SAND2014-1505C

The Derivation of Maximum Predicted Environments for Externally Carried Stores using a Small Number of Flight Tests*

Jerome S. Cap
Sandia National Laboratories
Program and Test Integration Department

Melissa C' de Baca
Sandia National Laboratories
Program and Test Integration Department

Troy J. Skousen
Sandia National Laboratories
Program and Test Integration Department

ABSTRACT

When defining a system's response to a field environment it is desirable to gather sufficient data to be able to derive the Maximum Predicted Environment (MPE) using a statistical model. Unfortunately, when it comes to the vibration responses produced during external captive carry, it is rare to have data from more than one or two flights. Furthermore, while a typical flight test will include 6-10 distinct related events (for example straight and level flight) for which responses were measured, those events usually represent different altitudes and airspeeds. This paper describes the derivation of a parametric model that allows the analyst to extrapolate the event specific responses to a single flight condition of interest, thereby creating a homogeneous ensemble for use in developing a statistical model.

OVERVIEW

One almost never has enough flight data with which to develop a statistical model of the flight environment. Without such a model, it is virtually impossible to define component test specifications with any confidence in the corresponding conservatism. Therefore, the goal of this study is to create a parametric model that takes the response data from a collection of test events measured at different velocities and altitudes and extrapolates those data points to a homogeneous ensemble of responses based on a single flight condition. This homogeneous ensemble of responses is then used to generate a Maximum Predicted Environment (MPE) responses having a one-sided 99% probability of occurrence with a 90% confidence interval (the P99/90 MPE) for use in generating component test specifications.

The analysis can be broken up into five steps:

1) Compilation of available flight data.

2) Screening of data and the removal of suspect data.

3) Generation of the parametric model.

4) Extrapolation of flight data to desired intensity level.

5) Creation of the P99/90 MPE.

AVAILABLE FLIGHT DATA

The parametric model used in this analysis is based on the fact that the dynamic pressure, Q, is considered to be a good first order indicator of the intensity of the vibratory response of an externally captive carried store when

* Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under Contract DE-AC04-94AL85000

exposed to straight and level flight conditions [Refs 1 and 2]. Therefore, the flight data will be sorted according to the value of Q associated with each set of flight conditions.

The vibratory response of an externally captive carried store varies with the nature of the flight conditions. For purposes of discussion, we chose to divide external carry flight into three phases:

1) High speed, low altitude straight and level flight (assumed to have a value of $Q > 400$ lb/ft^2).

2) Low speed, high altitude straight and level flight (assumed to have a value of $Q < 400$ lb/ft^2).

3) Maneuvers, which are assumed to be a function of both Q and the angle of attack (AOA).

The focus of this paper will be the high speed, low altitude straight and level flight with the intent to extrapolate the available flight data to the maximum value of Q (Max Q) for the aircraft of interest.

Ideally, the ensembles used to generate the MPE response would consist of many flights for each unique aircraft configuration and test condition of interest. This would make it relatively straight forward to generate a statistical model with which to estimate the P99/90 MPE. Unfortunately, it is rare that such a homogeneous database of test conditions exists. Indeed, the only reason that Sandia has acquired data from multiple flights is that we have measured data for one of our stores each time an aircraft was modified over the years. Therefore, what we actually have is a collection of 6-10 test conditions (each having a different set of flight parameters (altitude and airspeed) for each of 4-8 flights (each flight test representing a slightly different aircraft configuration).

Typically, only one of the test conditions for each flight was flown at or near to the Max Q conditions. If one were to use only 4-8 data points to generate a P99/90 MPE, the resulting scale factors associated with estimating high probabilities of occurrence and confidence intervals for such a small ensemble would drive the resulting MPE to unacceptably high levels [Ref 3]. The alternative is to merge the data from flights involving modified configurations of the same aircraft to increase the size of the ensemble. Although those differences will introduce increased variability and hence raise the P99/90 MPE, it is felt that the increased variability associated with minor changes in the aircraft configuration represent a smaller effect than the adjustments associated with high probability and confidence estimates for a small ensemble size.

The data used in this analysis are presented in the form of 1/6th octave bandwidth Acceleration Spectral Densities (ASDs). The use of octal bandwidth ASDs is intended to reduce the variance error (i.e., the hash) that can be present in high-resolution constant bandwidth ASDs given the relatively short duration data segments available from flight tests. The choice of 1/6th octave bands is considered to be the appropriate bandwidth for reducing the variance error while still preserving the true resonant response of the ASDs given the damping characteristics of our store [Ref 4].

IDENTIFYING AND REMOVING DATA OUTLIERS

Due to the span of years over which the data were acquired, the gage locations and/or naming conventions may have changed slightly. However, experience has shown that conducting statistical analyses on inhomogeneous data sets can lead to unrealistic predictions. This meant that the primary challenge in creating the raw data ensembles was how to "scrub" the ensembles to insure that each spectrum did indeed belong in the ensemble.

Therefore, the underlying data analysis reports were carefully studied and the raw ASD ensembles were reviewed. Any ASD that clearly did not appear to be in family was completely removed from the ensembles (it is likely that either the accelerometer was moved slightly or that the lowpass filters used to process the data may have changed over time). Alternatively, in at least one instance two ensembles having names that suggested a similar location were combined since they appeared to be in family. Figure 1 shows an example of an ensemble where there were clearly out of family ASDs at high frequency.

If the individual spectra had indeed come from the same flight conditions the screening process could have ended with this initial review. However, the extrapolation process described below tended to magnify even small variations within the ensemble since it scaled some spectra more than the others were.

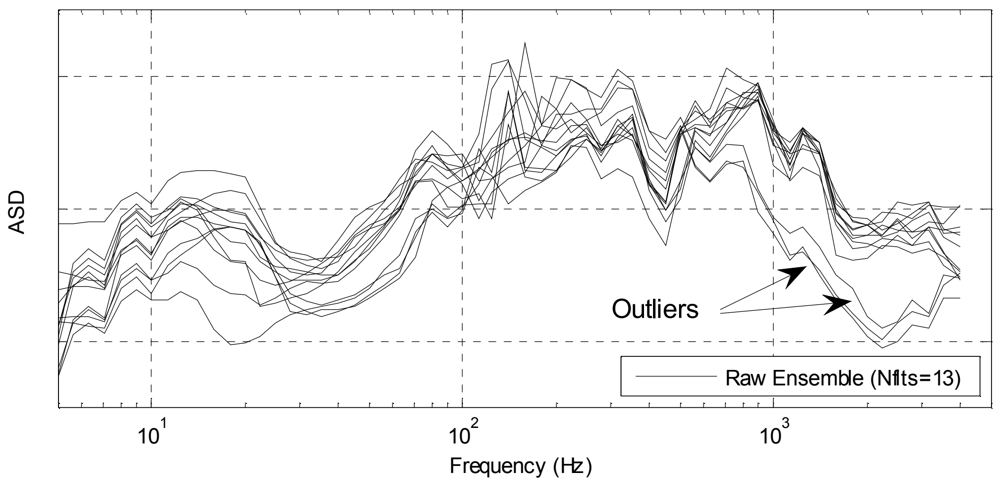

Figure 1: Example of Major Outliers in Raw Response Ensemble

Therefore, the extrapolated ASDs were examined a second time for more subtle occurrences of out of family response. Any of the scaled ASDs that did not fall in with the majority of the ASDs were edited to remove the offending narrowband deviations. The editing process began by first identifying any out of family ASDs that were clearly higher than the ensemble. The narrowband portions of those ASDs that exceeded the ensemble were removed (in the Matlab databases the suspect data points were replaced with NaN). The process was then repeated for ASDs that were clearly too low. In order to be conservative, very few high outliers were removed. Instead, the emphasis was placed on low points (typically resulting from an ASD that did not exhibit a resonant response at the same frequency as the other ASDs).

The rational for making these corrections can be explained as follows. The presence or absence of a resonant mode in a particular subset of ASDs suggests two separate populations. Had we performed separate analyses on both populations it would be highly unlikely that the statistically generated MPE for the lower amplitude population would exceed the MPE of the higher population. Furthermore, the statistical analysis described later in this paper adds additional margin to the MPE if fewer data points are available thereby compensating for the removal of the suspect data points. Figure 2 shows an example of extrapolated ASDs for which narrowband outliers were removed.

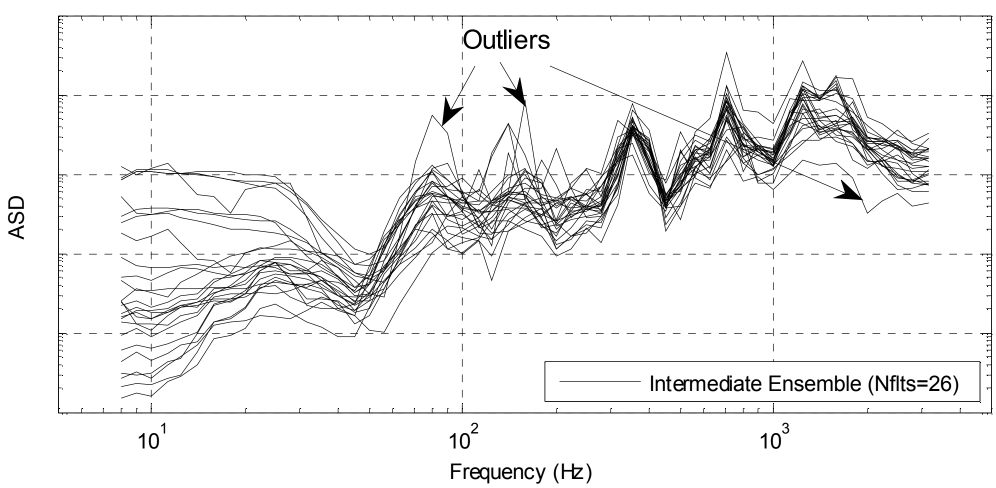

Figure 2: Example of Minor Outliers in Scaled Ensemble

GENERATION OF PARAMETRIC LEAST SQUARE MODEL

This raw ensemble is still not useable because the dynamic pressures are all different. However, based on the aforementioned relationship between the dynamic pressure, Q, and the root mean square acceleration (Grms), the decision was made to develop a linear least squares (LSQ) model, having a slope m_{MN} and y-intercept b_{MN}, as shown in equation (1). This equation represented our initial attempt to extrapolate all of the available straight and level flight data for the same phase of flight.

$$G = m_{MN} Q + b_{MN} \tag{1}$$

If all of the data fell exactly on the LSQ line then one could infer that there was no flight-to-flight variability in the data and the response spectra at any given value of Q could be computed algebraically. However, when we attempted to fit a first order (linear) least squares curve to the data, there were variations between the measured Grms values and the curve. The differences between the raw data and the least squares fit curve are assumed to represent the flight-to-flight variability.

Of course, true flight-to-flight variability should be defined as the variations seen when flying duplicate units of the same store design on several aircraft of the exact same type. The flight-to-flight variability seen in our ensembles also include other “phantom” sources of variability:

1) Problems with the acceleration measurements come primarily from limitations in the accelerometer accuracy.

2) The most probable source of error involves the identification of the flight conditions (altitude and airspeed), which are used to compute the dynamic pressure. It was difficult to ascertain those parameters with any degree of accuracy for the older fighter aircraft because the they had to be read off the Heads up Display (HUD) video, and often only one significant digit could be read. Furthermore, due to the tedious nature of viewing the HUD, we only identified a single set of conditions for each event. Were it possible, the better approach would be to continuously record the flight parameters for the entire time segment for which the accelerometer data were being measured. The average value of Q would be used, much in same manner that the ASD is an average of multiple blocks of data taken from the segment.

3) As mentioned earlier in this paper, the fact that the data comes from several different configurations of the aircraft will also introduce variability.

4) It is also recognized that test conditions flown near Mach 1 should have higher vibration responses than test conditions having a similar dynamic pressure but with a lower or higher Mach number. A cursory review of the Mach numbers associated with each test condition did not show a distinct trend so no attempt was made at this time to account for the effects of Mach number in the LSQ model.

However, all of these sources of uncertainty should result in a more conservative estimate of the variation and hence in the resulting MPE so we are willing to live with them.

One final issue regards the decision to base the parametric model on the overall Grms values instead of the frequency-by-frequency spectral values. The decision was based on the fact that the raw ensemble of spectra generally had the same spectral shape (indicating that the results would be approximately the same regardless of the approach used).

For the remainder of this section the data from a single data channel measured during flights on a single aircraft type will be used to provide graphic examples of the various concepts. Figure 3 shows the raw edited version of the ensemble (which by default still includes the minor outliers).

The initial approach for computing the least squares fit used the Matlab function “polyfit” which allows for the possibility of a non-zero y-intercept. In many cases, the y-intercept was quite large. It is recognized that the value of the Grms at Q=0 is not zero (engine noise prevents it from being zero). However, large values of the y-intercept were assumed to be due to one or more noisy data points and/or a lack of data (i.e., too narrow of a range for Q coupled with too few data points can lead to a poor estimate of the curve fit). Whatever the root cause, the decision was made to constrain the y-intercept to always be zero. The reader can visualize this operation if they think of rearranging the formula shown in equation 1 to solve for m_{MN} for the case where b_{MN} is zero. However, this approach was actually implemented in Matlab using the left matrix divide operation (Q\G) to derive the mean slope, m_{MN}.

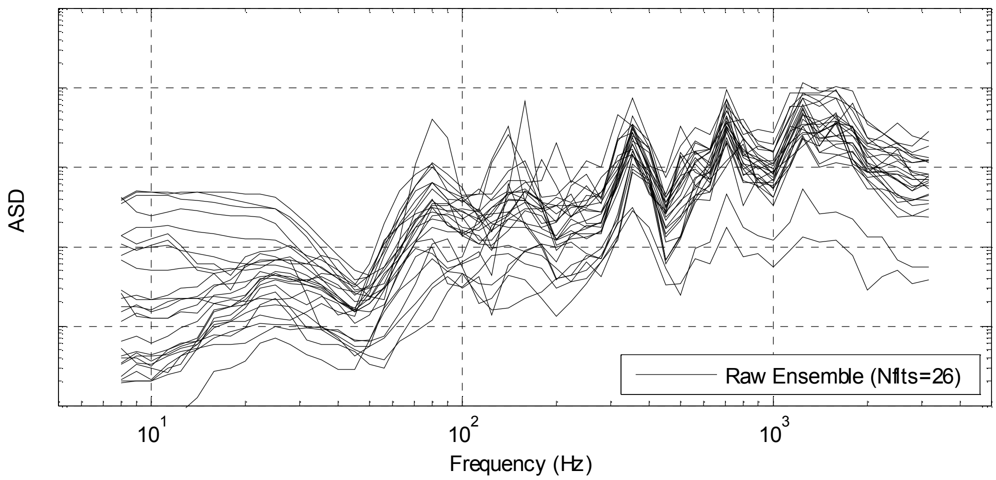

Figure 3: Example of Raw Edited Response Ensemble

Figure 4 compares the mean least squares fit Grms, G_{MN}, against the ensemble of raw Grms values, G_R, as a function of Q.

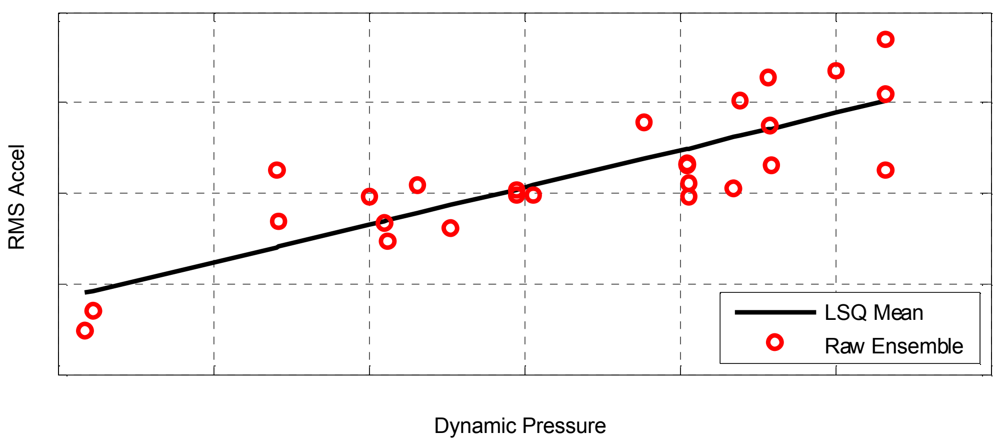

Figure 4: Example of Raw and Least Square Fit of Grms versus Q

However, while it is easy to see from this plot that the overall trend in the data is being captured, the differences between the raw data and the mean curve often appeared to increase with increasing values of Q. If this effect is not accounted for in the parametric model, any extrapolated results will be less accurate. Therefore, an additional first order least squares fit was generated between the dynamic pressure and the absolute values of the differences between the raw data and the mean curve as shown in equation (2) (again forcing the y-intercept to be zero). Figure 5 shows the ensemble of the absolute values of the differences compared with the least squares fit having a slope of m_{DF}.

$$\Delta G = m_{DF} Q \tag{2}$$

As an aside, an alternative model in which the differences in the Grms values were normalized by the mean Grms was investigated, but that did not produce credible results.

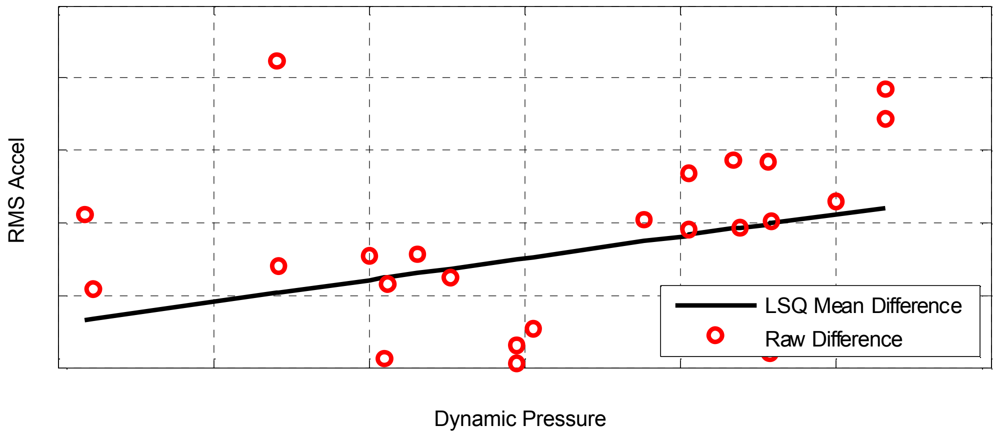

Figure 5: Example of Raw and Least Square Fit Grms Differences versus Q

EXTRAPOLATION OF ENSEMBLES

The next step is to use the parametric model to extrapolate the original rms responses, G_R, from the original values of Q (denoted as Q_R), to the new value of the rms responses, G_E, corresponding to the desired value of Q (denoted as Q_E). It is worthwhile go through the algebra associated with the process of implementing equations (1) and (2) using a systematic approach.

The first step is to split the value of G_R into two components: 1) the mean value, G_{RM} (the point on the mean LSQ curves associated with Q_R), and 2) the corresponding difference between G_R and G_{RM} (denoted as G_{RD}), as shown in equation {3}.

$$G_R = G_{RM} + G_{RD} \qquad (\text{where } G_{RD} = G_R - G_{RM}) \qquad (3)$$

G_{RM} and G_{RD} are then extrapolated to the desired value of the dynamic pressure, Q_E, using equations (3) and (4) respectively. The reader should note that since the slope of the rms difference LSQ curve, m_{DF}, was based on the absolute values of the differences, it is necessary to use the appropriate sign convention when applying it, hence the "a" and "b" forms of equation (4). The sanity check on this concept is that for a positive value of m_{MN}, the extrapolated version of the difference, G_{ED}, should be further away from the mean than G_{RD} was, regardless of the polarity of G_{RD}.

$$G_{EM}=G_{RM}+m_{MN}(Q_E-Q_R) \qquad (3)$$

$$G_{ED} = G_{RD} + m_{DF}(Q_E - Q_R) \quad (\text{for } G_{RD} > 0) \qquad (4a)$$

$$G_{ED} = G_{RD} - m_{DF}(Q_E - Q_R) \quad (\text{for } G_{RD} < 0) \qquad (4b)$$

The last step is to combine the extrapolated values of the mean and difference as shown in equation (5).

$$G_E=G_{EM}+ Q_{ED} \qquad (5)$$

Figure 6 shows the extrapolated Grms values plotted against the original Grms values and the least square mean curve.

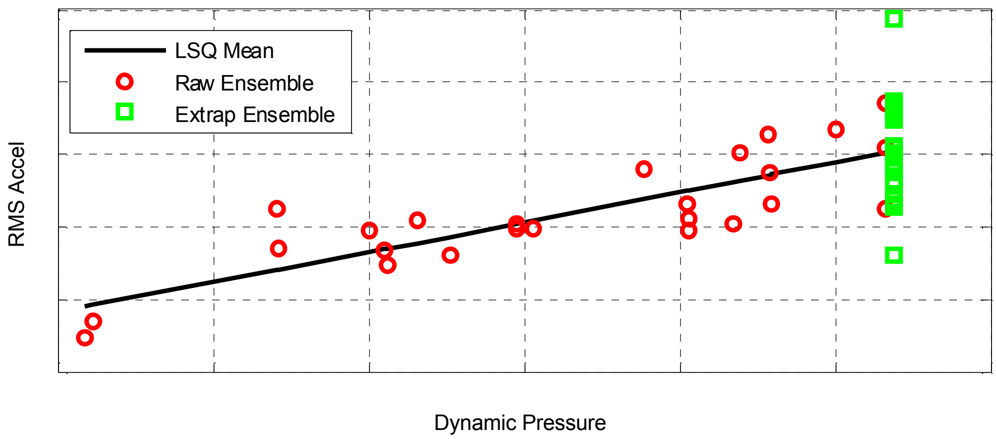

Figure 6: Raw, Least Square Fit, and Extrapolated Grms versus Q

A check was performed to determine whether or not the data were lognormal distributed. Figure 7 presents the distribution free Cumulative Distribution Function (CDF) for the log of the extrapolated Grms values, log(G_E), (denoted as "Extrap Ens") against the theoretical CDF for a normal distribution having the same mean and standard deviation as that of *log*(G_E). While the fit is not necessarily ideal (and perhaps this should not be too surprising given the small size of the ensemble), the fact that there is some agreement between the two CDFs is seen as a sign that the process is plausible.

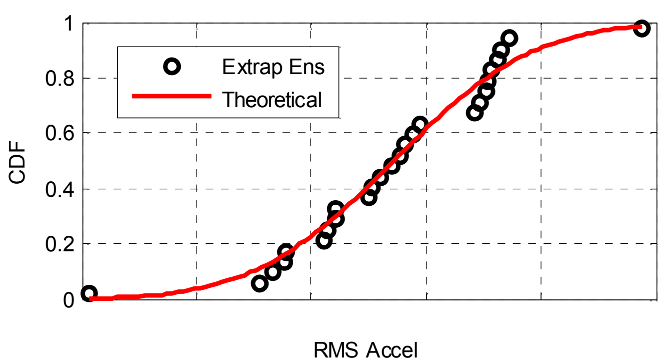

Figure 7: CDF of Extrapolated GRMS Responses

DERIVATION OF MAXIMUM PREDICTED SPECTRA

Given the set of scaled Grms values, G_E, the raw ASDs were adjusted until their Grms values matched the scaled Grms values. The next step was to perform the final narrowband editing discussed earlier in this paper. Figure 8 shows the extrapolated ASDs (denoted Extrap Ens) for the maximum dynamic pressure. If the process is valid, one would expect the overall levels of the individual ASDs to go up while the spread in the extrapolated ASD ensemble should become tighter with respect to the raw ensemble shown in Figure 3 owing to the fact that the variation associated with the different values of Q has been removed.

The final step was to generate the statistical measure of the MPE. Given the relatively small number of data points, the decision was made to employ a parametric statistical model. Based on commonly recognized practices for aerospace spectral data, it was assumed that the ASDs, S_{XX}, would be lognormal distributed (i.e., the logarithm of the raw ASDs, S_{YY}=log(S_{XX}) will be normally distributed). Therefore, the mean, S_{XXM}, is defined by the formula in equation (7) while the statistical upper limit, S_{XXU}, is defined by the formula in equation (8) where the values of k are tabulated in Reference [3] and "std" denotes the standard deviation.

$$S_{XXM} = 10^{[mean(S_{YY})]} \tag{7}$$

$$S_{XXU} = 10^{[mean(S_{YY})+k*std(S_{YY})]} \tag{8}$$

As a side note, the reader should remember that since minor narrowband outliers were removed from the ensemble there can be different numbers of valid ASD values for each frequency. Therefore, equations (7) and (8) had to be applied frequency line by frequency line.

Two statistical estimates were generated in order to better understand the conservatism of the process, the 99% one-sided probability of occurrence with both 50% and 90% confidence intervals (P99/50 and P99/90 respectively). The resulting mean and MPE spectra (denoted as "Mean", P99/50, and P99/90 respectively) are compared against the extrapolated ensembles in Figures 8.

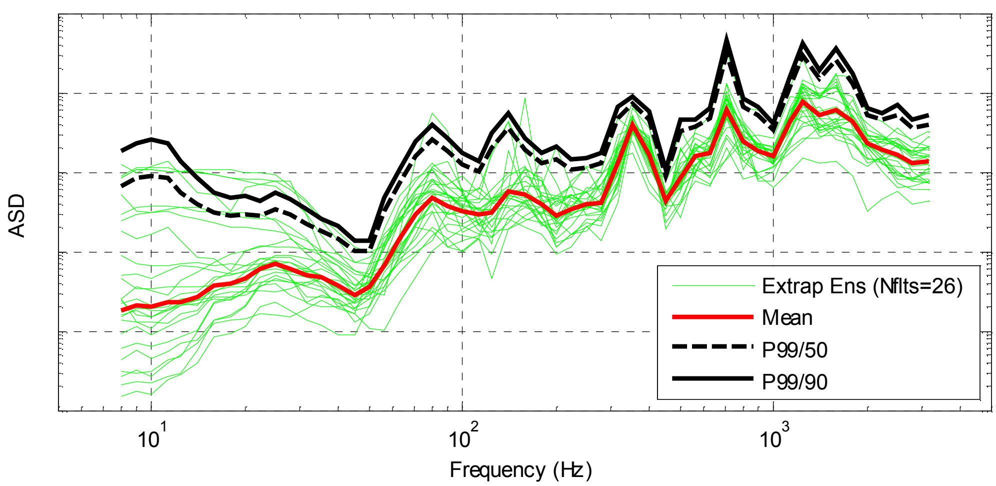

Figure 8: Example of Extrapolated Ensemble versus Mean and MPE

CONCLUSIONS

The process described in this paper is believed to represent a plausible methodology for deriving the MPE spectra for external captive carry flights when there are insufficient data points for the desired flight condition but at least 10-15 test points for the same phase of flight (for example high speed straight and level flight).

Our plan is to implement enhancements to the process for future flight testing, beginning with a change in how test conditions are chosen for flight tests to gather test points over the entire range of possible flight conditions and direct measurement of the "as flown" flight conditions.

Some effort will be made to determine if the parametric model should be modified to account for the Mach number.

Lastly, assuming that sufficient maneuver data can be gathered, an attempt will be made to extrapolate those responses with the goal of generating a statistically significant MPE.

REFERENCES

[1] Mil-Std-810G, “Environmental Engineering Considerations and Laboratory Tests”, 31 October 2008.

[2] Technical Report AFFDL-TR-71-158, “Vibration and Acoustic Test Criteria for Captive Flight of Externally Carried Stores.”

[3] OUO document NASA-HDBK-7005; Issued December, 2000; “Dynamic Environmental Criteria.”

[4] Piersol, A. G., “Optimum Resolution Bandwidth for Spectral Analysis of Stationary Random Vibration Data”, J. Shock and Vibration, Vol. 1, Issue 1, pp. 33-43, 1993/1994.