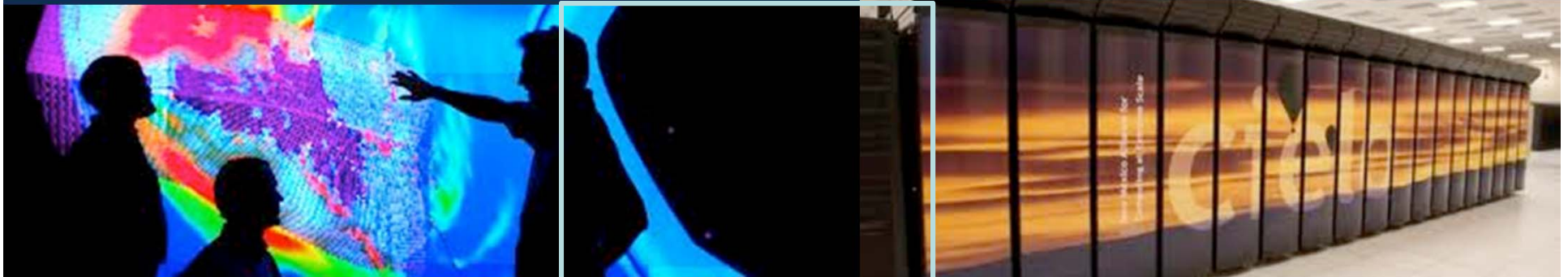


Exceptional service in the national interest



IMAC 2014 Keynote: Dynamics of Coupled Systems: Maybe We Are Not As Good As We Think

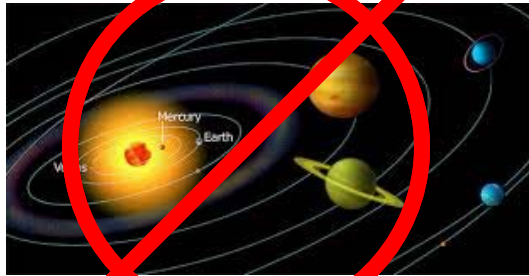
Dan Segalman

Sandia National Laboratories
For the moment!

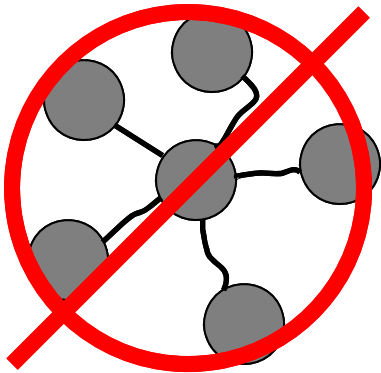


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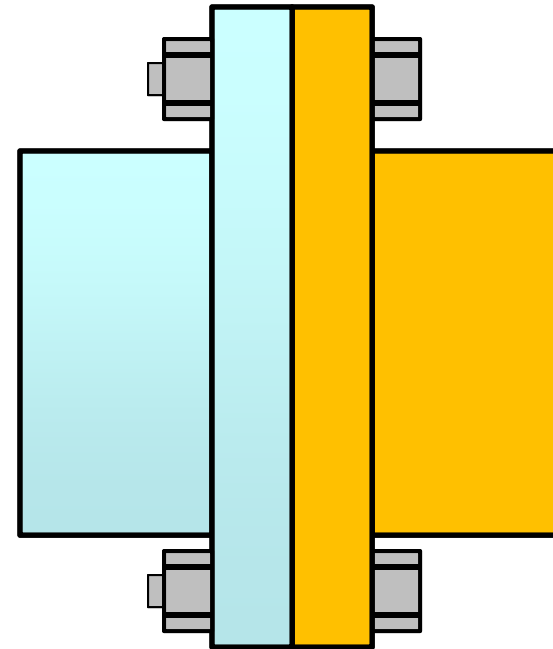
What I Mean by Multi-Body Dynamics for this Talk



NOT Celestial Mechanics



NOT Tethered Systems



Substructures attached to each other via standard connections, such as screws, bolts & nuts, tape joints, ...

Modal Analysis Links

Finite Element Structural Dynamics & Reality

Experiment:

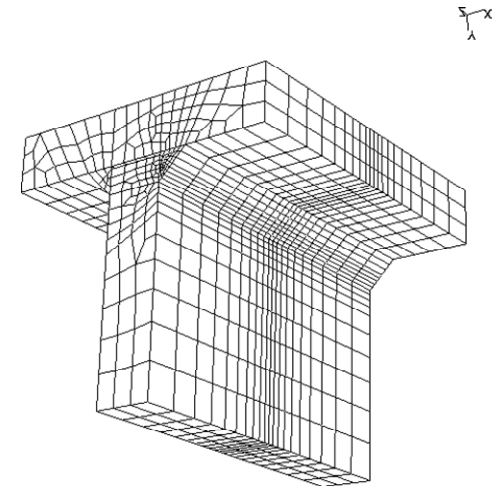
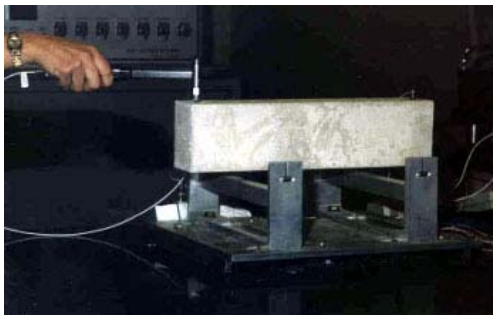
- Accelerations (Including spatial filtering, time domain, and frequency domain)
- Strains
- ...



Modal
Interpretation



Structural
Dynamics
Model



(Among other things, Modal analysis is used to assure the absence of major mesh errors..)

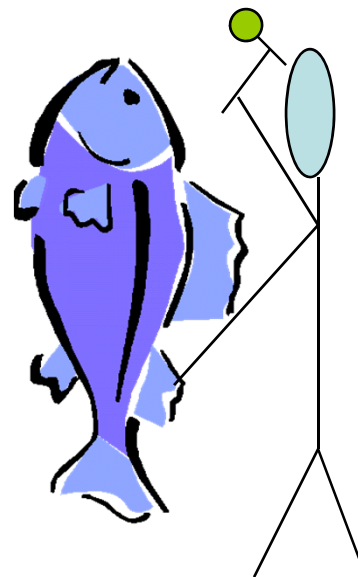
Traditional Approach to Structural Dynamics

Build full structure or subsystem and test in modal lab at relevant amplitudes

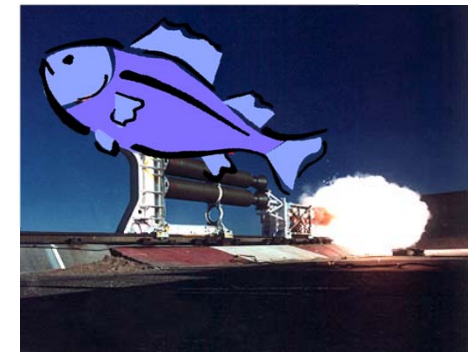
Systems test is performed on updated model



Analyst creates coarse mesh of model putting tunable springs at interfaces and postulating proportional/modal damping



Analyst tunes joint stiffness and modal damping to match test. Predictions follow.



Some Words About How We Currently Develop Structural Dynamics Models

Elements of Process

- Assume system to be linear
- Represent each joint DOF as a linear spring
- Build and test a prototype structure
- Tune the spring stiffnesses to match frequencies
- Tune modal (or more complicated) damping to match damping of structure

Let's Test Our Basic Understanding

Consider two plates: $T_B = 2 T_A$

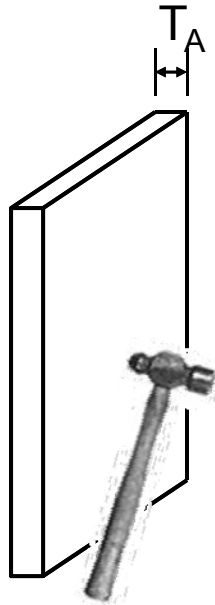


Plate A

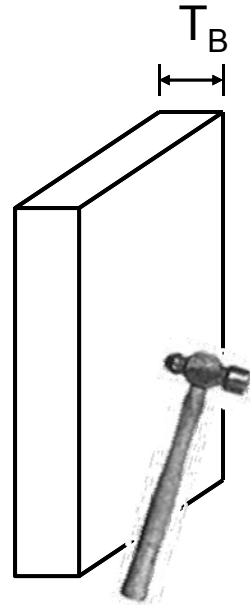
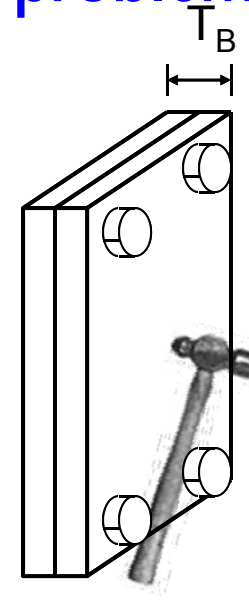


Plate B



Consider our first multi-body problem

$$K \approx T^3 \ \& \ M \approx T \ \Rightarrow \ \omega^2 \approx T^2$$

Let's try this out!

6 This we understand!

Let's test this one!

There is something puzzling here!

Observations

- We can predict the vibration response of monolithic pieces of metal with great precision.
We can do this with a modest number of solid and acoustic elements. (This includes acoustic radiation, Zener damping, and the miniscule bulk damping,)
- That we must go to experiment to select damping parameters means that we do not understand the damping mechanisms.
- We shall never be able to improve our ability to predict the behavior of real structures without a much better understanding of the underlying physics of interfaces.

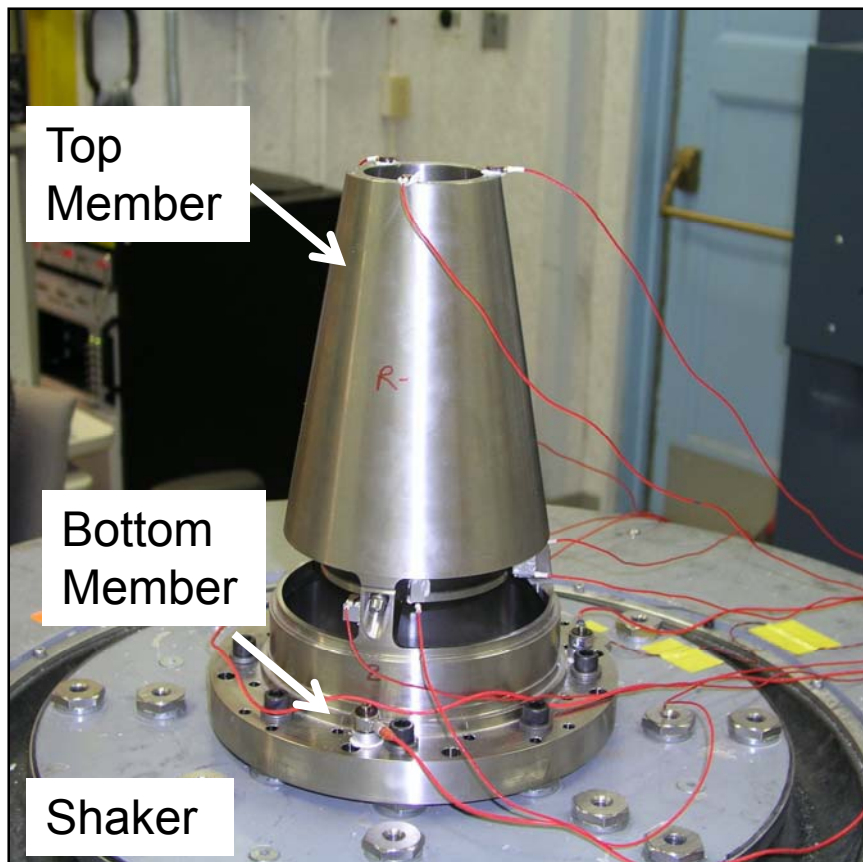
About Being Predictive

- If you have to build the full structure in order to predict structural response, then you are not predictive.
- There is a distinction between forward prediction and backwards prediction.

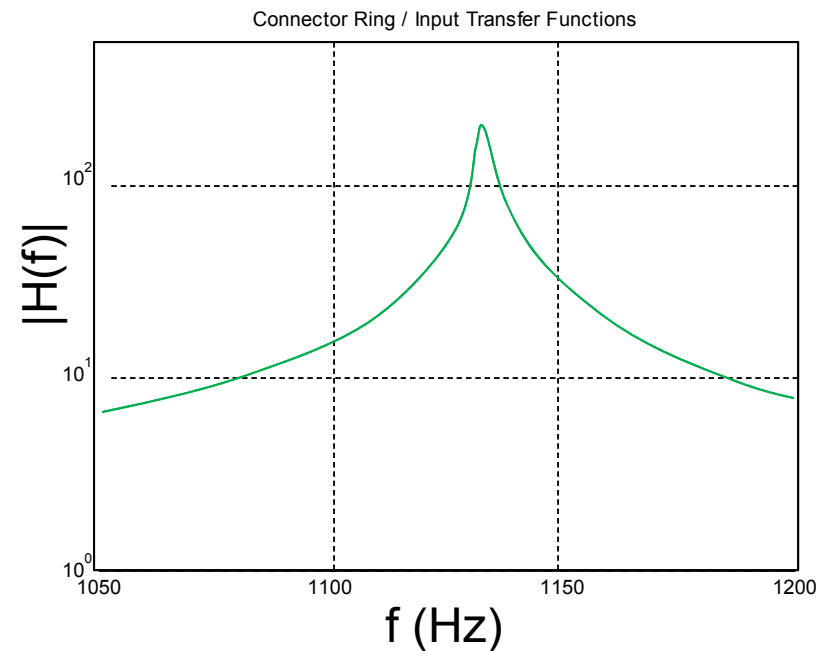
Significance of NOT being predictive

- For instance, consider test design. We employ transfer functions from our test device to the locations on our specimen to design our control signal.

Simple Structure with Only 2 Parts: Frequency Response Functions With Increasing Input Level



Shaker/Controller systems are *designed* to be linear and so that a unique transfer function exists for each linear specimen.



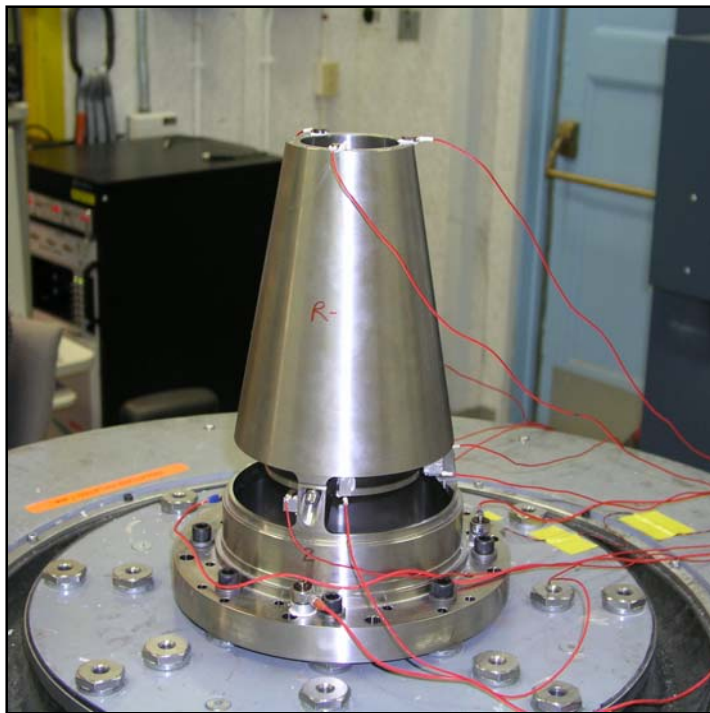
Nonlinearity of the structure affects how a system containing it looks to a controller.

Credit Dan Gregory & Brian Resor, SNL

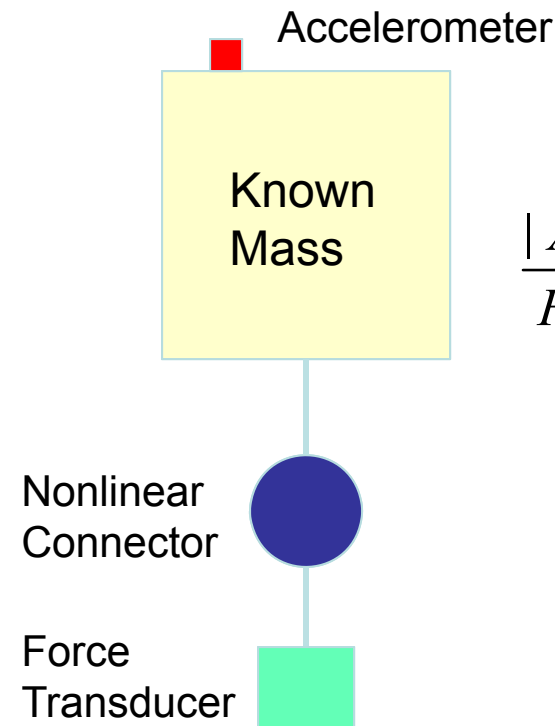
This Illustrates that:

Linear System
+ Nonlinear System
= Nonlinear System

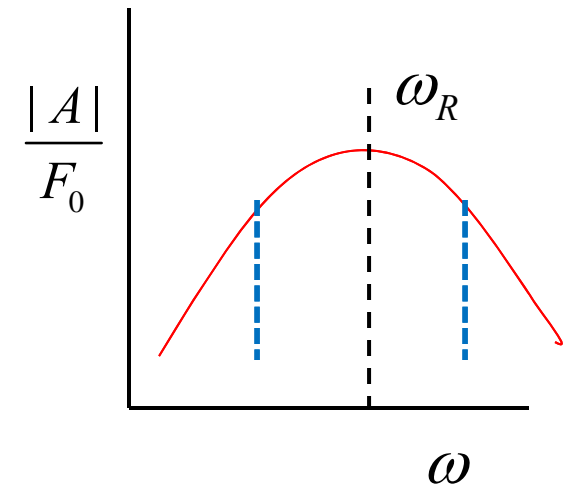
Properties of Our 3-Legged Structure From Harmonic Sweep at Fixed Load Amplitude.



$$F = F_0 \sin \omega t$$



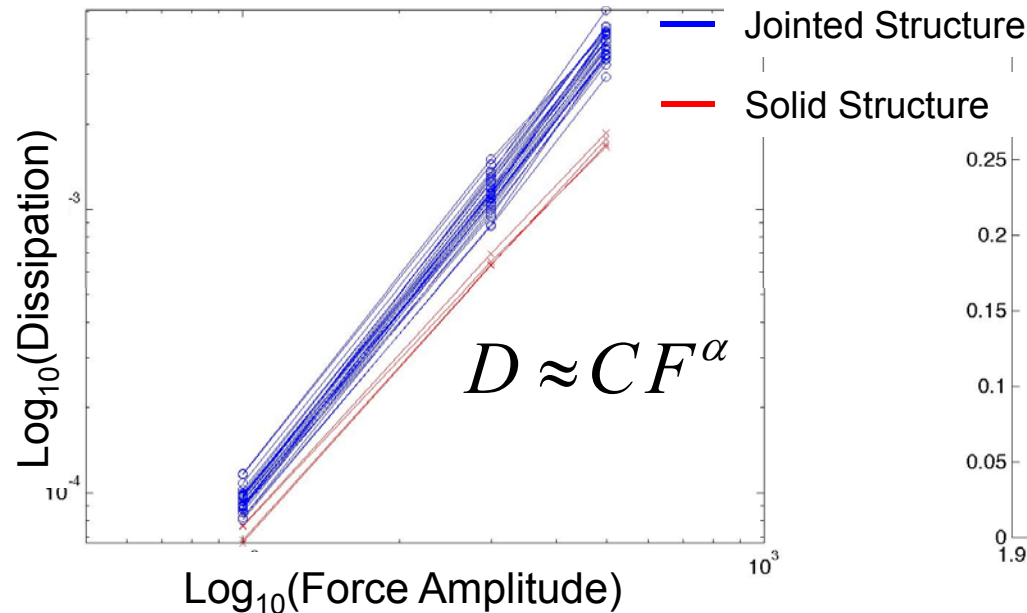
$$F = F_0 \sin \omega t$$



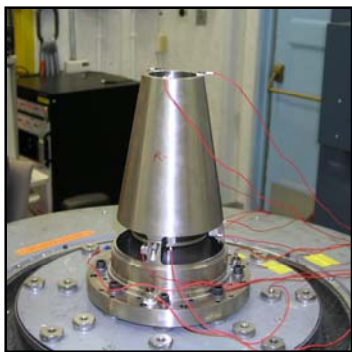
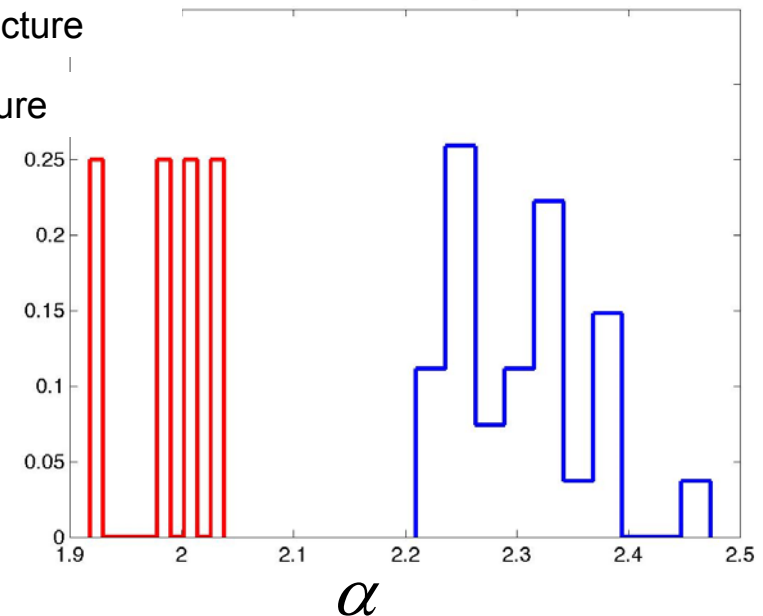
⇒ Equivalent **stiffness** and **damping** of nonlinear connector

Properties of Our 3-Legged Structure From Harmonic Sweep at Fixed Load Amplitude.

Dissipation vs Force Amplitude



Power-Law Slope

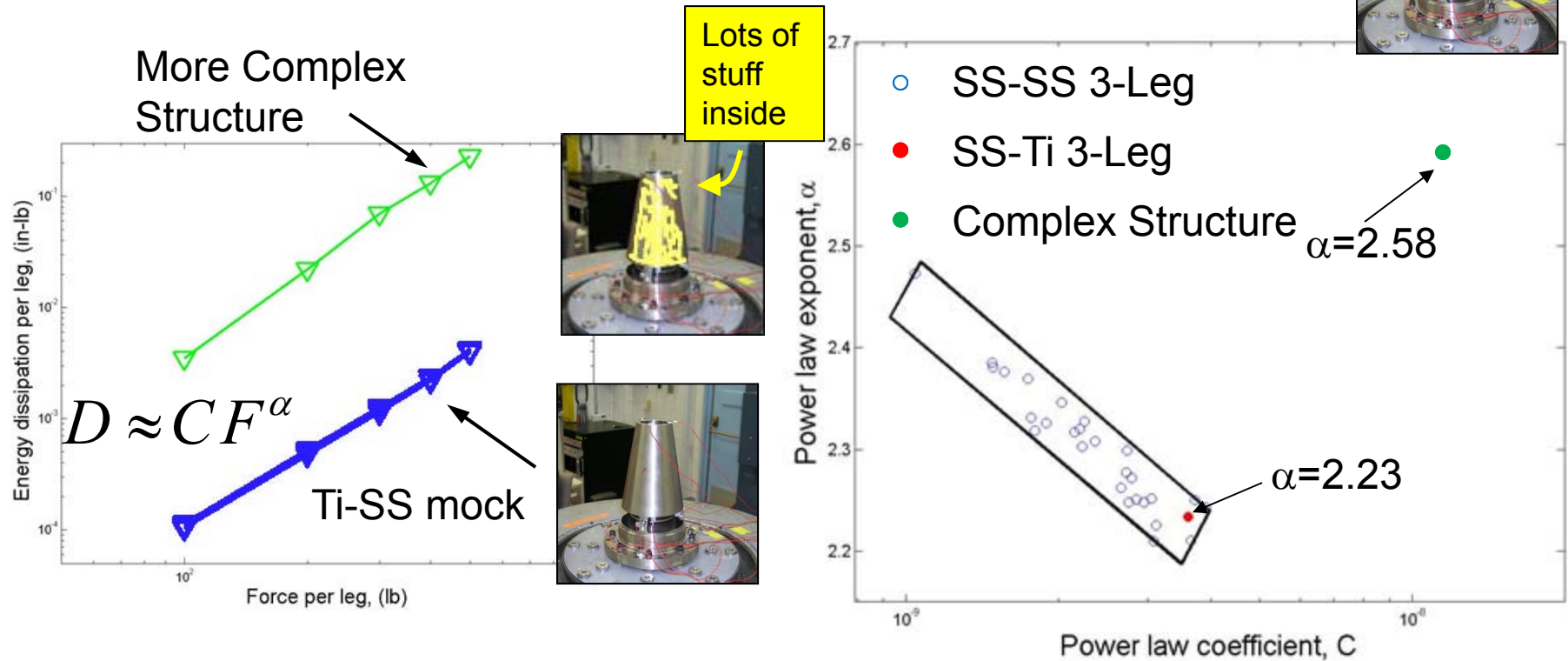


A Power-Law relationship between force amplitude and dissipation

- Linear structures dissipate energy with power law slope of exactly 2.
- The unjointed structure dissipates as if linear.
- The jointed structure is qualitatively nonlinear.

What Happens When We Add a LOT More Joints?

Energy Dissipation of More Complex Systems is Significantly Different

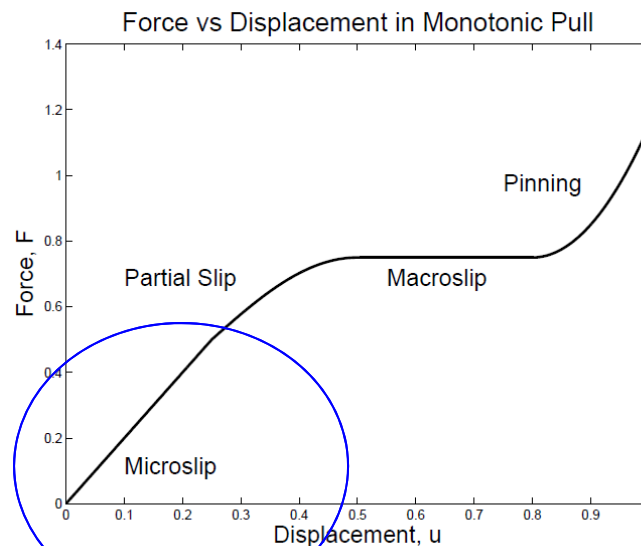
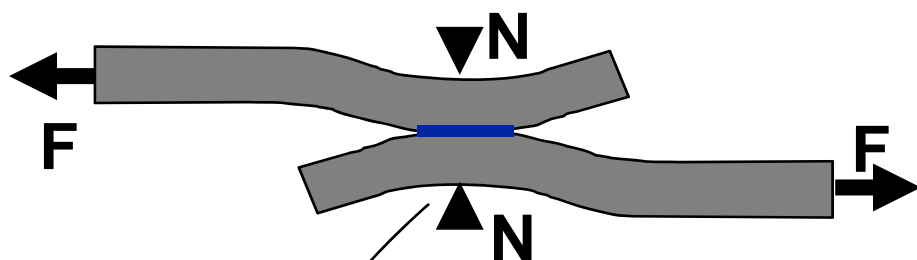
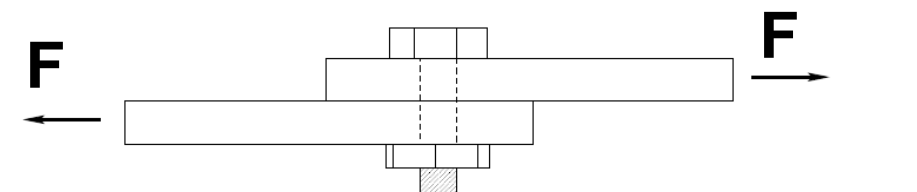


More complex structure is even more non-linear than the simpler structure: **More Joints = More Nonlinearity**

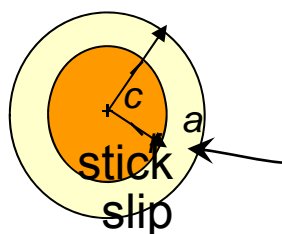
Conclude

- The nonlinearity is associated with frictional interfaces/joints.
- The strongest nonlinearity seems to be associated with damping.
- The more joints, the more nonlinearity – as measured by α .
- To deal with this nonlinearity of structures, we must know something about nonlinearity of the joints.

About Mechanical Joints



Structural Dynamics Regime



Slip Annulus

Contact Patch

The problem is intrinsically multi-scale

Size of Structure

>>

Size of Contact Patch

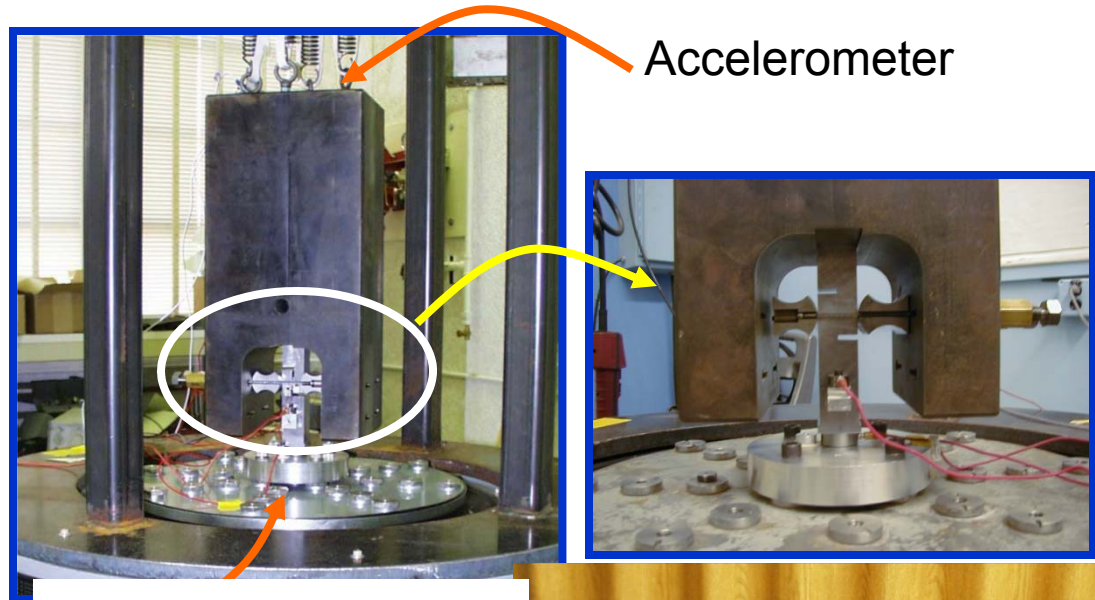
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Thickness of slip annulus

Experimental Micro-slip Setups

Key Elements

- Harmonic experiments minimize signal to noise issues.
- Use of jointed and corresponding solid specimens to identify the contributions of the interface to dynamic response.
- Design of experimental apparatus to minimize impact of mounting interfaces.
- Use of many specimens to assess intrinsic variability and assembly/disassembly effects.

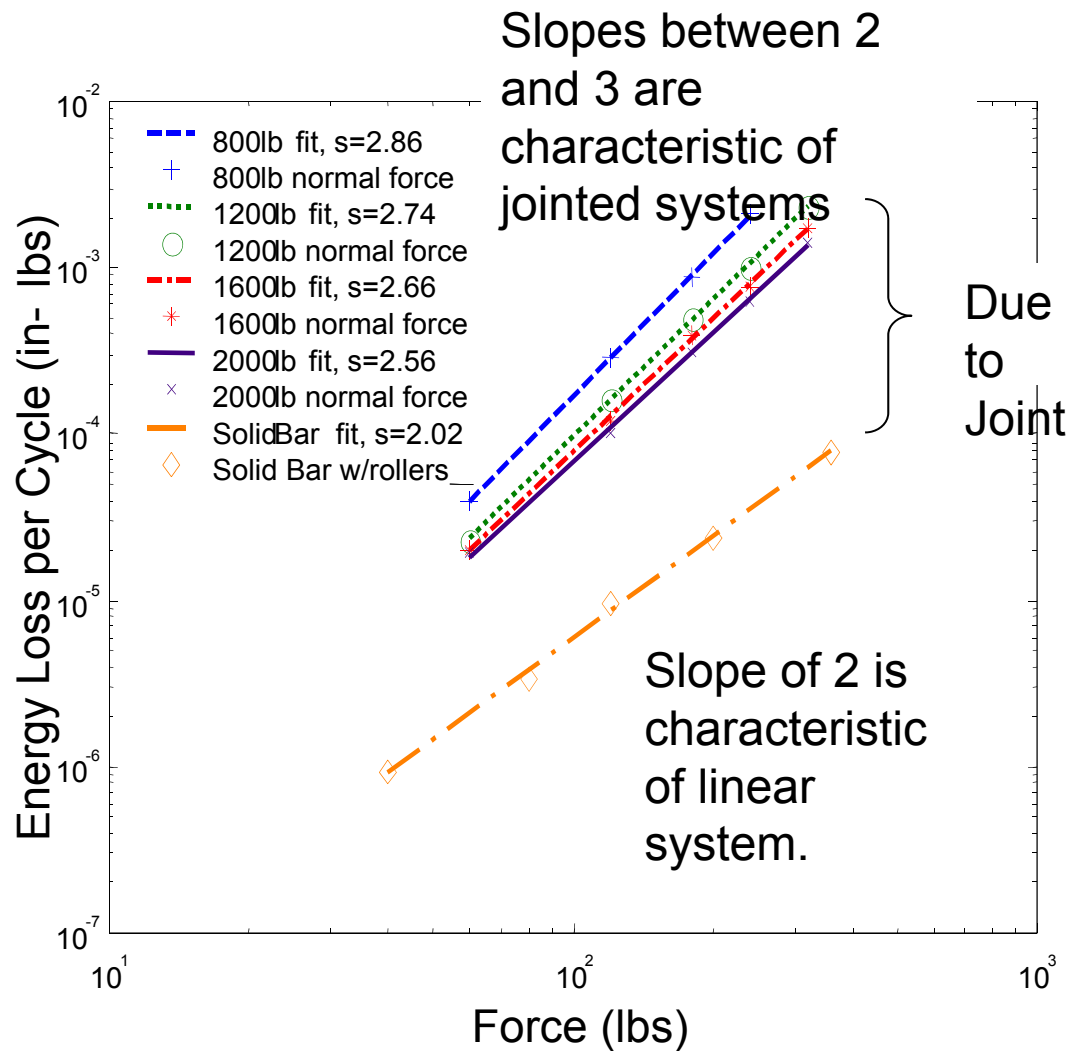


Accelerometer

Force Transducer



Energy Loss per Cycle for Simple Shear Loading



The Beginning of an Approach to Accommodate Joint Nonlinearities

What would be the first step to bring more physics into the analysis?

- Explicitly account for the joint nonlinearity
- Place a joint model at the location of the actual joint.

Strategy

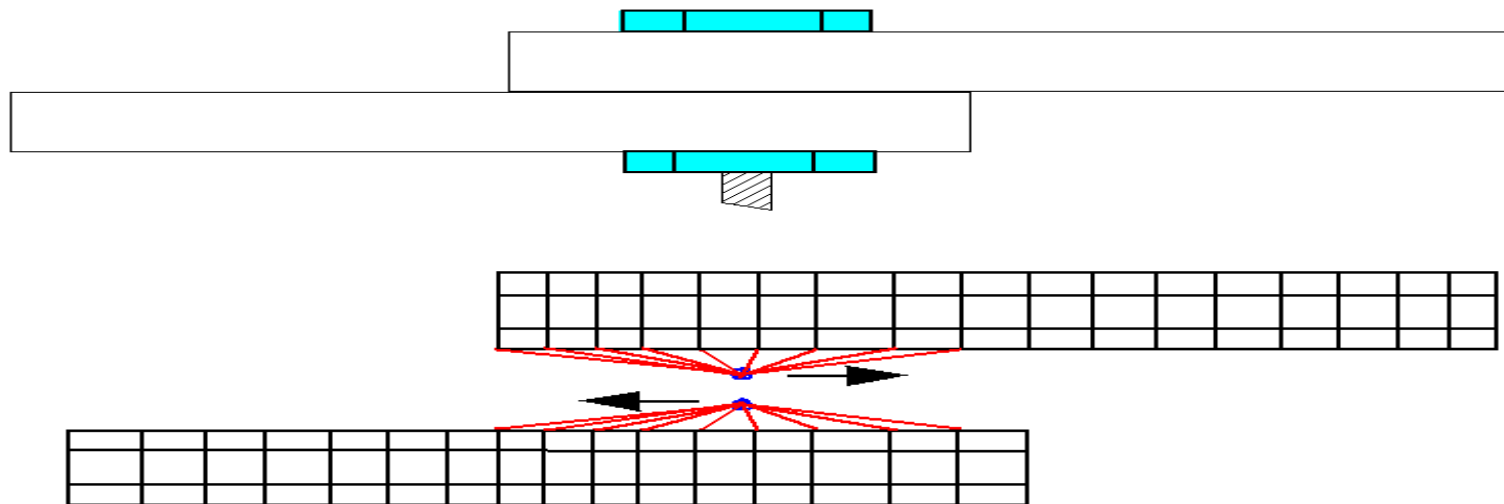
- Represent the whole joint with a small number of scalar constitutive models.
- Determine the parameters of these models either from micro-modeling or from experiments on individual joints.

D.J. Segalman ASME Journal of Applied Mechanics, V. 72, 752 (2005)

D.J. Segalman, Structural Control and Health Monitoring
V. 13, Issue 1, (2006)

If We Can Model Joints, Maybe We Can Model Jointed Structures

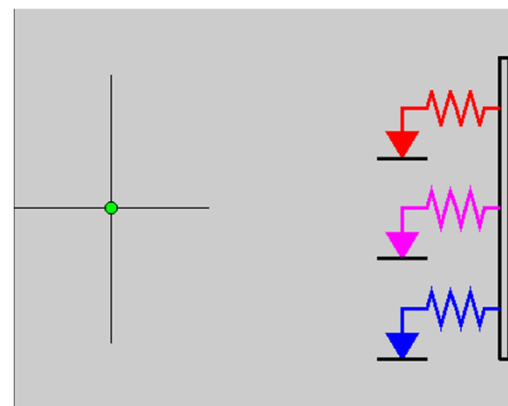
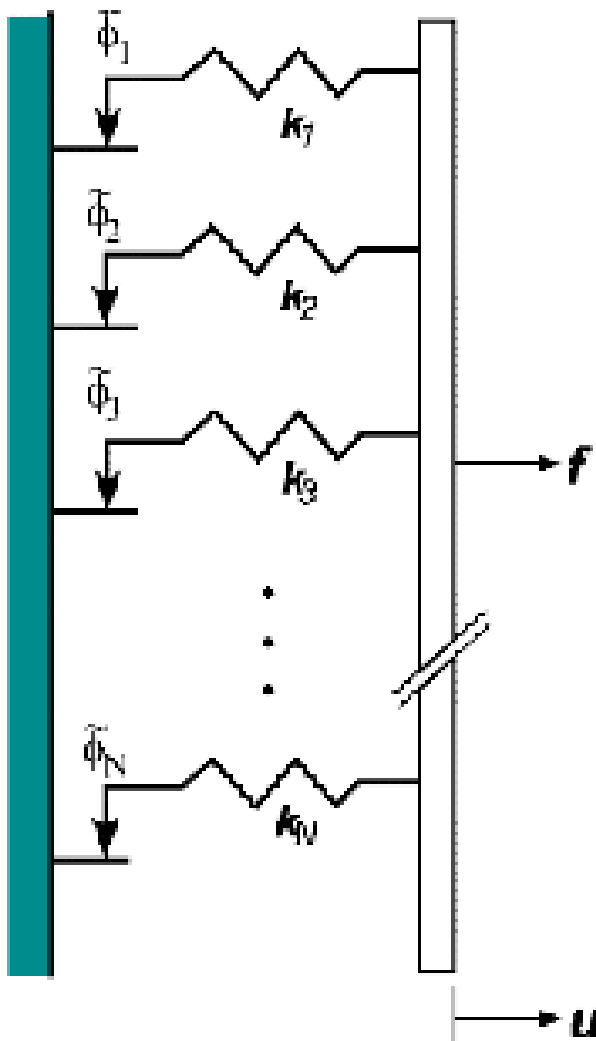
Simplest Approach: Simulation Using Whole-Joint Models



- MPC nodes on each side of interface
- Relate relative displacements via a scalar constitutive equation.

Example Constitutive Equation

Parallel-Series Iwan Model



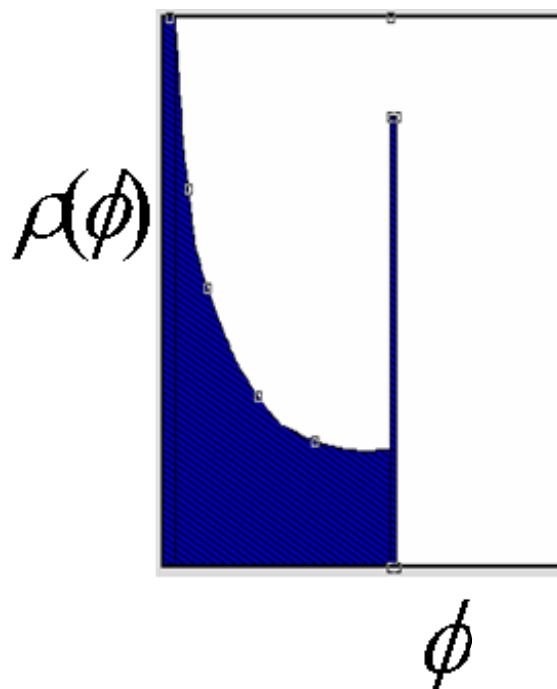
$$\dot{x}(t, \phi) = \begin{cases} \dot{u} & \text{if } |u - x(t, \phi)| = \phi \text{ and } \dot{u}(u - x(t, \phi)) > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$f(t) = \int_0^\infty \rho(\phi)[u(t) - x(t, \phi)] d\phi$$

The population distribution $\rho(\phi)$ of sliders of strength ϕ are chosen to yield power-law dissipation & macro-slip at large load.

A Four-Parameter Iwan Distribution

$$\rho(\phi) = R\phi^\chi (H(\phi) - H(\phi - \phi_{\max})) + S\delta(\phi - \phi_{\max})$$



- Nearly linear behavior at low amplitude.
- Power-law energy dissipation
- Physically reasonable
- Tractable

Parameters map into

- Macro-slip force
- Low-load stiffness
- Dissipation slope a small load
- Tangent stiffness at inception of macro-slip

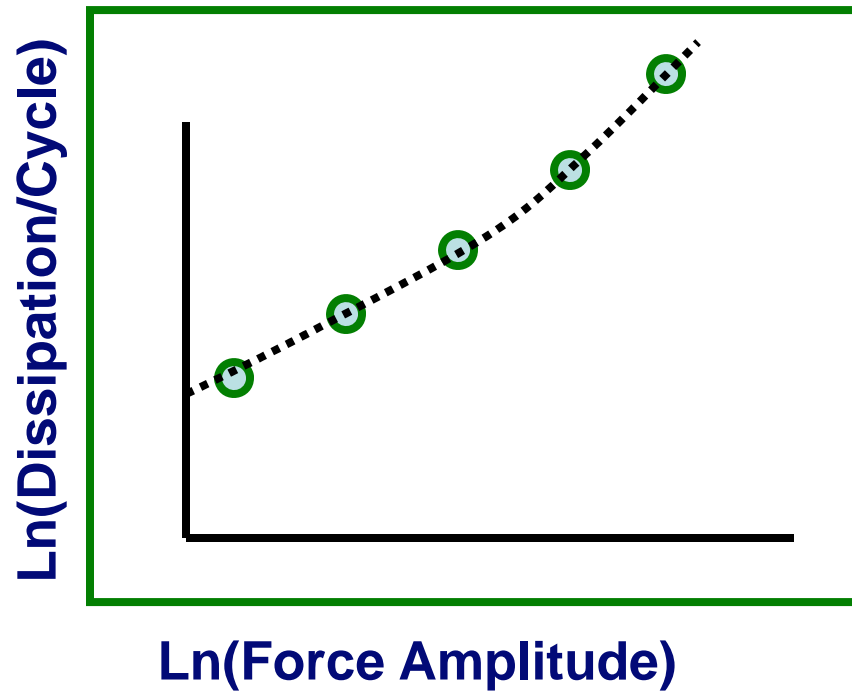
$$F_S$$

$$K_T$$

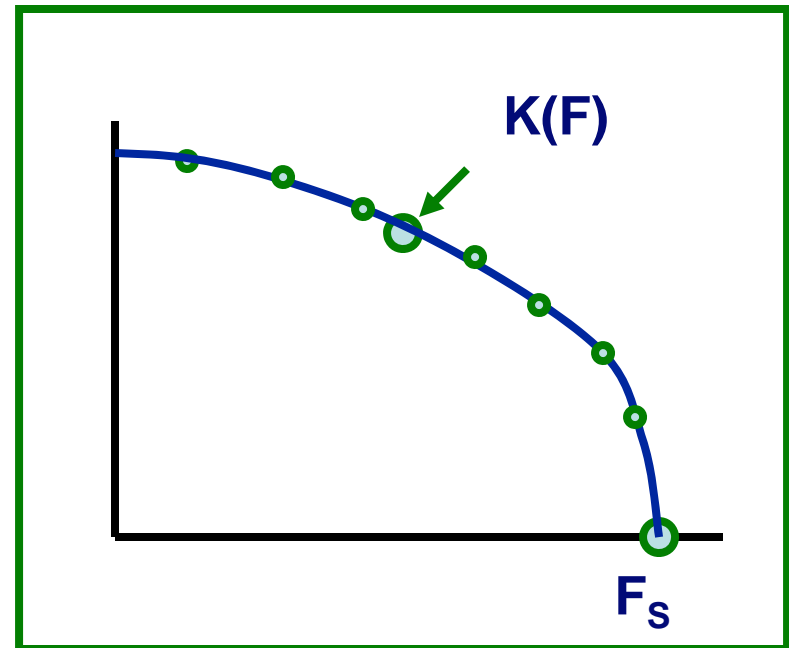
$$\chi = \alpha - 3$$

$$\beta$$

Determining Joint Parameters: Measured Properties



2 Equations

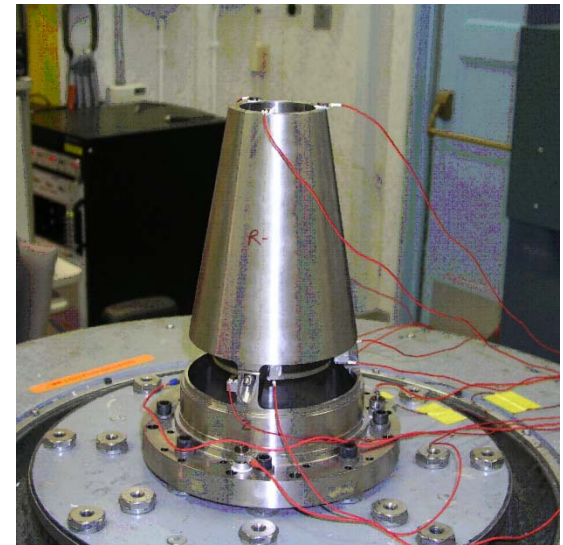
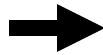
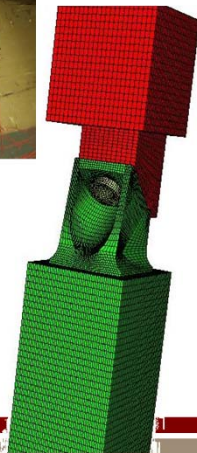
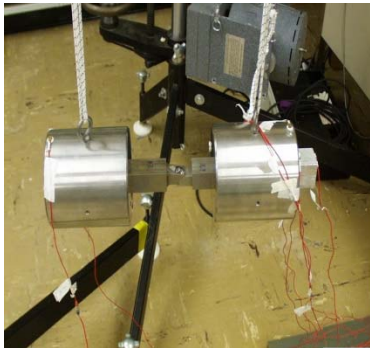


Force Amplitude

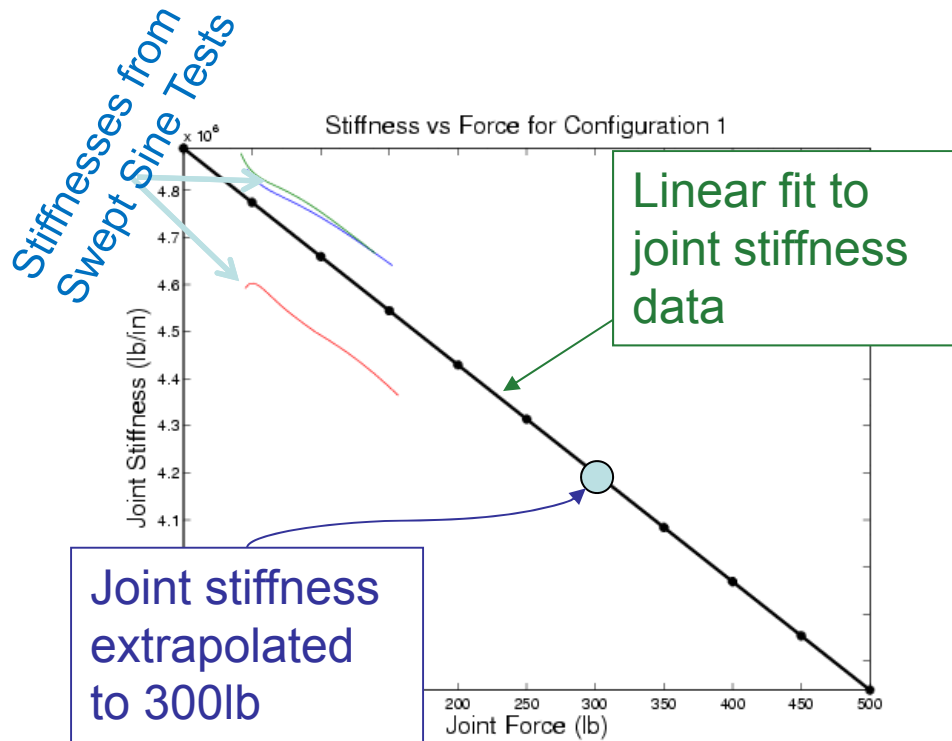
1 Equation

Macro-Slip Force, F_s , is the 4th condition.

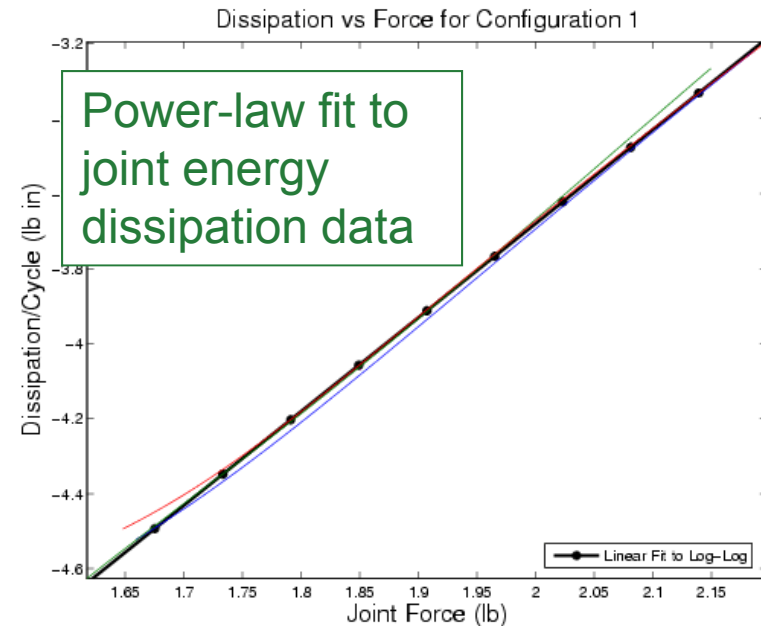
Calibration of Individual Joints to Predict Dynamics of 3-Legged Structure



Plot Joint Stiffness and Dissipation as Functions of Joint Force



Joint Stiffness

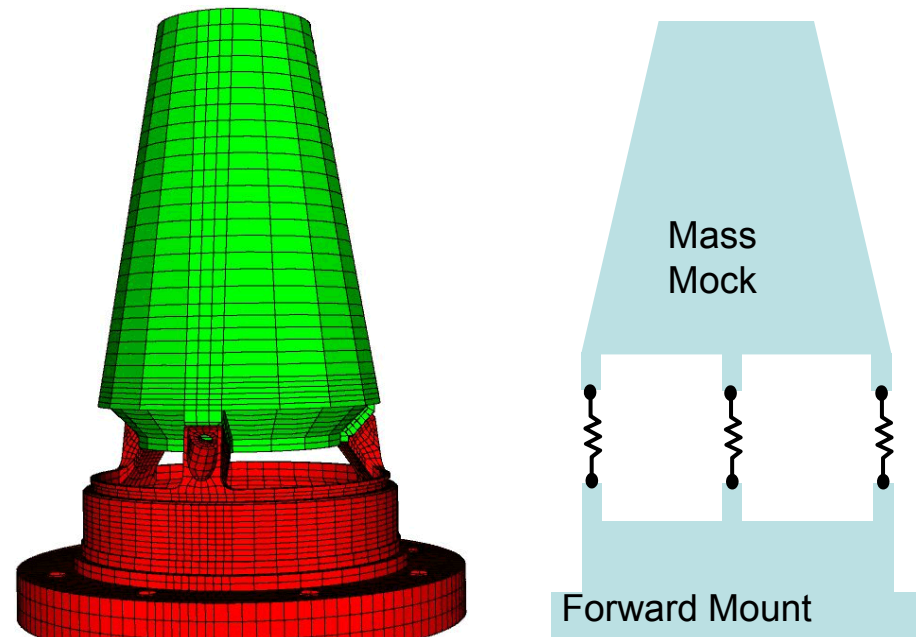


Joint Dissipation

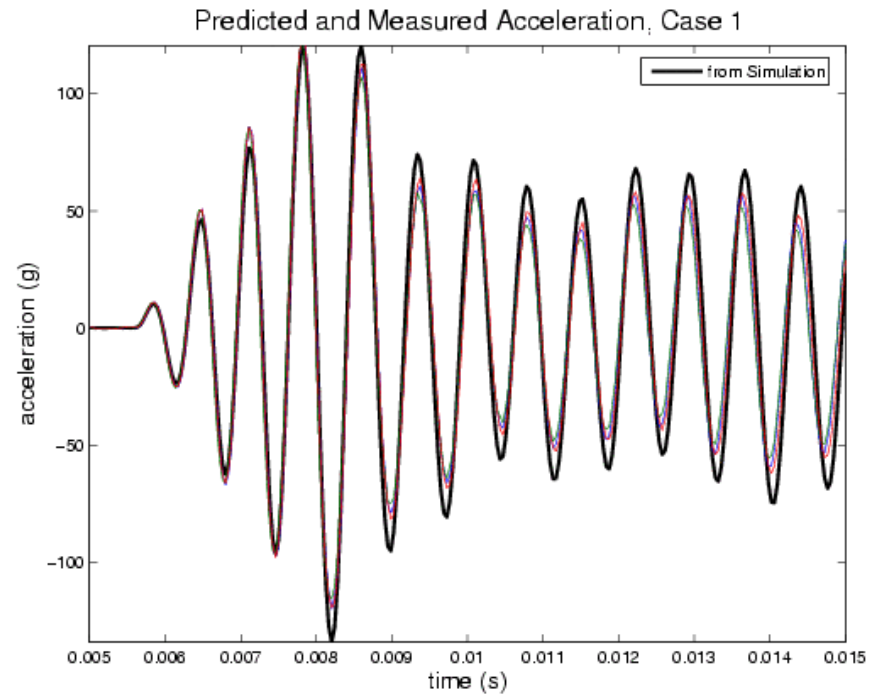
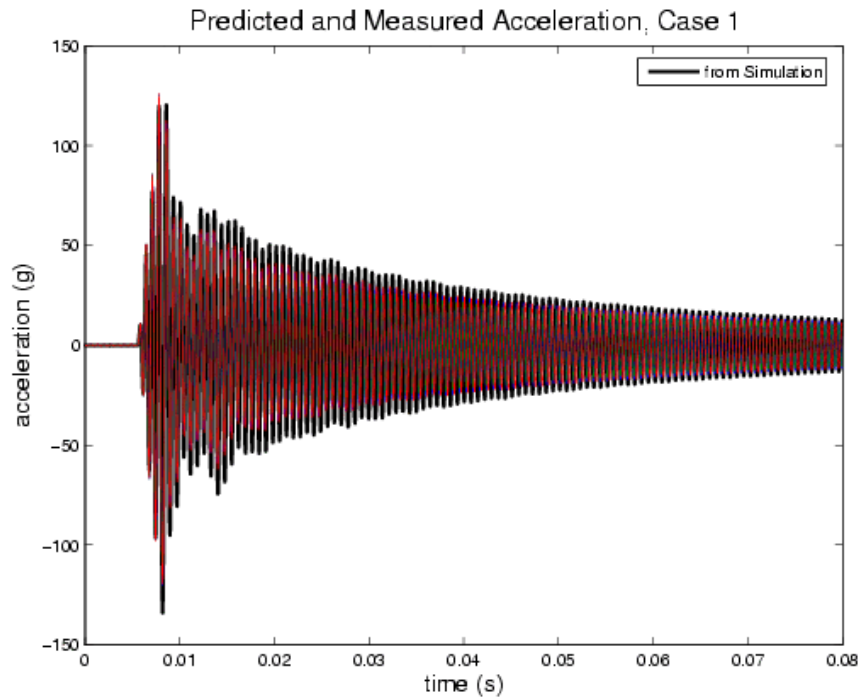
Model Parameters are selected to match the stiffness at 300lb force and to match the apparent power-law dissipation.

Predictions with Joint Model

- Employ 4-parameter model at joint
- Represent the rest of the structure with linear finite elements
- Excite base sufficiently to cause macro-slip.

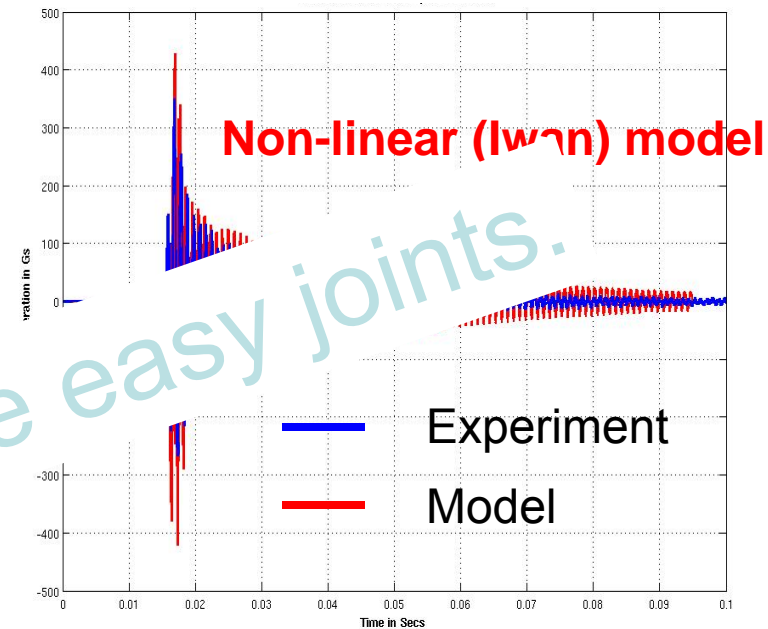
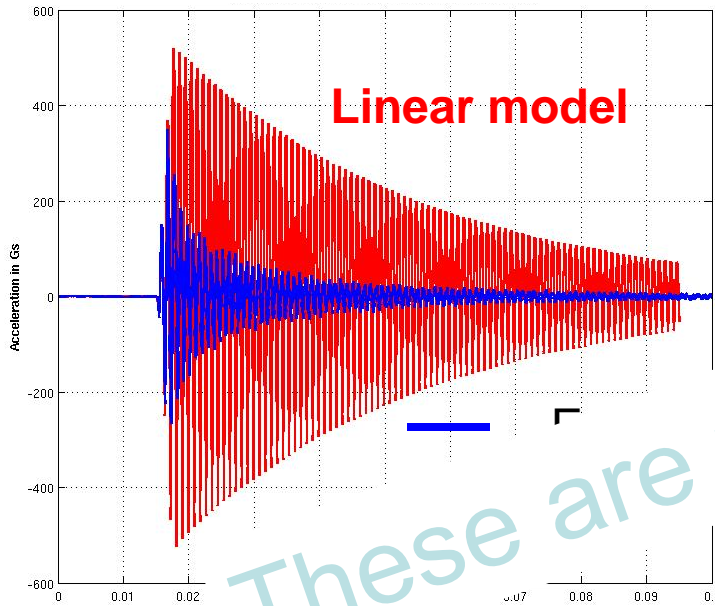


Blast Simulation for Configuration 1



Explicit incorporation of a joint model can significantly improve the quality of predictions.

Predictions for Axial Base Excitation that Entails Macro-Slip



These are the easy joints.

Explicit incorporation of a joint model can significantly improve the quality of predictions.

Obvious Limitations of This Approach

- ❑ These joint models are extremely primitive
 - This approach employed One-Dimensional models. Actual joints generally have at least 6 degrees of freedom and they are generally coupled.
 - Though this constitutive model captures dissipation well, it does not do as well with stiffness vs force
 - More complex constitutive models employ more parameters, making parameterization much more difficult.

- ❑ Must calibrate every joint/interface in system – this is generally intractable.

- ❑ Generally hard to incorporate in to finite element calculations. (We have done it, but not with ease.)

Structural Vibrations and Joints Like

What successful model descriptions actually are one-dimensional?

Why **Modal Analysis**, of course!

Recalling the Structures have a Joint-Like Response, Let's Develop a Nonlinear Modal Approach

Assumptions

1. Modal forces excite only corresponding modal responses. (Modes remain independent)
2. Modal coordinates evolve according to some simple nonlinear constitutive model.
3. The nonlinear modal constitutive response resolves to linear in the limit of small loads.

D.J. Segalman, "A Modal Approach to Modeling Spatially Distributed Vibration Energy Dissipation", Sandia Report SAND2010-4763, August 2010

Mathematically

$$M\ddot{u} + C\dot{u} + K_{\infty}u = F_N + F_X$$

K_{∞} = Stiffness matrix with joints absent

F_X = Externally Forces

F_N = Forces Due to Joints

$$M\ddot{u} + C\dot{u} + (K_{\infty} + K_T)u = \Delta F_N + F_X$$

K_T = From Low-Load Joint Stiffnesses

ΔF_N = Remaining, Nonlinear Part of Joint Response

Transform to Modal Coordinates $u = \Phi \alpha$

$$\ddot{\alpha} + \text{diag}(2\zeta_k \omega_k) \dot{\alpha} + \text{diag}(\omega_k^2) \alpha = \Phi^T \Delta F_N + \Phi^T F_X$$

So far, using only the standard simplifications.

Recall “Baseless” Assumptions

Even at large amplitude, modes do not couple

$$\ddot{\alpha} + \text{diag}(2\zeta_k \omega_k) \dot{\alpha} + \text{diag}(\omega_k^2) \alpha = \Phi^T \Delta F_N + \Phi^T F_X$$

$$F_M = \Phi^T \Delta F_N = \{f_n(\alpha_n(s), s = -\infty \text{ to } t)\}$$

Each model force depends only that the history of that modal displacement

Modal response – particularly damping – is joint-like

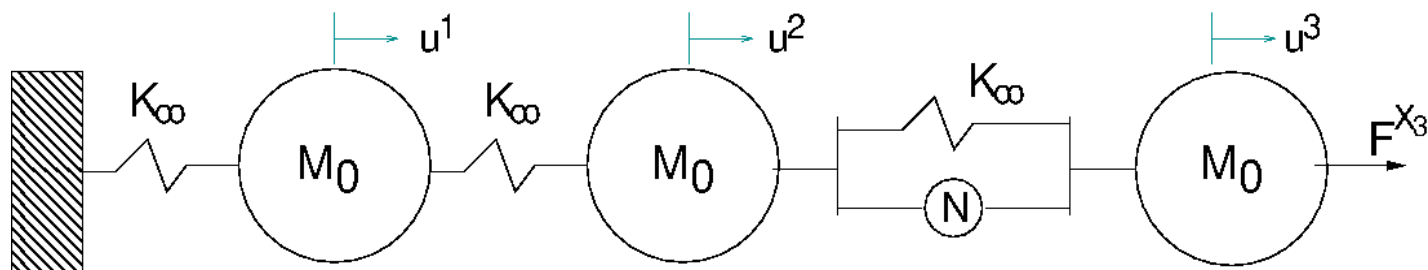
$$\{F_M\}_n = \{\Phi^T \Delta F_N\}_n = \int_0^\infty \rho_n(\phi) \lambda_n(t, \phi) d\phi$$

where

$$\dot{\lambda}_n = \begin{cases} \dot{\alpha}_n & \text{if } \dot{\alpha}_n (\alpha_n - \lambda_n) > 0 \quad \& \quad |\alpha_n - \lambda_n| = \phi \\ 0 & \text{otherwise} \end{cases}$$

Note ρ_n characterizes nonlinear response of mode n

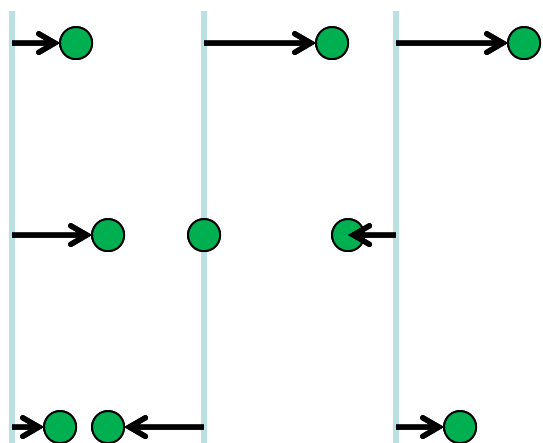
Try this out on a toy problem



$$M\ddot{u} + K_0 u = F_X + \Delta F_N$$

$$K_0 = \begin{bmatrix} 2K_\infty & -K_\infty & 0 \\ -K_\infty & 2K_\infty + K_T & -K_\infty - K_T \\ 0 & -K_\infty - K_T & K_\infty + K_T \end{bmatrix}$$

Eigen modes, y

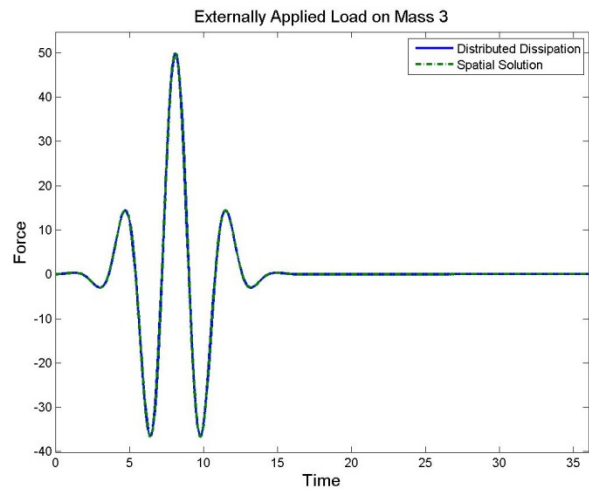
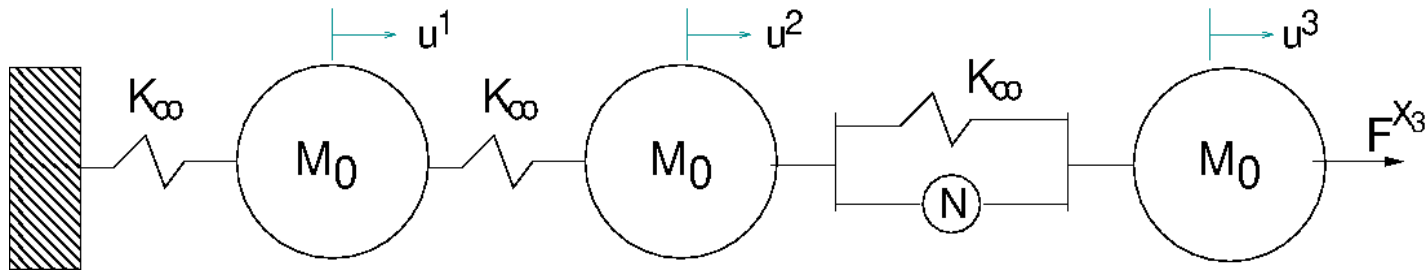


Deduce parameters of ρ_n in terms of ρ and eigen modes y_n

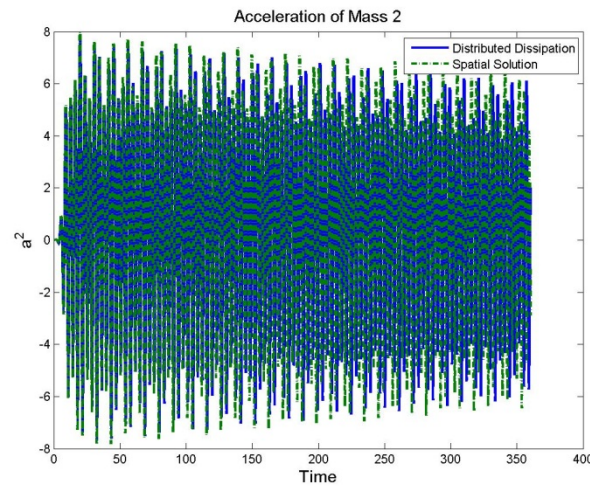
$$\chi_n = \chi \quad \beta_n = \beta \quad K_{\infty_n} = y_n^T K_\infty y_n$$

$$F_{S_n} = (y_n^3 - y_n^2) F_S \quad K_{T_n} = (y_n^3 - y_n^2)^2 K_T$$

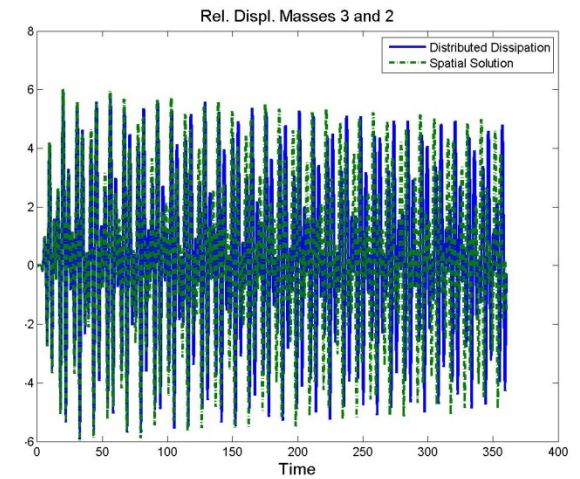
Tailor Force to Excite Third Mode



Force on Mass 3



Acceleration of Mass 2



Relative Displacement of Mass 2 & Mass

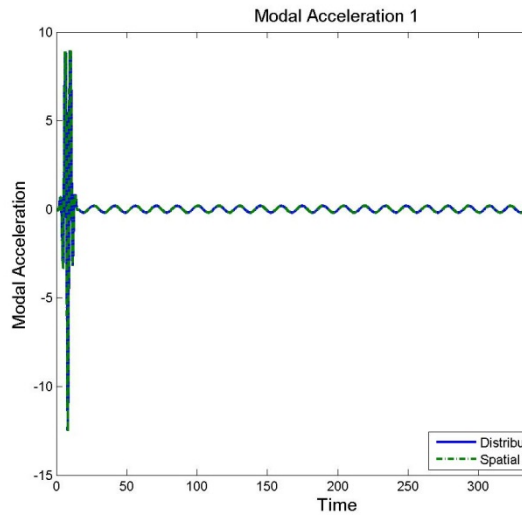


NL Modal Solution

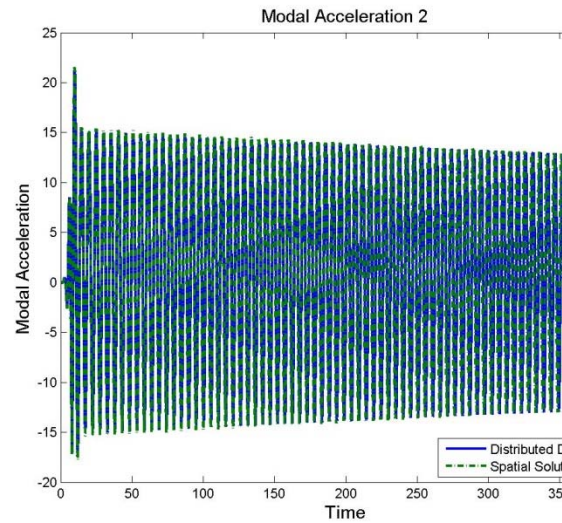


Full Spatial Solution

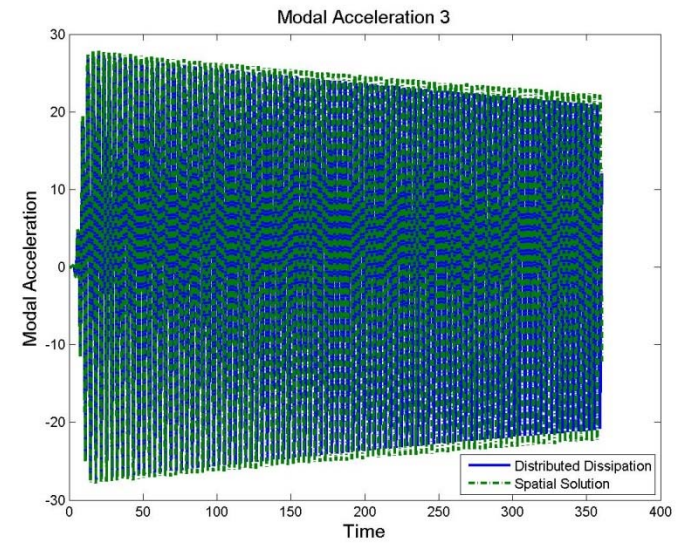
Modal Accelerations



Acceleration of Mode 1



Acceleration of Mode 2



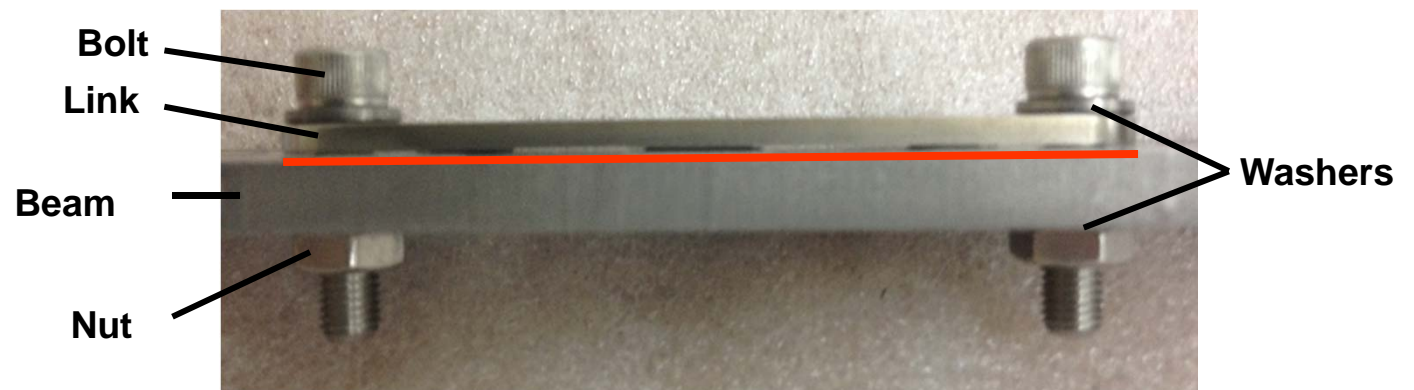
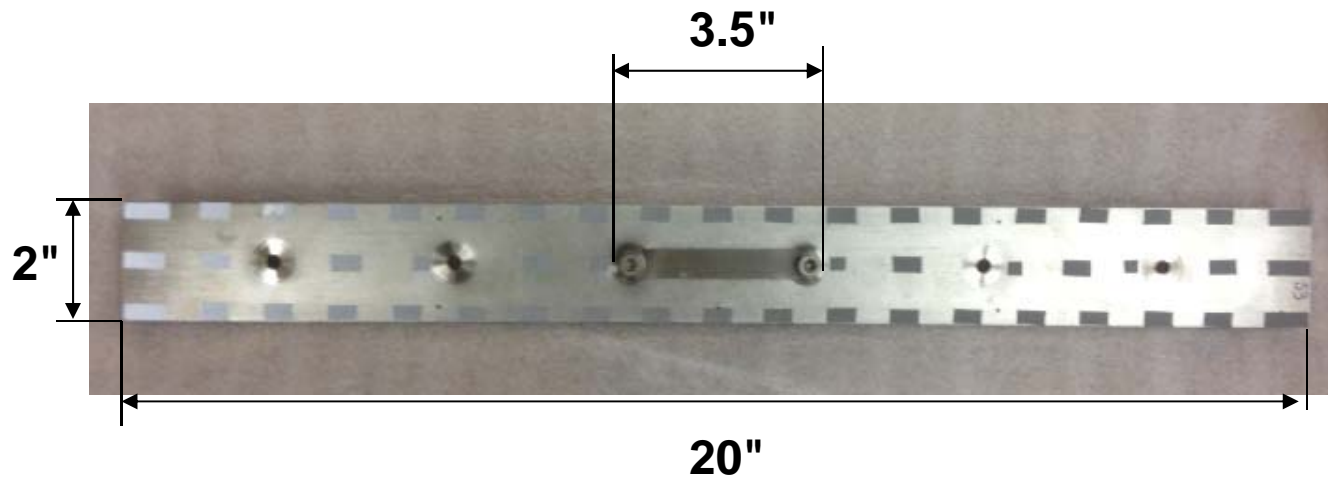
Acceleration of Mode 3

———— NL Modal Solution - - - - - Full Spatial Solution

Full Spatial Solution

Results are not always this clean

Test this on a more realistic problem: beam with a link attached by bolts.



37

Credit Brandon Deaner and James Allen, U Wisconsin

B. J. Deaner, M. S. Allen, M. J. Starr, and D. J. Segalman, "Investigation of Modal Iwan Models for Structures with Bolted Joints," presented at the International Modal Analysis Conference XXXI, Garden Grove, CA USA, 2013.

Deduce Modal Iwan Parameters Experimentally

The problem is separating response of individual modes.

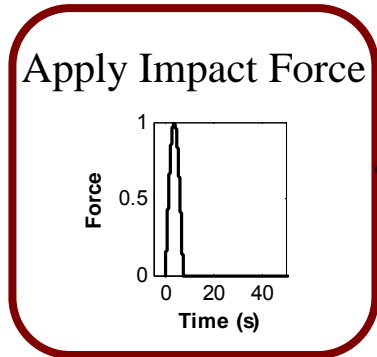
- Generally, more than one mode are excited at a time.
- It is hard to de-convolve each modal component in a nonlinear system

This is addressed with modern tools

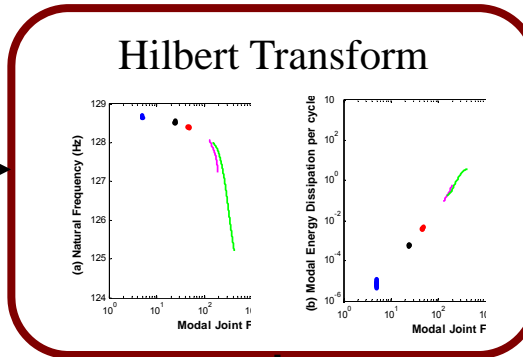
- Hilbert-Huang
- Empirical Mode Decomposition (Vakakis et al)

Comparison procedure for discrete and modal simulations.

Discrete Simulation: F_S, K_T, χ, β



Convert physical response to modal

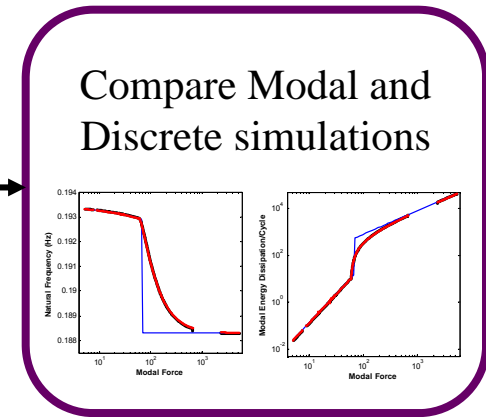
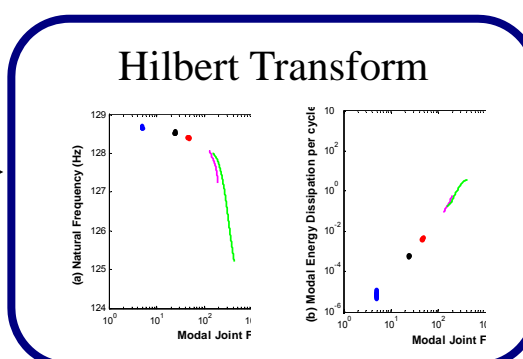
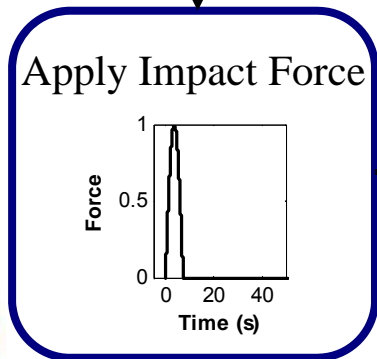
$$q = \Phi^{-1}x$$


Optimize Analytical Model to frequency and energy dissipation to each mode.

$$\text{Min } f$$

Modal Simulation:

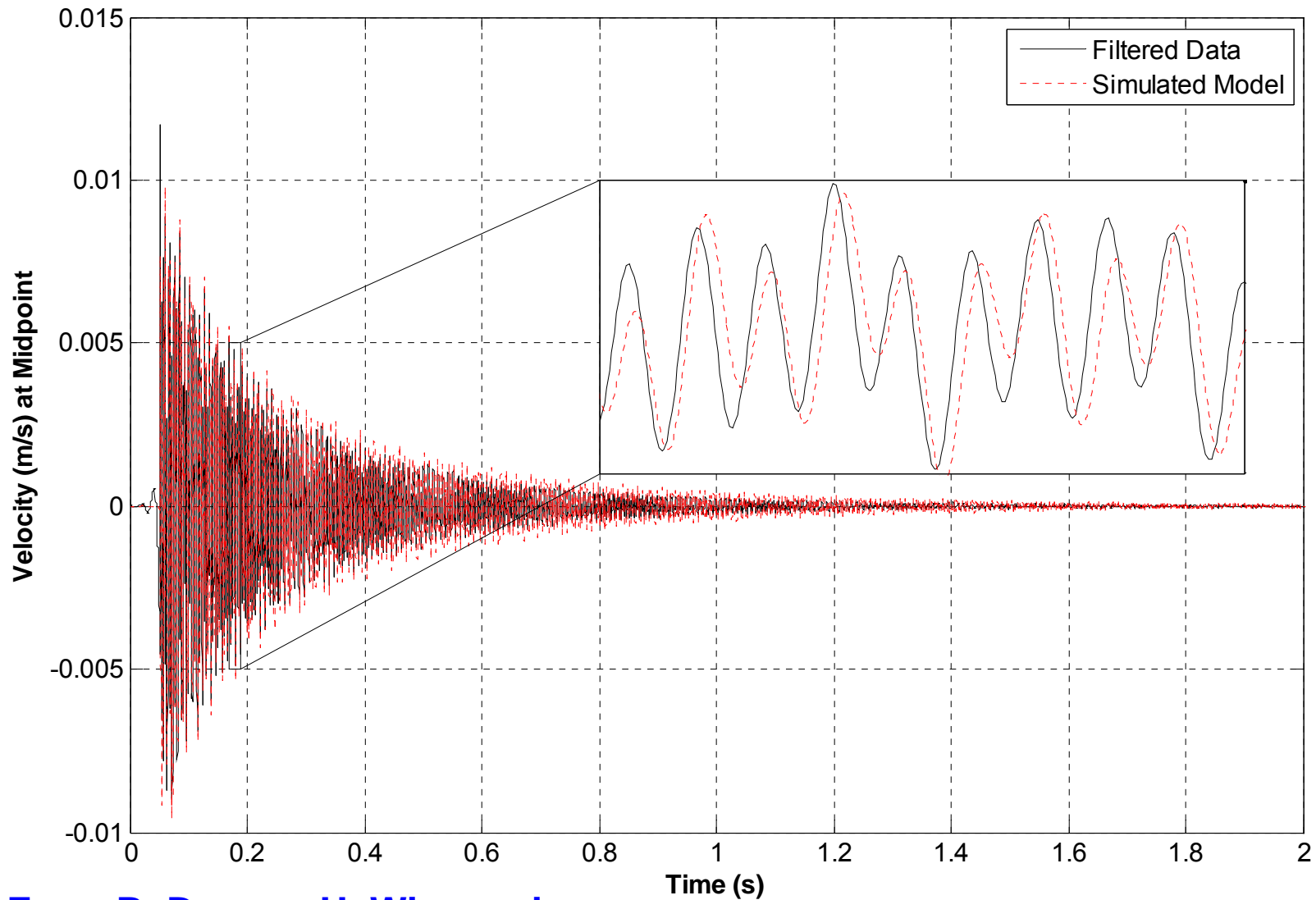
$$\hat{F}_S^l, \hat{K}_T^l, \hat{\chi}^l, \hat{\beta}^l$$



From B. Deaner,
U. Wisconsin



The response can be simulated with a few SDOF modal simulations.



From B. Deaner, U. Wisconsin

Where Does This Leave US?



1. Strong experimental evidence of nonlinearity of jointed structures.
2. Process for modeling individual joints and incorporating into finite element models for the full structure. (All still primitive.)
3. Capability capture that nonlinearity in a modal representation for the full structure
4. A really perplexing question:
If structures are so nonlinear, why is it that we are so successful using linear structural dynamics?
A likely answer:
We are successful when we calibrate our linear models – particularly modal damping – using experiments in the load amplitudes we want to predict.

How good are we at coupled systems?

1. We do OK when we use linear structural dynamics on problems of very small load amplitude.
2. We also do OK when we calibrate our model at the load amplitudes of application.
3. There may be serious issues when merging subsystem models without accounting properly for interfaces between those subsystems.
4. When we have to predict over a range of amplitudes, we need to incorporate the nonlinearities explicitly.
5. We are still learning!

Backup Slides

$$D = \oint F v dt$$

