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Development of Model for Consistent Geologic Material Failure

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Overview

- **Kayenta Material Model**
  - Pressure-dependent yield stress
  - Softening response
  - Spatial variability
- **Failure Response Modeling**
  - Brittle material response
  - Dilation of failed material under shear
- **Calibration Approaches for Kayenta Model**
- **Conclusions**

Kayenta Development

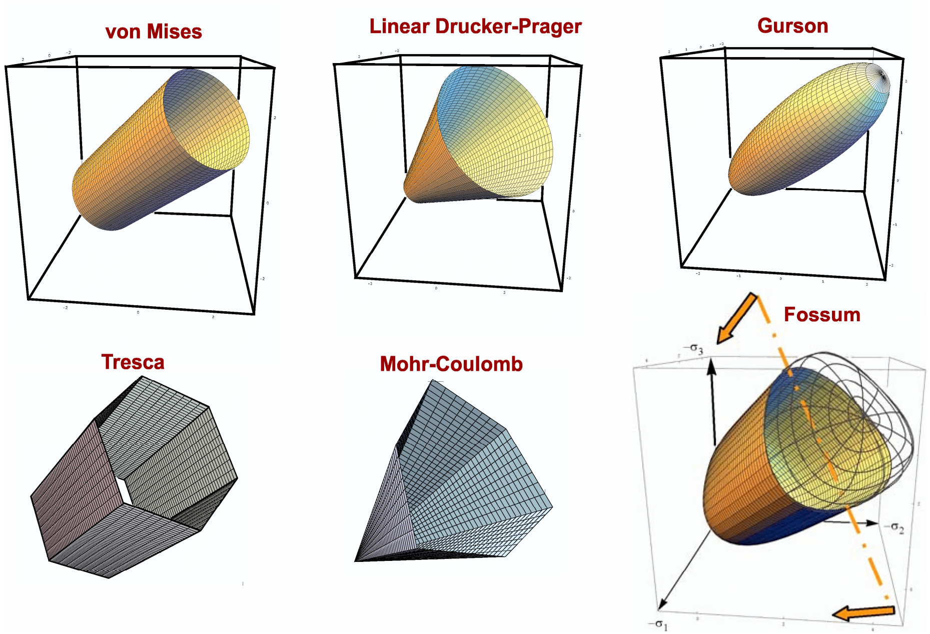
- **Acknowledgements**
  - Sandia National Laboratories
    - Arlo Fossum (original model developer -- now at BP)
  - University of Utah
    - Rebecca Brannon (model developer) and students

- **A. Fossum began development of geologic material model (1995) with many of these attributes.**
- **Subsequent development was pursued by R.M. Brannon and O.E. Strack which resulted in the Kayenta material model.**
- **Currently under joint development with University of Utah**
- **Active implementation and use in selected Sandia finite element and finite volume codes**

Kayenta Material Model

Key features

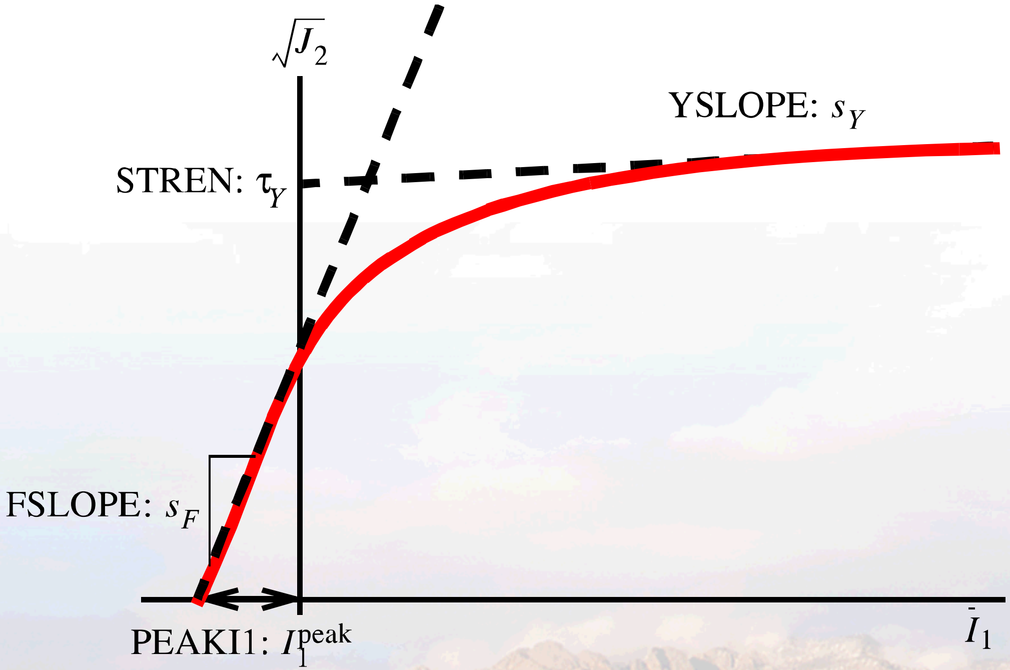
- 3 invariant, mixed hardening, continuous surface cap plasticity
- Pressure and shear dependent compaction of pores
- Strain-rate independent or strain-rate sensitive yield surface
- Nonlinear kinematic hardening accounting for Bauschinger effect
- Varying TXE/TXC strength ratio through third invariant dependence
- Peak shear threshold marking onset of softening and for fully damage material

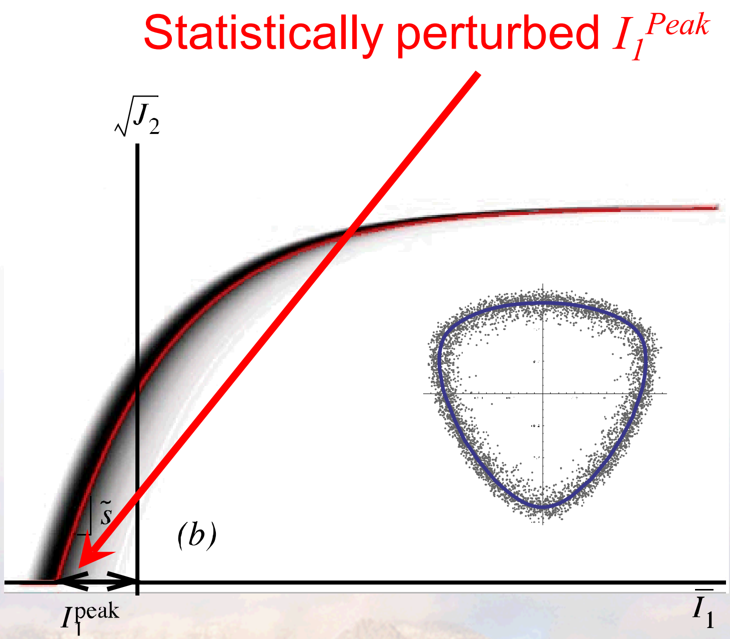


Kayenta Material Model

Parameterization of yield surface through yield function variables

- I_1^{Peak}, $S_{F,}$ $\tau_{Y,}$ S_Y

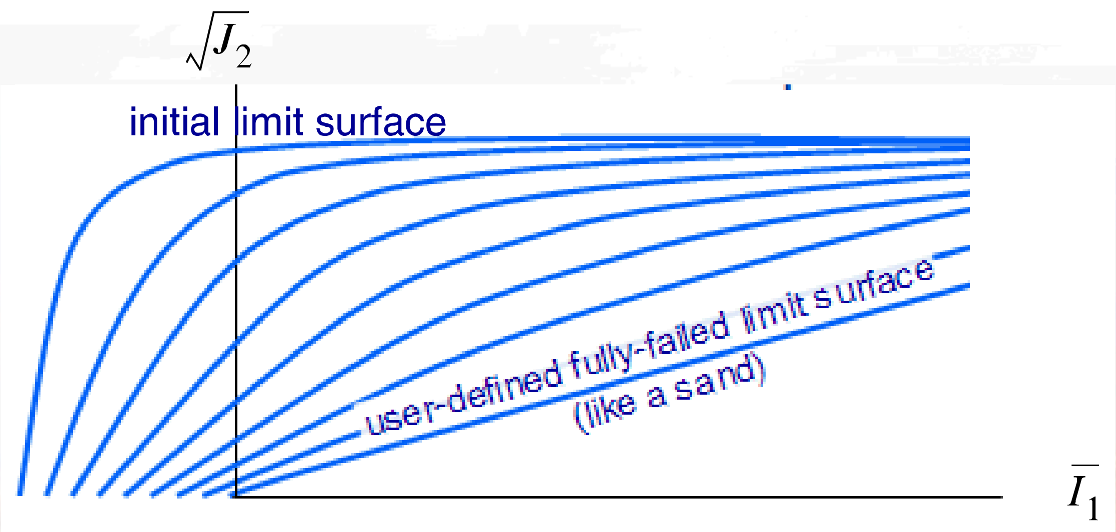




Kayenta Material Model

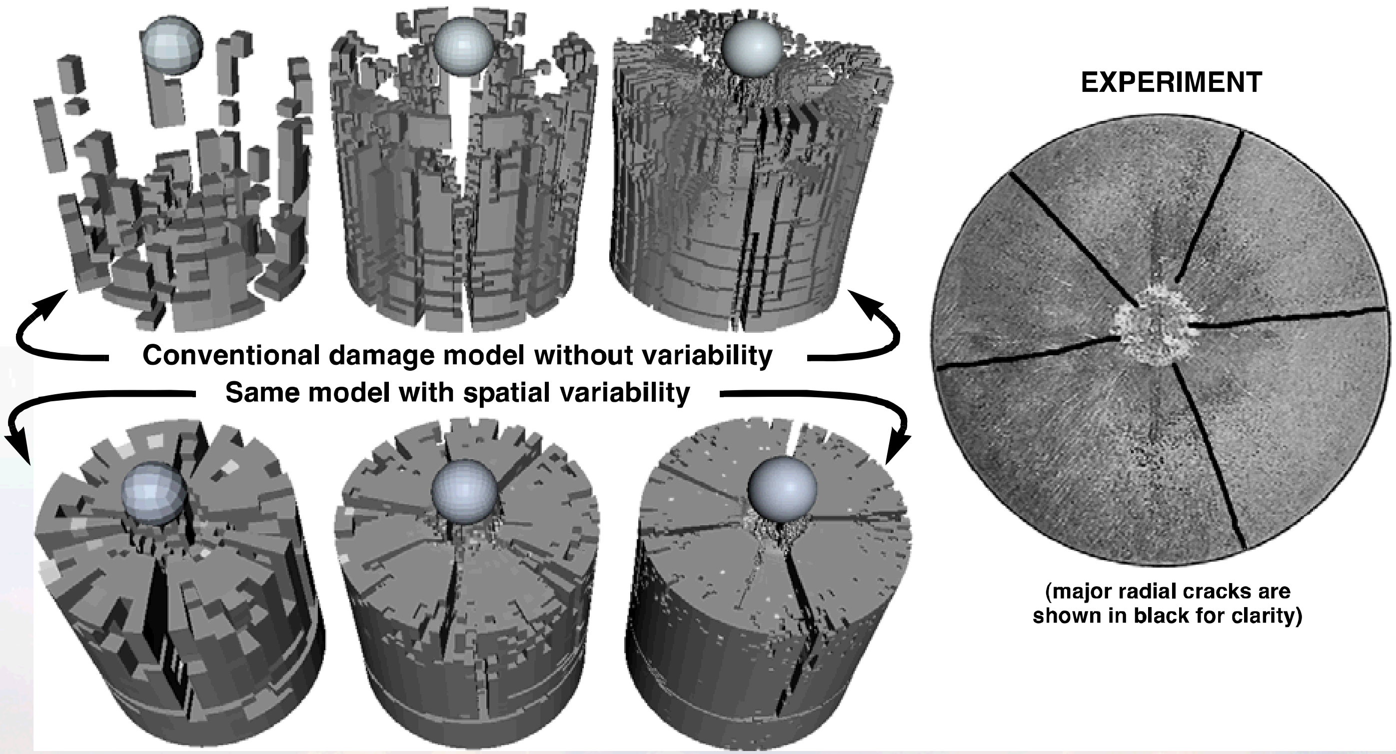
Softening response

$$I_1^{\mathrm{Peak}} = (1-D) I_{10}^{\mathrm{Peak}} + D I_{1F}^{\mathrm{Peak}}$$
$$S_F = (1-D) S_{F0} + D S_{FF}$$
$$\tau_Y = (1-D) \tau_{Y0} + D \tau_{YF}$$
$$S_Y = (1-D) S_{Y0} + D S_{YF}$$



Kayenta Material Model

Sphere indentation with softening

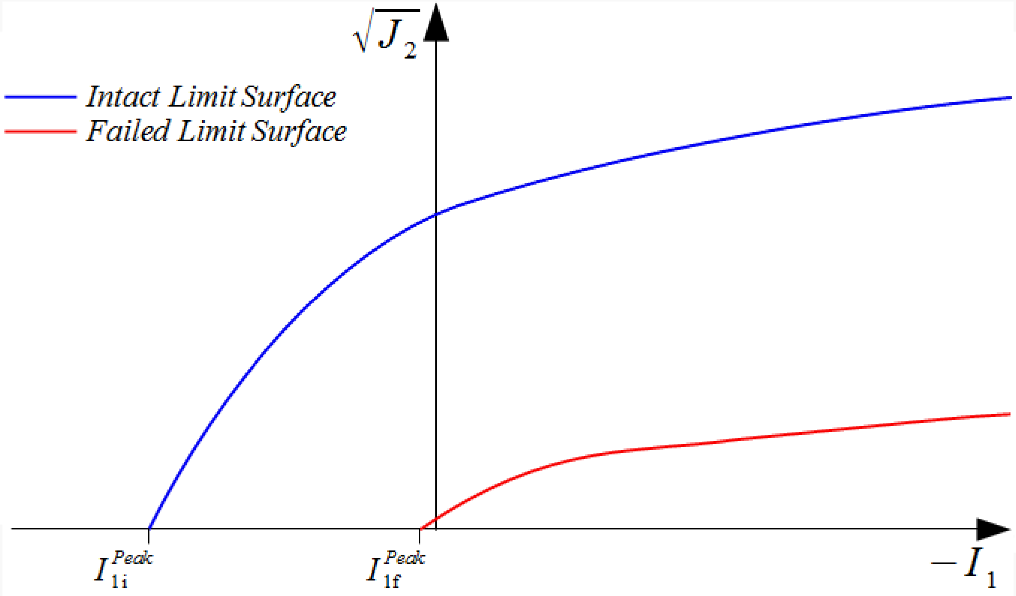


Kayenta with EOS

- Thermodynamic quantities needed by Kayenta are bulk and shear moduli.
- For low strain rates, these quantities are sufficient. However, at higher strain rates, compressions or temperatures a mechanical equation of state is useful for computing material response.
- Other thermodynamic quantities needed to complete a finite element cycle: pressure, sound speed and temperature.
- Thus, Kayenta should be paired with an equation of state model (i.e., Mie-Grüneisen or tabular EOS) to provide the material description required by the code.

Kayenta with Void Insertion Model

- The void insertion model tensile limit, corresponds to I_1^{Peak}
- As the material fails, the tensile limit for void insertion reduces.
- Void insertion model with Kayenta seeks to avoid stress states where the mean stress is below $Peak\ I_1$
- In mixed material cells (i.e., gas and solid), the Kayenta material can often be subjected to excessive expansion. Void insertion keeps I_1 from becoming too tensile and failing Kayenta.



$$I_1^{Peak} = (1-D)I_{1o}^{Peak} + DI_{1f}^{Peak}$$

$$D \equiv Damage, \quad vi \equiv \text{after void insertion}$$

$$P(\rho) < -I_1^{Peak}, \quad P_{vi}(\rho_{vi}) = -I_1^{Peak}$$

$$\rho_{vi}\phi_{vi} = \rho\phi \quad \text{(for mass conservation)}$$

$$\phi_{vi} = \frac{\rho}{\rho_{vi}}\phi, \quad \phi_{vi} < \phi \quad \text{(volume fraction after void insertion)}$$

Deformation and Density Consistency

- **Void insertion reduces material volume, raising density (from A to B until the threshold pressure is reached.**
- **Newton iteration to find ρ_A**

$$P(\rho_A) = P_A < P_{frac}$$

$$P(\rho_B) = P_{frac}$$

$$\rho_i = \rho_{i-1} + \frac{P_{frac} - P_{i-1}}{dP/d\rho}$$

- **Deformation rate modification for void insertion.**

$$\mathbf{D} \equiv \text{Deformation rate}$$

$$\Delta\varepsilon_{vol} = tr(\mathbf{D})\Delta t$$

- **Because density modified by void insertion, need to compute consistent volume change (trace of deformation rate).**

$$\Delta\varepsilon_{vol} = \varepsilon_{volB} - \varepsilon_{volA} = \ln\left(\frac{\rho_A}{\rho_B}\right)$$

$$tr(\tilde{\mathbf{D}}) = \frac{\Delta\varepsilon_{vol}}{\Delta t}$$

$$\tilde{\mathbf{D}} = Deviator(\mathbf{D}) + \frac{1}{3} tr(\tilde{\mathbf{D}})\mathbf{I}$$

Dilatation

- We assume that the equation of state model provides the pressure calculation for the material.
- However, for a failed material, Kayenta will calculate a dilated state, modeling expansion or increased pressure due to crushed material motion under shear.
- When the material dilates, the pressure form the equation of state is no longer consistent with the mean stress computed by the Kayenta model.
  - Material density decreasing while mean stress increasing
- Recognize that dilatancy results from the interaction among failed (crushed) material particles. Volume will expand, implying formation of voids in the material matrix.
- The pressure then would be representative of the density of the solid material (i.e., without the voids caused by the dilation).

Resolution of Dilation Effects

- To bring the equation of state model into consistency with the dilated state computed by Kayenta, a solid density is computed for the dilated mean stress.
- The solid density will be greater than the material density which includes the voids due to the volumetric expansion of the dilating material.
- Using an iterative scheme similar to void insertion, the equation of state is iterated on density until the dilated pressure is obtained. The resulting density is considered the solid density.

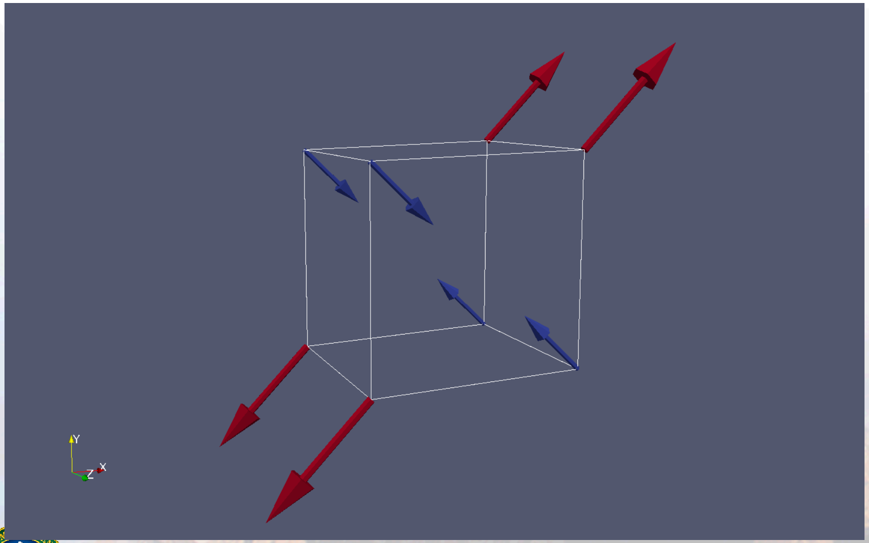
$$\rho_i = \rho_{i-1} + \frac{(-I_1 - P_{i-1})}{dP/d\rho}$$

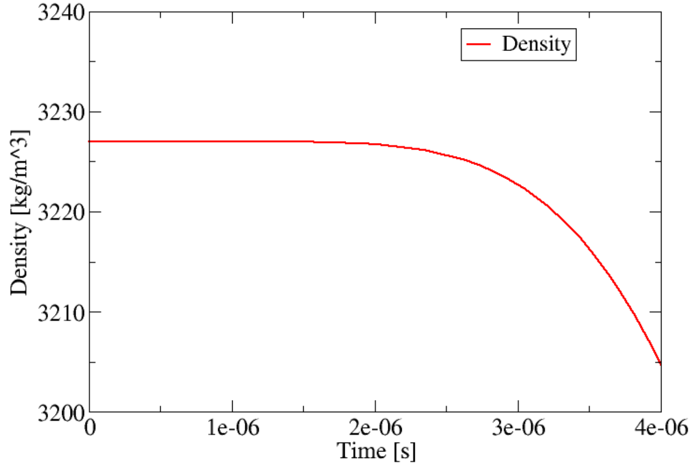
$$\text{Convergence when } P = -I_1$$

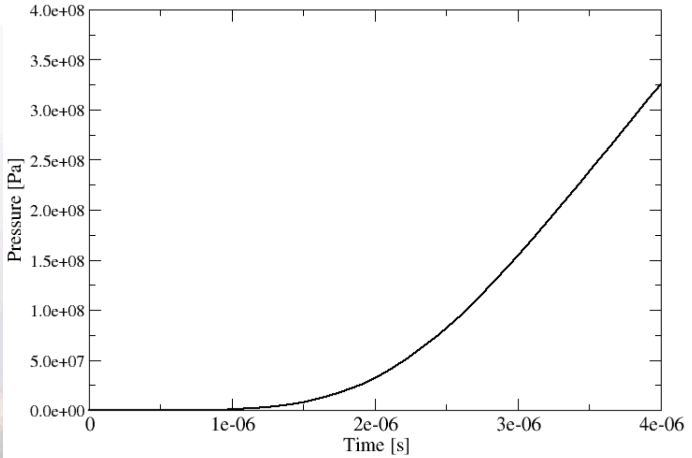
$$\text{Solid density}: \rho_s = \rho_i$$

Demonstration of Dilatation of Failed Material in Kayenta

- Single element subjected to shear forces with gradually increasing magnitude.
- Pressure increases, density decreases, indicating dilatation.
- Coherence (1-Damage), decreases.
- Solid density diverges from density.

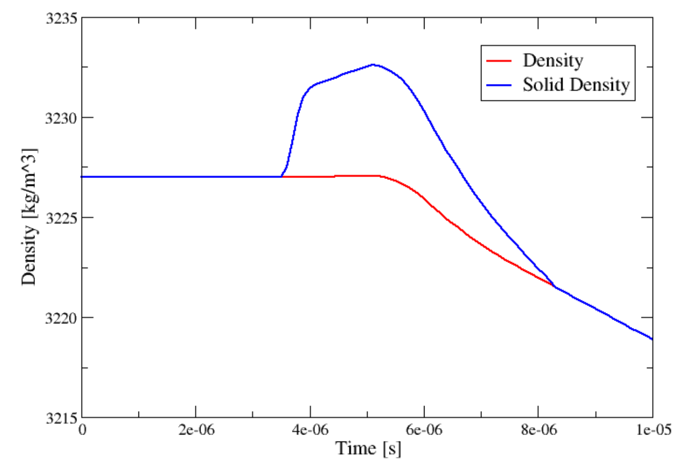


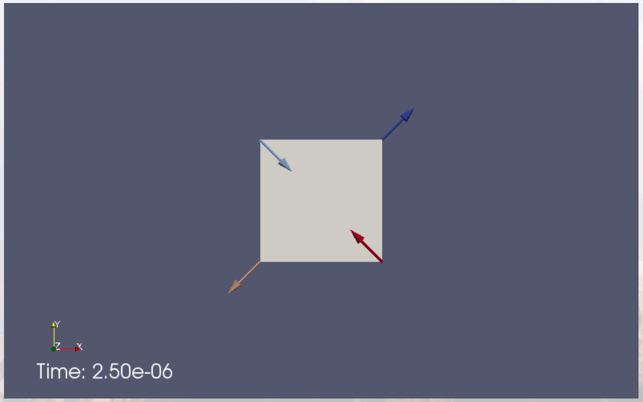


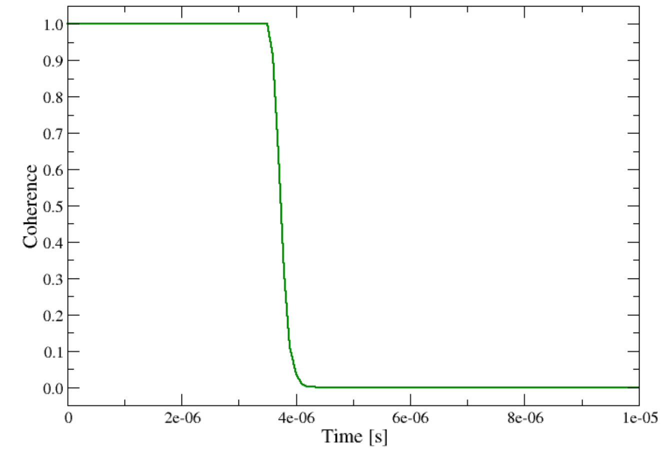


Unloading of Failed Material from Dilated State

- Single element loaded in shear (up to 5 μs), then extended vertically through 8 μs.
- Material dilates upon failure (coherence drops).
- Unloading causes solid density to reduce until it reaches material density.

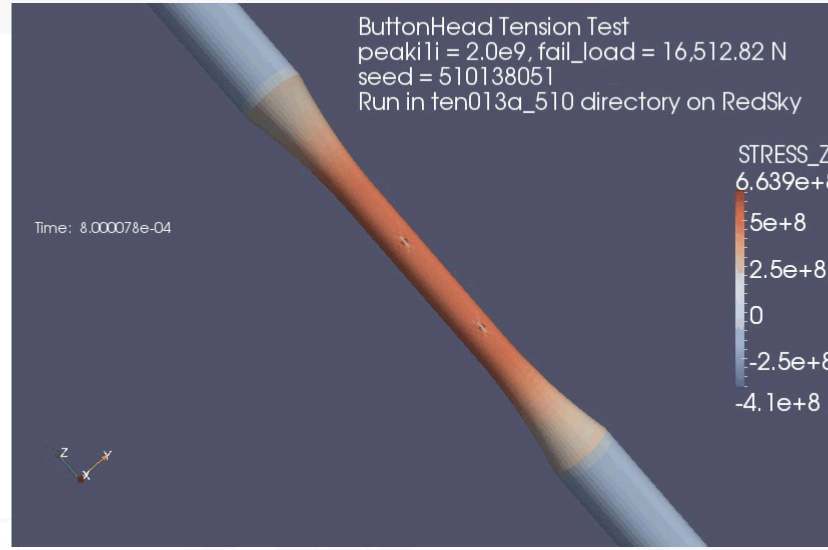


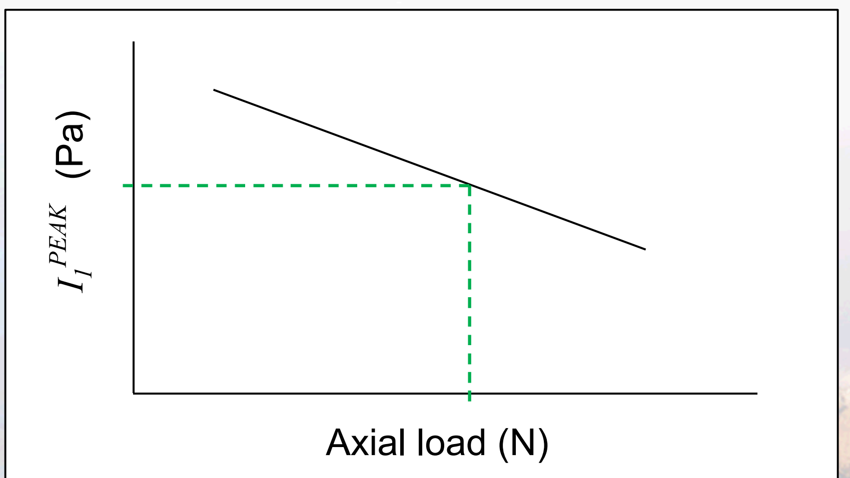


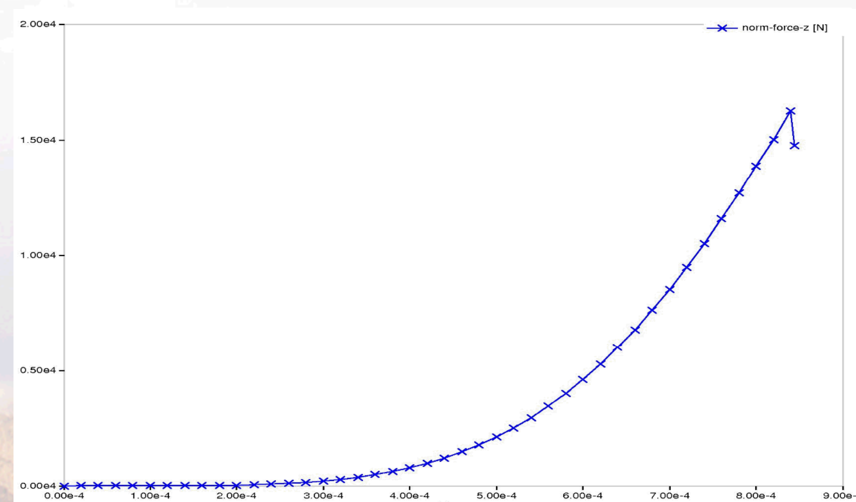


Demonstration of Tensile Failure of Brittle Material with Kayenta

- Button head tension test
- Cylindrical test specimen with tapered ends and button head for mounting in testing apparatus.
- Test provides maximum tensile load
- Series of calculations for a set of spatially variable cases run to tensile failure.
- I_1^{PEAK} selected from results of calculations over a series of I_1^{PEAK} trials

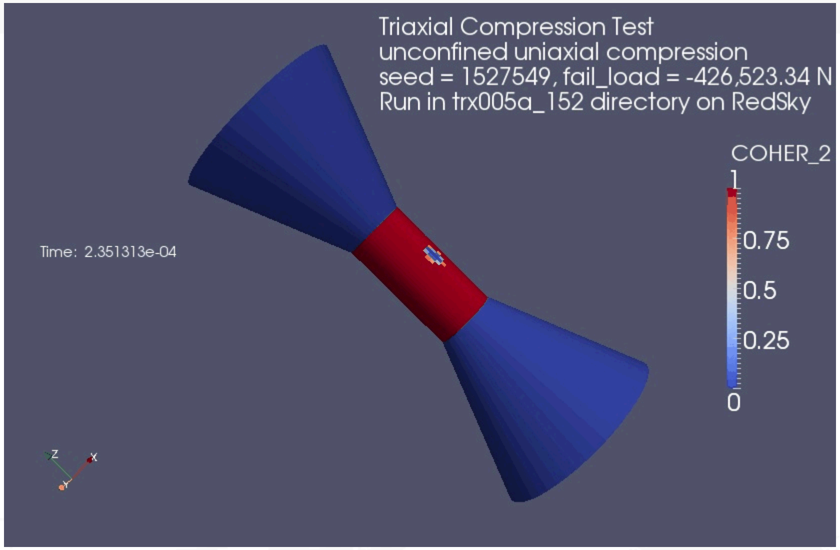


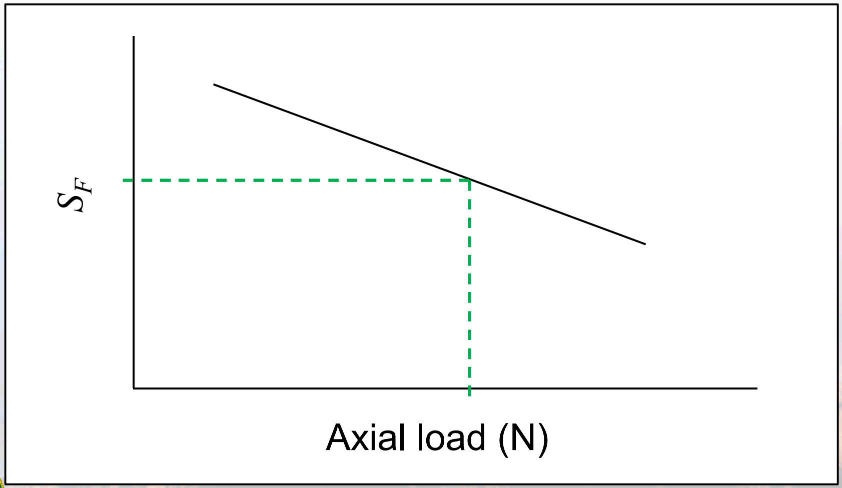


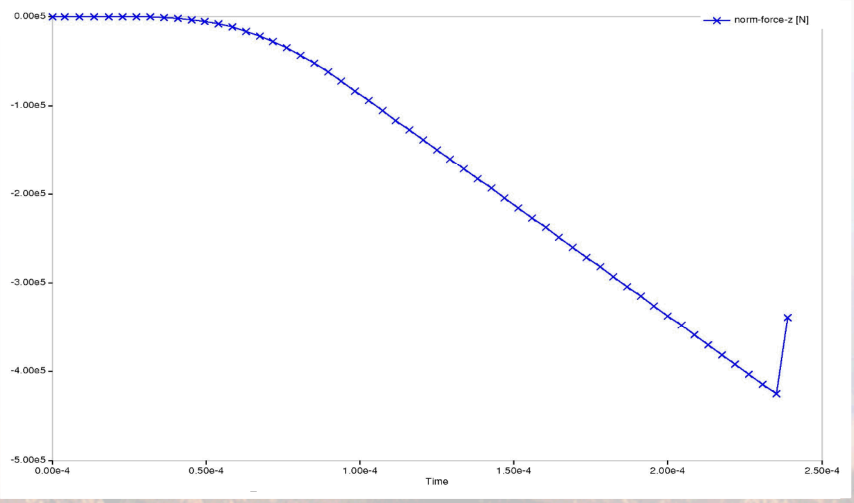


Unconfined Compression Response

- **Triaxial, Unconfined Compression**
- **Test specimen between tapered platens with a range of confining pressure**
- **Loading to failure (loss of bearing capacity) allows identification of limit surface.**
- **For unconfined case, failure related to $S_{F,}$ (fslope)**







Concluding Remarks

- Kayenta material model successfully integrated with mechanical EOS and void insertion submodels.
- Capability to manage tensile failure and dilatation
- Spatial variability used to simulate experimental results with failure localization. Provide methodology for calibrating material parameters from experimental data.