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A Bayes Network Approach to Uncertainty Quantification in Hierarchically-Developed Computational Models

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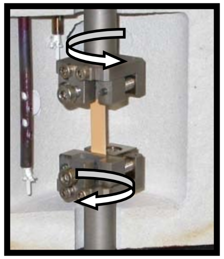
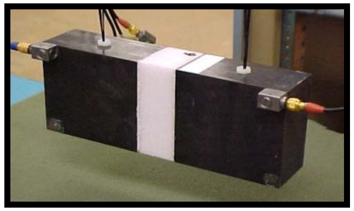
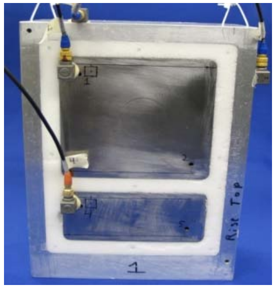
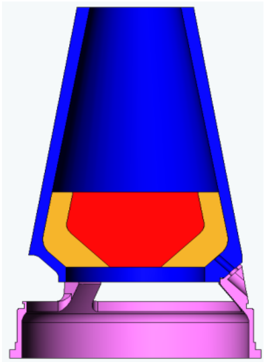
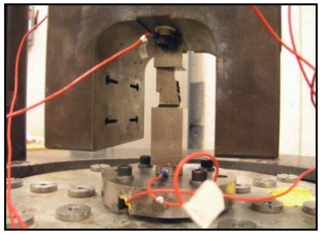
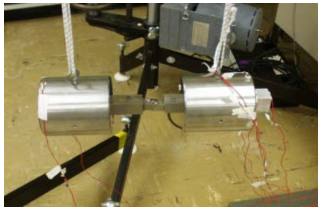
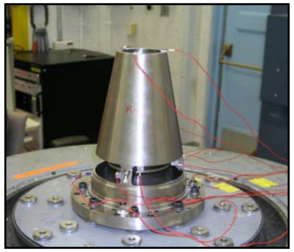
Outline of Presentation

- Motivation
- Problem description
- Hierarchical system model
- Bayes network approach
- Numerical example and results
- Summary

Motivation

- High consequence systems need to be qualified despite lack of system level data
- Component and sub-system data can usually be obtained
- Modeling and simulation used in lieu of system level data
- System level models are often built in a hierarchical manner (i.e. building block approach)
- There is a need to incorporate all available data into a system level analysis
- Sources of uncertainty need to be identified and propagated to the system level response of interest

Structural Dynamics Example

	Level 0	Level 1	Level 2	System level
Foam				
	Material characterization	Component level	Sub-system level	
Joints				

Hardware data and photos courtesy of Sandia National Laboratories

System complexity → Increases

Sources of uncertainty → Increase

Amount of real data → Decreases

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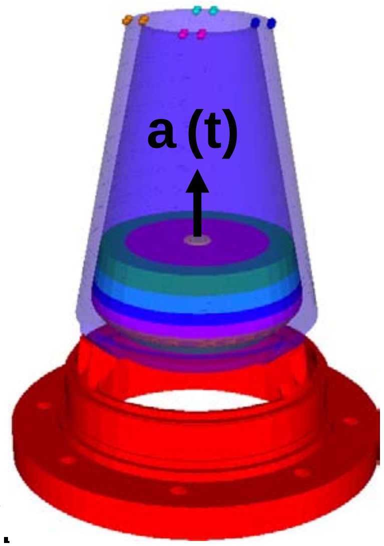
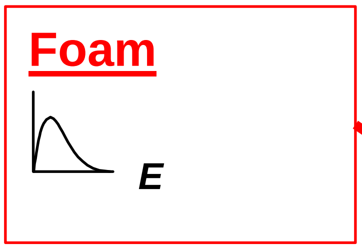
Structural Dynamics Example



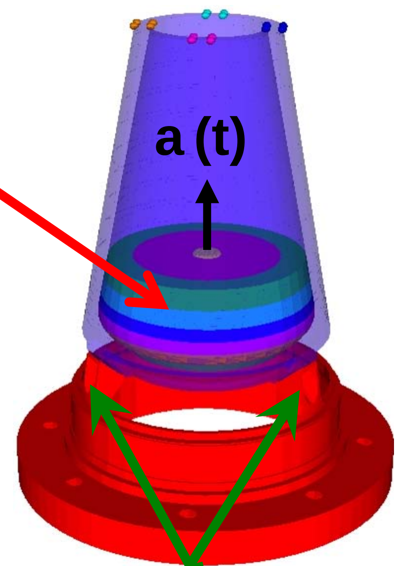
- Multi-part aerospace component.
  - Conical part connected to lower base via three bolted connections
  - Inner mass is encapsulated in foam.
- We are interested in *the peak acceleration of the inner mass*
- There is uncertainty associated with:
  - Material property of the foam: modulus of elasticity, E.
  - The behavior of the bolted connection described by a 3 parameter model (Smallwood model)
    - Linear stiffness, K_{lin}
    - Non-linear stiffness, K_{non}
    - Degree of nonlinearity, n_{pow}
- The simulation code used is Salinas.
- The UQ codes used are Matlab and WinBUGS.

Uncertainty Quantification

Aleatoric Uncertainty



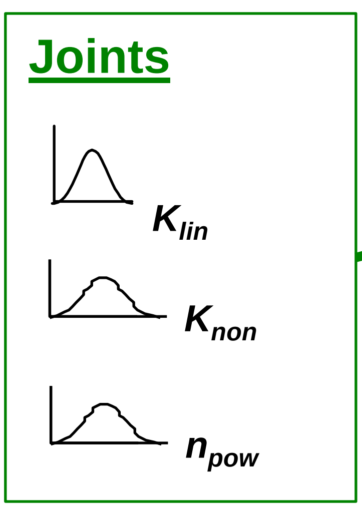
System level model

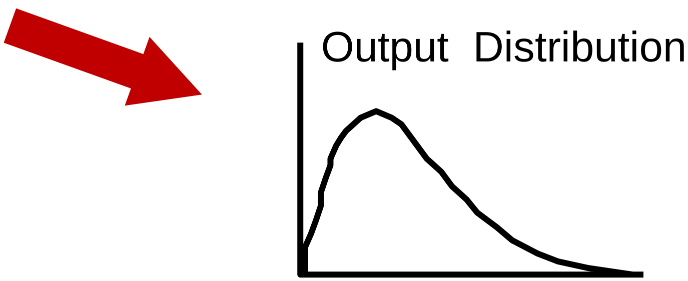


Aleatoric Uncertainty

- It arises from:
  - Unit/unit variability of nominally identical systems
  - Experimental setup/setup
- Treated with probability theory

Aleatoric Uncertainty





Motivation for Bayesian Analysis

"The main purpose of statistical theory is to derive from observations of a random phenomenon an inference about the probability distribution underlying this phenomenon." –Christian Robert

Bayesian analysis allows us to formally combine

Earlier understanding of a phenomenon

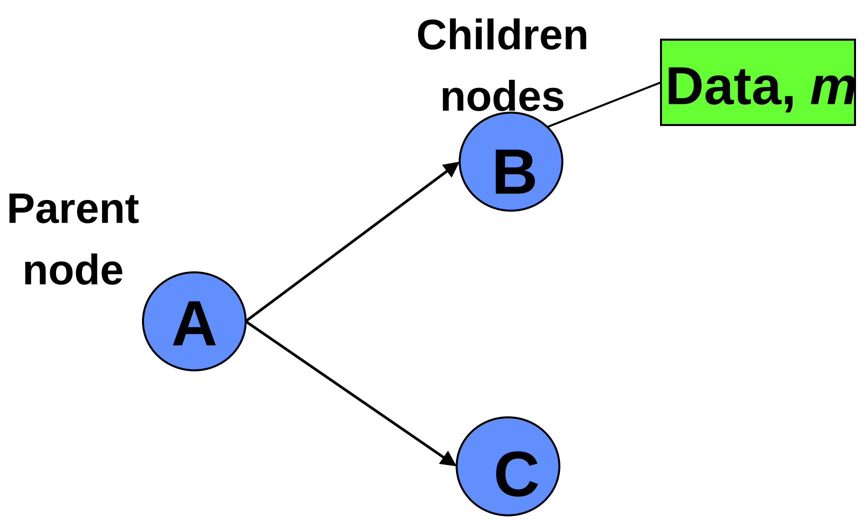
Currently measured data

Updated degree of belief

Marginal and joint "belief" information about estimated parameters can be obtained.

Bayes Networks

- Directed acyclic graphical representations of random variables (nodes) and conditional probabilities (arcs) between nodes
- Updates the probability distribution for all the nodes, given some observations
- Hierarchical modeling can be accommodated



Why Bayes networks?:

- Data at different levels can be incorporated
- Response data can be of different origin (acceleration, density measures, etc)
- Uncertainty can be quantified and propagated

Bayes Network Implementation

- Need to specify prior distributions for parent nodes
  - Usually non-informative prior (Normal w/zero mean, large variance)
  - If some data is available, use it to define prior (either full distribution or statistics)
- When data is available, joint PDF of Bayes network can be updated using Bayes theorem

Prior PDF

Posterior PDF

Likelihood function

$$f(\theta \mid D) = \frac{\pi(\theta) f(D \mid \theta)}{\int_{-\infty}^{\infty} \pi(\theta) f(D \mid \theta) d\theta}$$

➢ Integration in the denominator is difficult; use Markov chain Monte Carlo (MCMC) with Gibbs sampling instead

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Bayes Network Implementation

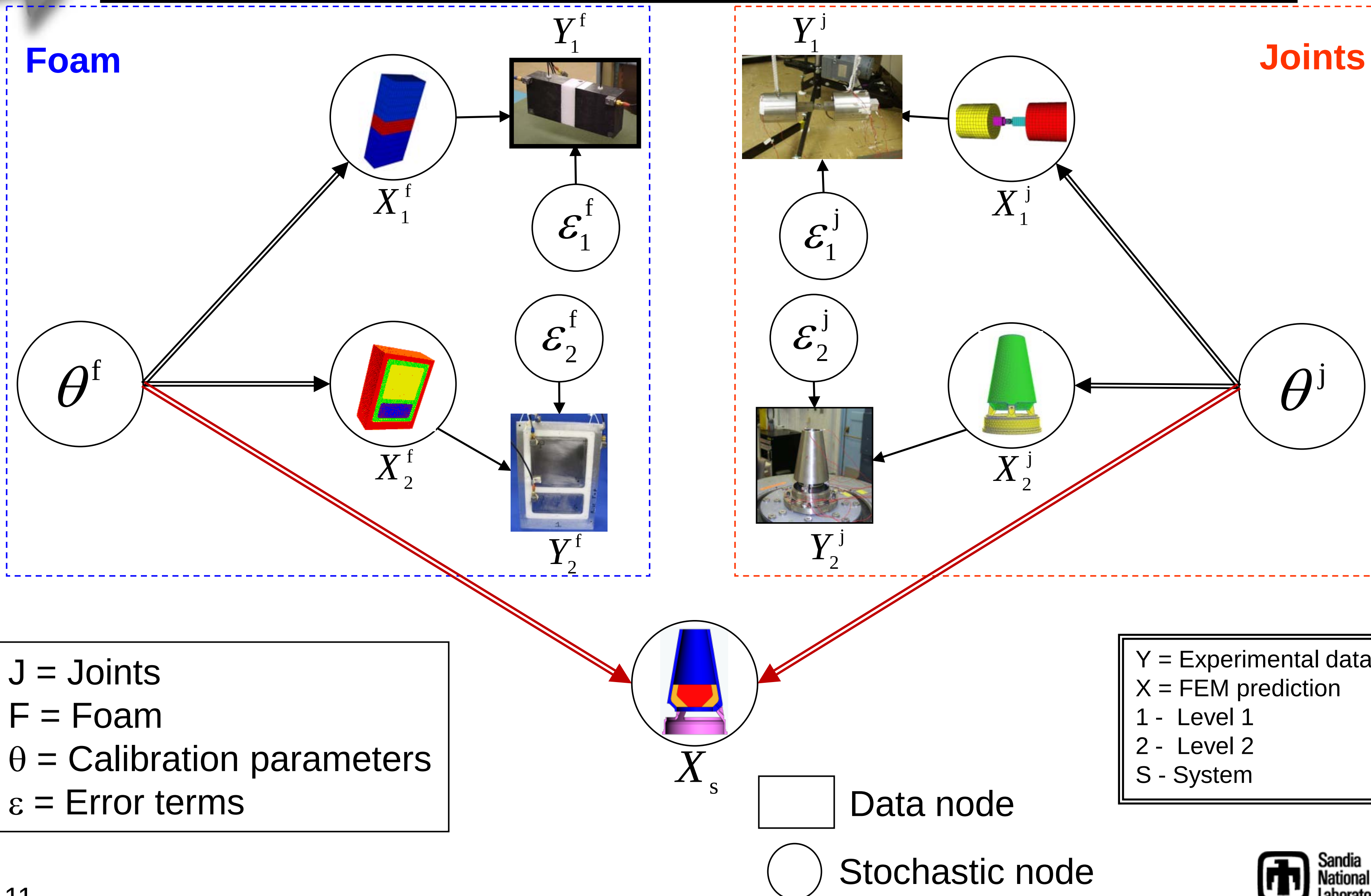
- Markov Chain Monte Carlo
  - Establishes transition probabilities such that the stationary distribution is the target distribution

- Gibbs Sampling
  - Sampling scheme to carry out MCMC
  - Conceptually for a Bayes net (BN), a Gibbs sample is proportional to all the terms containing a particular node in the joint PDF
  - From the simple BN shown previously, joint PDF is:

$$f(U,m) = f(A)\,f(B \mid A)\,f(C \mid A)\,f(m \mid B)$$

then Gibbs sample of node B will be given by: $B \sim f(B \mid A)\,f(m \mid B)$

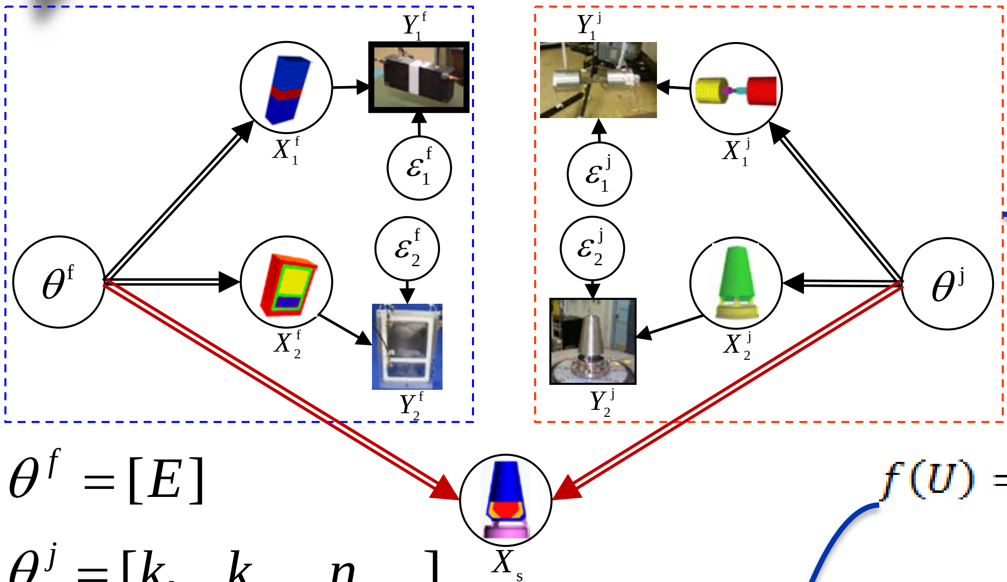
Bayes Network Implementation



J = Joints
F = Foam
θ = Calibration parameters
ε = Error terms

Y = Experimental data
X = FEM prediction
1 - Level 1
2 - Level 2
S - System

The Math Behind the Net



$$\theta^f = [E]$$

$$\theta^j = [k_{lin}, k_{non}, n_{pow}]$$

Construct joint PDF of the BN – note links between nodes represent conditional probabilities

$$f(U) = f(k_{lin}) * f(k_{non}) * f(npow) * f(E) * * f\left(Y_1^f \middle| g_1(E), \epsilon_1^f\right) * f\left(Y_2^f \middle| g_2(E), \epsilon_2^f\right) * f\left(Y_1^j \middle| h_1(k_{lin}, k_{non}, npow), \epsilon_1^j\right) * f\left(Y_2^j \middle| h_2(k_{lin}, k_{non}, npow), \epsilon_2^j\right)$$

If data is available, then PDF of each parent node can be updated via Bayes Theorem

$$f\left(k_{lin,} k_{non,} npow, E \middle| Y_{1\&2}^j, Y_{1\&2}^f\right) = \frac{f(U)}{\int f(U)\, d(k_{lin})\, d(k_{non})\, d(npow)\, d(E)}$$

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Bayesian Calibration Example
Foam

$$f(U) = f(k_{lin}) * f(k_{non}) * f(npow) * f(E) * f\left(Y_1^f \middle| g_1(E), \epsilon_1^f\right) * f\left(Y_2^f \middle| g_2(E), \epsilon_2^f\right) * f\left(Y_1^j \middle| h_1(k_{lin}, k_{non}, npow), \epsilon_1^j\right) * f\left(Y_2^j \middle| h_2(k_{lin}, k_{non}, npow), \epsilon_2^j\right)$$

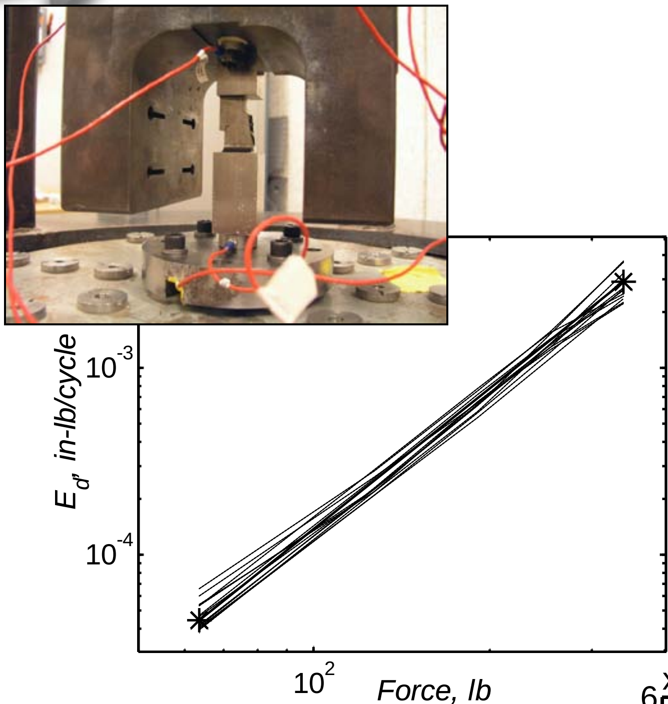
- Prior distribution on *E, f(E)*
  - Prior knowledge is included
  - Gaussian distribution assumed

- Likelihood function
  - Incorporates data, model prediction and error term
  - Normally distributed

- Error function, ε
  - Term includes experimental variability and model error
  - Normally distributed, zero mean and calibrated variance

$$Y_1^f = g_1(E) + \varepsilon_1^f \quad \left\{ \begin{array}{l} f(Y_1^f \mid g_1(E), \varepsilon_1^f) \sim N(\{Y_1^f - g_1(E)\}, \varepsilon_1^f) \\ \varepsilon_1^f \sim N(0, \sigma) \end{array} \right.$$

Prior Distribution
Joint Parameters

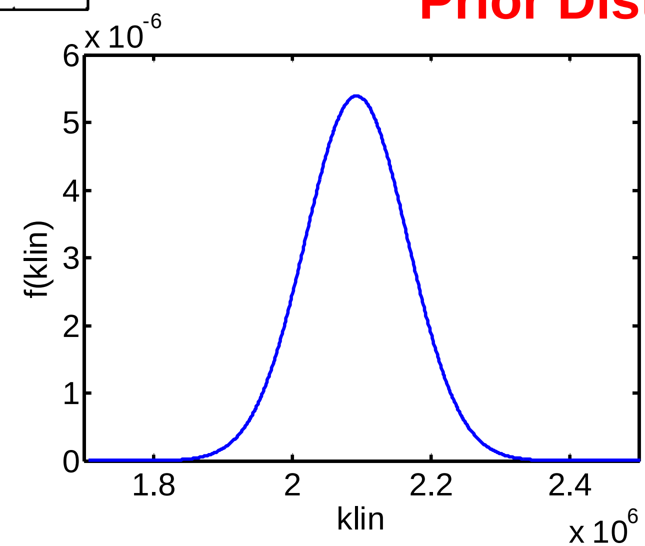


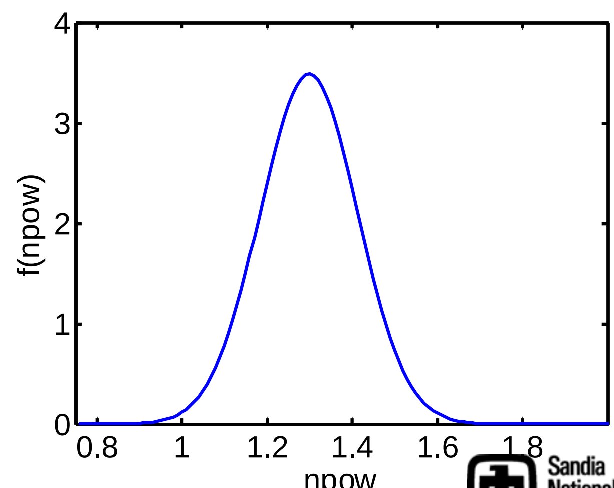
Smallwood model

$$F_j = klin\left(d_j - d_i\right) - knon\left(d_j - d_i\right)^{npow} + F_i$$

- Data from sinusoidal excitation used to calculate energy dissipated, E_d vs. force, F
- Slope of each curve of E_d vs. F, in log-log space, is parameter *npow*. *klin* and *knon* are obtained also found from curves

Prior Distributions





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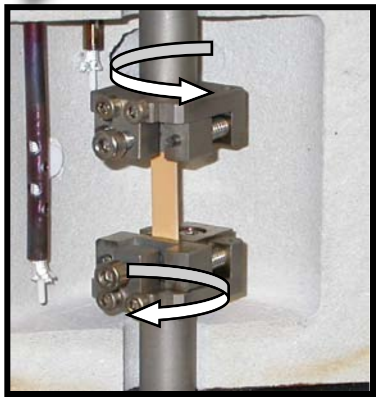
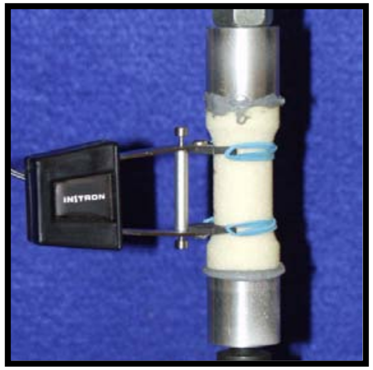
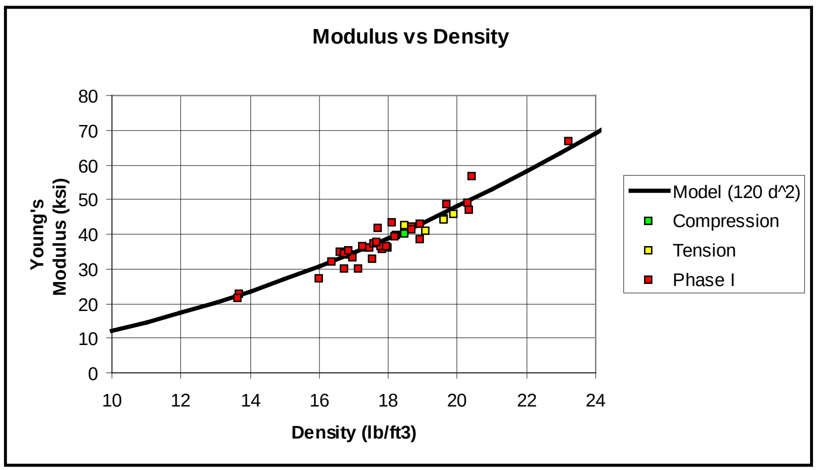
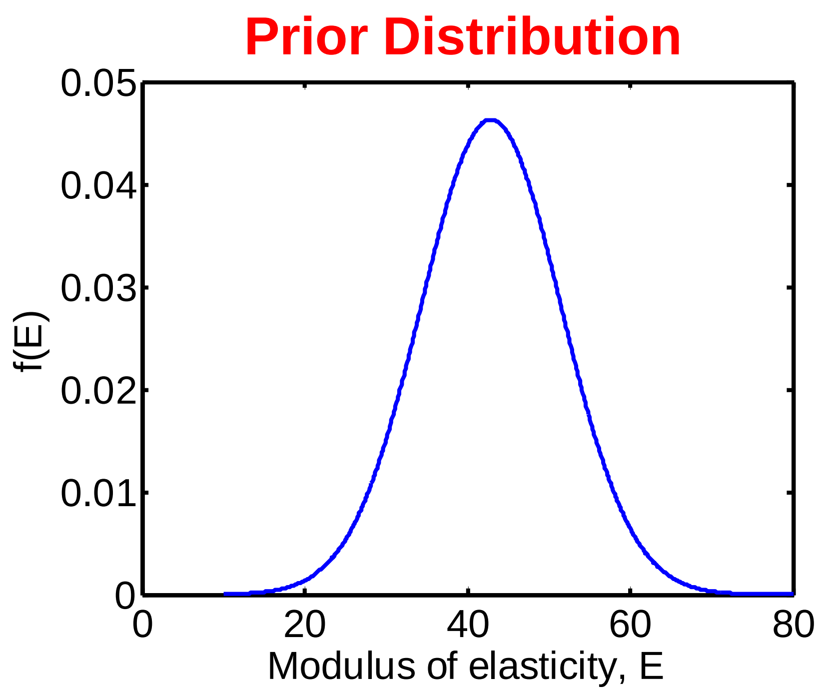
Prior Distribution
Foam Parameters





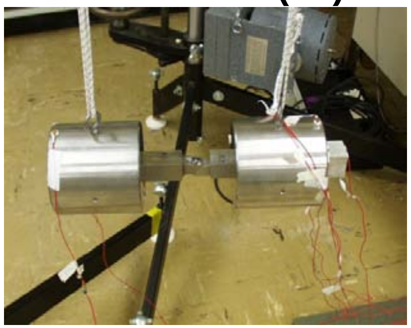
- Data from torsion, tension and compression experiments are used to obtained data of modulus of elasticity, E versus density, ρ

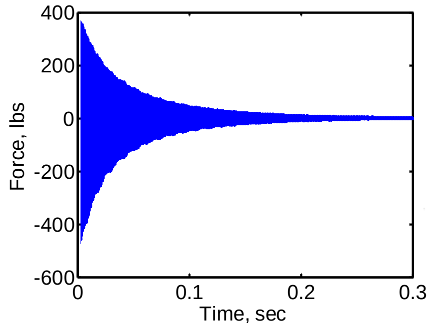


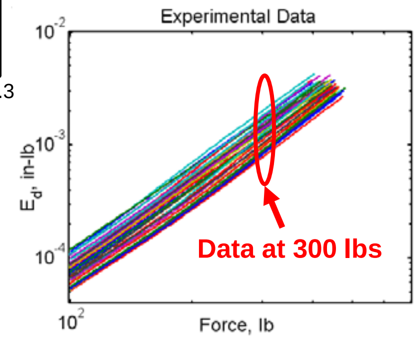


Available data for updating – Joints

Dumbbell (L1)

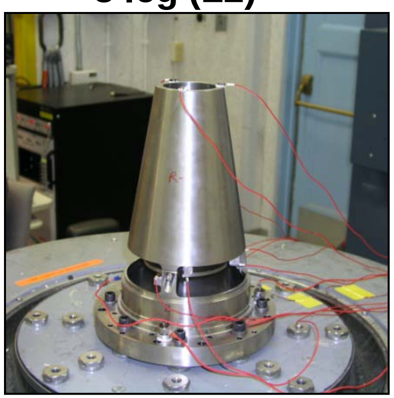


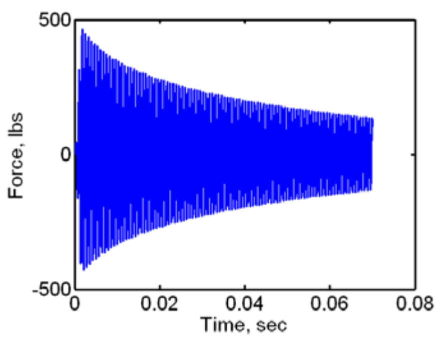


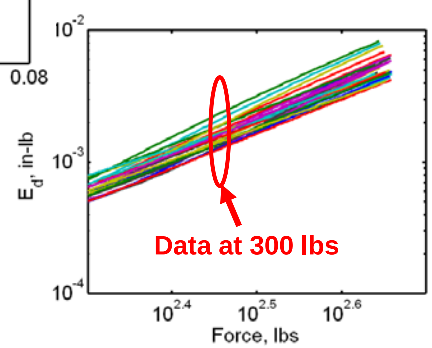


- Data from hammer impact tests
- 45 tests (acceleration time histories are recorded)
- Energy dissipated at 300 lbs force is calculated from accel. time histories

3 leg (L2)





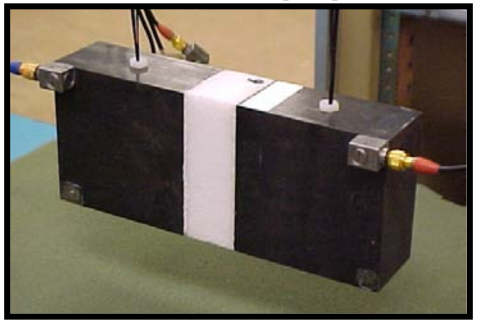


- Data from shaker tests using wavelet input
- 27 tests (acceleration time histories are recorded)
- Energy dissipated at 300 lbs force is calculated from accel. time histories

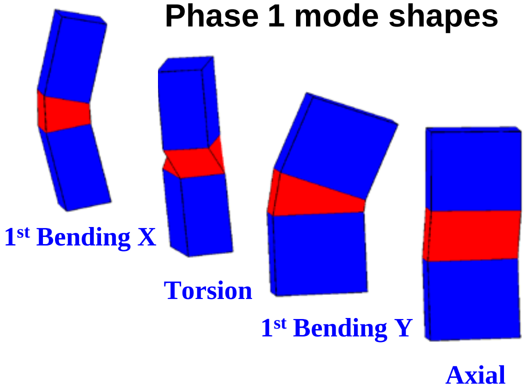
(these are the *Y*ʲ in the BN slide)

Available data for updating – Foam

Phase I (L1)



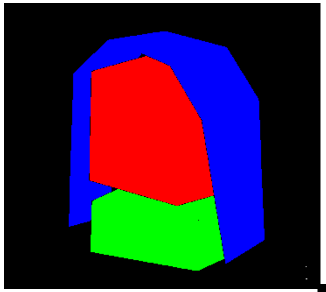
Phase 1 mode shapes



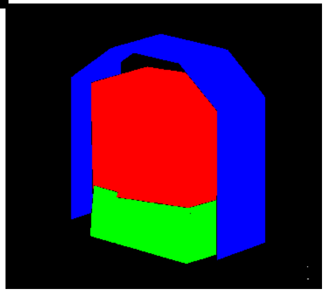
- Data from modal tests
- 6 tests (acceleration time histories are recorded)
- Modal frequencies of axial mode are used

Phase II (L2)

Torsion



Axial



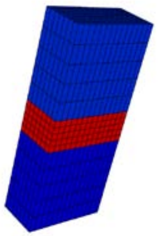
- Data from modal tests
- 3 tests (acceleration time histories are recorded)
- Modal frequencies of axial mode are used

(these are the Y^f in the BN slide)

Available Models
(these give the “X’s” in the BN slide)

Foam

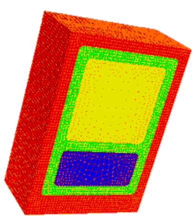
Phase I (L1)



$$X_1^f = g_1^f\left(\theta^f\right)$$

Calculated axial frequency

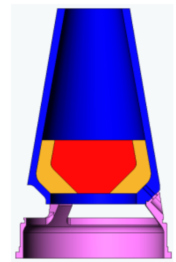
Phase II (L2)



$$X_2^f = g_2^f\left(\theta^f\right)$$

Calculated axial frequency

System



$$X_s = g_s\left(\theta^j, \theta^f\right)$$

Calc. peak acceleration

$$\theta^f = [E]$$

$$\text{E} = \text{Modulus of elasticity}$$

$$\theta^j = [k_{lin}, k_{non}, n_{pow}]$$

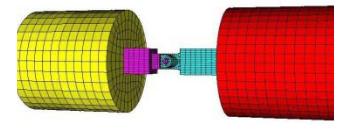
$$k_{lin} = \text{linear stiffness}$$

$$k_{non} = \text{nonlinear stiffness}$$

$$n_{pow} = \text{degree of nonlinearity}$$

Joints

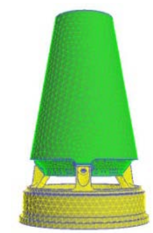
Dumbbell (L1)



$$X_1^j = g_1^j\left(\theta^j\right)$$

Calculated energy dissipated

3 leg (L2)



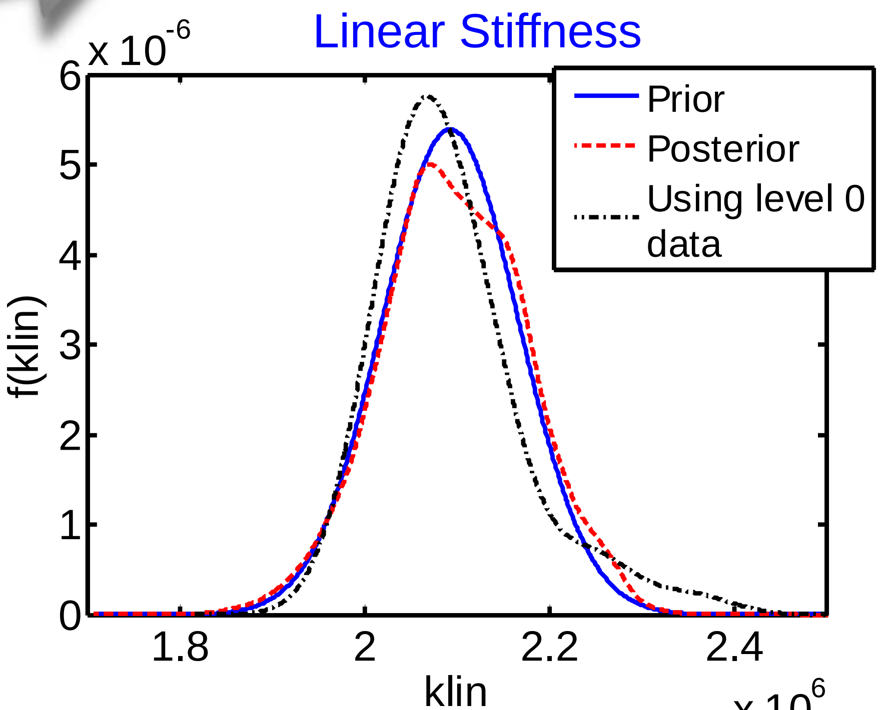
$$X_2^j = g_2^j\left(\theta^j\right)$$

Calculated energy dissipated

Note: Gaussian process models are used in lieu of finite element models for computational efficiency

Results

Joint Parameters

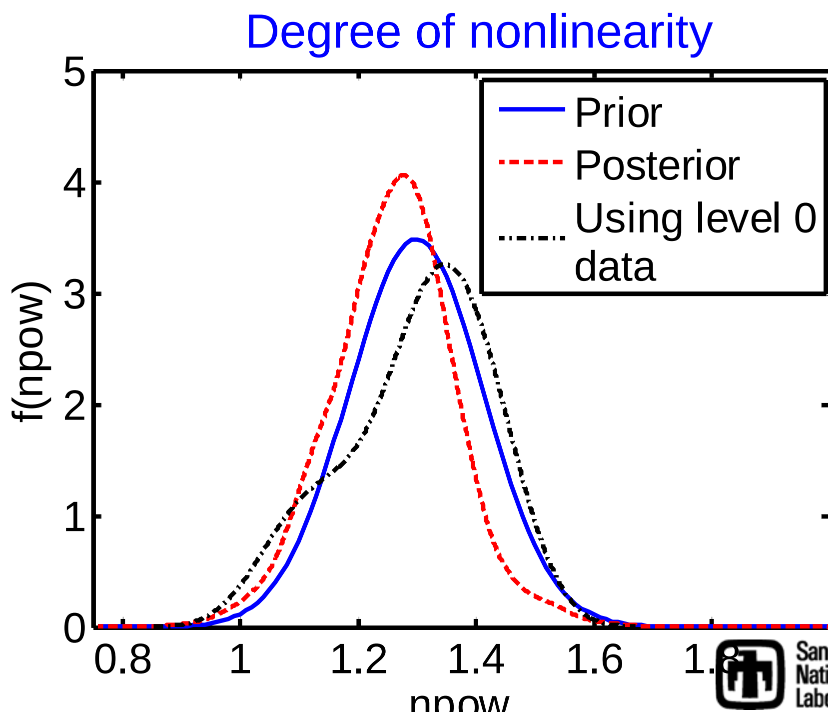


<u>Linear Stiffness</u>

- Comparison with parameters estimated from level 0 data
- Posterior reflects available data at levels 1 & 2
- When all data is included, posterior is similar to parameters estimated from level 0 only

<u>Degree of nonlinearity</u>

- Comparison with parameters estimated from level 0 data
- Posterior reflects available data at levels 1 & 2
- After updating, the variance is smaller relative to using level 0



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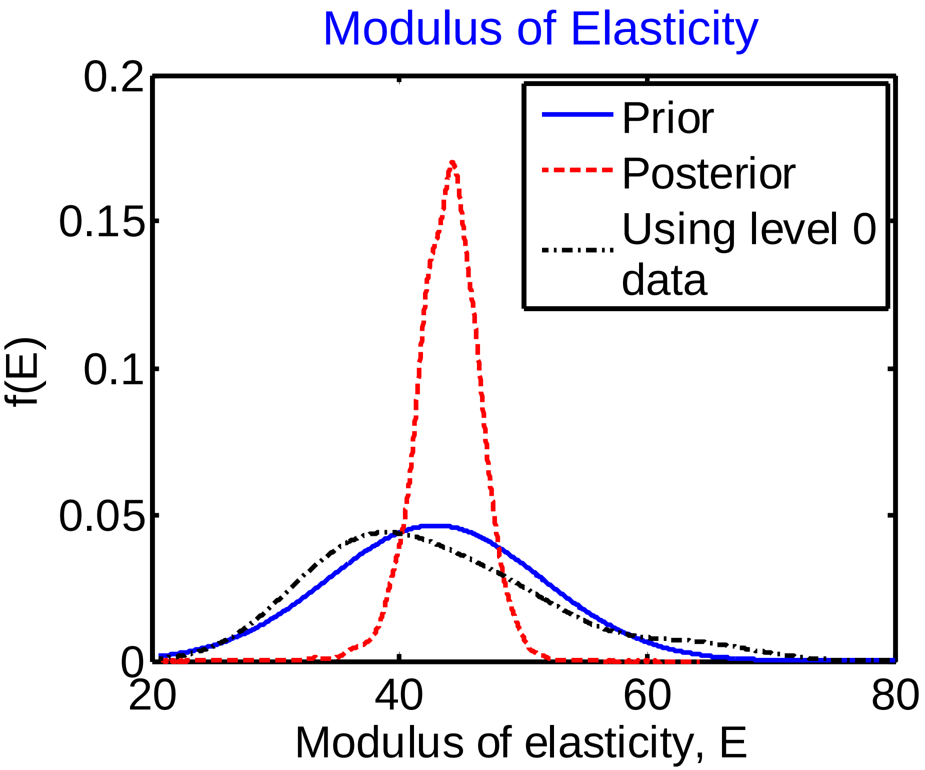
Results
Foam Parameter

Modulus of elasticity

- Comparison with parameter estimated from level 0 data
- Posterior reflects available data at levels 1 & 2
- When all data is included, the variance in the parameter is smaller than when using level 0 only
- This establishes a more “realistic” range for this parameter given the available data



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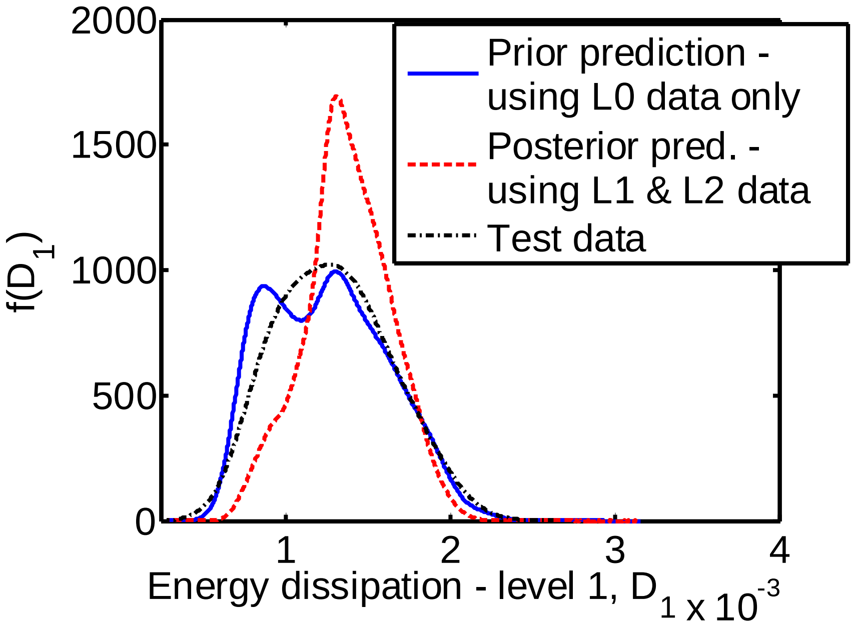
Results

Joint and Foam Predictions

Energy dissipated at Level 1



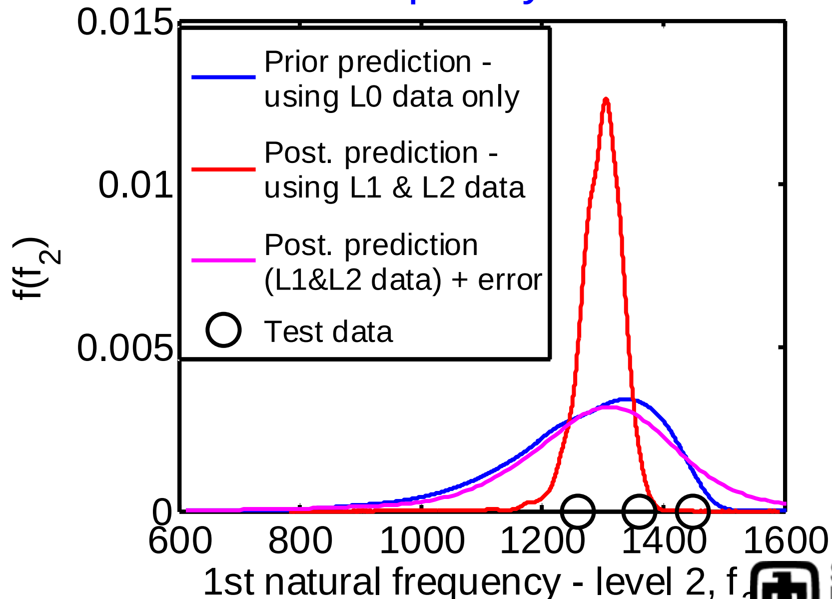
Energy dissipated (L1)

- This prediction is based on 5,000 samples
- Posterior distributions of parameters are used for evaluation
- Red dotted line is the prediction of the mean of this response
- Values are consistent with test data

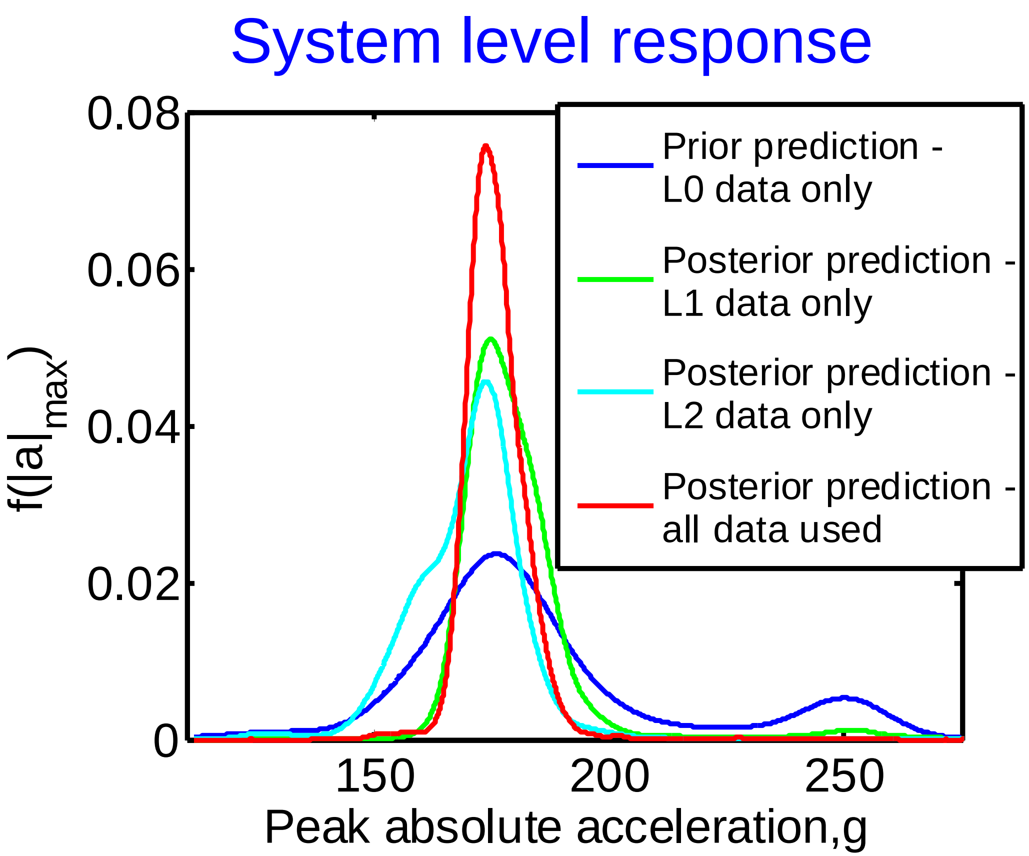
Axial Frequency (L2)

- Post prediction captures the mean behavior
- Post. prediction plus error agrees with the test data.
- Error term captures the discrepancy between the model mean behavior and the test data

Axial Frequency at Level 2



Results
System level response



- System level response is an extrapolated quantity
- Prediction uses posterior of all parameters updated with available data at levels 1 & 2
- Based on 5,000 MCMC samples of the parameters
- This gives an estimate of the uncertainty in the system level response

Summary

- A hierarchical system model was build to simulate a complex system.
- A Bayes network was used to incorporate all available data.
- Uncertainty was propagated through the hierarchical model up to the system level.
- The effect of knowledge (i.e. data) is reflected in a reduction in uncertainty.
- Errors were quantified.