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Inverse Source and Material Identification in Sierra-SD

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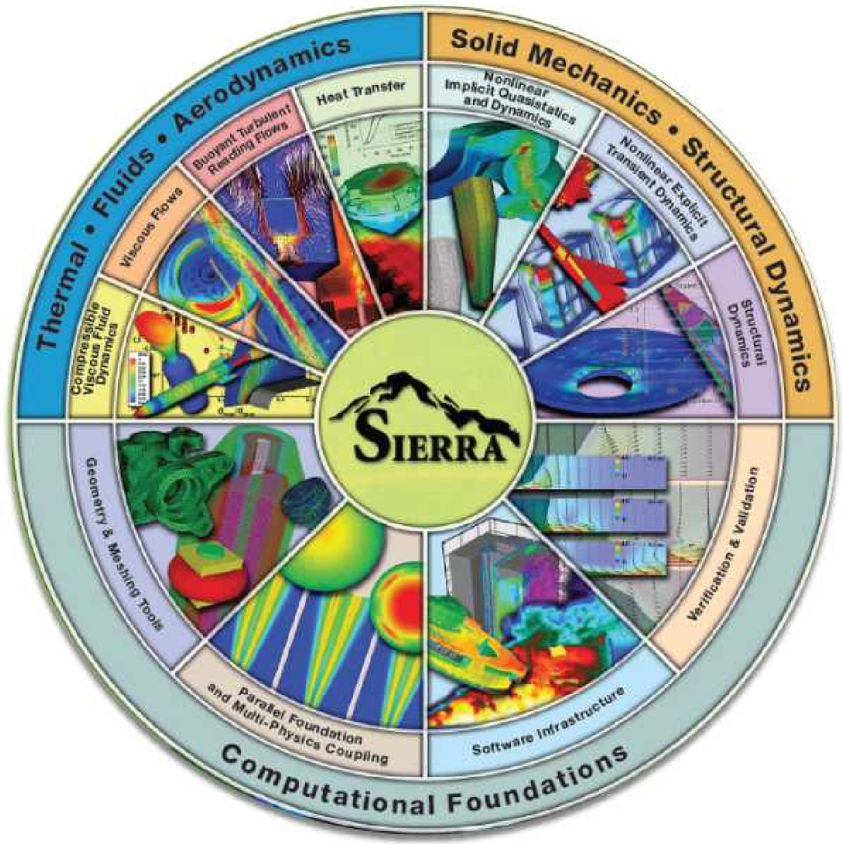
Outline

- Quick overview of Sierra Mechanics
- Quick overview of Sierra-SD(Salinas)
- Inverse problems in Sierra-SD
  - Source inversion
  - Material inversion
- Example applications of Sierra-SD

Overview of Sierra Mechanics

- **Goal: massively parallel coupled multiphysics calculations**
- **Modules for structural dynamics, solid mechanics, fluids, thermal, etc**



Overview of Sierra-SD (Salinas)

- Massively parallel implicit finite element analysis for structural dynamics and acoustics
- Scalable to thousands of processors, has been run on >10,000 processors
- Transient, direct frequency response (Helmholtz), modal analysis capabilities
- Embedded fully coupled structural acoustic capability

Inverse Problems- Motivation

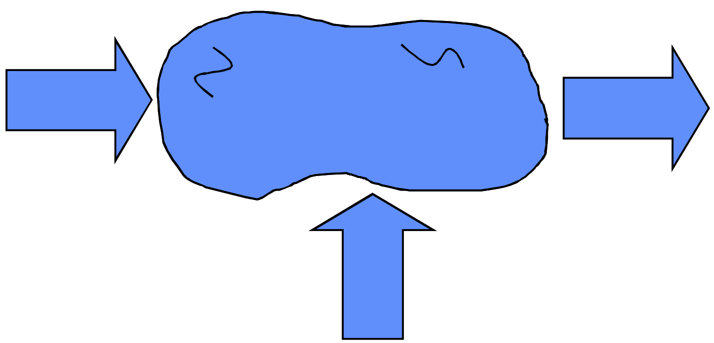
- Characterize energy sources from experimental measurements
  - acoustic testing of aerospace structures, damage or defect identification from acoustic emission, aeroacoustics
- Determine unknown material properties from experimental measurements
  - Model calibration, defect characterization
- For applications that involve complex geometries and/or sources, finite element modeling is needed for an accurate solution of the forward problem.
- Goal: leverage existing Sierra-SD massively parallel finite element technology developed for forward problems to solve the inverse problem.

Inverse Problems: The physical View

The direct or forward problem

External inputs (known)

e.g. forces, fluxes, etc.



System response (unknown)

e.g. displacements, temperature, concentrations, etc.

The System (known)

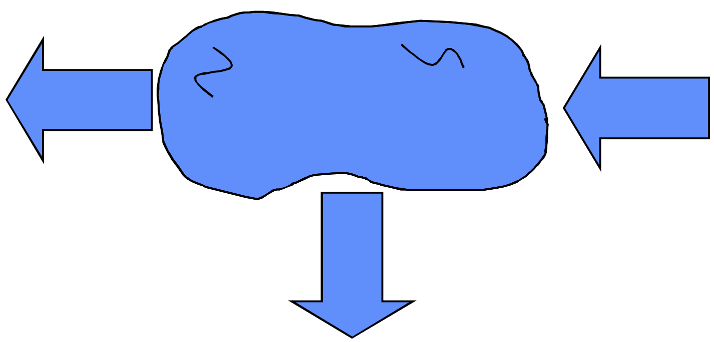
e.g. geometry, material properties, etc.

Inverse Problems: The physical View (2)

The inverse problem – general scenario

External inputs (unknown)

e.g. forces, fluxes, etc.



System response (partially known)

e.g. displacements, temperature, concentrations, etc.

The System (unknown)

e.g. geometry, material properties, etc.

Inverse Problems in Sierra-SD

- Emerging capabilities aimed at providing force, material, and shape inversion capabilities.
- All capabilities are parallelized
- Current capabilities:
  - Force/source inversion for acoustics and structures
    - Determines amplitudes of sources, given experimental response measurements
  - Material inversion in time and frequency domain
    - Determines material properties (elastic and viscoelastic), given experimental response measurements

Rapid Optimization Library (ROL)

— Optimization of differentiable simulated processes:
  - partial differential equations (PDEs)
  - differential algebraic equations (DAEs)
  - network equations (gas networks, electrical networks)

— Inverse problems, optimal design and control problems.

— The parameter/design/control spaces can be very large, often related to the size of the computational mesh (PDEs) or the size of the device network or graph (DAEs).

⇒ *Matrix-free, gradient-based, embedded methods.*

POC: Ridzal/Kouri (1441)

Rapid Optimization Library (ROL)

- **Unconstrained optimization:**
  Gradient descent, quasi-Newton (secant) methods, nonlinear CG, Gauss-Newton, Newton, with line-search and trust-region globalizations.

- **Equality constraints:**
  Sequential quadratic programming (SQP), with line-search and trust-region globalizations.

- **Inequality constraints:**
  For box constraints, use projected gradient and projected Newton methods. For general inequalities, use interior-point algorithms.

POC: Ridzal/Kouri (1441)

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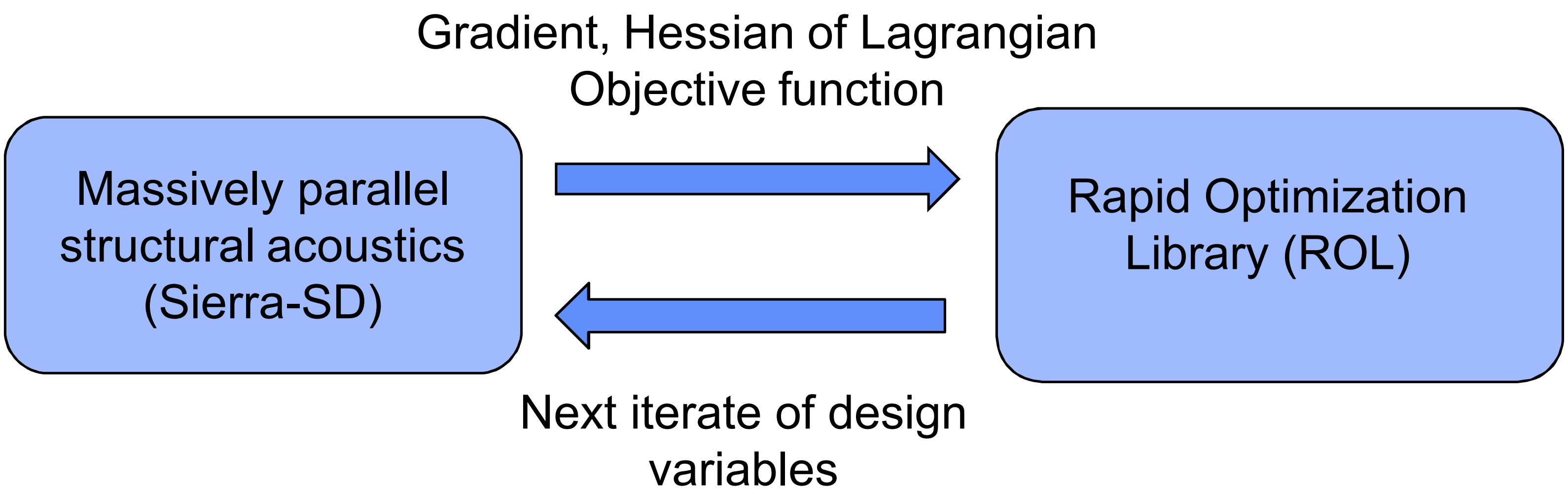
Interaction of Finite Element and Optimization Codes

Finite Element and Optimization Codes operate as independent entities

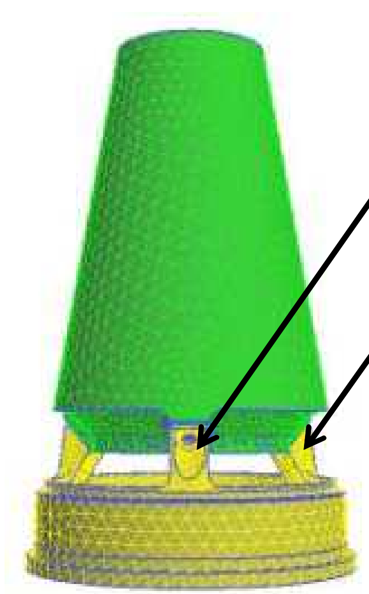


- The adjoint method is used to compute the gradients and Hessians

Use Cases for Material Inversion

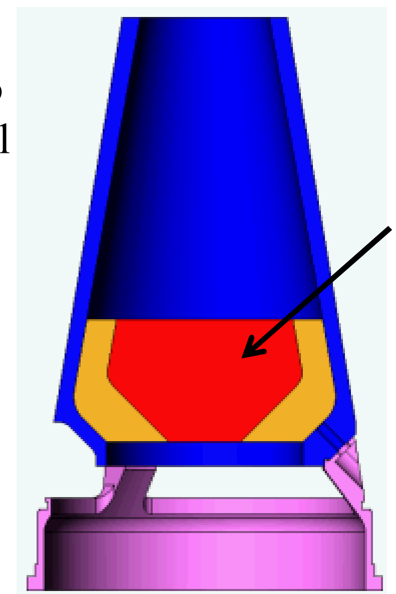
Both linear and nonlinear material inversion capabilities needed in NW community

Material inversion:
Nonlinear joints



- Unknown joint parameters to be derived from experimental data

Material inversion:
Viscoelastic foams



- Unknown distributed foam parameters to be derived from experimental data

Other needs: parameters for plasticity and other nonlinear material models

Use Cases for Source Inversion

Goal:

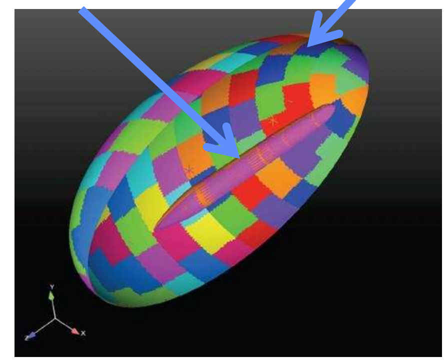
Solve inverse problem to obtain acoustic patch inputs that produce the given 17 experimental microphone measurements.

2 approaches:

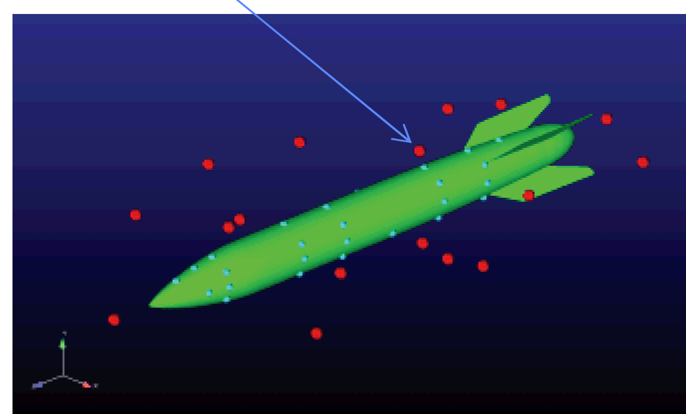
1. Frequency domain
   - forward problem is frequency sweep from 40Hz-4000Hz
2. Time domain
   - forward problem is implicit time integration with about 4000 time steps

Surface with 172 acoustic patches

Structure of interest



17 Microphone locations



Formulation of Source Inverse Problem

$$J(\{u\}_1, .., \{u\}_{N_f}) = \frac{\kappa}{2} \sum_{i=1}^{N_f} \left(\overline{\{u\}_i - \{u_m\}_i}\right)^T [Q] \left(\{u\}_i - \{u_m\}_i\right) + \mathcal{R}(\{p\})$$

Objective Function

$$\mathcal{L} = J + \sum_{i=1}^{N_f} \mathrm{Re}\left(\overline{\{w\}}_i^T \left([H(\omega_i)]\{u\}_i - \{F(\{p\})\}_i\right)\right)$$

Lagrangian

<u>KKT conditions:</u>

$$D_{\{w\}_j}\mathcal{L} \cdot \{\delta w\} = 0 \implies [H(\omega_j)]\{u\}_j = \{F(\{p\})\}_j$$

Forward problem

$$D_{\{u\}_j}\mathcal{L} \cdot \{\delta u\} = 0 \implies [H(\omega_j)]\{w\}_j = \kappa[Q]\left(\{u_m\}_j - \{u\}_j\right)$$

Adjoint problem

$$D_{\{p\}}\mathcal{L} \cdot \{\delta p\} = -\sum_{i=1}^{N_f} \mathrm{Re}\left((\overline{\{w\}}_j^T \left[\frac{\partial \{F(\{p\})\}_j}{\partial \{p\}}\right] \{\delta p\}\right) + D_{\{p\}}\mathcal{R}(\{p\}) \cdot \{\delta p\}$$

Gradient

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Dissipative Material Inversion in Frequency Domain

Generic formulation based on complex modulus

$$\underset{\boldsymbol{u},\boldsymbol{p}}{\text{minimize}} \quad J(\boldsymbol{u},\boldsymbol{p})$$ Objective function

$$\text{subject to} \quad \boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \boldsymbol{0}$$ PDE constraint

$$\mathcal{L}(\boldsymbol{u},\boldsymbol{p},\boldsymbol{w}) := J + \boldsymbol{w}_R^T \boldsymbol{g}_R + \boldsymbol{w}_I^T \boldsymbol{g}_I = J + \Re(\boldsymbol{w}^h \boldsymbol{g})$$ Lagrangian

$$\boldsymbol{\sigma}(\omega) = \boldsymbol{D}(\omega)\boldsymbol{\epsilon} = (b(\omega)\boldsymbol{D}_b + G(\omega)\boldsymbol{D}_G)\boldsymbol{\epsilon}(\omega)$$ Constitutive Law

Viscoelasticity

$$b(\omega) = b_R(\omega) + i b_I(\omega)$$
$$G(\omega) = G_R(\omega) + i G_I(\omega)$$

Block Proportional Damping

$$b(\omega) = b + i\omega\beta b$$
$$G(\omega) = G + i\omega\beta G$$

Dashpots

$$E_R = 0$$
$$E_I = \omega c$$

Source Inversion Methodology in Sierra-SD

- PDE-constrained optimization approach
  - Offers flexibility and extensibility
  - Applicable to time-domain, frequency-domain, and nonlinear problems. Can be tailored to each application.
  - Applicable to large numbers of design variables.
  - Allows significant code sharing with material inversion capability (backward time integrators for adjoint problems, experimental data manager, objective function, etc)
- Massively parallel finite element code Sierra-SD is used for solving the forward and adjoint problems.
- Optimization code ROL is used for solving the optimization problem.

Structural Acoustic Equations of Motion

acoustics

$$\nabla^2 \phi = \frac{1}{c^2}\ddot{\phi}, \ \text{in } \Omega_f \times (0,T)$$
$$\nabla \phi \cdot \boldsymbol{n}_f = -\rho_f \ddot{u}_n, \ \text{on } \partial\Omega_f^N \times [0,T]$$
$$\phi = 0, \ \text{on } \partial\Omega_f^D \times [0,T]$$
$$\phi(0,T) = 0, \ \text{in } \Omega_f$$
$$\dot{\phi}(0,T) = 0, \ \text{in } \Omega_f$$

solid mechanics

$$\nabla \cdot \boldsymbol{\sigma} = \rho \ddot{\boldsymbol{u}}, \ \text{in } \Omega \times (0,T)$$
$$\boldsymbol{\sigma n} = \boldsymbol{h}, \ \text{on } \partial\Omega^N \times [0,T]$$
$$\boldsymbol{\sigma} = \boldsymbol{D} : \nabla \boldsymbol{u}, \ \text{in } \Omega \times [0,T]$$
$$\boldsymbol{u} = \boldsymbol{0}, \ \text{on } \partial\Omega^D \times [0,T]$$
$$\boldsymbol{u}(0,T) = \boldsymbol{0}, \ \text{in } \Omega$$
$$\dot{\boldsymbol{u}}(0,T) = \boldsymbol{0}, \ \text{in } \Omega$$

Time domain

$$[M]\boldsymbol{a}(t) + [C]\boldsymbol{v}(t) + [K]\boldsymbol{u}(t) = \boldsymbol{f}(t)$$

Frequency domain (Helmholtz)

$$[H(\omega)]\boldsymbol{z}(\omega) = \boldsymbol{F}(\omega)$$
$$[H(\omega)] = -\omega^2[M] + i\omega[C] + [K]$$

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Structural Acoustic Equations of Motion

Fully coupled formulation

$$\begin{bmatrix} M_s & 0 \\ 0 & M_a \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} C_s & L^T \\ -L & C_a \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & K_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} = \begin{bmatrix} f_s \\ f_a \end{bmatrix}$$

Condensed notation

$$[M]\boldsymbol{a}(t) + [C]\boldsymbol{v}(t) + [K]\boldsymbol{u}(t) = \boldsymbol{f}(t)$$

We will use the condensed notation in following slides

Statement of Inverse Problem

Minimize objective function

$$J(\{\boldsymbol{u}\},\{\boldsymbol{p}\}) = \frac{\kappa}{2}\Big(\{\boldsymbol{u}\}-\{\boldsymbol{u}_m\}\Big)^{\mathrm{T}}[Q]\Big(\{\boldsymbol{u}\}-\{\boldsymbol{u}_m\}\Big)+\mathscr{R}(\{\boldsymbol{p}\}),$$

$\{\boldsymbol{u}\}$ State variables (displacement, pressure)

$\{\boldsymbol{u}_m\}$ Measured data (displacement, pressure)

$\{\boldsymbol{p}\}$ Unknown parameters (loads, material parameters)

$[Q]$ Weight matrix

Subject to equations of motion

$$[M]\boldsymbol{a}(t)+[C]\boldsymbol{v}(t)+[K]\boldsymbol{u}(t)=\boldsymbol{f}(t)$$

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Optimality conditions

- Optimality is obtained by setting derivatives of Lagrangian to zero
- We adopt a reduced space approach where we derive reduced gradients and Hessians from full space approach
- Reduced space approach can be derived from full space

Solution of Inverse Problem

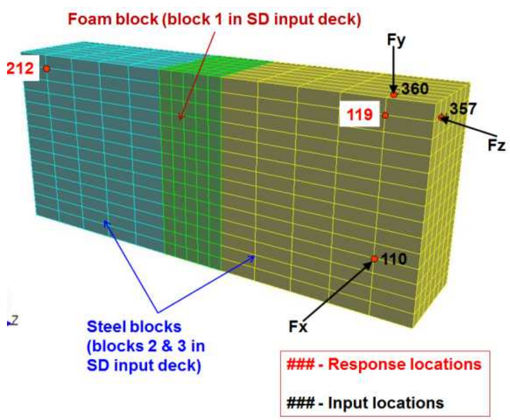
Do until tolerance < eps

- Solve forward problem
- ↓
- Solve adjoint problem
- ↓
- Compute gradients, Hessians
- ↓
- Optimization step
- ↓
- Receive design variable updates from optimization solver

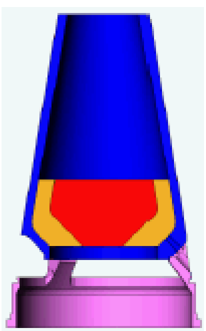
end

Application of Material Inversion (from A. Urbina)

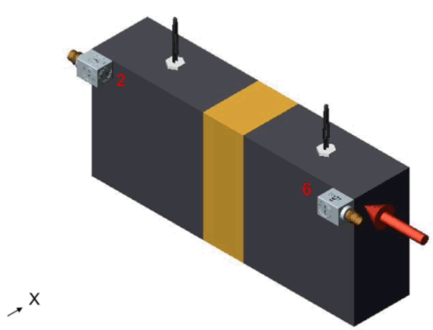
Characterizing viscoelastic foams

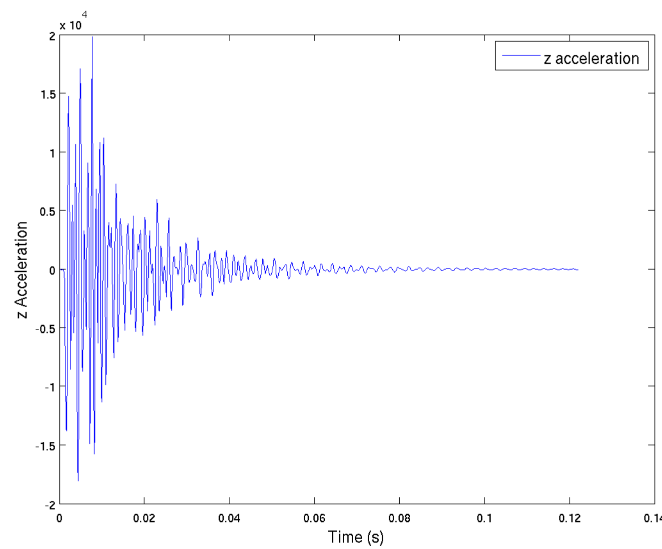


System



Experimental setup of foam phase 1 hardware





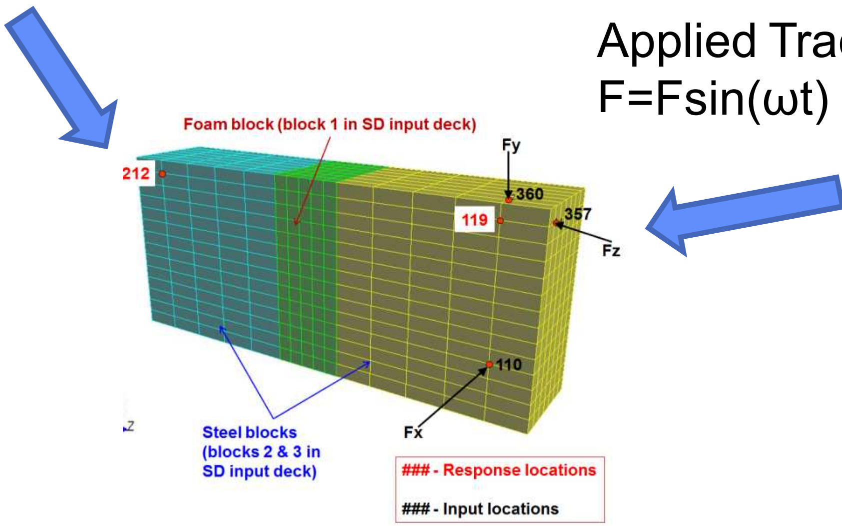
- Data is highly damped
- Requires viscoelastic material inversion

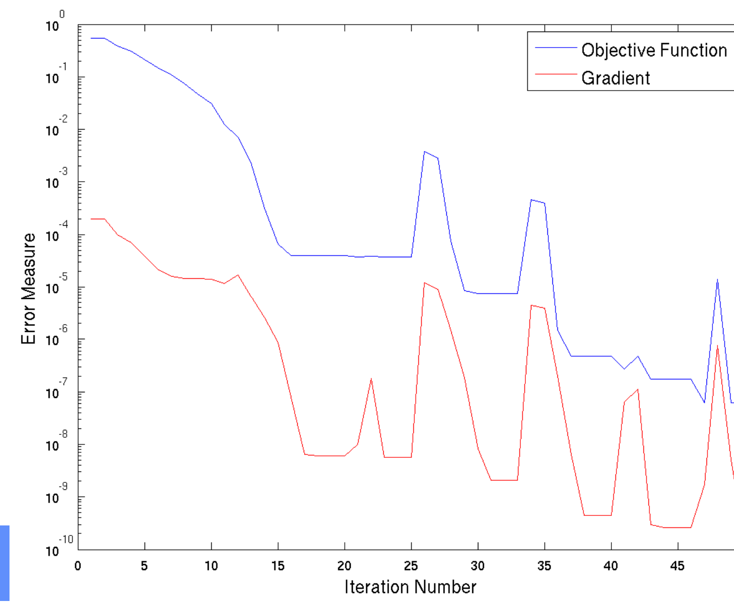
Application of Frequency Domain Viscoelastic Material Inversion

Characterizing viscoelastic foams

Measurement point

Applied Traction
F=Fsin(ωt)





	Exact	Computed
Real part of K	40000	38303.30
Imag part of K	0	1101.39
Real part of G	16000	16326.25
Imag part of G	5000	4950.58

Objective function and gradient reduced to acceptable level

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Source Inversion on B61

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Solve inverse problem to obtain acoustic patch inputs that produce the given 17 experimental microphone measurements.

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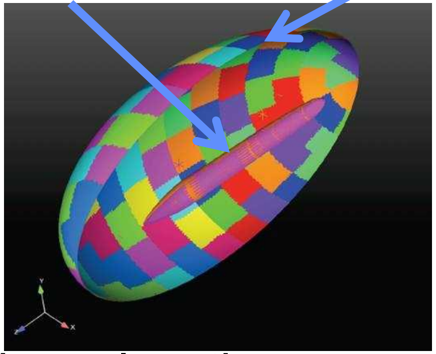
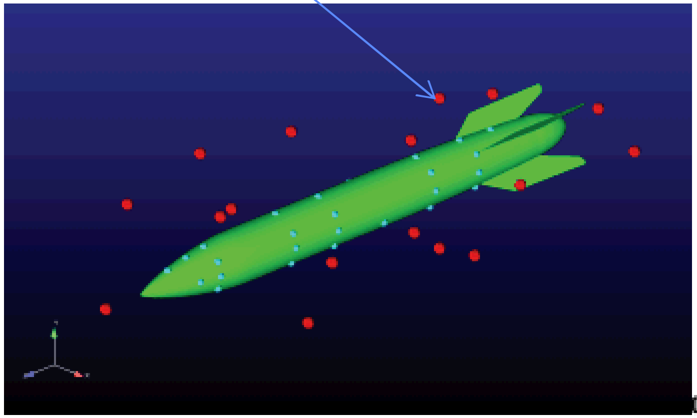
Structure of interest

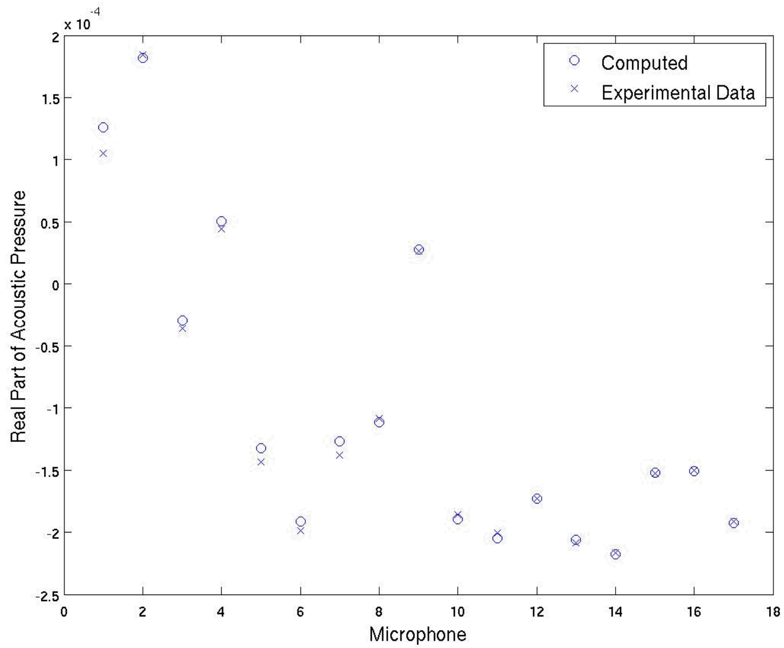


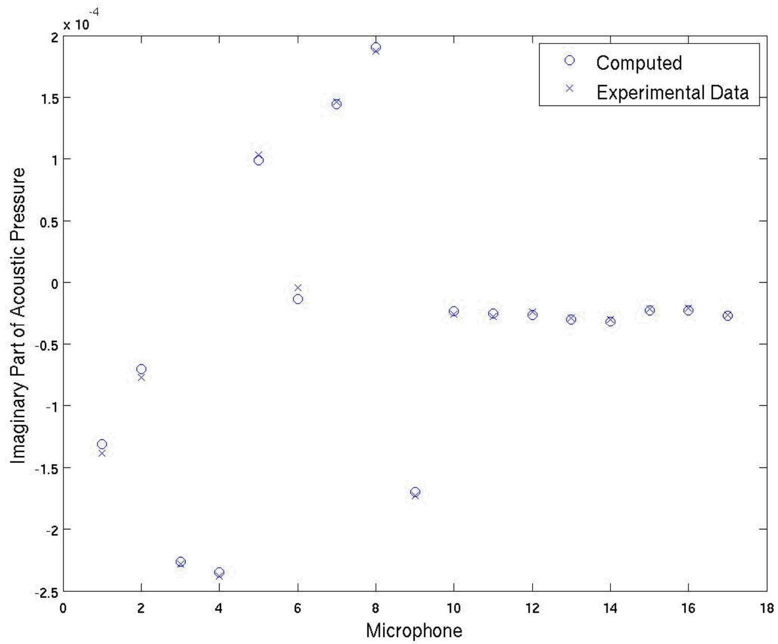
17 Microphone locations



Frequency Domain Source Inversion

Single Frequency Results





Real part of acoustic pressure

Imaginary part of acoustic pressure

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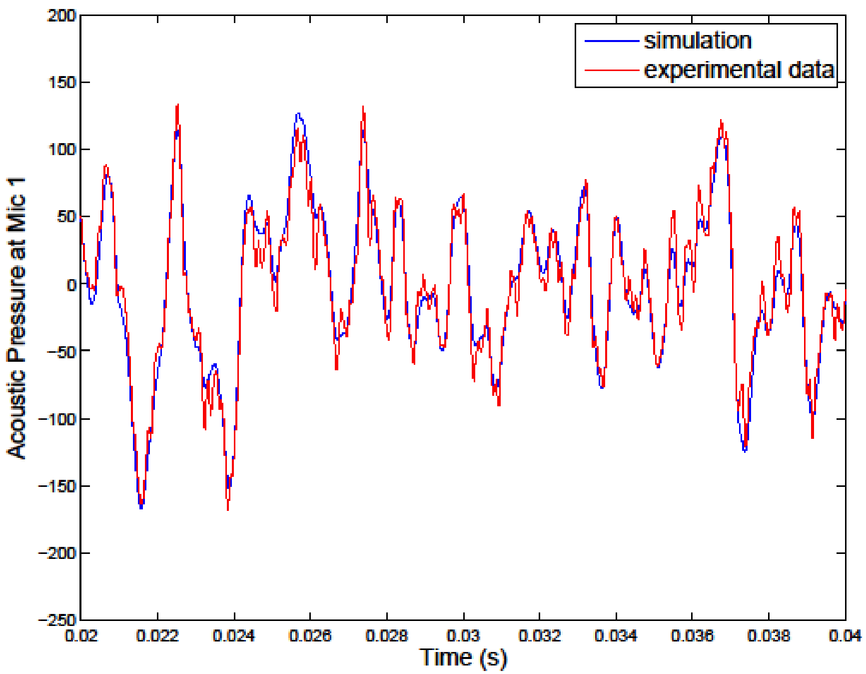
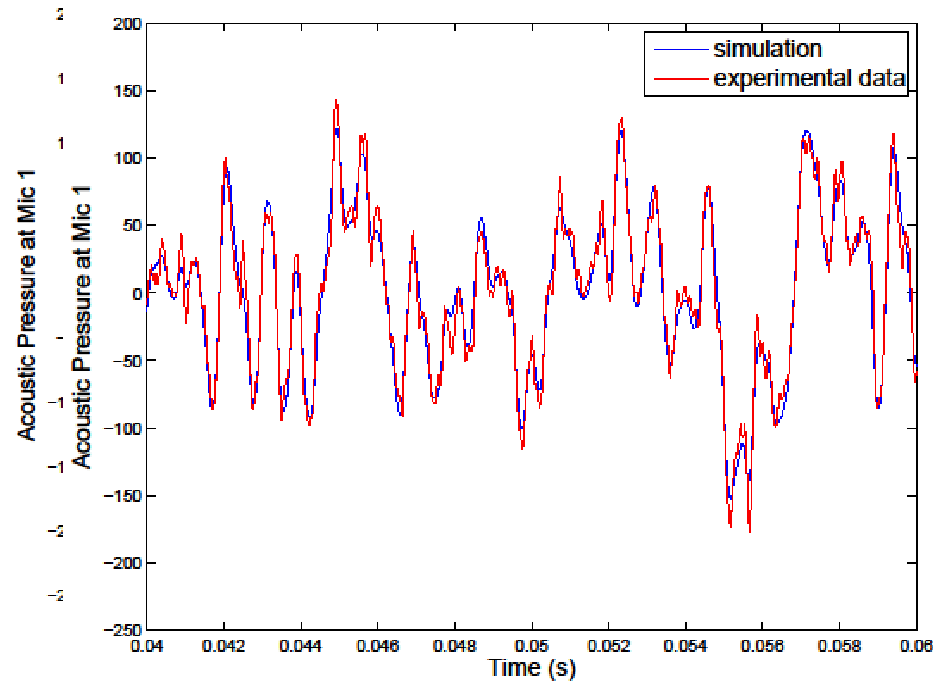
Time Domain Source Inversion

Results for Microphone 1



0.02(s) < t < 0.04(s)



0.04(s) < t < 0.06(s)

Conclusions

- Massively parallel finite element structural acoustics and optimization codes have been loosely coupled for the solution of source and material inversion problems.
- Adjoint methods have been implemented in Sierra-SD in both time and frequency domains.
- Applicable to large-scale models with many degrees of freedom.
- The method allows flexibility to work with both time and frequency domain, and nonlinear problems.
- Method has been applied to solve source and material inversion on problems of interest.