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Efficient Stochastic Finite Element Modeling Using Parameterized Reduced Order Models

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Motivation

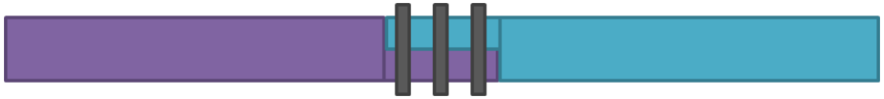
- A need to explore the effects of parameter uncertainty in FE models – this can constitute a large parameter space
- Exploring this large parameter space is very costly, both in terms of man hours and computation time
- These costs makes studying the uncertainty difficult
- If the number of meshes and/or the number of runs can be reduced, these uncertainty studies are attainable

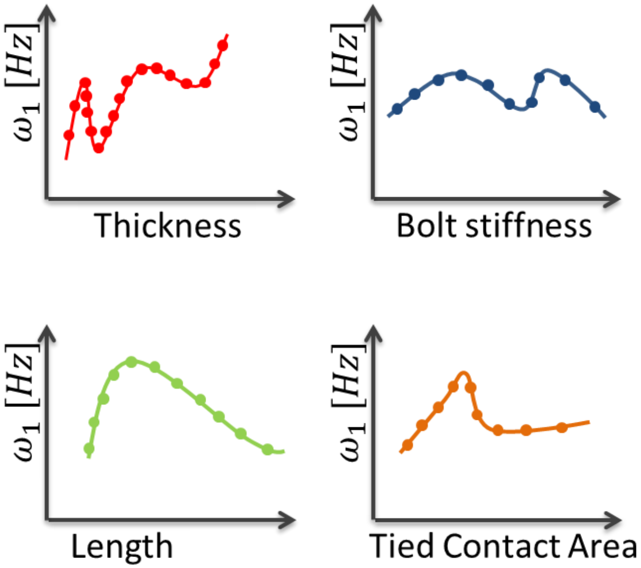
Our Method:

- Rather than mesh and run at each of many parameter values, approximate the system matrices using Taylor series
- To reduce computational cost, use Craig-Bampton reduced-order representations of these systems and approximate the CB matrices

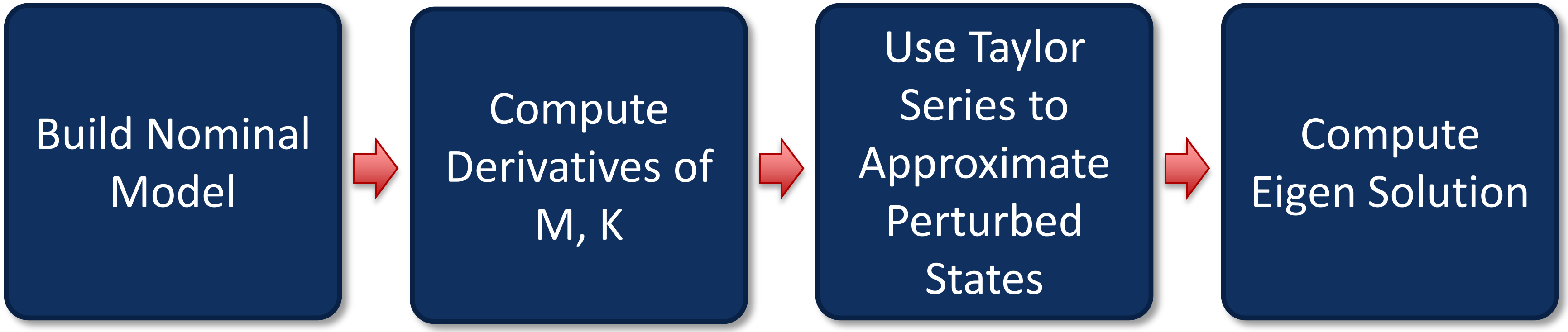
Example Sensitivity Study

- Need predictions to that account for uncertainty
- Want to explore the effect of these uncertain variables on the modes of the system
- Due to:
  - Manufacturing variability
    - Geometry
    - Assembly
    - Machining, welding
  - Insufficient property data
- Uncertainty in many variables:
  - Plate thickness
  - Material density
  - Material modulus





How this Works:


Build Nominal Model
Compute Derivatives of M, K
Use Taylor Series to Approximate Perturbed States
Compute Eigen Solution

Three Parameterization Methods:

Full DOF Parameterization

- Approximate the Full DOF System Matrices
- Saves meshing time
- No change in solution time

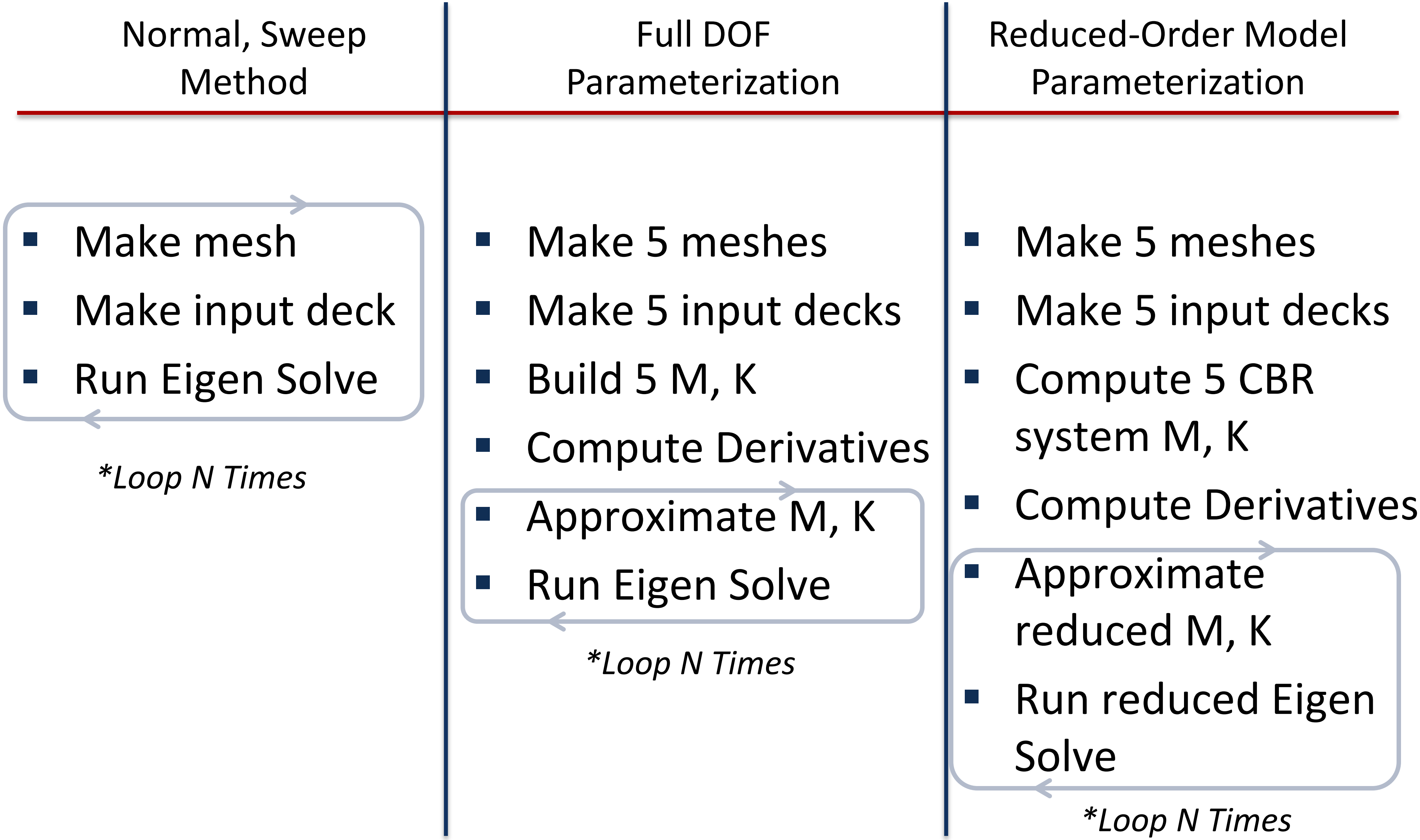
Reduced-Order Model Parameterization

- Approximate the CBR System Matrices
- Saves meshing time
- Saves solution time

Eigen Value & Vector Parameterization

- Approximate the eigen values and vectors based on nominal model
- Saves meshing time
- Saves solution time

Detailed Process:



Normal, Sweep Method

- Make mesh
- Make input deck
- Run Eigen Solve

**Loop N Times*

Full DOF Parameterization

- Make 5 meshes
- Make 5 input decks
- Build 5 M, K
- Compute Derivatives
- Approximate M, K
- Run Eigen Solve

**Loop N Times*

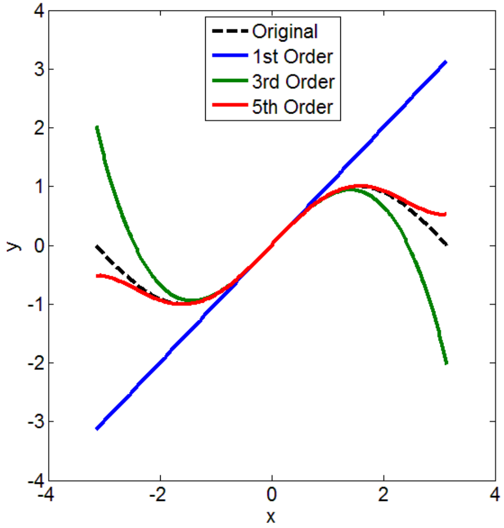
Reduced-Order Model Parameterization

- Make 5 meshes
- Make 5 input decks
- Compute 5 CBR system M, K
- Compute Derivatives
- Approximate reduced M, K
- Run reduced Eigen Solve

**Loop N Times*

Review of Taylor Series

- Series expansion function about a point
- Here, the "point" is the value of an entry in the Mass or Stiffness matrix for the Nominal model
- $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$
  - $f(x)$ = function of parameter x. $M_{1,5}(\rho)$, for example.
  - a = point $f(x)$ is expanded about
- Order of Taylor Series refers to the highest-order derivative
- More accurate approximation to true function with higher order



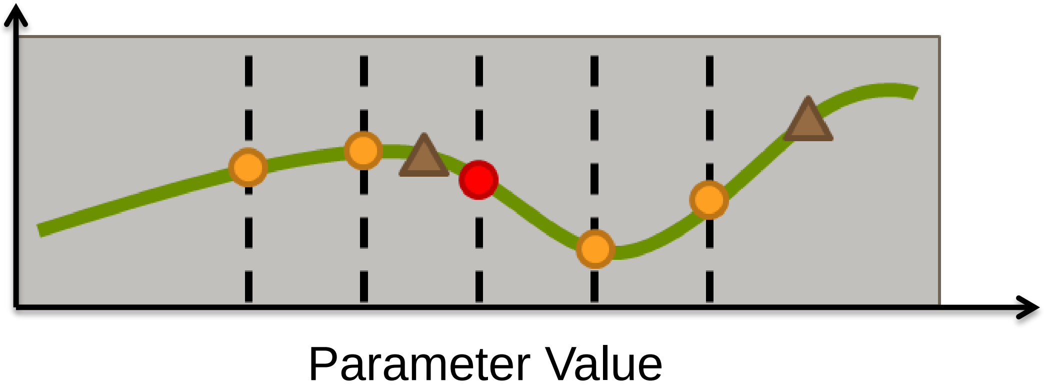
Review of Finite Differences

- Method for approximating derivatives of functions
- Here, the function is the entry of M,K as a function of parameter.
  - $M_{1,5}(\rho)$, for example.
  - Used Central Finite Differences here
- First Derivative: $f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$
  - h is the step size between nominal value and +/- values
- Higher Derivatives: $f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$
- Better approximations had with higher accuracy differences
  - First Derivative with 2nd order accuracy:
  - First Derivative with 4th order accuracy:
- But, higher derivatives require more meshes

Derivative:	Accuracy:	Number of Meshes:
1st	2	2
	4	4
	6	6
2nd	2	3
	4	5
	6	7
3rd	2	4
	4	6
	6	8
4th	2	5
	4	7
	6	9

Calibration Set: Points Required

- Calibration Set = Points Required for FD Calculations
- Central Finite Differences require several points above and below the Nominal point
  - Points (matrices) at the nominal, $f(a)$, and perturbed points, $f(a-2h),\ f(a-h),\ f(a+h),\ f(a+2h)$, etc.
- Each point requires the construction of the Mass & Stiffness matrices
  - Unique mesh and/or input deck parameters



Looking at Matrix Derivatives

- How does a Finite Difference work on a Matrix?
- Simply:
  - Entry-by-Entry → $M_{3,3}$ Entry at Nominal & Perturbed states
  - Calibration Set: $M_{3,3_{(-2h)}}$, $M_{3,3_{(-h)}}$, $M_{3,3_{(nom.)}}$, $M_{3,3_{(+h)}}$, $M_{3,3_{(+2h)}}$

$-h$ Point

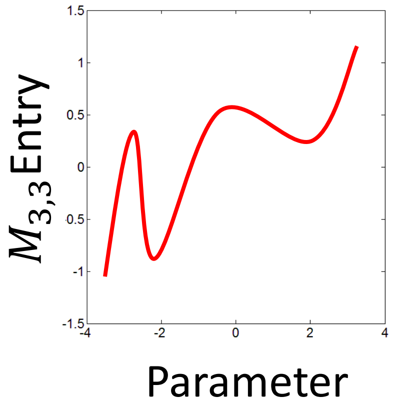
0.9	0	0	0	0
0	1.8	0	0	0
0	0	1.8	0	0
0	0	0	1.8	0
0	0	0	0	0.9

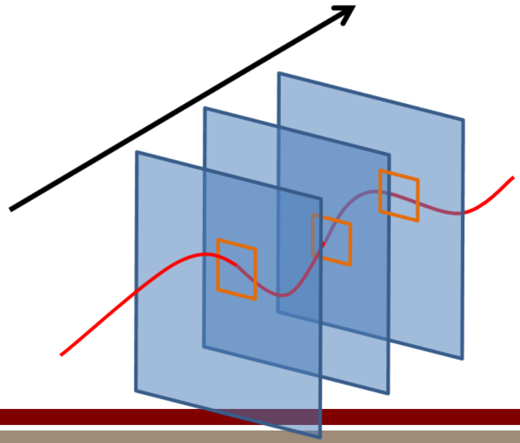
Nominal

1	0	0	0	0
0	2	0	0	0
0	0	2	0	0
0	0	0	2	0
0	0	0	0	1

$+h$ Point

1.1	0	0	0	0
0	2.2	0	0	0
0	0	2.2	0	0
0	0	0	2.2	0
0	0	0	0	1.1



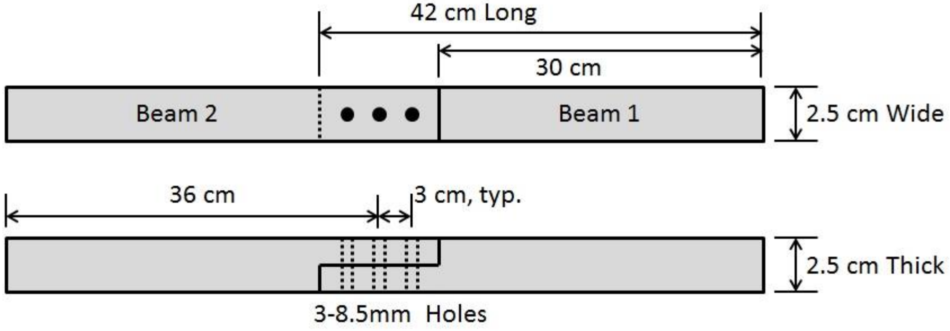


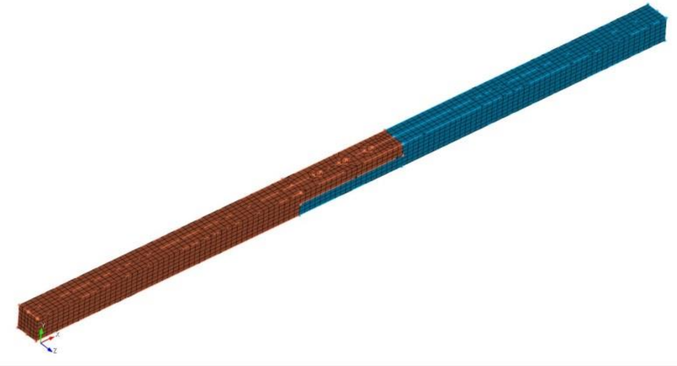
Entries in M, K From Taylor Series

- Each Entry in System Matrices is Approximated
  - $M_{3,3}(1.13x_{nom}) = M_{3,3}(x_{nom}) + M_{3,3}{}'(x_{nom})(\Delta_x) + \frac{M_{3,3}{}''(x_{nom})}{2}(\Delta_x)^2 + \cdots$
    - $\Delta_x = 0.13x_{nom}$
  - $K_{3,3}(1.13x_{nom}) = K_{3,3}(x_{nom}) + K_{3,3}{}'(x_{nom})(\Delta_x) + \frac{K_{3,3}{}''(x_{nom})}{2}(\Delta_x)^2 + \cdots$
- Note: since we're approximating each entry, resulting system matrices *may* be physically un-realizable

- Craig-Bampton Reduced Systems:
  - Components are reduced, resulting in reduced M_{CBR}, K_{CBR} matrices
  - These reduced M_{CBR}, K_{CBR} matrices are then approximated

Parameterized Beam Model



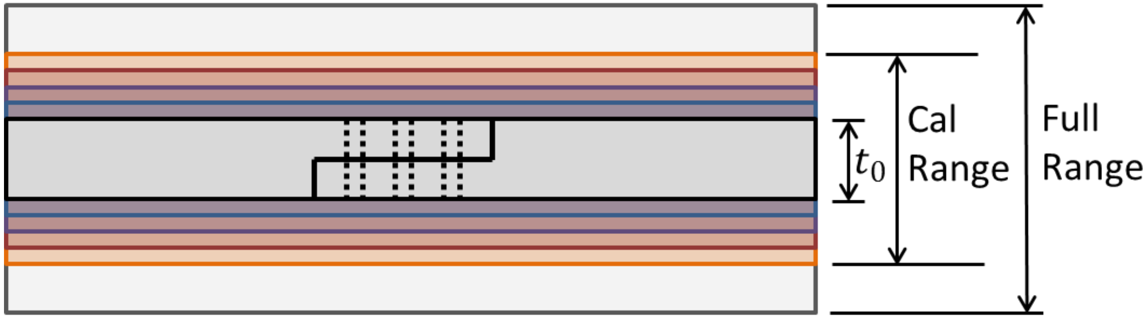


- 2 component system
- Components connected with springs representing bolts
- Each component can be reduced to a Craig-Bampton superelement
- Non-trivial size: 6192 nodes, 18576 DOF
- Parameters Explored:
  - Modulus, Density, Thickness

Creating the Calibration Set

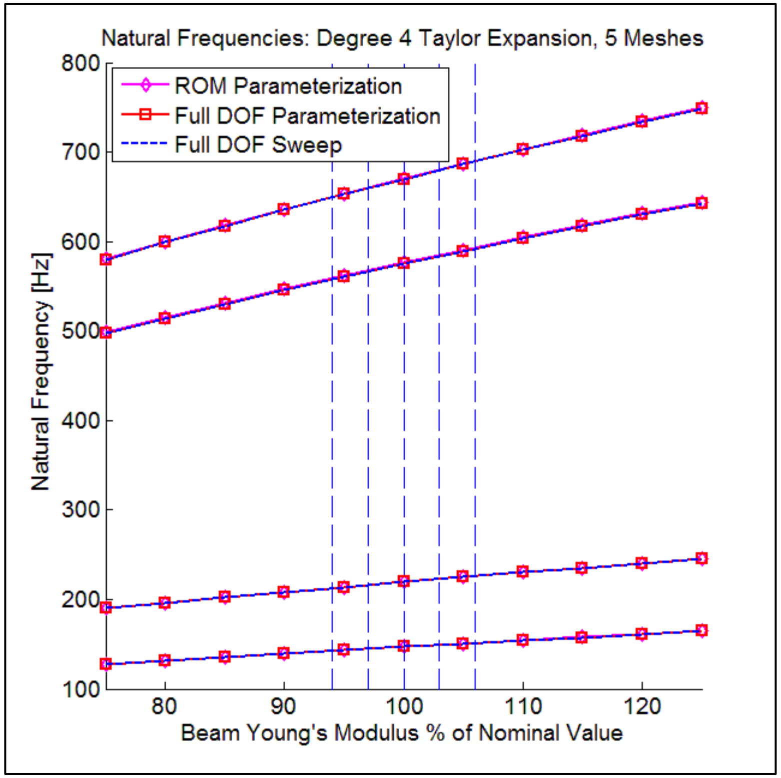
- For each Parameter:
  - 5 runs
    - 5 meshes & 5 input decks
- Here, used +/- 6% of nominal value for Calibration Set
- Parameter range explored to +/- 20%

Parameter	-2 h	-h	Nominal	+h	+2h
Density [kg/m3]	7398	7634	7870	8106	8342
Modulus [GPa]	197	204	210	216	223
Thickness [mm]	23.5	24.3	25.0	25.8	26.5

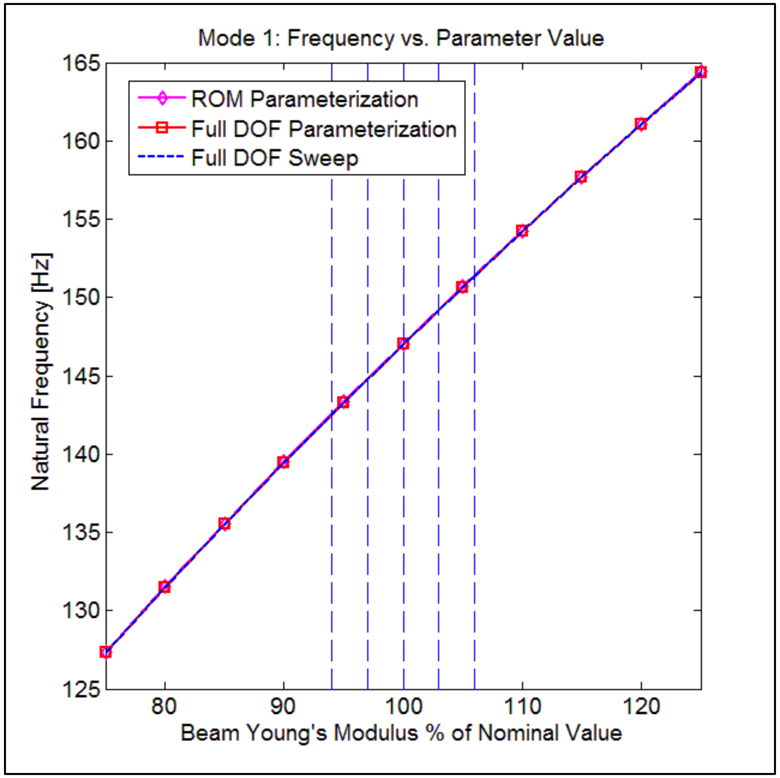


Parameter Effects on Modes

Parameterizing the Modulus:



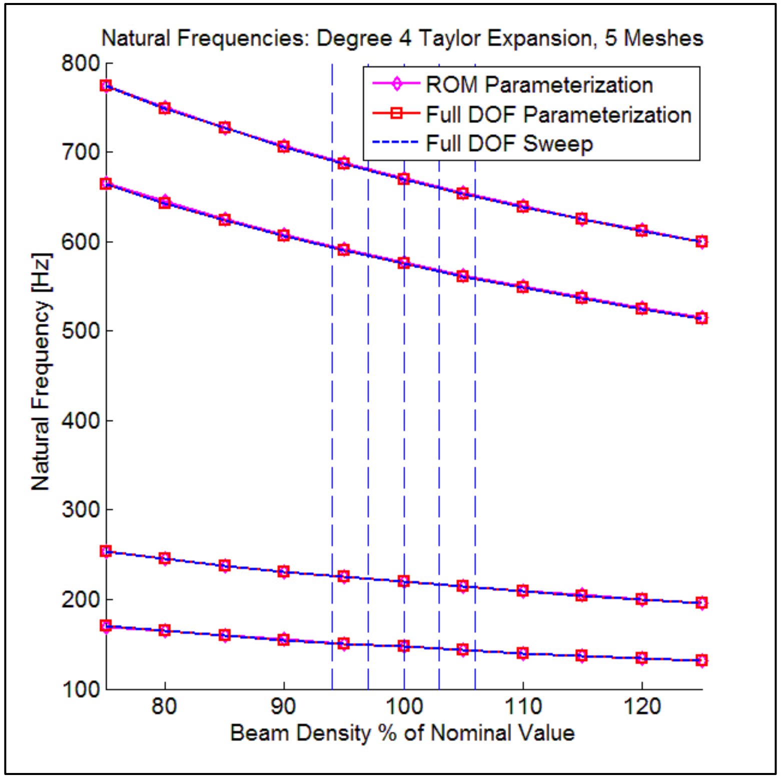
Four Modes



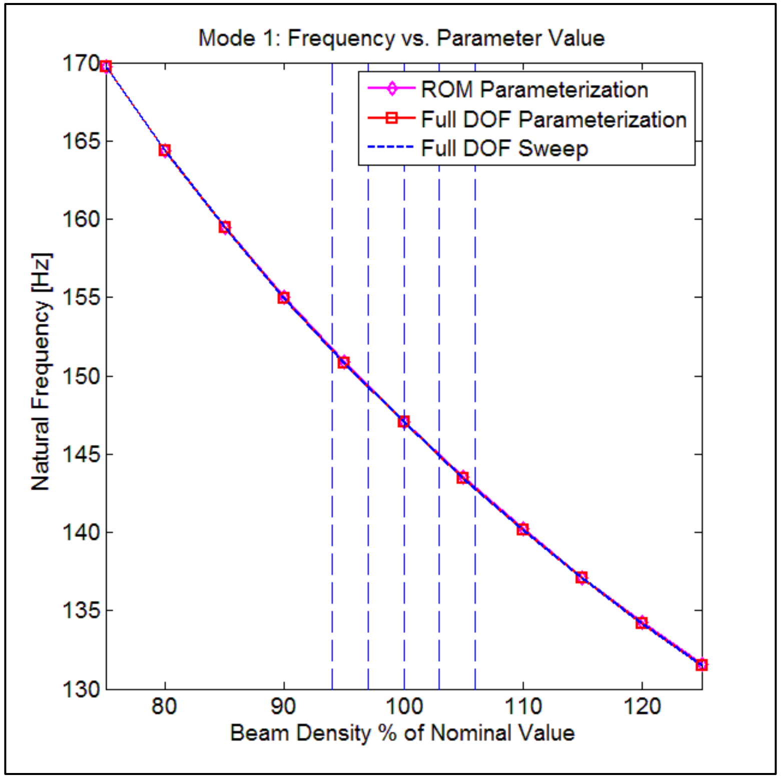
First Mode Only

Parameter Effects on Modes

Parameterizing the Density:



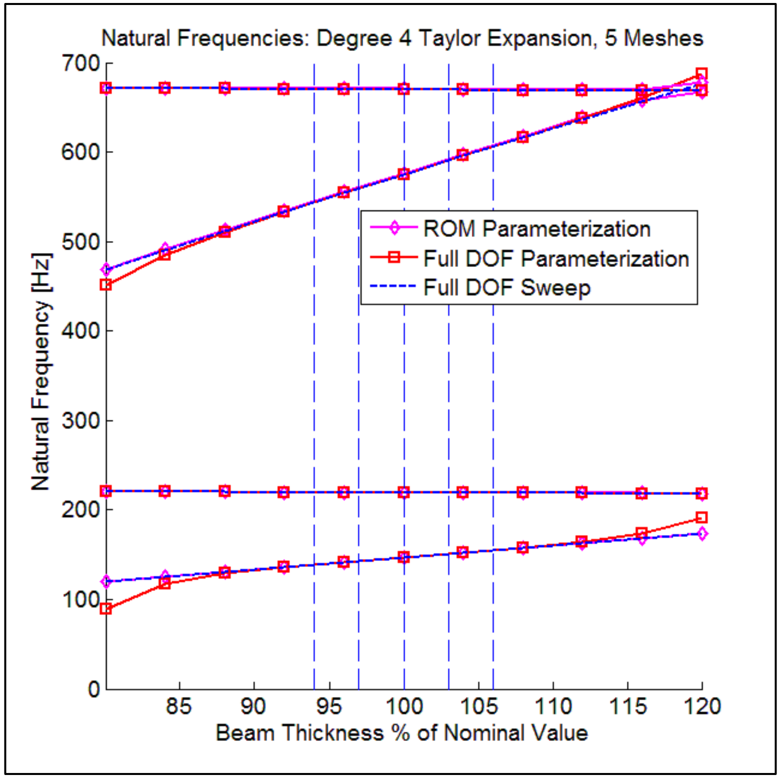
Four Modes



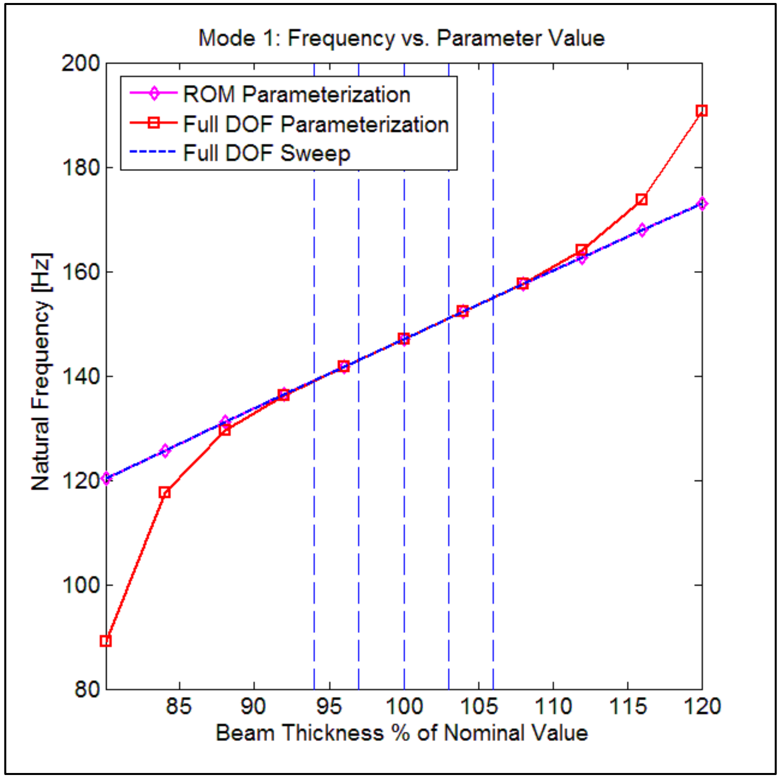
First Mode Only

Parameter Effects on Modes

Parameterizing the Thickness:



Four Modes



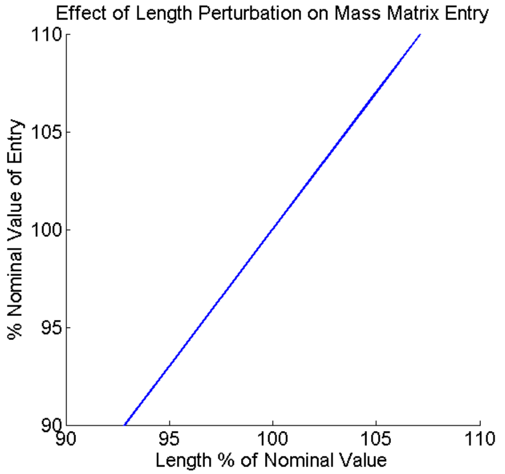
First Mode Only

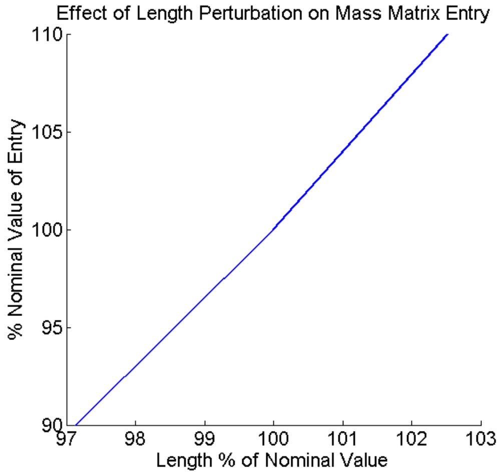
Effect on Mass, Stiffness Matrices

Full DOF Parameterization

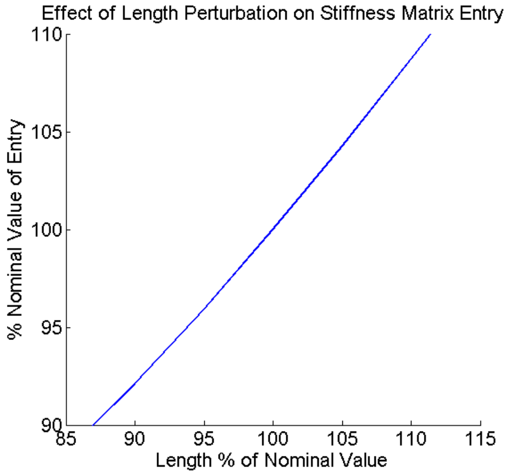
CBR Parameterization

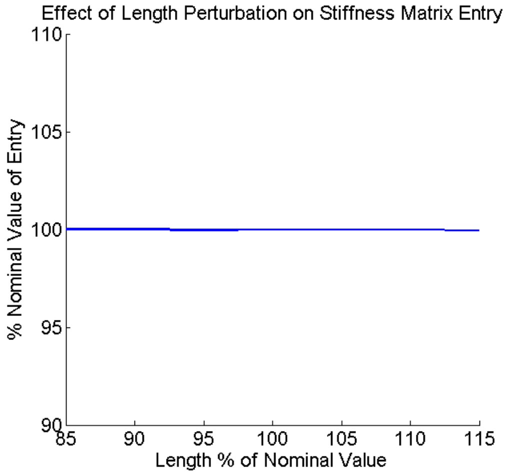
Mass Matrix





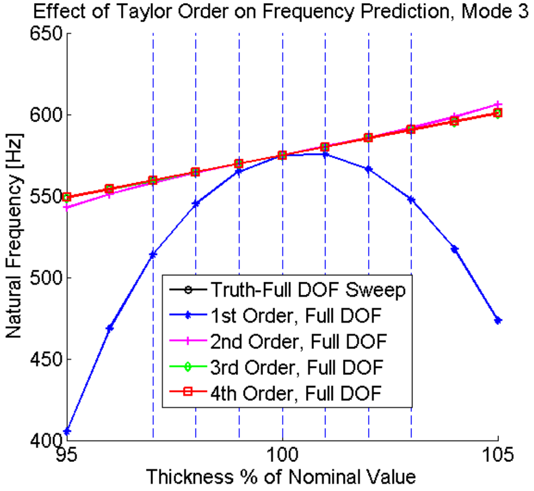
Stiffness Matrix

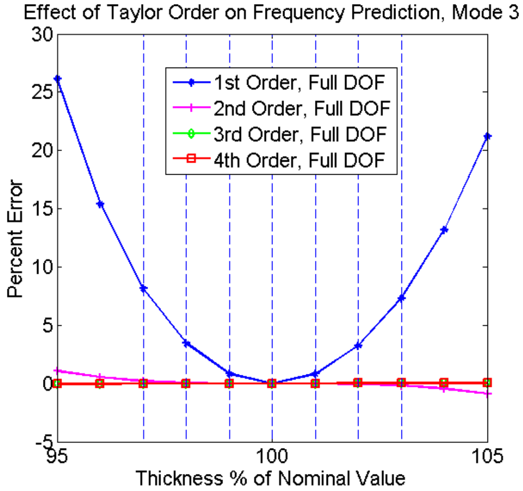




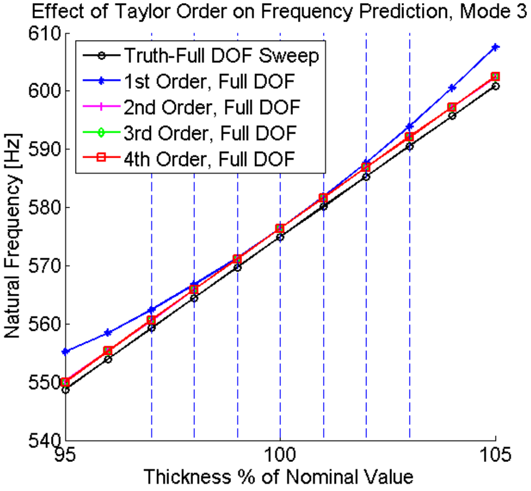
Effect of: Order of Taylor Series

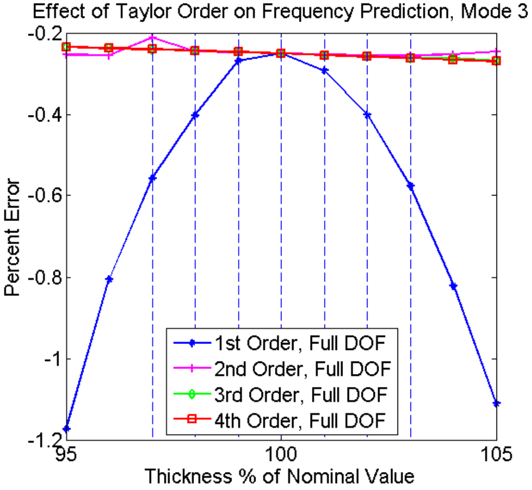
Full DOF Parameterization





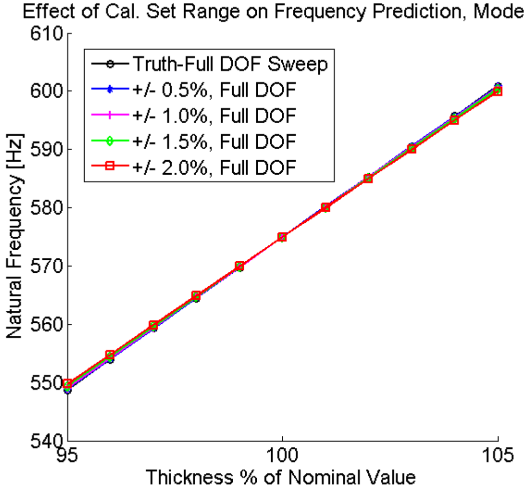
CBR Reduced DOF Parameterization

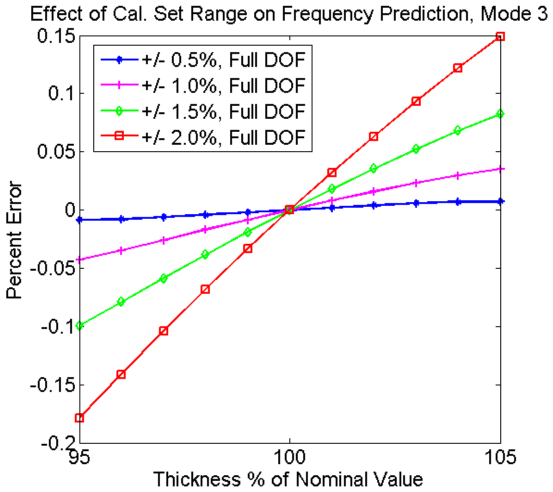




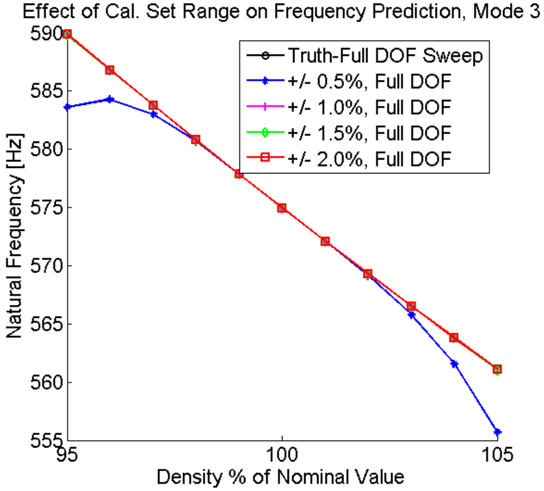
Effect of: Calibration Set Range

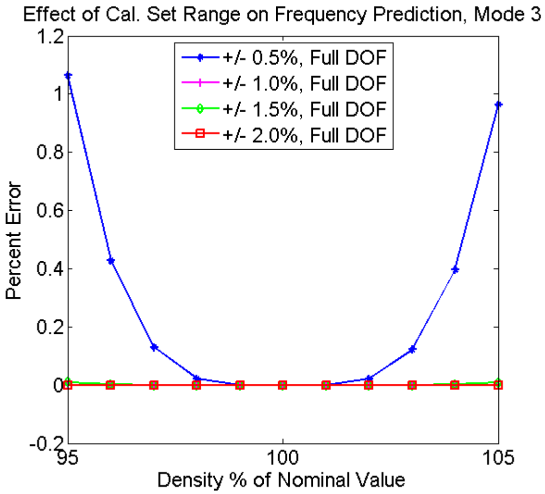
Changing Thickness





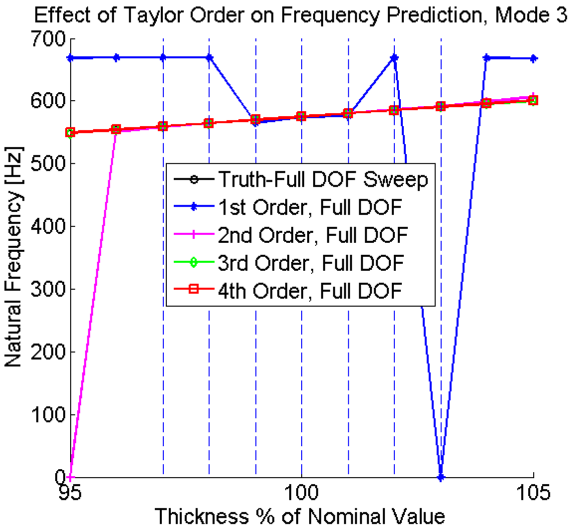
Changing Density



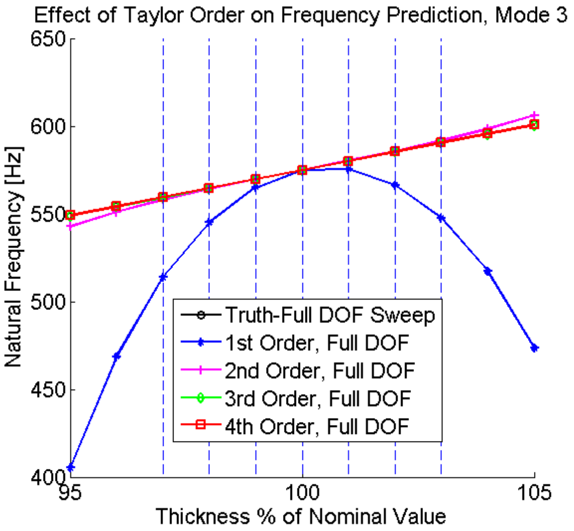


Practical Considerations: Mode Order

- With changes to the system matrices, mode order can change relative to the Nominal model
- To compare mode-by-mode to Nominal model, much compare like modes
- Use a MAC to find which modes are similar, THEN compare frequencies



Without MAC Filter

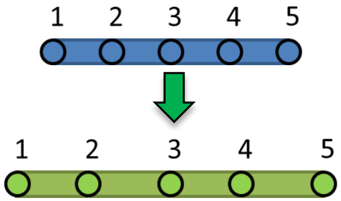


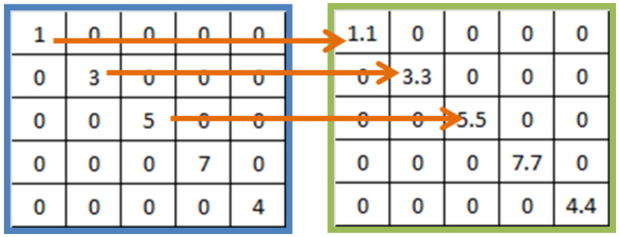
With MAC Filter

Practical Considerations: Node Order

- To compute derivatives, each M, K matrix in Cal. Set must be same size
- Nodes must stay in same order
  - Otherwise, computing derivatives with effective terms:
  - $M^{0}_{2,2}$, $M^{+1}_{5,5}$, $M^{-1}_{3,3}$

Consistent Numbering:

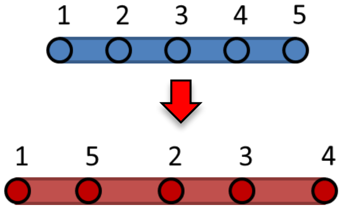


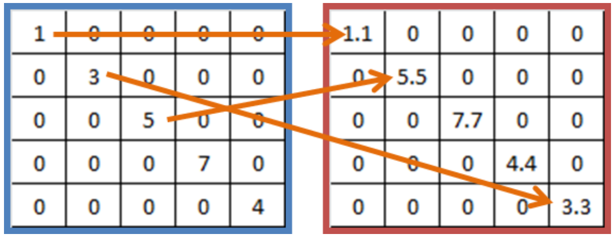


1	0	0	0	0
0	3	0	0	0
0	0	5	0	0
0	0	0	7	0
0	0	0	0	4

1.1	0	0	0	0
0	3.3	0	0	0
0	0	5.5	0	0
0	0	0	7.7	0
0	0	0	0	4.4

Inconsistent Numbering:





1	0	0	0	0
0	3	0	0	0
0	0	5	0	0
0	0	0	7	0
0	0	0	0	4

1.1	0	0	0	0
0	5.5	0	0	0
0	0	7.7	0	0
0	0	0	4.4	0
0	0	0	0	3.3

Cost Savings – Workflow Review

- "Normal" Method:
  - Make a mesh & run simulation at each perturbed point & parameter
- Parameterized Full-DOF Method:
  - Make 7 meshes & run 7 simulations
  - Approximate M, K at any other points
  - Compute eigen solution using these M, K
- Parameterized Reduced-Order Model Method:
  - Make 7 meshes & run 7 CBR solutions, making the reduced M, K
  - Approximate the reduced M, K at any other points
  - Compute eigen solution with reduced M, K
- Cost is even throughout with the "Normal" method
- Cost is all up front with the Parameterized methods

Cost Savings – Simple Example

- Say we want to explore 5 different variables, 200 points each
- 10,000 runs total
- Want to know effect on modes for each of these cases
- *ignoring meshing time*

	Solution Time [s]				
Method	1 Value	5 Values	100 Values	1000 Values	10000 Values
Normal, Full Sweep	2.2	11.0	220.0	2200.0	22000.0
ROM Parameterization	11.0	11.2	14.3	44.3	344.1

Conclusions

- Stochastic modeling of real systems is prohibitively expensive
  - Man hours: meshing, model setup
  - Computer hours: solving full DOF problems
- Methods here reduce these costs:
  - Only need enough meshes to compute FD derivative to desired order
  - By making approximation on CBR matrices, solution cost reduced
- Demonstration on non-trivial model is promising:
  - Reasonable agreement with Truth curve – frequency vs. parameter
  - The CBR parameterization looks more stable than full DOF
  - Implemented via post-processing in MATLAB
- More study required:
  - Effect of Calibration Set range

Questions?

Thanks again to my collaborators:

- M.R. Brake
- S.D. Topping,
- N.M. McPeek-Bechtold
- J.A. Fike,
- R.V. Field
- R. Dingreville

BACKUP SLIDES

Craig-Bampton Reduction