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An Active Thevenin Equivalent Network Approach to EMI/EMC Problems

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Introduction

Much of the previous EMI/EMC analysis associated with field coupling into and out of electronic circuits has assumed weak coupling between the linear electromagnetic problem and the often nonlinear circuit problem. In other words, the nonlinear circuit load is assumed to have no effect on the linear electromagnetic coupling. However, the changing impedances of the nonlinear circuit elements result in changing boundary conditions for the linear coupling and energy distribution problem. The various parts of these problems influence each other. Therefore, we need an efficient but accurate method of incorporating the changing electromagnetic (EM) coupling into the nonlinear circuit solution.

Due to its generality, multiport analysis is a common approach used for EMI/EMC and signal integrity analysis. For our purposes ports are essentially terminal pairs, defined at reference planes, that are physically close enough compared to the wavelength such that self-consistent voltages and currents can be defined. For printed circuits with lumped linear circuit elements, a multiport approach allows for the analysis of mixed signals via a broadband characterization of the entire printed circuit board. The coupling of external fields to printed circuit traces and transmission line networks is usually done with multiconductor transmission line theory. Unfortunately, this is not well suited for complicated printed circuit layouts. A Hybrid Scattering Matrix technique has been used to tackle these shortcomings (Bayram, 2007). However, this approach is difficult to implement and the results are difficult to compare directly with measurements. In the following section we will introduce an alternate approach, the Active Thevenin Equivalent Network Approach (ATHENA), which is an efficient and self-consistent (bidirectional) method for solving these problems.

Active Multiport Equivalent Network

Consider the general system shown in Fig. 1. We assume that the entire linear electromagnetic coupling problem exists external to the surface S_E, the solution to which can be obtained through any of a variety of analytical, computational or experimental techniques. These solutions can be obtained in either the time or frequency domain; however, we will assume for generality in this discussion that they are obtained in the frequency domain. Internal to S_E are equivalent lumped and distributed electric circuits and elements. These "*circuit loads*" can be linear or nonlinear and are connected to the linear EM coupling problem only through defined ports. The ATHENA approach is to develop an active Thevenin equivalent network for the EM coupling problem, connect this network with the circuit loads and use an appropriate circuit analysis algorithm (e.g. SPICE) to solve for the port voltages and currents. Once these solutions are obtained other quantities within in the circuit loads and the reradiated fields from the system can be readily determined.

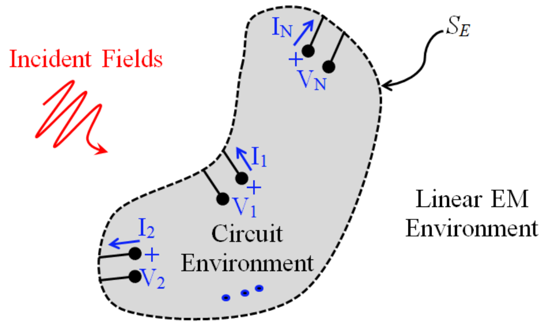

Figure 1: General N-Port Electromagnetic System.

To develop our approach, we apply the Lorentz reciprocity theorem to the EM system external to the surface S_E. We assume that at the ports (terminal pairs) within S_E quasi-static voltages and currents (V_m and I_m) can be defined. Incident electromagnetic fields are represented as sources in our equivalent network for the EM system and by reciprocity can be related to radiated fields from currents at the ports. After some analysis and assuming that the EM system is reciprocal, we obtain an active multiport equivalent network for the original EM coupling problem, as shown in Fig. 2. The voltage at each port (V_m) is defined in terms of the port currents (I_m) and equivalent voltage sources (V_m^{OC}) at each port as

$$V_m = \sum_{n=1}^{N} Z_{mn} I_n + V_m^{OC}, \qquad m = 1, 2, \cdots, N \,, \tag{1}$$

where Z_{mn} are the open-circuit impedance parameters for the network and V_m^{OC} is the open-circuit voltage at port m due to an incident electromagnetic wave. In matrix form,

$$[\mathrm{V}] = [\mathrm{Z}][\mathrm{I}] + \left[\mathrm{V}^{\mathrm{OC}}\right]. \tag{2}$$

Here $[\mathrm{V}]$, $[\mathrm{I}]$, and $\left[\mathrm{V}^{\mathrm{OC}}\right]$ are the $N \times 1$ column vectors for the port voltages, currents and open-circuit sources, respectively, and $[\mathrm{Z}]$ is the $N \times N$ impedance matrix for the electromagnetic circuit. This represents the active multiport Thevenin equivalent network for the linear EM system.

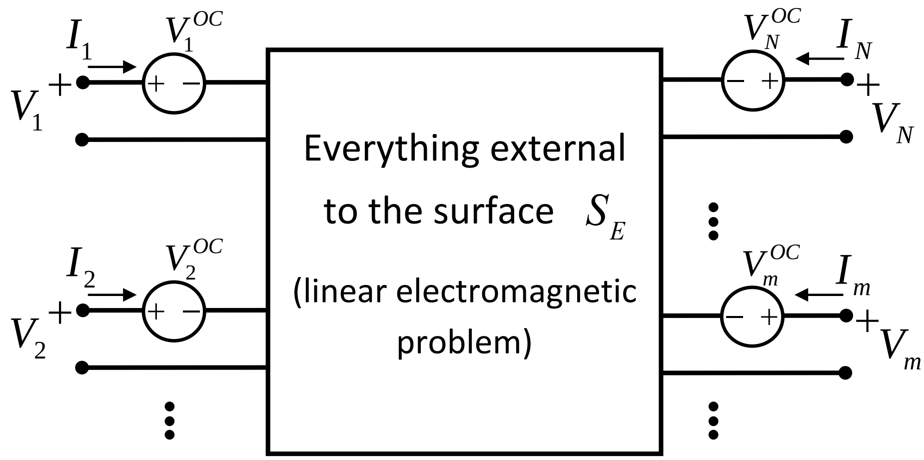

Figure 2: Active Thevenin Equivalent Network.

The open-circuit voltage at port *m* due to an incident EM wave is

$$V_m^{OC} = \frac{1}{j\omega\mu_o}\left(\mathbf{e}_o \cdot \mathbf{e}_m^T(\theta,\varphi)\right). \tag{3}$$

Here $\mathbf{e}_m^T$ is the normalized radiated electric field (normalized by $e^{-jk_0 r}/4\pi r$) due to a unit current driving port *m* and $\mathbf{e}_o = E_o\,\hat{\mathbf{p}}$, where $\hat{\mathbf{p}}$ is the specific incident electric field component of interest and E_o is the complex amplitude of the incident plane wave at the origin. Implicit to this expression, and hence the multiport network model, is that for an arbitrary elliptically polarized incident wave, two multiport network solutions are required (one for each orthogonal component of the incident field). This definition is also a statement of the reciprocal relationship between the transmit and receive patterns of a linear, reciprocal antenna.

The simplest way to determine V_m^{OC} is to open-circuit all the network ports, drive port *m* with a unit current source, determine the $\hat{\mathbf{p}}$ component of the radiated electric field, and then substitute it into (3). In general, if the ports are not open-circuited, driving port *m* with a current I_{mm} will produce non-zero currents at all the other ports (I_{mn}, current at port *n* due to current applied to port *m*), each of which will contribute to the total radiated field as

$$\boldsymbol{E}_{T,m}(\theta,\varphi) = \sum_{\mathrm{n}=1}^{\mathrm{N}} I_{mn}\mathbf{e}_n^T(\theta,\varphi)\ , \qquad m = 1, 2, \cdots, N\ . \tag{4}$$

Inverting this expression and substituting into (3) we obtain, in a general form,

$$\left[\mathrm{V^{OC}}\right] = \frac{1}{j\omega\mu_o}\mathbf{e}_o \cdot [I]^{-1}[\boldsymbol{E}_T(\theta,\varphi)] = \frac{1}{j\omega\mu_o}[I]^{-1}\mathbf{e}_o \cdot [\boldsymbol{E}_T(\theta,\varphi)]\ . \tag{5}$$

The primary purpose in developing ATHENA is to allow for the analysis of the circuits with a variety of load conditions, including nonlinear loads. To achieve this we have generalized the approach in (Liu, 1977) to analyze, in the time domain, arbitrary circuit loads driven by an active multiport network. Taking the inverse Laplace transform of (2) yields

$$[v(\mathrm{t})] = [z(t)]\otimes[i(t)] + [v^{OC}(t)]\,. \tag{6}$$

In this equation ⊗ is a convolution operation. To help understand exactly what this matrix expression represents, consider the example of a 2-port network. In this case, assuming that $i_m(t) = 0$ for $t \leq 0$ and expanding (6) yields

$$\begin{bmatrix} v_1(\mathrm{t}) \\ v_2(\mathrm{t}) \end{bmatrix} = \begin{bmatrix} \int_0^t z_{11}(t-\tau)i_1[v_1(\tau)]d\tau + \int_0^t z_{12}(t-\tau)i_2[v_2(\tau)]d\tau \\ \int_0^t z_{21}(t-\tau)i_1[v_1(\tau)]d\tau + \int_0^t z_{22}(t-\tau)i_2[v_2(\tau)]d\tau \end{bmatrix} + \begin{bmatrix} v_1^{OC}(t) \\ v_2^{OC}(t) \end{bmatrix}. \tag{7}$$

In the limit as $\delta \rightarrow 0$, and suppressing the dependence of current on voltage for the sake of space, we obtain

$$\begin{bmatrix} v_1(\mathrm{t}) - \tilde{z}_{11}(0)i_1(\mathrm{t}) - \tilde{z}_{12}(0)i_2(\mathrm{t}) \\ v_2(\mathrm{t}) - \tilde{z}_{21}(0)i_1(\mathrm{t}) - \tilde{z}_{22}(0)i_2(\mathrm{t}) \end{bmatrix} = \begin{bmatrix} \int_0^{t^-} z_{11}(t-\tau)i_1(\tau)d\tau + \int_0^{t^-} z_{12}(t-\tau)i_2(\tau)d\tau \\ \int_0^{t^-} z_{21}(t-\tau)i_1(\tau)d\tau + \int_0^{t^-} z_{22}(t-\tau)i_2(\tau)d\tau \end{bmatrix} + \begin{bmatrix} v_1^{OC}(t) \\ v_2^{OC}(t) \end{bmatrix}, \tag{8}$$

where

$$\tilde{z}_{mn}(0) = \lim_{\delta\rightarrow 0}\int_{t-\delta}^{t} z_{mn}(t-\tau)d\tau \approx z_{mn}(0)\Delta t\,. \tag{9}$$

and Δt is the time step in a discretized implementation of this approach. These expressions can be readily generalized for N-ports, discretized and implemented as an N-port device in a circuit code, such as SPICE.

Example Results

Here are two examples that involve a two-port circuit. Although the circuit is the same in both instances, calculations are performed for the linear load and the nonlinear load cases. The circuit, a distributed excitation example from (Paul, 2008), consists of a single thin wire line over a ground plane illuminated by a normally incident plane wave. The geometry of the problem is shown in Fig. 3. The 20mil diameter wire is 1m long and suspended 2cm above the ground plane. For all the calculations the circuit load for Port 2 is a 1kΩ load resistor. For Port 1, two different loads are considered: a linear 500Ω resistor and a nonlinear diode.

The exciting electric field, shown in Fig. 3, is a plane wave incident normal to the ground plane and aligned with the transmission line axis. Its magnitude increases linearly from zero to its maximum value in 1ns. For the resistive load case, the maximum amplitude is 1V/m. For the diode load case, the maximum amplitude is increased to 100V/m in order to provide enough excitation to turn on the diode.

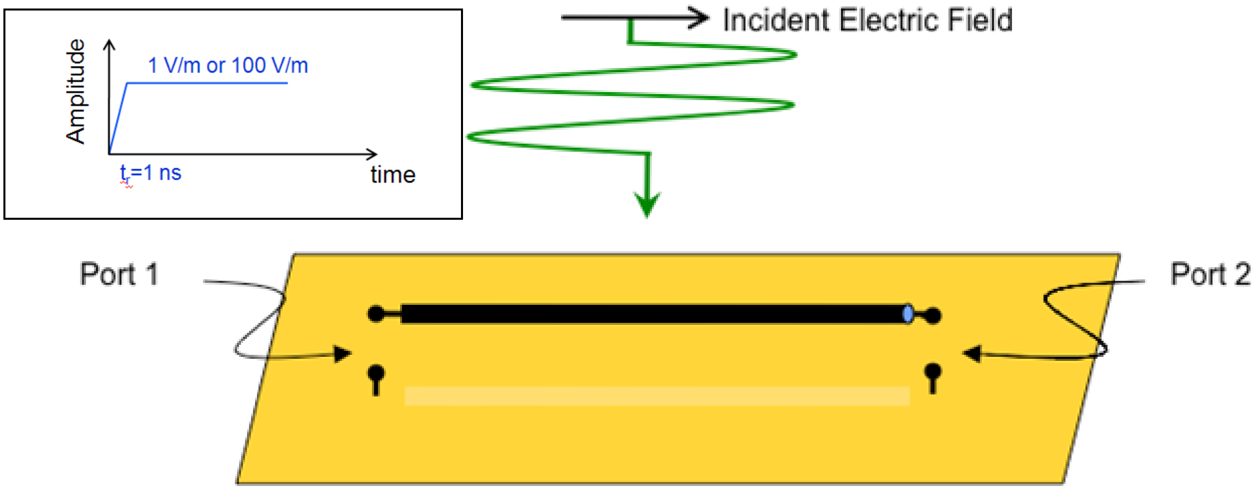

Figure 3: Two-Port Representation of a Single-Wire Transmission Line over a Ground Plane.

To obtain the two-port ATHENA device for this problem, we need to determine $[z]$ and $[v^{OC}]$ at the port terminals shown in Fig. 3. To solve for these quantities a frequency domain full-wave electromagnetic simulation using the method-of-moments wire code NEC is used. The frequency spectrum is calculated from 1MHz to 3000MHz in 1MHz steps. An inverse FFT was used to obtain the time domain representations and the open source code *ngspice* augmented by an ATHENA device is used to solve the resulting system.

Linear Resistor Loads

Figure 4 shows the Port 1 voltage when the circuit loads are linear resistors, Port 1 load R_{L1} = 500Ω and Port 2 load R_{L2} = 1000Ω. The structure of the resulting time domain waveforms is easily understood. The incident field excites the line along its entire length simultaneously due to its normal angle of incidence. Since the incident electric field is polarized along the wire, the voltage at the Port 1 end goes negative with respect to the ground plane as the field pulls positive charge from the ground through the 500Ω resistor. The voltage induced at the right-hand end is positive as charges flow through the 1000Ω resistor toward ground. Each of these terminal voltages launches a voltage wave traveling along the line at the speed of light. These waves reach the opposite ends of the line 3.3ns later and are reflected. Since both of the termination resistors exceed the ~303Ω line impedance, the reflection coefficients are positive. These solutions are in excellent agreement with the time domain solutions in (Paul, 2008).

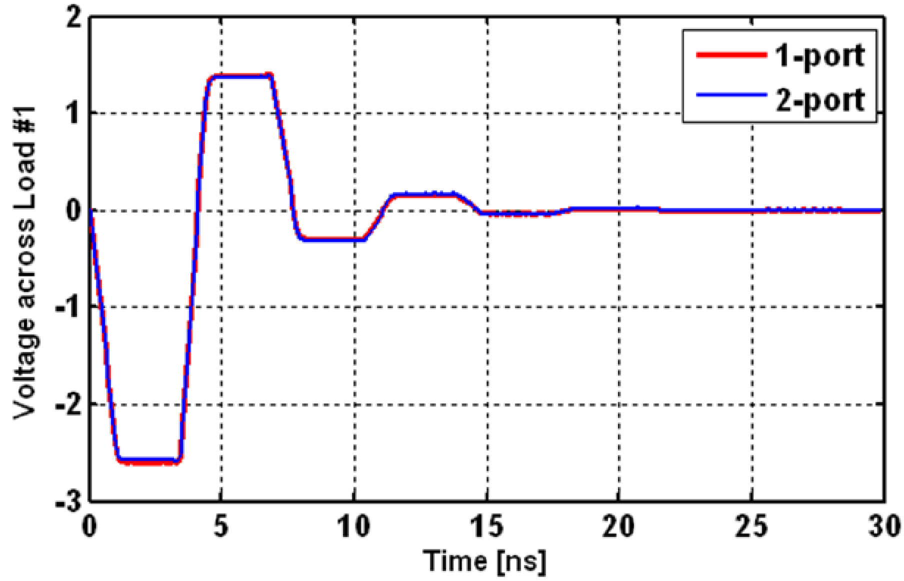

Figure 4: Port 1 voltage when ports are terminated by 500Ω and 1kΩ resistors. Comparison is made with between 1 and 2-port ATHENA models.

Non-Linear Diode Load

For the final example, the two-port model with a nonlinear diode load terminating Port 1 and a 1000Ω resistor terminating Port 2 is considered. For the diode,

$$i_1 = -I_S\left(e^{v_1/nV_T} - 1\right), \tag{10}$$

where $I_S = 10^{-15}$A, $V_T = 25$mV, and $n = 1$. Figure 5 shows a plot of the Port 1 voltage. We observe from this response that when the voltage initially goes negative, the diode is biased off, so the load is essentially an open circuit. Thus, v^{OC} and the diode response track each other. When the voltage wave excited at the Port 2 end of the transmission line arrives at Port 1, the voltage swings positive until the diode turns on and clamps the voltage to the diode's saturation value. This results in a negative reflection coefficient, since the impedance of the saturated diode is much lower than the transmission line impedance. The negative wave launched at this time reappears each time v^{OC} swings positive because it is reflected back from the Port 2 end.

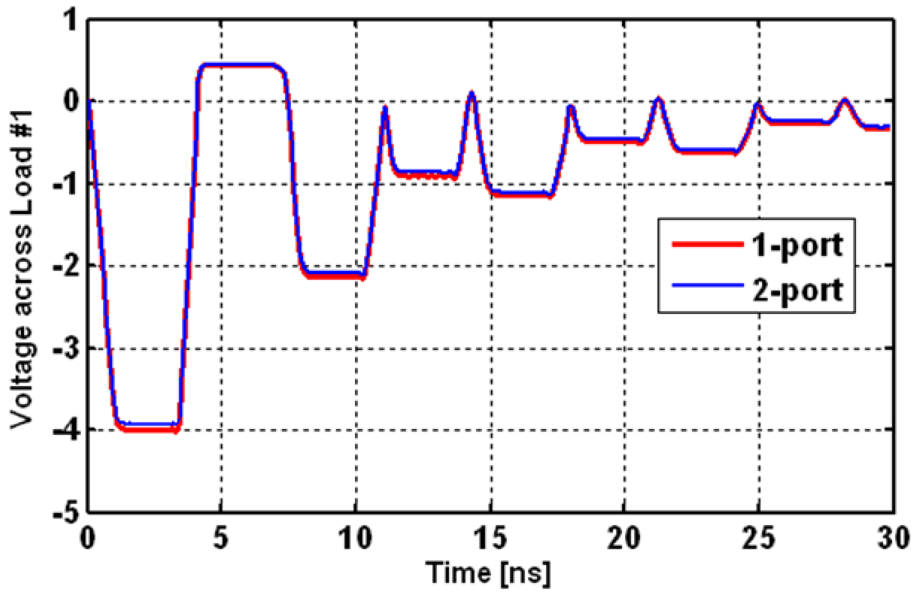

Figure 1: Port 1 voltage when Port 1 is terminated by a diode and Port 2 by a 1kΩ resistor. Comparison is made with between 1 and 2-port ATHENA models.

Summary

In this work, we have established the theoretical foundation of the Active Thevenin Equivalent Network Approach (ATHENA), an efficient approach for incorporating EM fields self-consistently into circuit simulations. We have defined and run test and validation problems in simple circuit codes to debug and validated the algorithm for one and two port devices with both linear and nonlinear loads. The work has also been extended and validated for general N-port networks. A multiport ATHENA device has been developed to run in the open-source Spice *ngspice*. Other circuit solvers can be employed with this approach, but a new circuit device will have to be added to each to represent the multiport equivalent network.

References

Bayram, Y. and J. L. Volakis. 2007. *Hybrid S-parameters for Transmission Line Networks with Linear/Nonlinear Load Terminations Subject to Arbitrary Excitations*. IEEE Trans. Microwave Theory and Techniques, Vol. 55, No. 5 (May), pp. 941-950.

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