Substructuring

Beginning with all the substructures concatenated together utilizing the physical degrees of freedom with connection forces **g**

$$\mathbf{M}\ddot{\overline{\mathbf{u}}} + \mathbf{C}\dot{\overline{\mathbf{u}}} + \mathbf{K}\overline{\mathbf{u}} = \overline{\mathbf{f}} + \overline{\mathbf{g}}$$

$$\mathbf{B}\overline{\mathbf{u}} = \overline{\mathbf{0}}$$

$$\mathbf{L}^T \mathbf{g} = \overline{\mathbf{0}}$$

$$\mathbf{g} = -\mathbf{B}^T \overline{\boldsymbol{\lambda}}$$

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \boldsymbol{\lambda} \end{bmatrix} + \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \boldsymbol{\lambda} \end{bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{Z} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix}$$

Frequency Based Substructuring with Dual Formulation

$$\begin{bmatrix} \mathbf{Z} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix}$$

Replace Z^{-1} with frequency response function H, after several steps one can show

$$\overline{\mathbf{u}} = \mathbf{H}\overline{\mathbf{f}} - \mathbf{H}\mathbf{B}^T(\mathbf{B}\mathbf{H}\mathbf{B}^T)^{-1}\mathbf{B}\mathbf{H}\overline{\mathbf{f}}$$

The transmission simulator method modifies the constraint as

$$\hat{\mathbf{B}}\overline{\mathbf{u}} = \overline{\mathbf{0}} \quad \text{with} \quad \hat{\mathbf{B}} = \boldsymbol{\Psi}^{BD+}\mathbf{B}$$

$$\overline{\mathbf{u}} = \mathbf{H}\overline{\mathbf{f}} - \mathbf{H}\hat{\mathbf{B}}^T(\hat{\mathbf{B}}\mathbf{H}\hat{\mathbf{B}}^T)^{-1}\hat{\mathbf{B}}\mathbf{H}\overline{\mathbf{f}}$$

Frequency Based Substructuring Practical Implementation

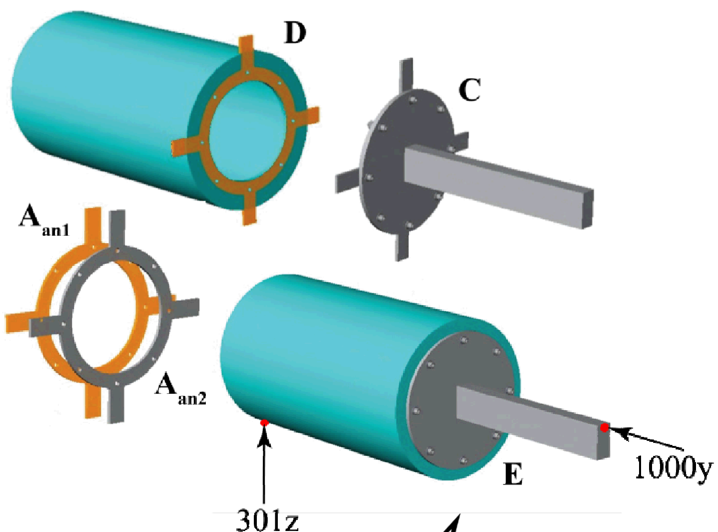
Typically including the entire matrix on a PC is too slow so we reduce the size of the problem down to the rows of interest. Two of the most useful sets of rows are

Input on substructure C Output on D

$$\mathbf{HT}_{ri} = \mathbf{HD}_{rc}\left(\mathbf{HD}_{cc} + \mathbf{HC}_{cc}\right)^{-1}\mathbf{HC}_{ci}$$

Input on substructure C Output on C

$$\mathbf{HT}_{ri} = \mathbf{HC}_{ri} - \mathbf{HC}_{rc}\left(\mathbf{HD}_{cc} + \mathbf{HC}_{cc}\right)^{-1}\mathbf{HC}_{ci}$$



$$\mathbf{HD}_{rc} = \mathbf{HD}_{rp}\mathbf{R}_{TS}^{T+}$$

$$\mathbf{HD}_{cc} = \mathbf{R}_{TS}^{+}\mathbf{HD}_{pp}\mathbf{R}_{TS}^{T+}$$

$$\mathbf{HC}_{cc} = \mathbf{R}_{TS}^{+}\mathbf{HC}_{pp}\mathbf{R}_{TS}^{T+}$$

$$\mathbf{HC}_{ci} = \mathbf{R}_{TS}^{+}\mathbf{HC}_{pi}$$

$$\mathbf{HC}_{rc} = \mathbf{HC}_{rp}\mathbf{R}_{TS}^{T+}$$