

# An Approach to QMU for Computational Analyses Involving Limited Resources

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# QMU Background -- Relevant Definitions

**Quantification of Margins and Uncertainties** (QMU) is a scientific methodology that identifies relevant nuclear-warhead parameters and quantifies, using available experimental and computational tools, the margin of that parameter relative to its failure point and the uncertainties associated with the parameter.

... .

**Aleatory Uncertainty** arises from an inherent randomness in the properties or behavior of the system under study.

**Epistemic Uncertainty** derives from a lack of knowledge about the appropriate value to use for a quantity that is assumed to have a fixed value in the context of a particular analysis.

... . NNSA Policy Letter: NAP-XX, Draft 5/1/07 (Taken from Helton, SAND2009-3055)



# Focus for QMU Analyses: Addressing Requirements

**REQUIREMENT:** We need to be  $YY\%$  certain (confident) that at least  $XX\%$  of the unit population will yield a response lower than the threshold  $T$ .

The values of  $XX$  and  $YY$  and the comparisons 'at-least' versus 'at-most' and 'lower' versus 'upper' are parameters of the requirement.

This can often be approached using tolerance bound methodology

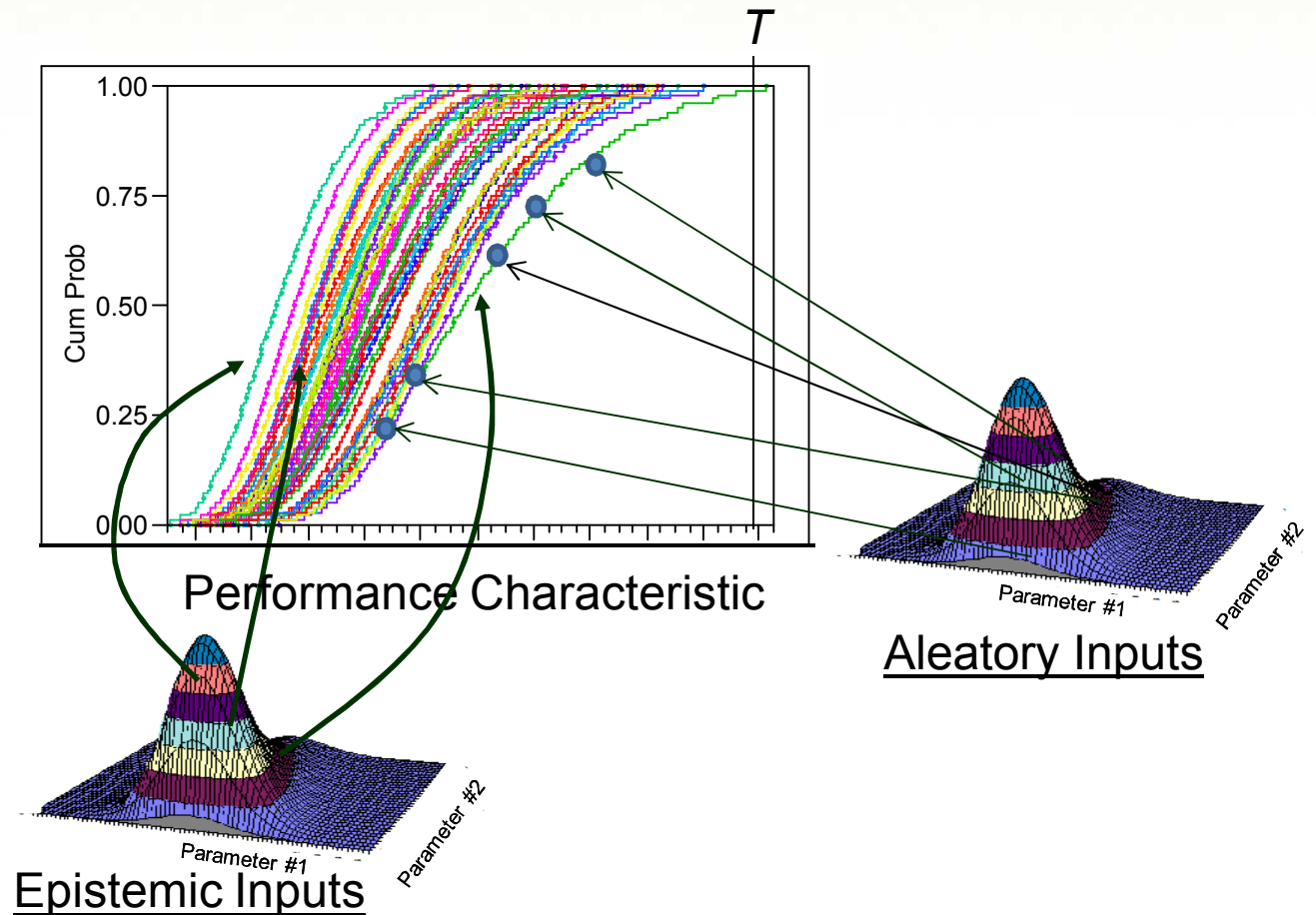
# Probability-of-Frequency Approach

## Basic Idea for Computational Simulation Response Data

Also referred to as the

“Second-Order Sampling Method” –

referring to the process for sampling

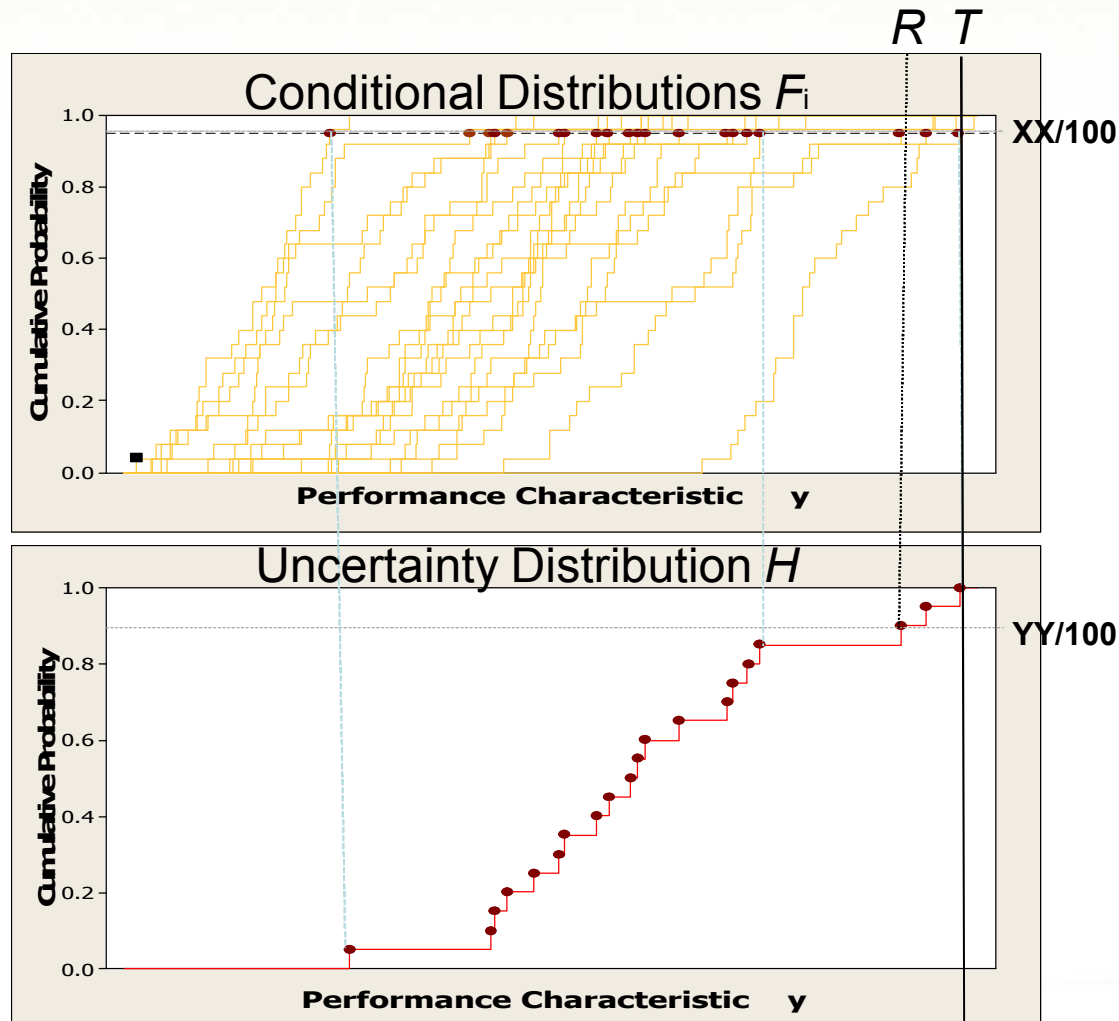


Helton, JC. Quantification of Margins and Uncertainty: Conceptual and Computational Basis. *Reliability Engineering and System Safety*; (2011) 96, 976-1013.



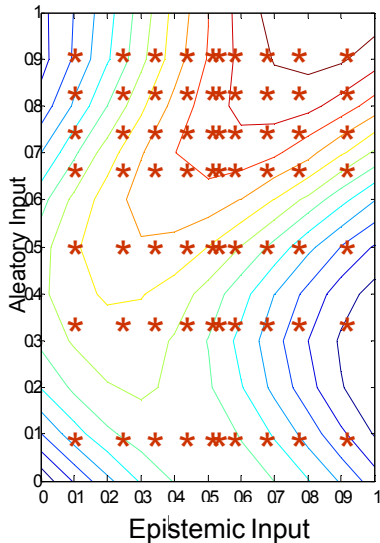
# Probability-of-Frequency Approach

## Conditional and Uncertainty Distributions



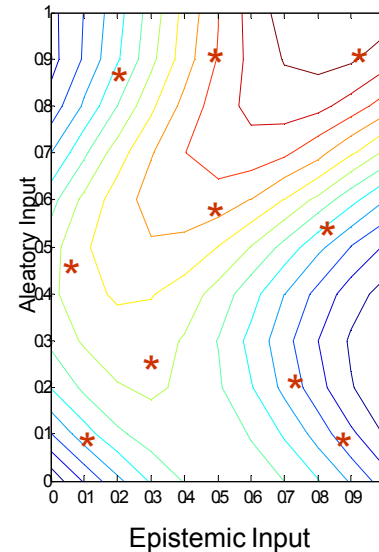
# How can we Choose Simulation Points for an Efficient Analysis – Response Surfaces

## Sampling Process



Second Order Sampling Process is actually applied to the sampling algorithm

## Response Surface

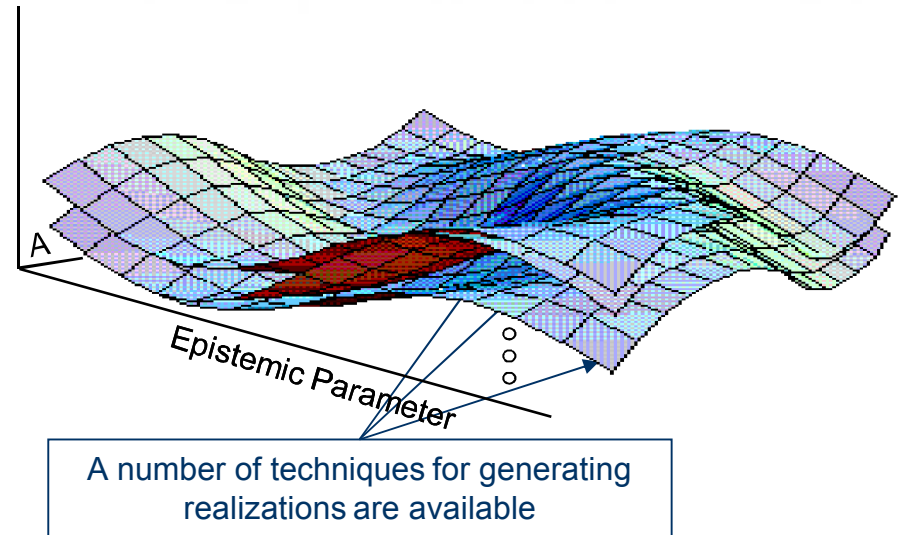


Second Order Sampling Process is utilized after a surrogate model is constructed

Problem is the uncertainty in the response surface is not accounted for.

# An Approach to Accommodating Limited - Simulation Uncertainty – Response Modeling

- Used in the Geosciences in 2 and 3 dimensions – Called Stochastic Simulation
- Can be treated as a discrete probability measure (equal probability associated with each “realization”)
- Provides an approach for propagating response uncertainty into uncertainty in an arbitrary performance characteristic



Deutsch CV, Journel AG. *GSLIB: Geostatistical Software Library and Users Guide Second Edition*. New York: Oxford University Press; 1998.

Gotway CA, Rutherford BM. In Armstrong M, Dowd P eds. *Geostatistical simulations: proceedings of the geostatistical simulation workshop, Fontainebleau, France, 27-28 May 1993*. Dordrecht: Kluwer Academic Publishers 1994.

Rutherford B. A response-modeling alternative to surrogate models for support in computational analyses. *Reliability Engineering and System Safety* 2007;91:1322-30.

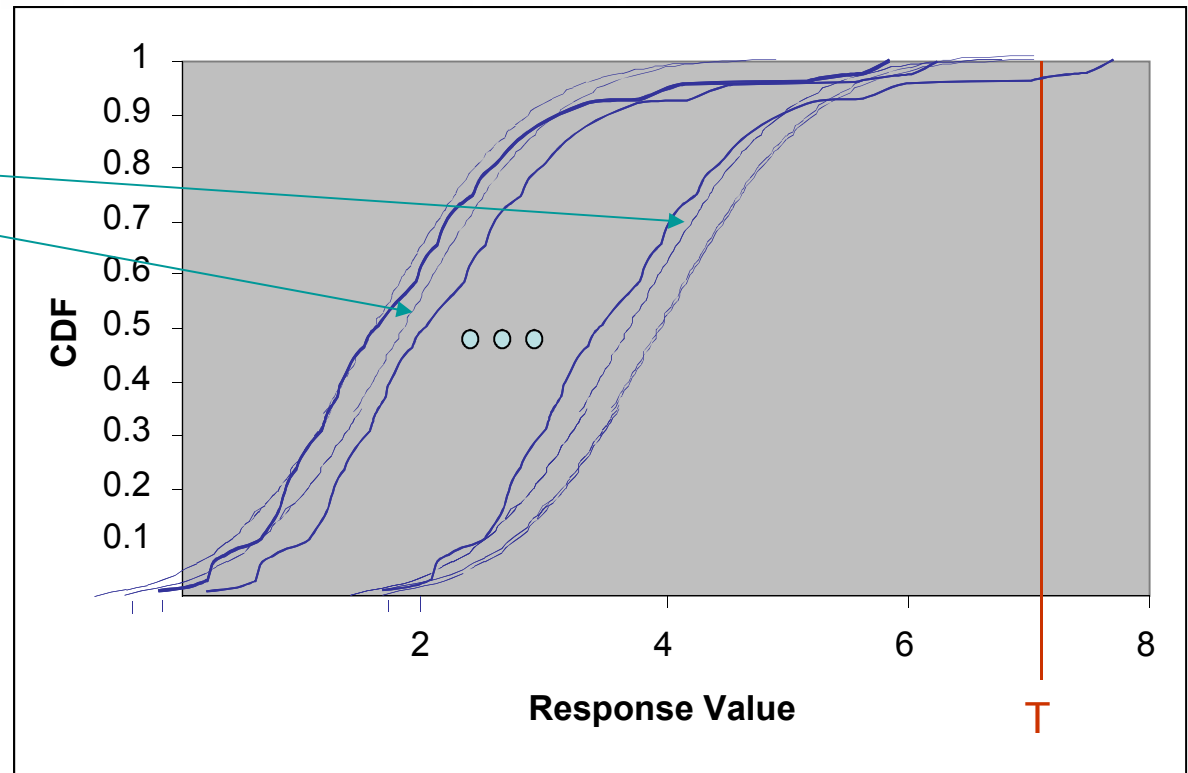


# Probability of Frequency Approach

## Using Response Modeling

Each set of epistemic parameters would result in several ( $n_r$ ) CDFs

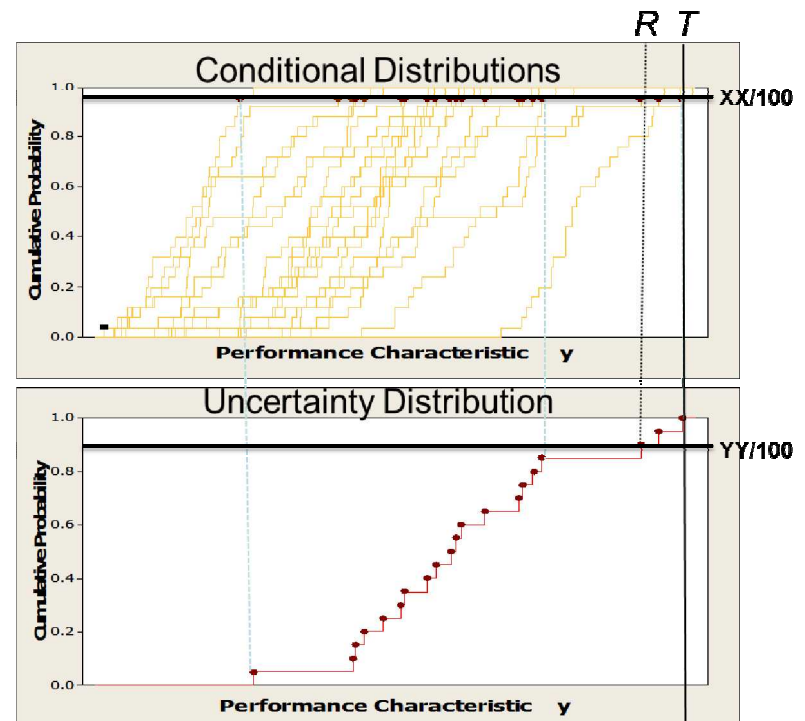
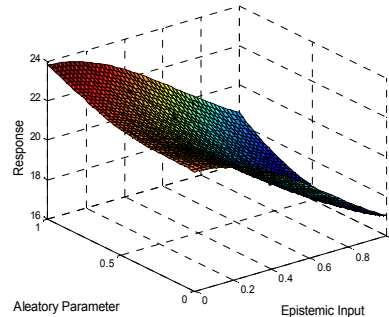
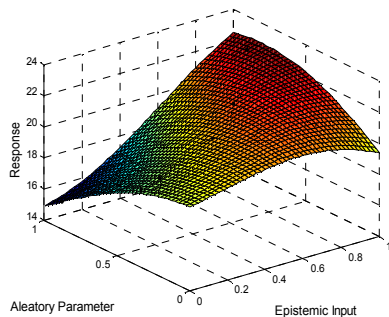
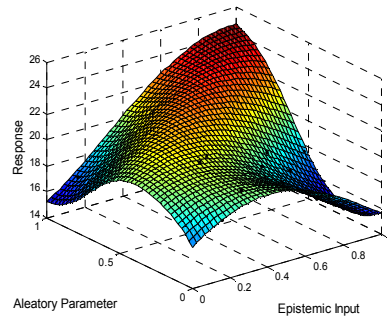
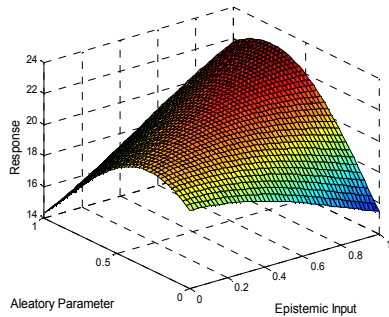
The additional Uncertainty reflects our uncertainty in the Response Surface



# Test Problem(s)

Test over a Number of Randomly Generated  
“True” Surfaces

What We Would Hope to Accomplish



Want to be consistently close to the correct confidence (ie. the true  $R$  value)



# Results – for 9 and 16 Point Simulations – Average Results over 300 Problems

9 Simulations		Mean	Std. Dev.	Abs. Diff.
Response Modeling		85%	.16%	.25 units
Response Surface		81%	19%	.27 units
Sampling		78%	.28%	.48 units

16 Simulations		Mean	Std. Dev.	Abs. Diff.
Response Modeling		86%	.14%	.14 units
Response Surface		80%	19%	.14 units
Sampling		79%	.25%	.35 units



# Summary

- For experimental data, tolerance bounds often provide a straightforward solution to “the requirements”.
- The probability-of-frequency construct can be used for this same purpose for computational simulation data where there is additional epistemic uncertainty.
- When a limited number of simulations can be run there is additional uncertainty resulting from estimating the response through sampling or fitting a response surface. Response modeling can capture much of this additional uncertainty.

