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Engineering optical forces in waveguides and cavities based on optical response.

Peter T. Rakich[1], Zheng Wang[2], & Milos A. Popovic[2]

[1] *Applied Photonic Microsystems Group, Sandia National Laboratories, Albuquerque NM, 87123*

[2] *Research Laboratory of Electronics Massachusetts Institute of Technology, Cambridge MA 02139*

Photonics West, San Francisco CA, Jan 2010

Outline:

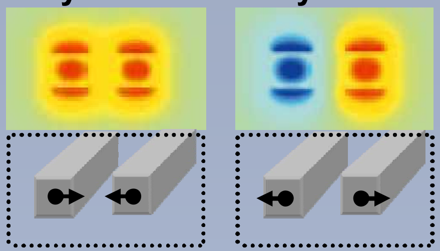
- Force computation/design in optomechanics.
- Introduce the response theory of optical forces (RTOF) [1].

  ➔ Yields simple analytical framework for optical forces.
- Describe conceptual basis of RTOF.
- Illustrate simplified design with RTOF.

[1] Peter T. Rakich, Miloš A. Popovic, and Zheng Wang, "General treatment of optical forces and potentials in mechanically variable photonic systems," Opt. Express 17, 18116-18135 (2009)

Optomechanics: Recent Work

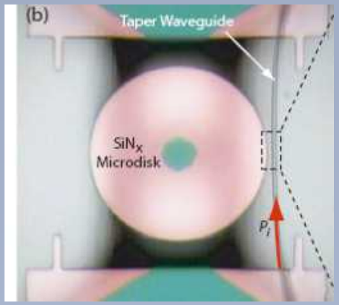
Symmetric Asymmetric



Attractive force Repulsive force

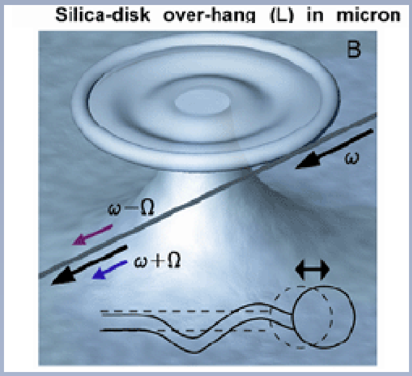
Povinelli, et. al. Opt. Lett. 30,3042 (2005).

Optical microdisk



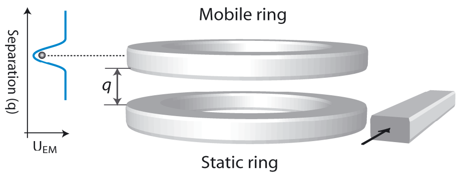
Eichenfield, M. *et al.* Nature Photon. 1, 416–422 (2007).

Lipson



Vahala kippenberg Karmon Groups:

Dual microring geometry:



Tang

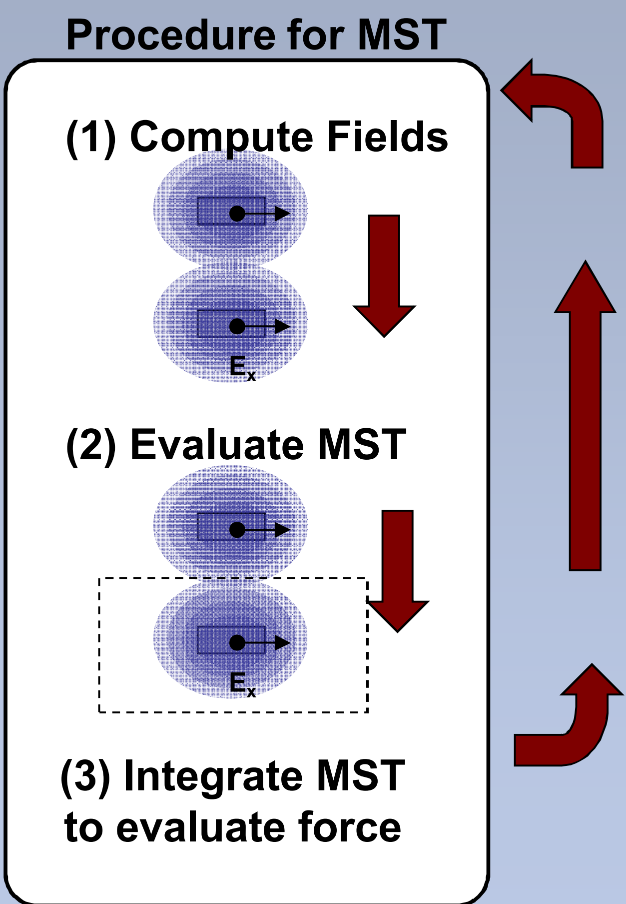
Mizrahi

Notomi

- **Variety of optomechanical systems studied.**
- **For applications: Goal is to enhance/tailor the optical forces.**
- **Topic of this presentation: How to understand and model such systems?**

Q: What is the status of force computation & design in optomechanics today?

• Maxwell Stress Tensor is widely used method for force computation. but there's no well established method for force design. ➔ this is the problem that we address.



<u>Pros of MST:</u>

- Straight forward recipe.
- Proven.

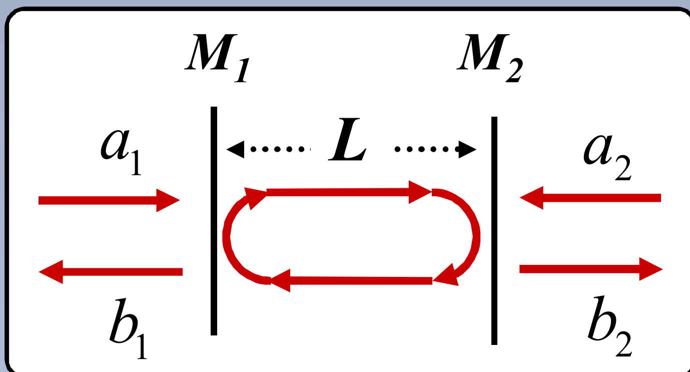
<u>Cons of MST:</u>

- Computationally costly
  ➔ requires full field calculation.
- Offers little intuition/insight.
- Difficult to design/synthesize forces with MST.
- No simplifications apparent with MST.
- No way to identify similarities/patterns between various systems

Response theory of optical forces (RTOF)

- **Optical force can be computed from the mechanically variable optical response.**
- **How is the mechanically variable optical response defined?**

Two-port system: Fabry-Perot

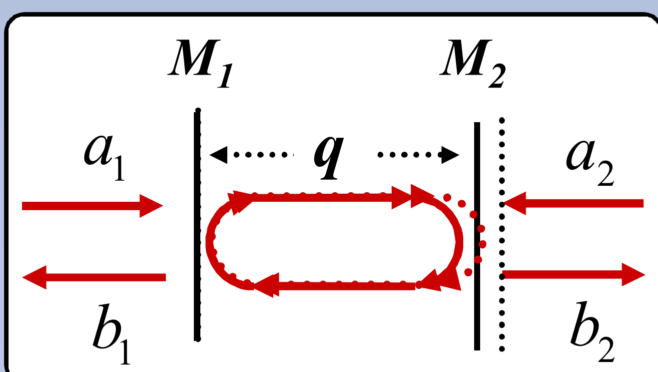


Fabry-Perot: Two-port system

If L = constant:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \left[\tilde{S}\right] \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Optomechanically variable **Two-port system**



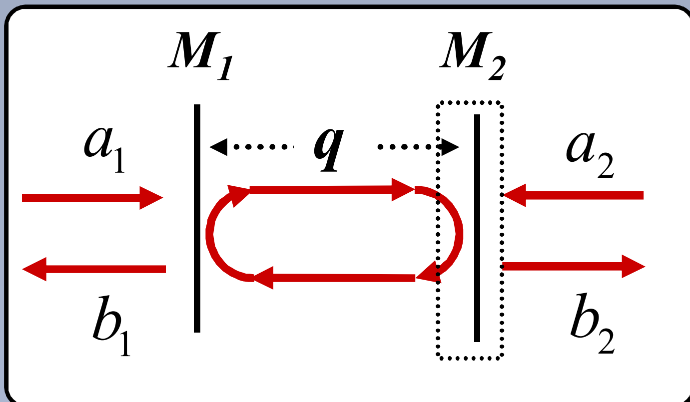
Mechanically variable optical response:

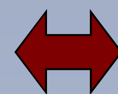
$$\begin{bmatrix} b_1(q) \\ b_2(q) \end{bmatrix} = \left[\tilde{S}(q)\right] \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\varphi_{o,k}(q) = \arg\left[\tilde{b}_k(q)\right]$$

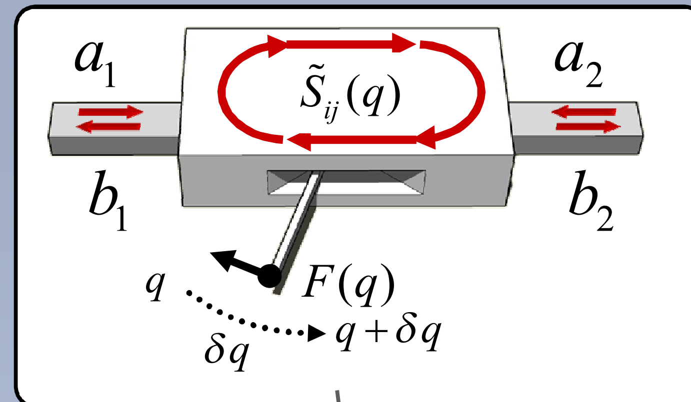
Forces from optical response:

Fabry-Perot: Two-port system

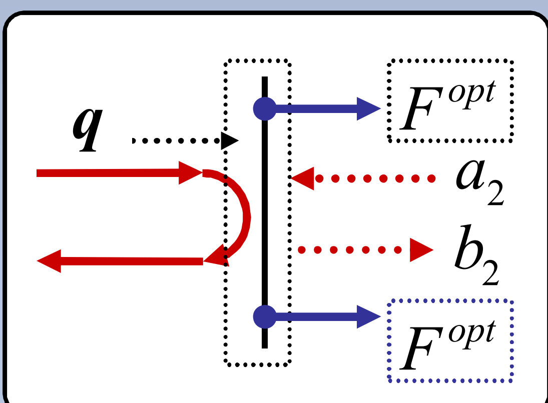




Generalized Mech. variable two-port:



Optical forces on Mirror:



Two-port system response

$$\begin{bmatrix} b_1(q) \\ b_2(q) \end{bmatrix} = \left[\tilde{S}_{ij}(q)\right] \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\varphi_{o,k}(q) = \arg\left[\tilde{b}_k(q)\right]$$

Forces computed from response:

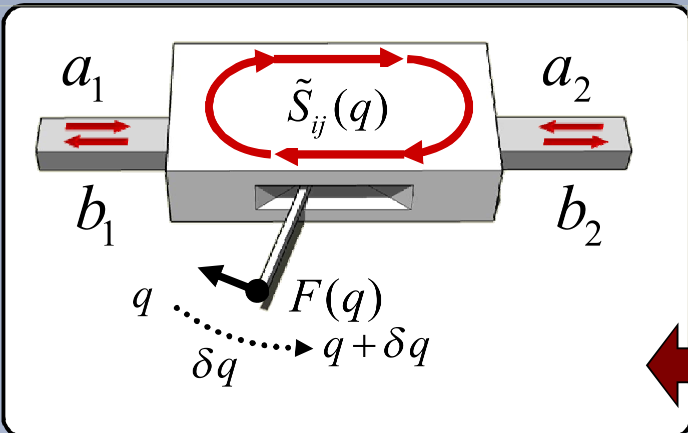
Optical force expressed on mechanical degree of freedom, *q*, given by:

$$F_q(q) = \frac{1}{\omega} \sum_k \left|b_k(q)\right|^2 \cdot \frac{d\phi_{o,k}(q)}{dq}$$

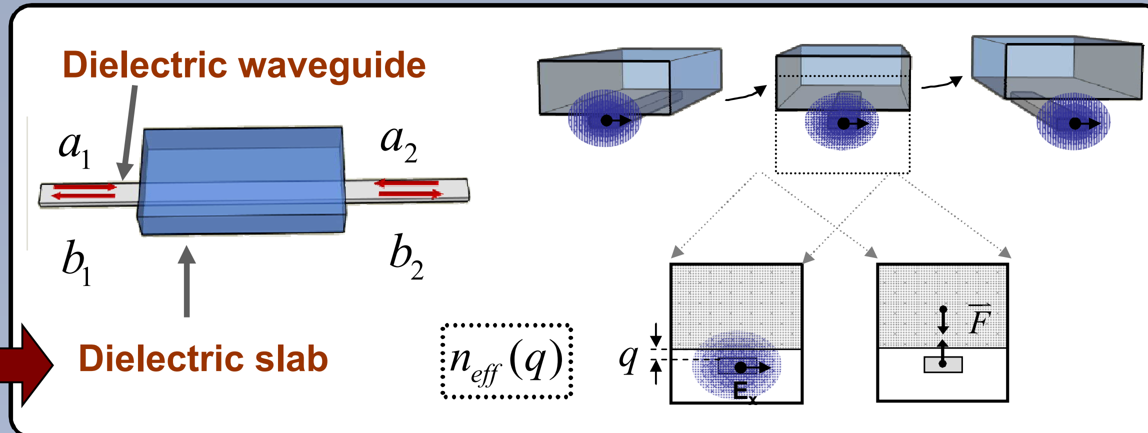
Fabry Perot is a simple example, but concept is very broadly applicable…

Generality of the response theory of optical forces:

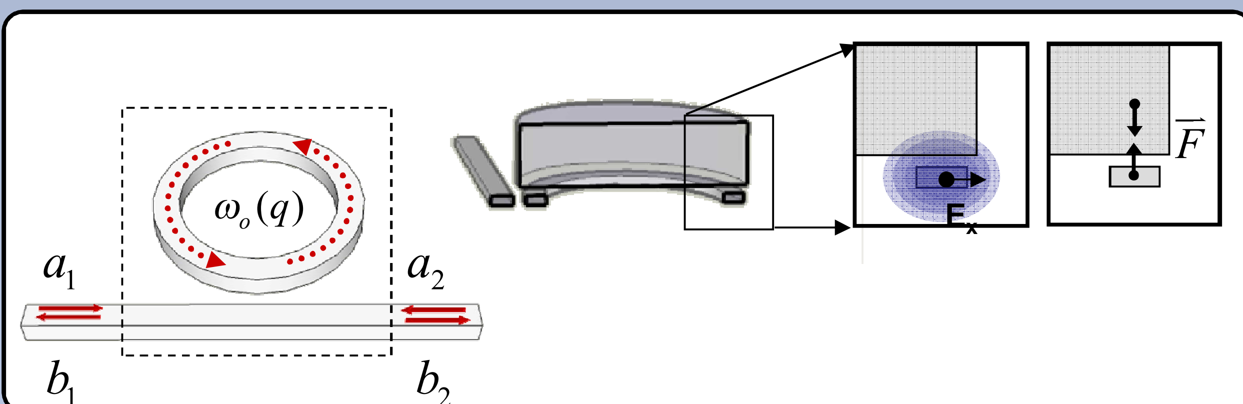
Generalized Mech. variable two-port:



Waveguide segment under dielectric perturbation:



Optomechanically ring resonator All-pass filter:



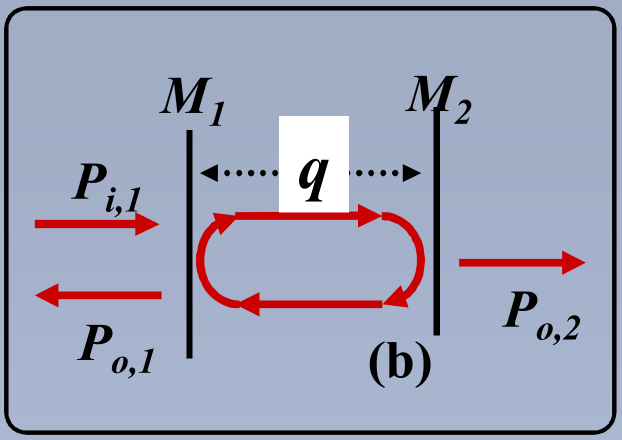
Force and potential are computed from RTOF.

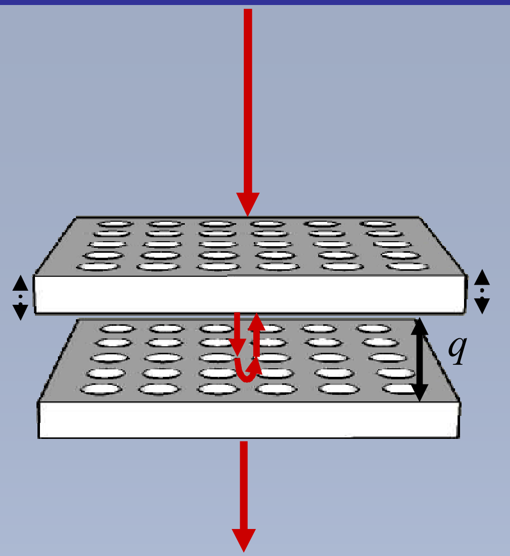
Why would is the potential useful?

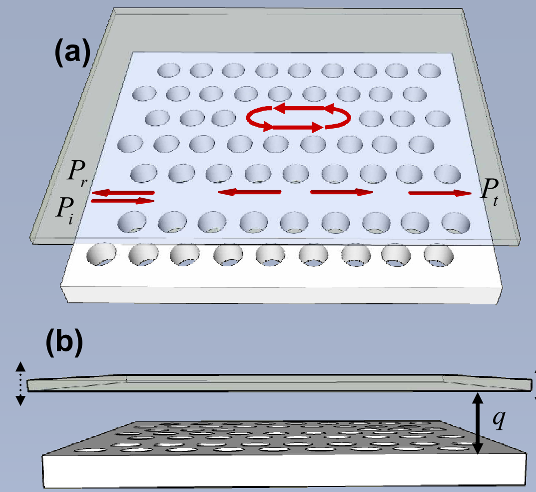
$$F_q(q) = \frac{1}{\omega} \sum_k |b_k(q)|^2 \cdot \frac{d\phi_{o,k}(q)}{dq}$$

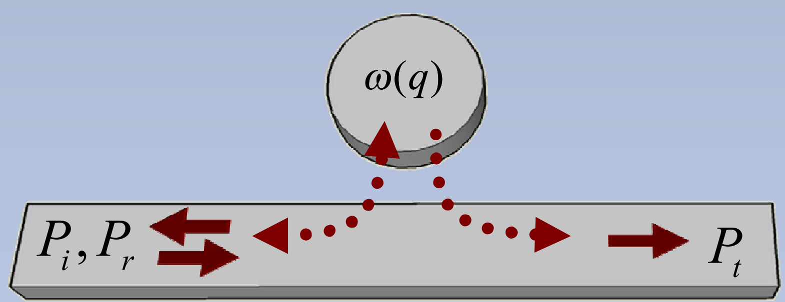
Link behavior of similar optical systems:

Radiation Pressure





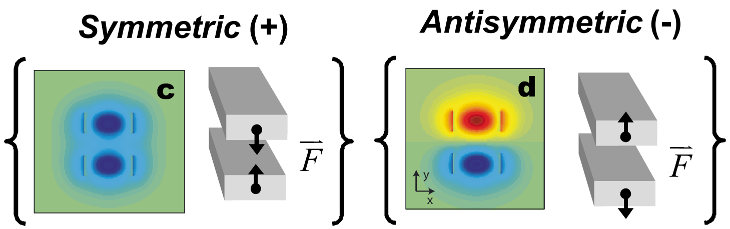


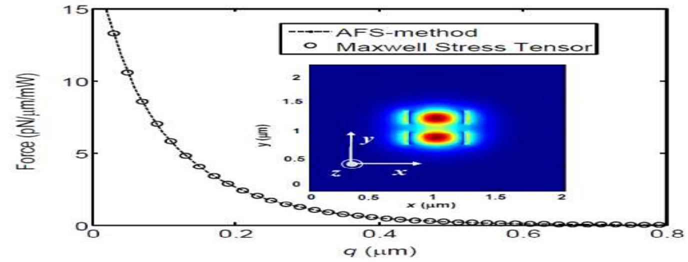


- Systems look quite different, but their response can be identical.
  - All are two-port resonant systems (ω_o)
  - $\omega_o(q)$
  - $H(\omega_o(q))$
- Despite similarities, Maxwell stress tensor requires complete reformulation of problem electromagnetically.

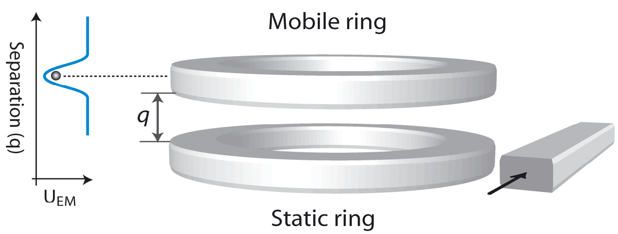
Design of new functionalities

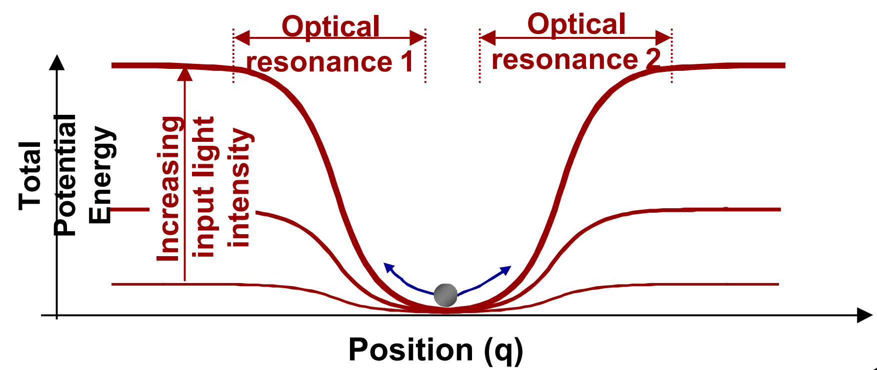
Design for maximum optical force



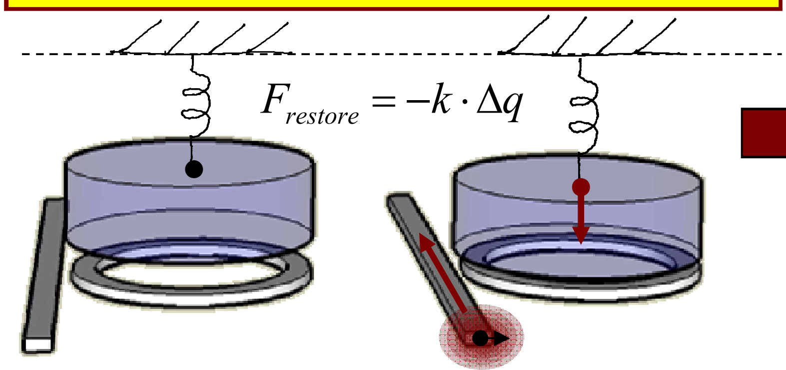


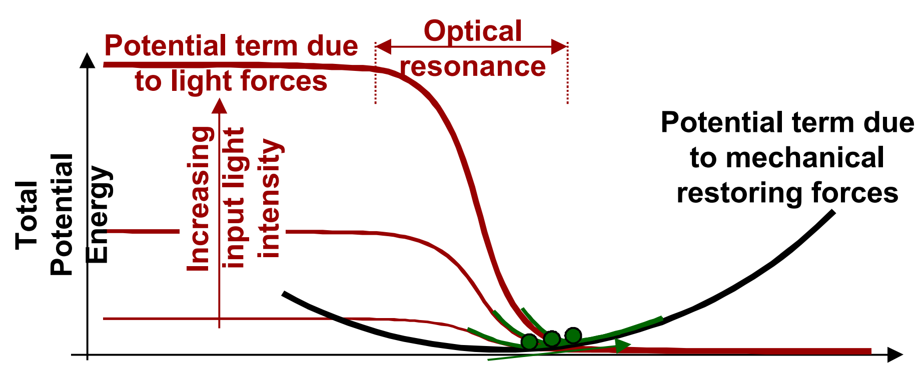
Design poential well ➔ trapping & manipulation





Design nonlinearity ➔ Bistable switching



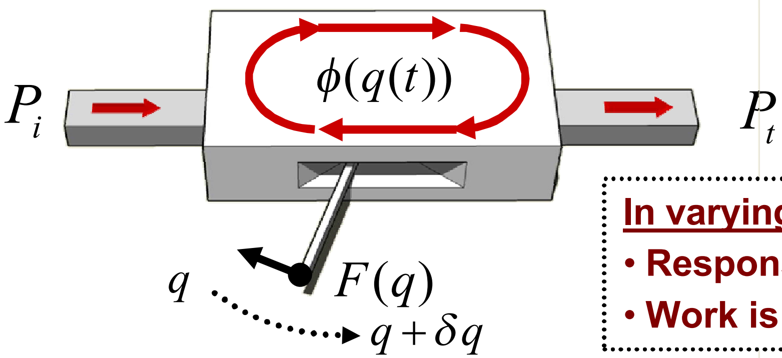


Conclusion: Light-intensity-dependent equilibrium state (●) in total (sum) potential

(2) Conceptual basis for RTOF

What is the basis for RTOF? First must consider conservation of energy in optomechanically variable systems:

Smoothly time-varying optomechanical system:



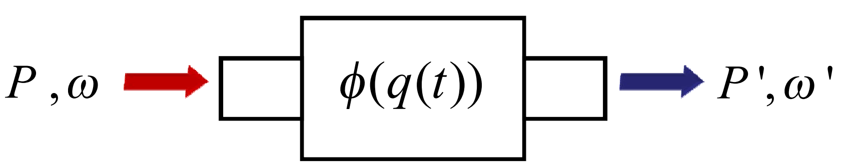
In varying q:

- **Response changes.**
- **Work is done against F^{opt}.**

For simplicity, we assume:

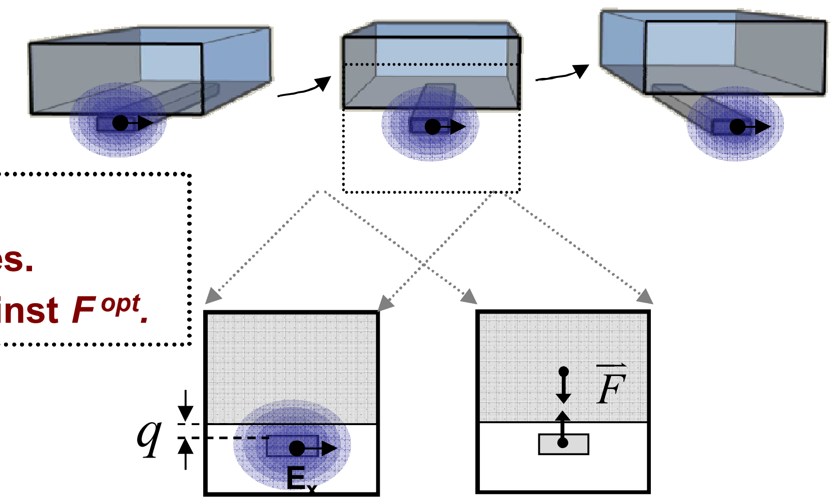
- **Device = Reflectionless one-port**
- **No Losses ➔ Coordinate, *q*, effects phase.**
  (System =optomechanical phase modulator)

$$exp[-i(\omega t)] \rightarrow exp[-i(\omega t - \phi(q))]$$



For time-varying *q*, frequency & power modified.

Simple example of reflectionless one-port:



$$n_{eff}(q)$$

$$\delta W = F^{opt} \cdot \delta q$$

Energy conservation:

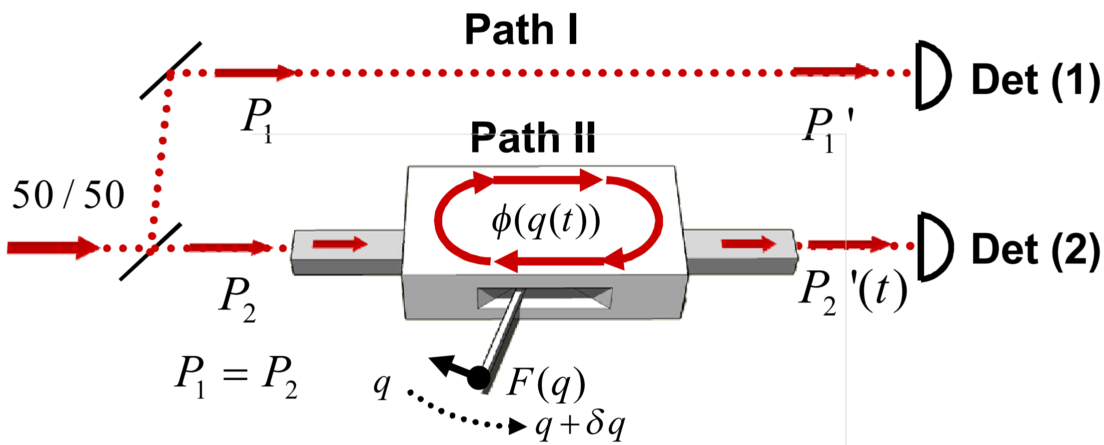
$$\delta U_{EM} \rightarrow \delta U_{EM} = \delta W$$

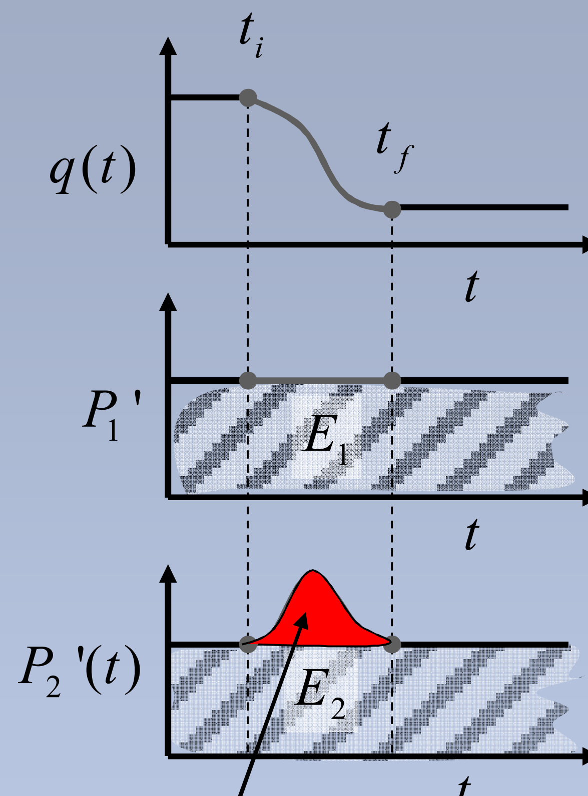
$$\delta U_{EM} \approx N\hbar \cdot \delta\omega$$

How do we formulate ΔU_{em} *more rigorously*?

A closer look at energetics of time-varying system:

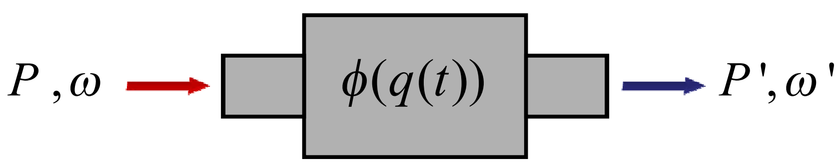
Thought experiment Illustrating Energetics





$$exp[-i(\omega t)] \rightarrow exp[-i(\omega t - \phi(q))]$$

(Lossless one port) = (Optomechanical phase modulator)



Change in energy carried by EM wave:

$$\Delta U_{EM} = \int (P' - P) \cdot dt$$

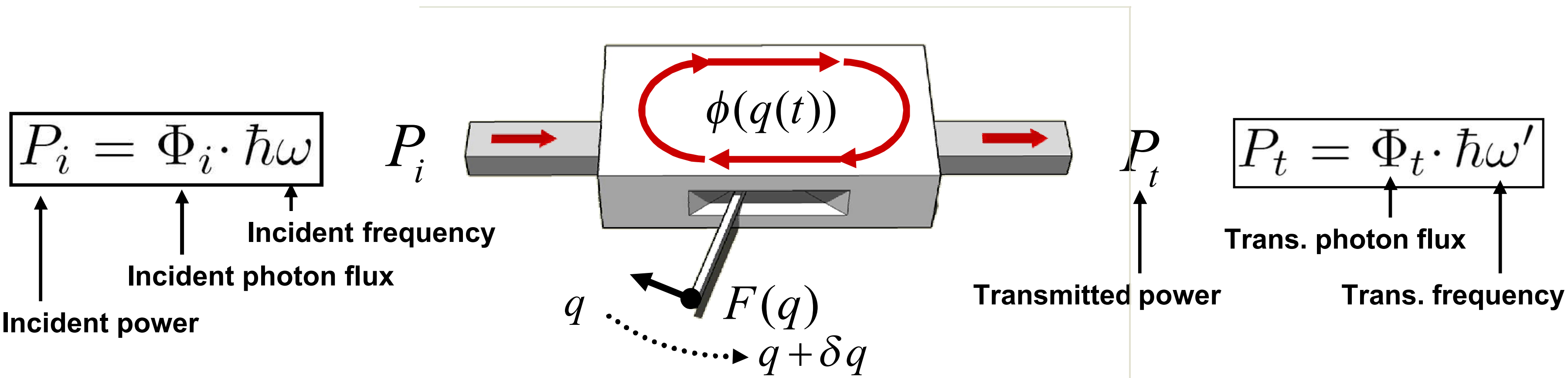
Work done against optical forces:

$$W = \int F^{opt}(q) \cdot dq$$

How to compute the change in EM energy?

Great simplification through use of the photon construct:

(1) Relate the incident power to photon energy and photon flux.
(2) Assume that *q(t)* evolves slowly.
(3) Assume that photons are conserved.



If *q(t)* evolves slowly, it is reasonable to assume:

$$\Phi_i = \Phi_t = \Phi$$ **(Photon flux, conserved)**

$$\omega' = \omega + \delta\omega(t) = \omega - \dot{\phi}\left(q(t)\right)$$

$$\Delta U_{EM} = \int \left(P' - P\right) \cdot dt = \Phi\hbar \cdot \int \delta\omega(t) \cdot dt = -\Phi\hbar \cdot (\phi_f - \phi_i)$$

How to compute the change in EM energy?

Great simplification through use of the photon construct:

$$\Delta U_{EM} = \int \left(P' - P\right)\cdot dt = \Phi\hbar \cdot \int \delta\omega(t)\cdot dt = -\Phi\hbar\cdot(\phi_f - \phi_i)$$

fixed variable

Since $\Delta U_{EM} = W$

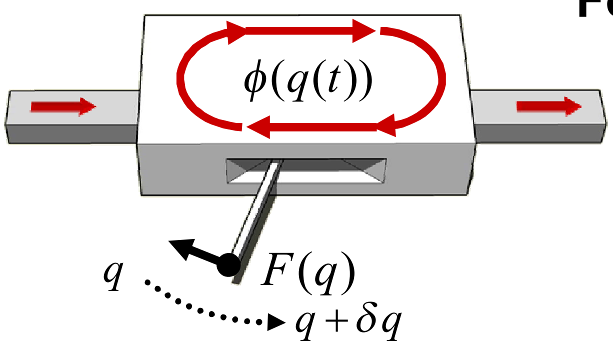
Potential:

$$U^{eff}(q) = -\Phi\hbar\cdot\phi(\omega, q)$$

Force:

$$F(q) = \Phi\hbar\cdot\left(\frac{\partial\phi}{\partial q}\right)$$

$$F(q) = -\left(\frac{\partial U_{eff}}{\partial q}\right)$$



Result:

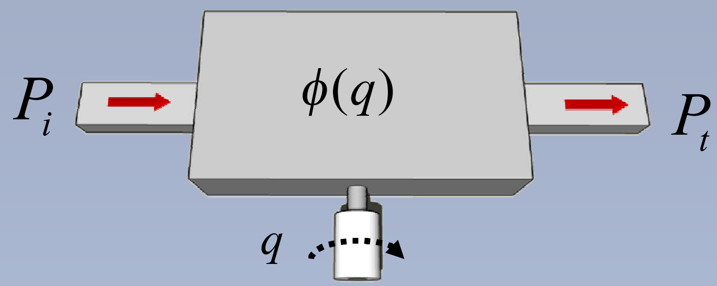
Given $\phi(q)$ we can compute the optical force on coordinate, q.

Multi port relations Here:

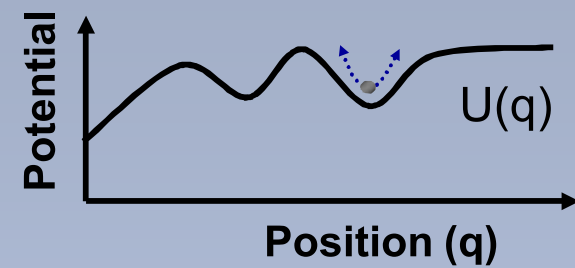
Extension to multi-port systems.

Lossless single-port, with a single mechanical degree of freedom, q.
(Input flux (Φ) and frequencies (ω) are assumed to be fixed).

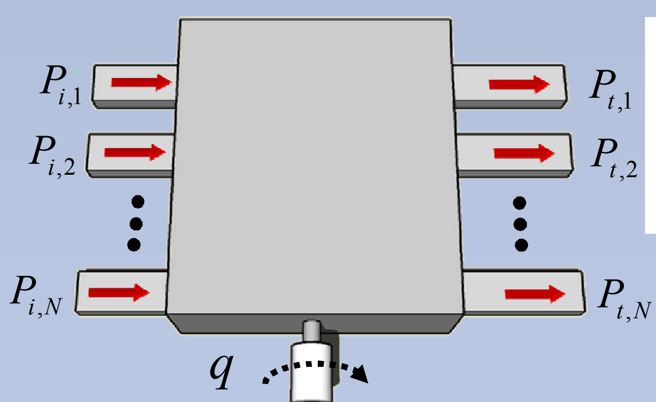
$$U_{eff}(q) = -\Phi \cdot \hbar\phi(q)$$



- Allows direct potential synthesis!
- Phase synthesis already known [1].



Lossless multi-port, with a single mechanical degree of freedom, q.
(Input fluxes (Φ_i's) and frequencies (ω_i's) are assumed to be fixed).



$$U_{eff}(q) = -\hbar \cdot \int \left[\sum_k \Phi_{t,k}(q) \cdot \frac{d\phi_{t,k}(q)}{dq} \right] \cdot dq$$

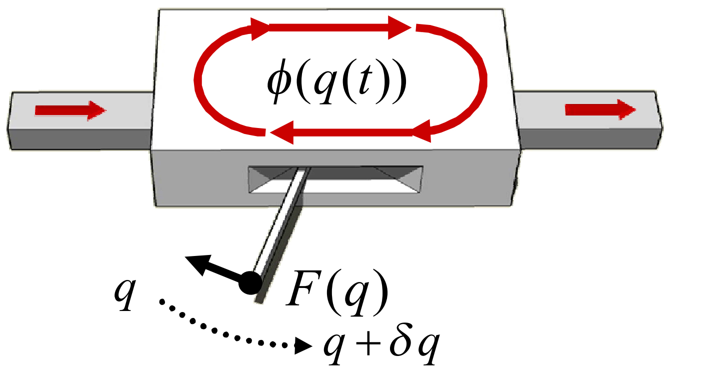
Phase response

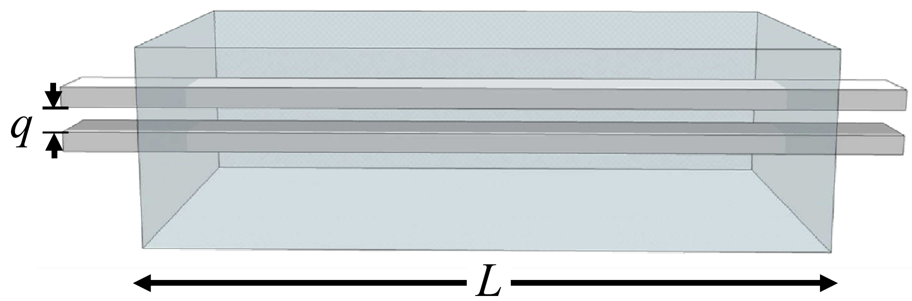
Photon flux (k^{th} output port)

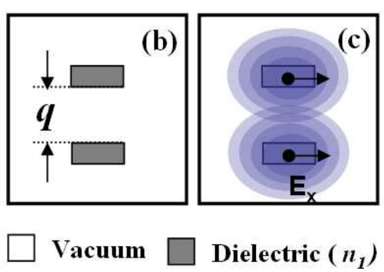
[1] Madsen, C. K. & Zhao, J. H. *Optical flter design and analysis* (Wiley, New York, 1999).

(3) Comparison of RTOF with Maxwell Stress Tensor methods.

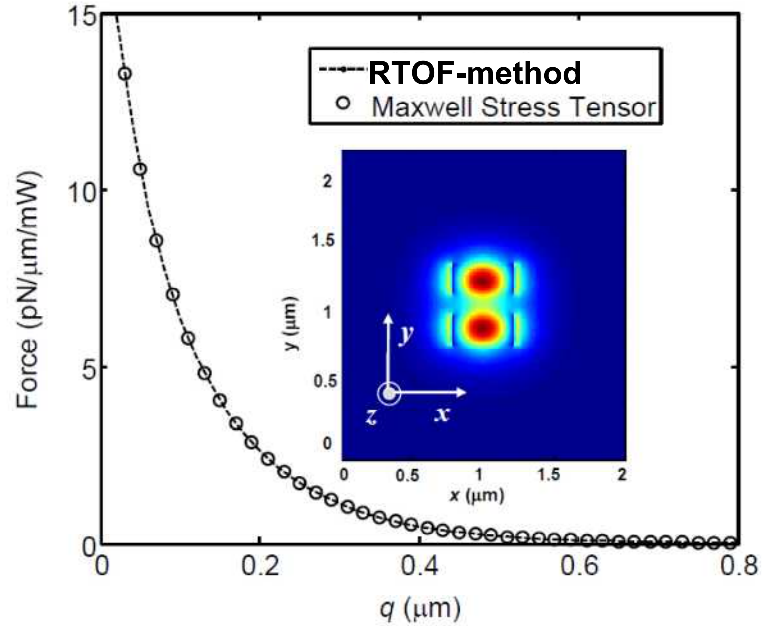
Comparison of MST with RTOF: Numerical Example







$$n_{eff}(q)$$

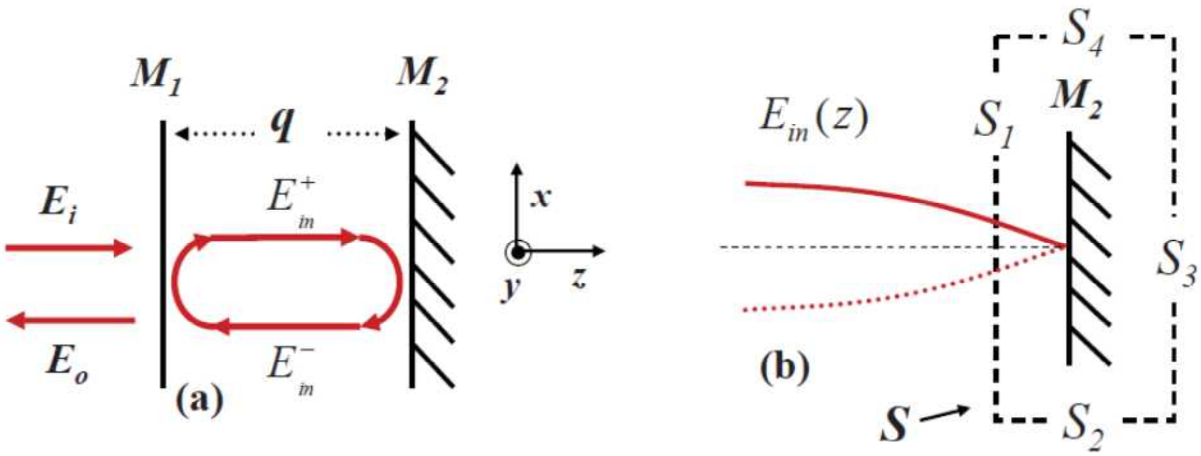


Analytical expression for Force (RTOF):

$$\phi(\omega,q)=\frac{\omega}{c}\cdot n_{eff}(\omega,q)\cdot L$$

$$F_q^o(\omega,q)=L\cdot\frac{P}{c}\frac{\partial n_{eff}}{\partial q}$$

Comparison of MST with RTOF: Analytical Example



RTOF method

$$\phi(\omega,q) = tan^{-1}\left[\frac{(1-r^2)sin(\delta)}{2r-(r^2+1)cos(\delta)}\right]$$

$$F_q(q) = \Phi\cdot\hbar\cdot\frac{d\phi(q)}{dq}$$

$$\frac{F_q(\omega,q)}{A} = -\frac{E^2}{\mu_o c^2}\left[\frac{(1-r^2)}{2r\cdot cos[2q(\omega/c)]-(r^2+1)}\right]$$

MST method

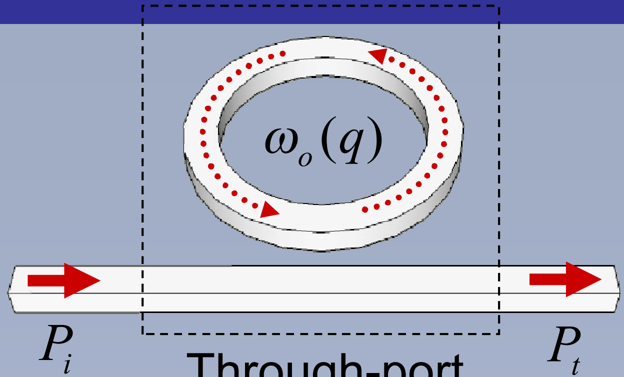
$$\vec{F} = \oint_S \overleftrightarrow{T}\cdot d\vec{a}$$

$$T_{i,j} = \varepsilon_o[E_iE_j-(1/2)\delta_{i,j}|E|^2]+[B_iB_j-(1/2)\delta_{i,j}|B|^2]/\mu_o$$

$$\frac{F_q(\omega,q)}{A} = -\frac{E^2}{\mu_o c^2}\left[\frac{(1-r^2)}{2r\cdot cos[2q(\omega/c)]-(r^2+1)}\right]$$

(4) Simplified design via RTOF.

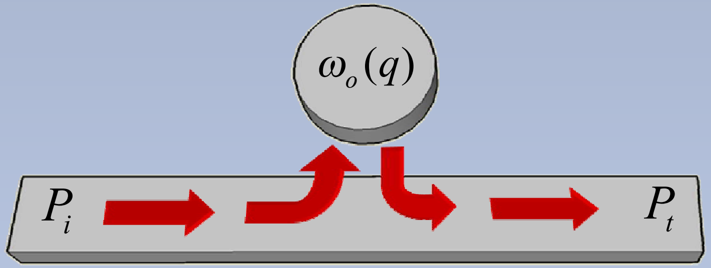
Example: All-pass filter.



$$\phi(q) = 2\cdot \arctan[(\omega - \omega_o(q))\tau_t]$$

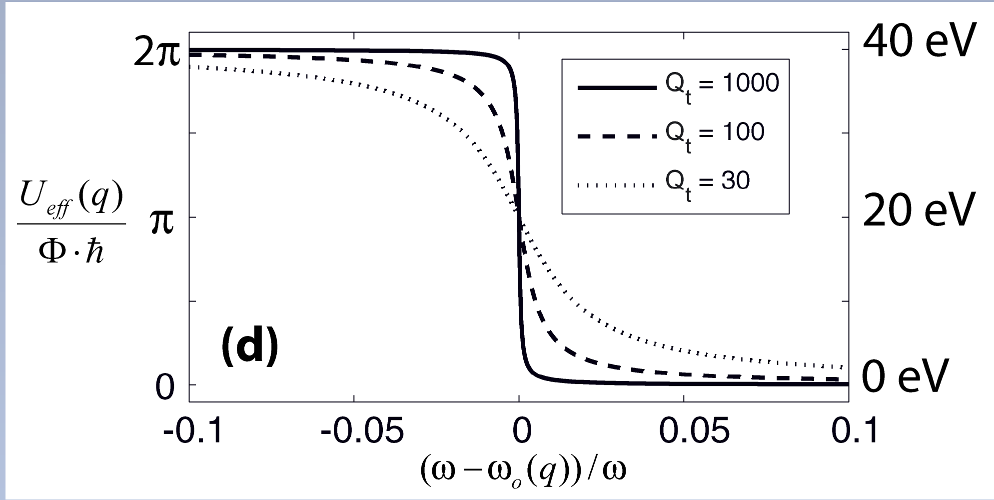
$$U_{eff}(q) = -\Phi\cdot \hbar\phi(q)$$

C.M.T. Model of microring:



$$\frac{d}{dt}a = i\omega_o a - \frac{1}{\tau}a + i\kappa s_i$$

$$\kappa = \sqrt{2/\tau_t}$$



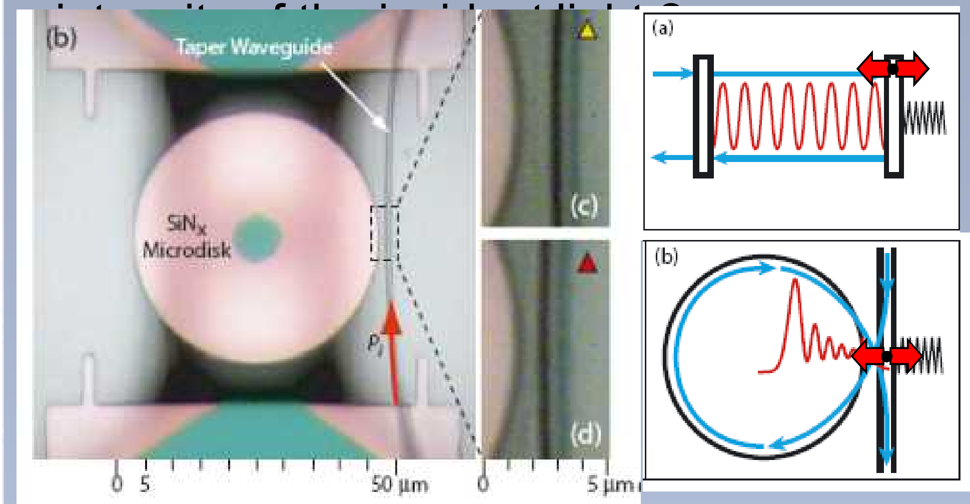
Potential makes "jump" of

$$\Delta U_{eff} = \Phi\cdot \hbar\cdot 2\pi$$

Independent of cavity Q!

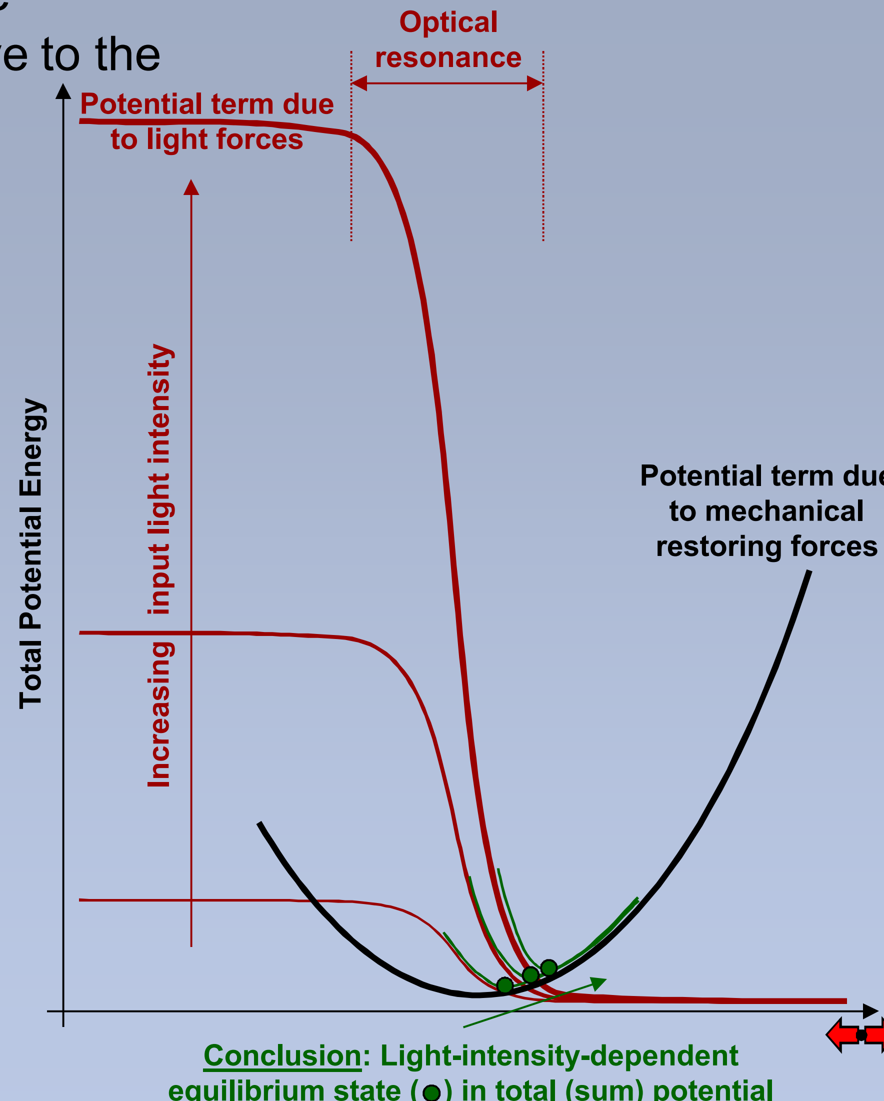
Bistable microring switch.

- Q: Why is the equilibrium state of the bistable optomechanically variable system so sensitive to the



Eichenfield, M. *et al.* Nature Photon. 1, 416–422 (2007).

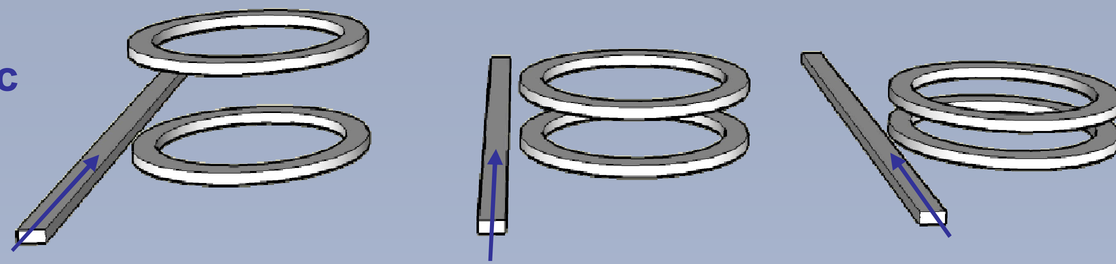
- A: Because half of the potential-well is produced by light, while the other half of the potential well is created by the mechanical restoring force (i.e. a spring of some sort.)
- As a result, the equilibrium position of the system is highly intensity dependent.



Design of optical potential wells with RTOF:

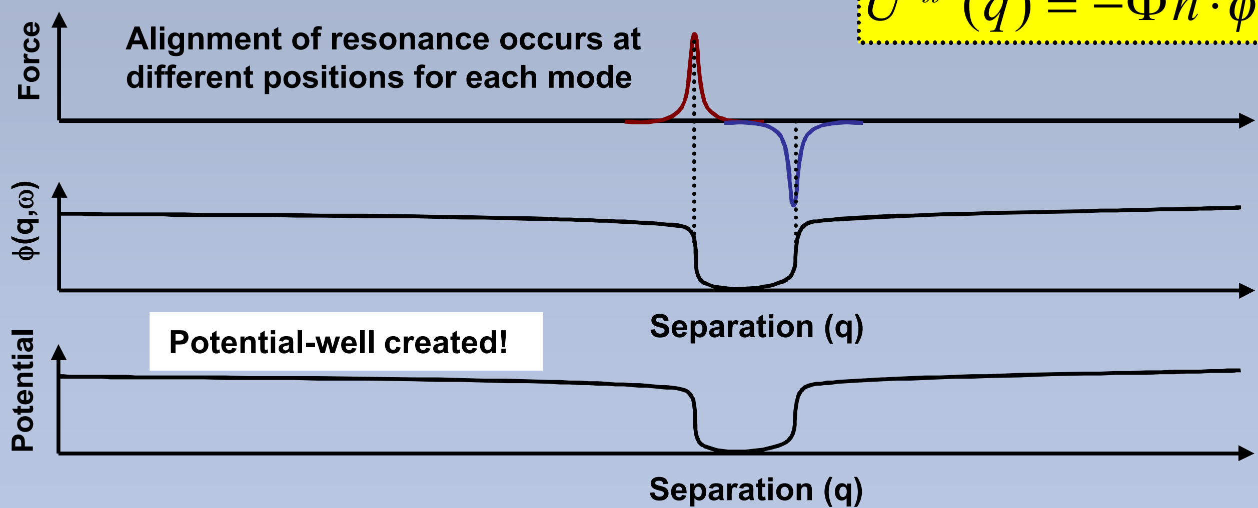
Examine phase response as *q* varies for fixed ω:

Monochromatic excitation at frequency ω



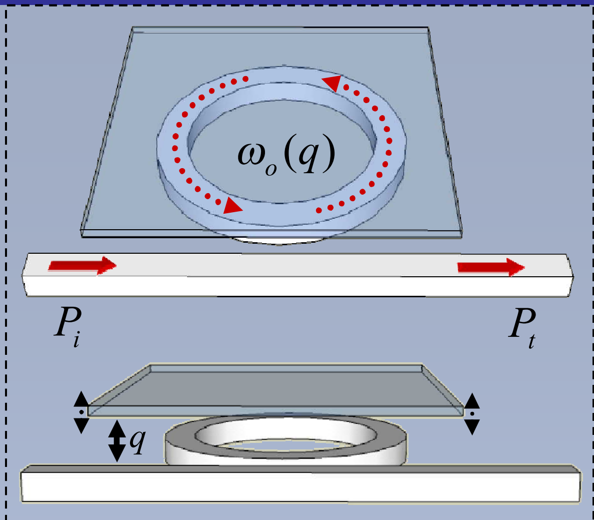
fixed variable

$$U^{eff}(q) = -\Phi\hbar\cdot\phi(\omega, q)$$

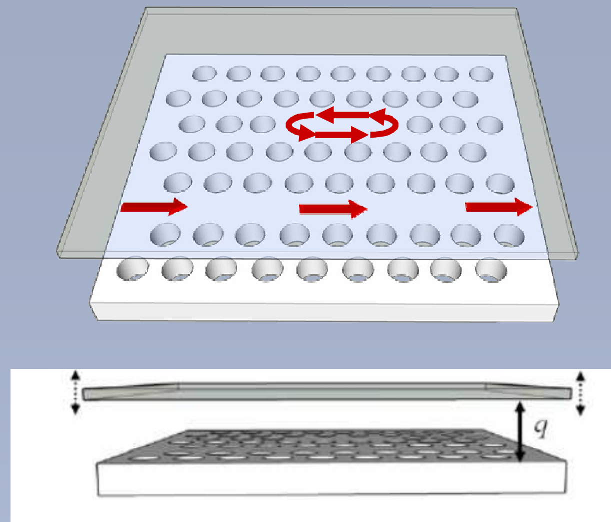


For more details see Nat photonics paper.

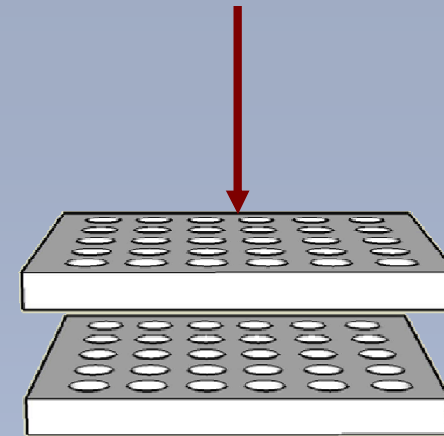
Analyze any system representable by CMT!



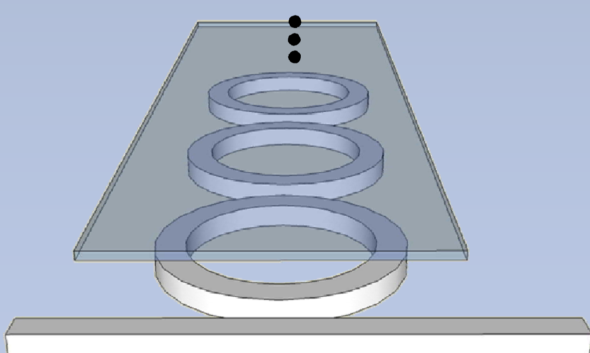
Rakich et. al. Opt. Lett. 31, 1241-1243 (2006).

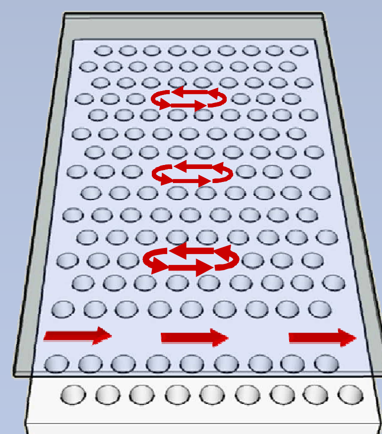


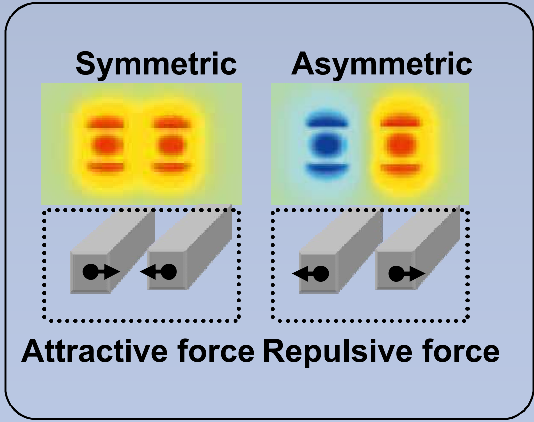
M. Notomi, et. al.PRL 97(2), 023903 (2006).



Suh et al, Opt. Lett., vol. 28, p1763 (2003)







Povinelli, et. al. Opt. Lett. 30,3042 (2005).

Conclusions:

New formalism, “Response theory of optical forces” or RTOF enables:

Derivation of the force and potential from mech variable optical response.

→ Synthesize potentials.

→ Unify the behaviour of systems.

→ Simple conceptualization of forces & tractable analytical models.

Can apply this theory provided that:

→ Response can be written simply (ideally finite number of ports).

→ System is lossless.

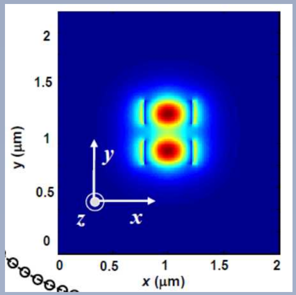
Further possibilities:

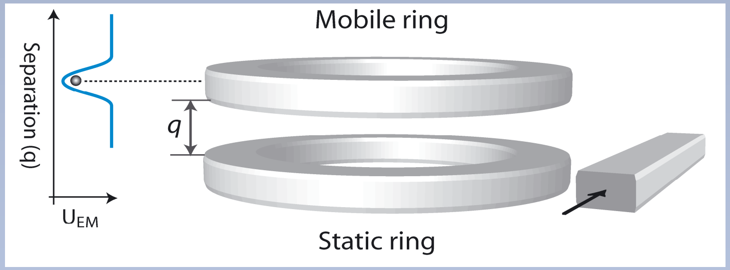
Can treat systems with losses in many cases provided the source of the losses is known and can be assigned to an output port.

Backup slides follow:

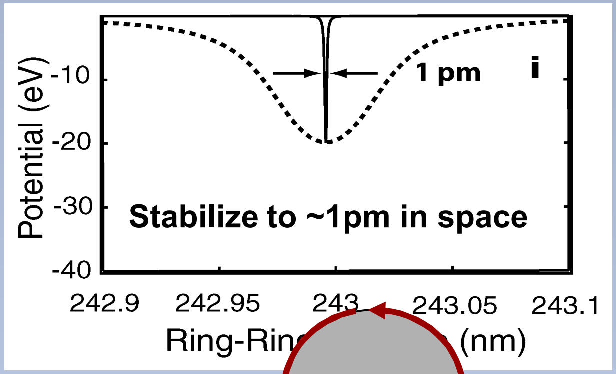
Design of new functionalities

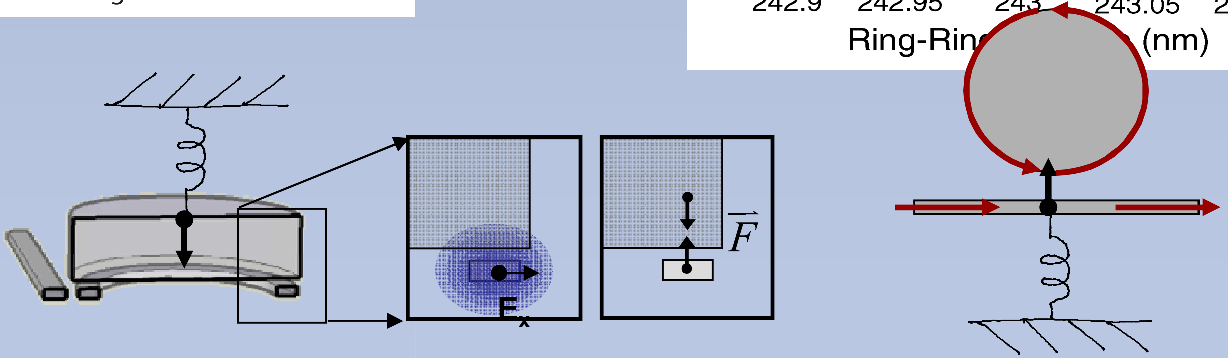
- Design for maximum force
- Design for function
  - ➔ potential well
  - ➔ Optical bistability

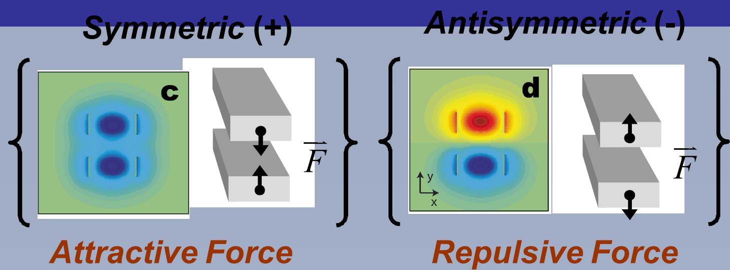


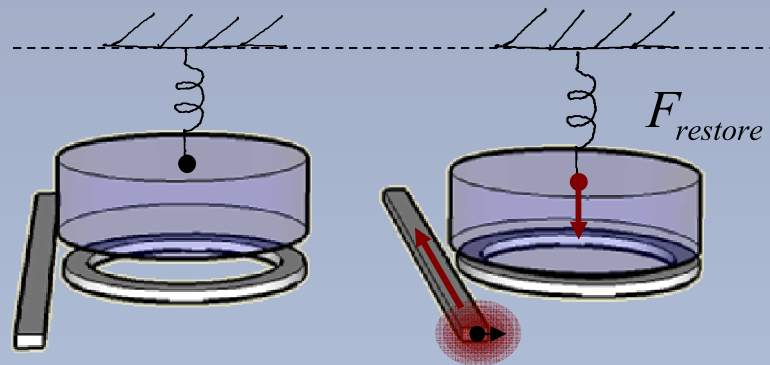


Ultra-precise control of bodies

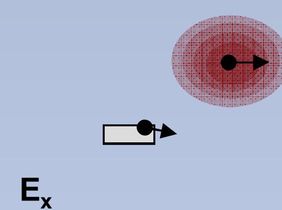


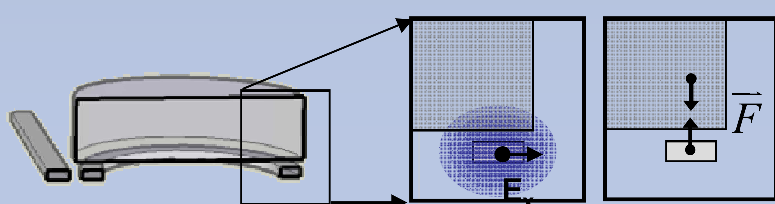


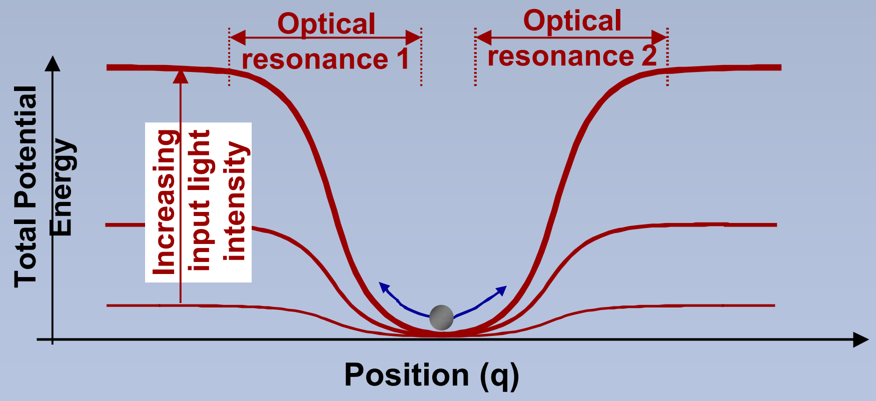


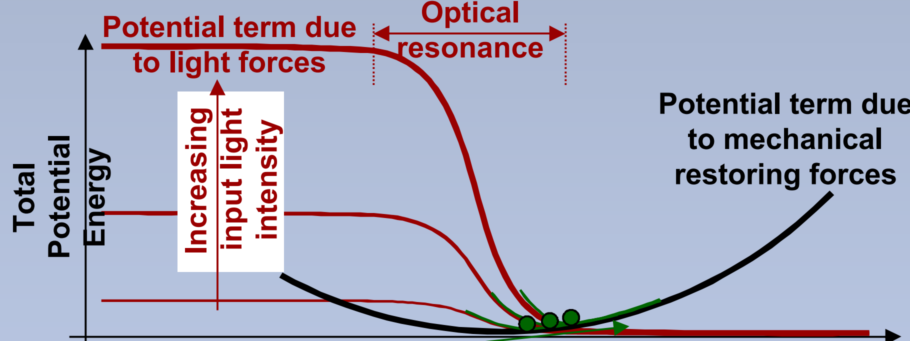


$$F_{restore} = -k \cdot \Delta q$$



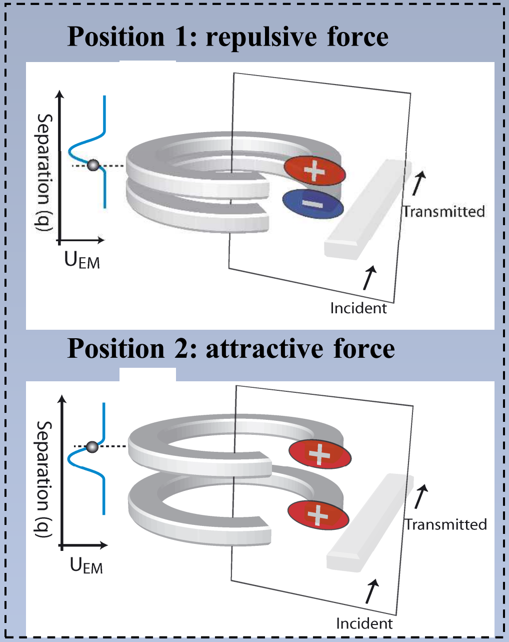



Optical
resonance 1
Optical
resonance 2
Total Potential Energy
Increasing input light intensity
Position (q)

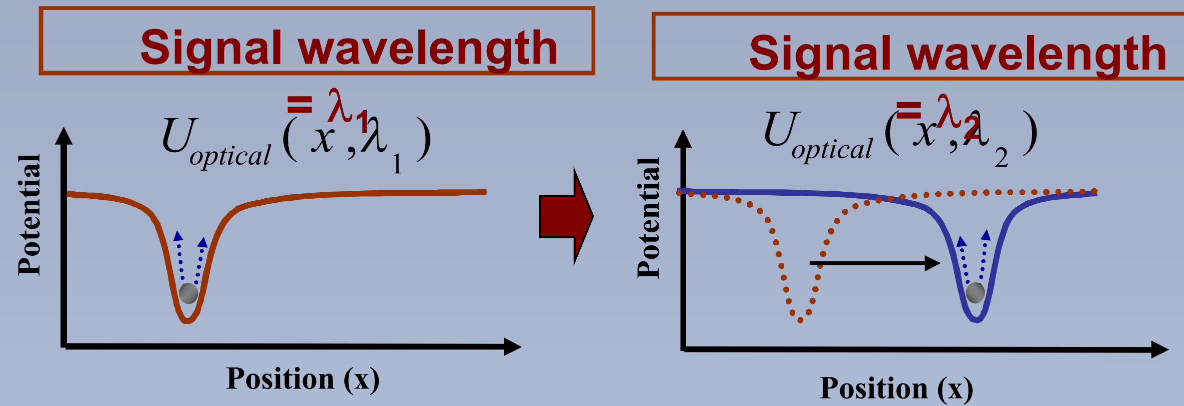


Conclusion: Light-intensity-dependent equilibrium state (●) in total (sum) potential

For example:

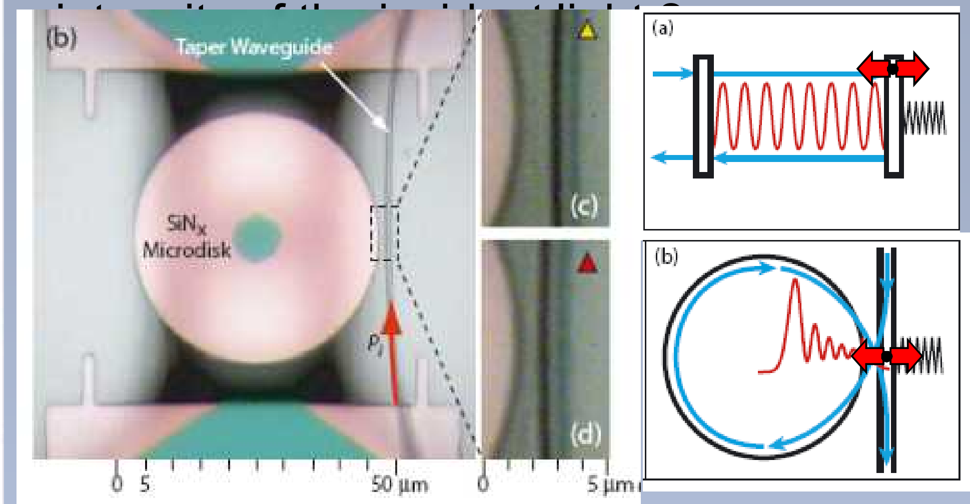


Synthesis of optical force profiles for sophisticated functionality

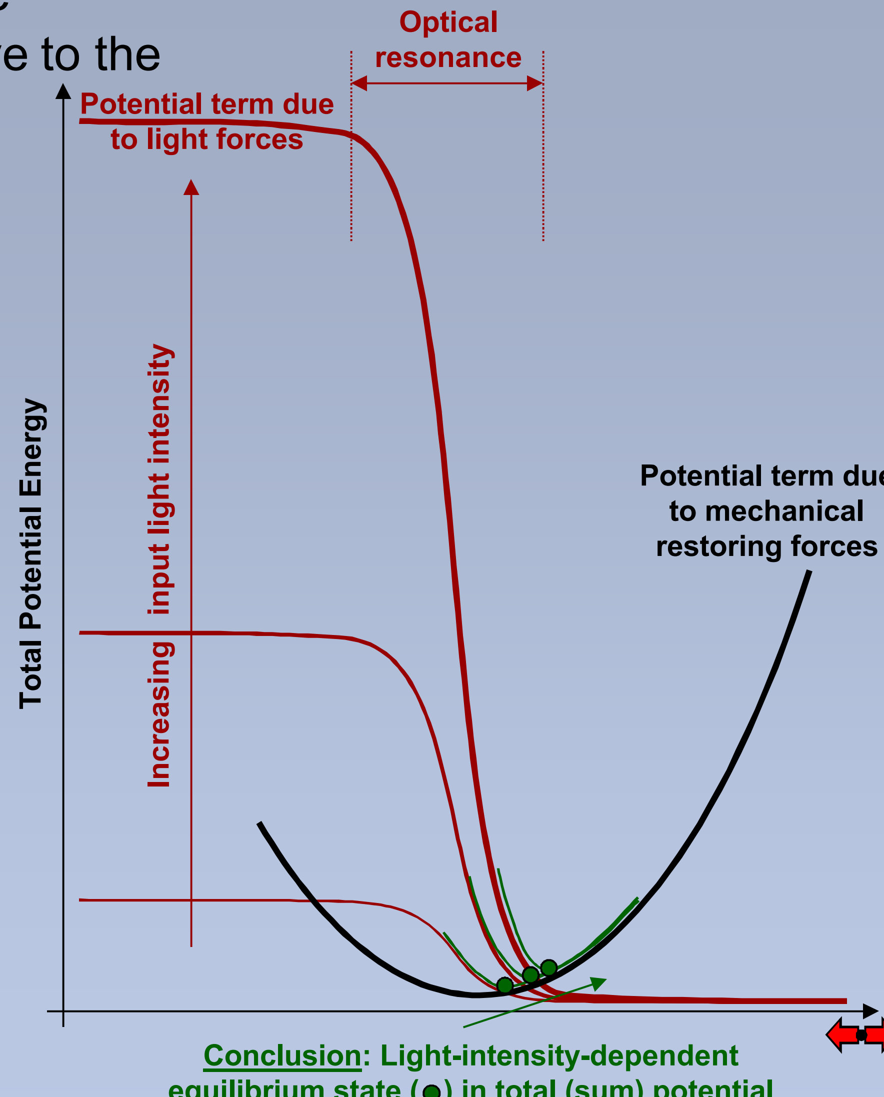


Amplitude-Invariant Equilibrium States: Frequency Control

- Q: Why is the equilibrium state of the bistable optomechanically variable system so sensitive to the



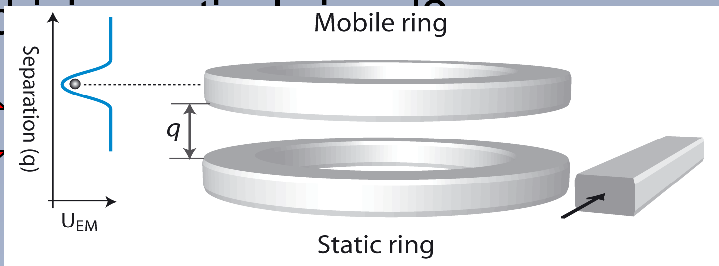
Eichenfield, M. *et al.* Nature Photon. 1, 416–422 (2007).



- A: Because half of the potential-well is produced by light, while the other half of the potential well is created by the mechanical restoring force (i.e. a spring of some sort.)
- As a result, the equilibrium position of the system is highly intensity dependent.

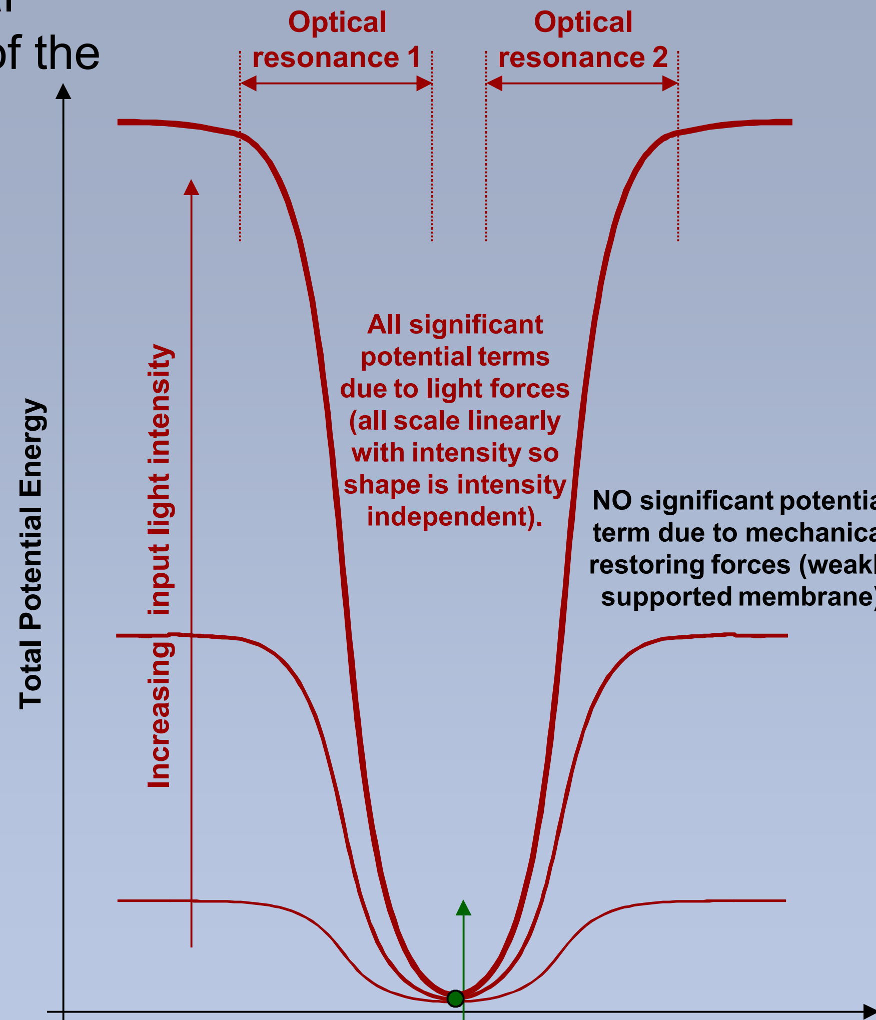
Amplitude-Invariant Equilibrium States: Frequency Control

Q: Why is the equilibrium state of an all-optical potential-well independent of the intensity of the



A: Because both sides of the potential well are created by the same incident optical signal. Therefore, as the intensity of the light changes, both sides of the potential-well change in the same way.

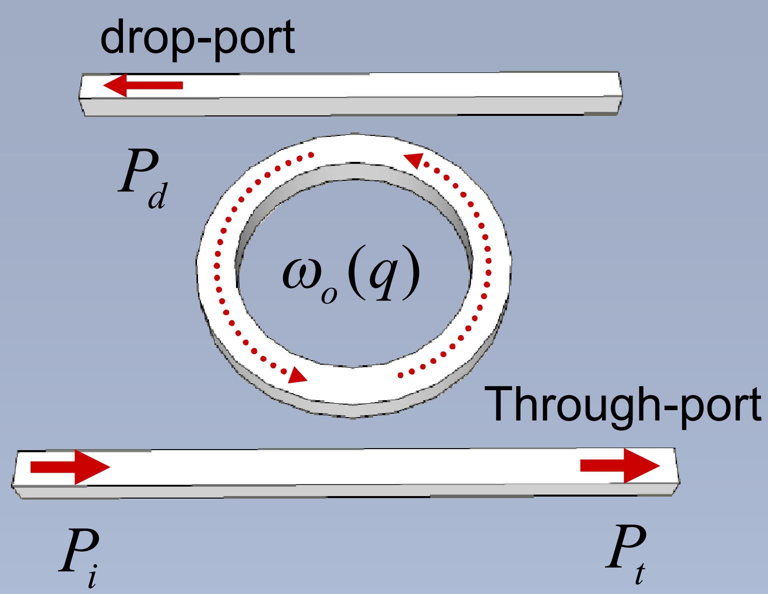
- As a result, the equilibrium position of the system has no intensity dependence.

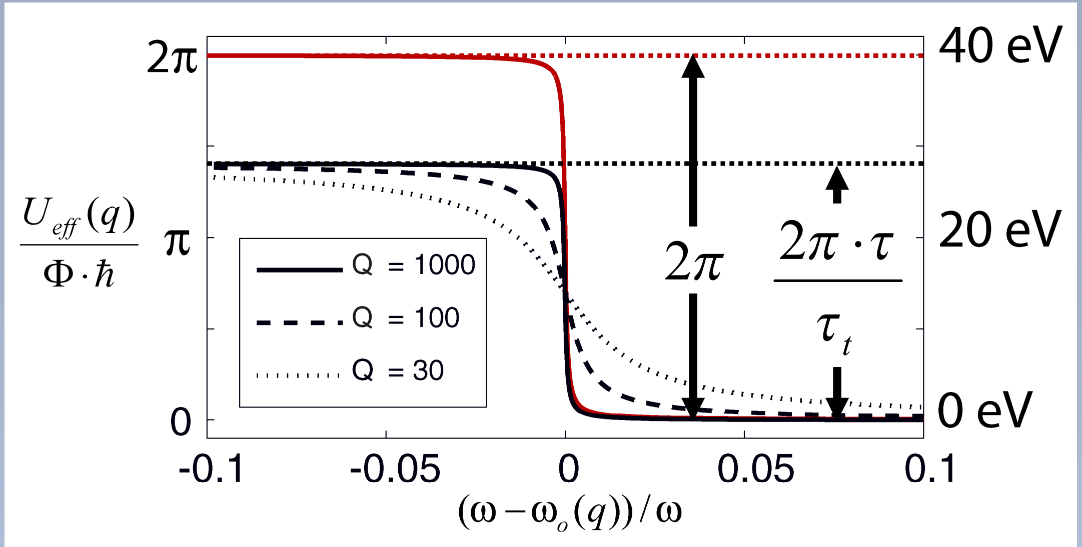


<u>Conclusion</u>: Light-intensity-INDEPENDENT equilibrium state (●) in total (sum) potential

Example: Multi-port ring resonator system.

$$1/\tau = 1/\tau_t + 1/\tau_d$$





Single-port system: $U^o_{eff} = \hbar \cdot \Phi \cdot 2 \cdot \arctan[(\omega - \omega_o(q))\tau_t]$

Multi-port system: $U^o_{eff} = \hbar \cdot \Phi \cdot \frac{2\tau}{\tau_t} \cdot \arctan[(\omega - \omega_o(q))\tau]$

Same relation for N-ports: $1/\tau = 1/\tau_t + 1/\tau_1 + 1/\tau_2 \ldots + 1/\tau_N$

Outline:

- Discuss direction of optomechanics.
- Describe the response theory of optical forces (RTOF)*.

  ➔ Yields simple analytical framework for optical forces.
- Illustrate simplified design with RTOF.

[*] Peter T. Rakich, Miloš A. Popovic, and Zheng Wang, "General treatment of optical forces and potentials in mechanically variable photonic systems," Opt. Express 17, 18116-18135 (2009)

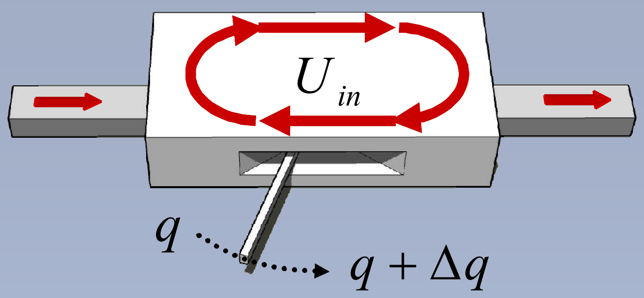
Abstract: Engineering optical forces in waveguides and cavities based on optical response

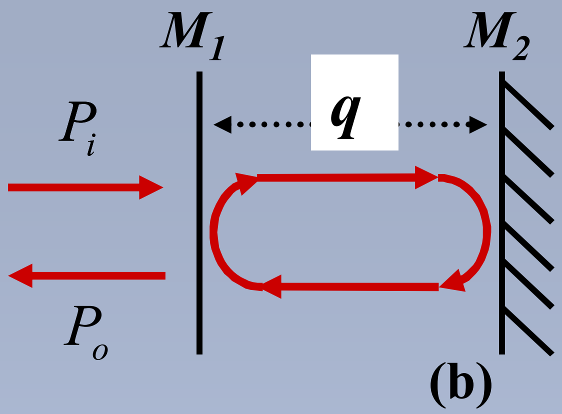
- Paper 7579-52 of Conference 7579Date: Sunday, 24 January 2010 Time: 2:00 PM – 2:30 PM

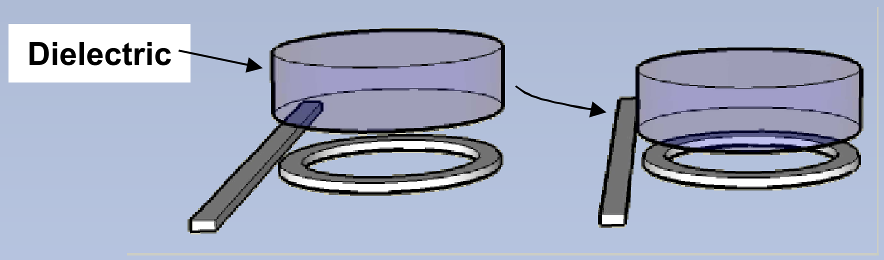
- Author(s): Peter T. Rakich, Sandia National Labs. (United States); Zheng H. Wang, Milos A. Popovic, Massachusetts Institute of Technology (United States)

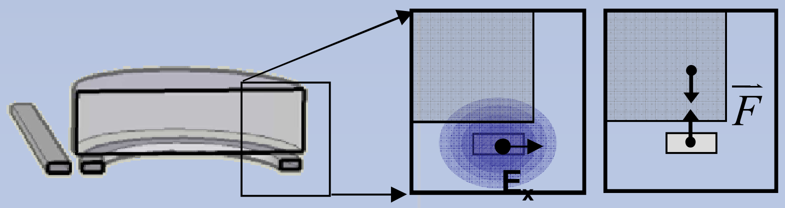
- We present a new treatment of optical forces, revealing that the forces in virtually all optomechanically variable systems can be computed exactly and simply from only the optical phase and amplitude response of the system. This treatment, termed the response theory of optical forces (or RTOF), provides conceptual clarity to the essential physics of optomechanical systems, which computationally intensive Maxwell stress-tensor analyses leave obscured, enabling the construction simple models with which optical forces and trapping potentials can be synthesized based on the optical response of optomechanical systems. Furthermore, we discuss novel signal processing applications developed through use of this formalism.

Various lossless reflectionless one-port systems to consider:

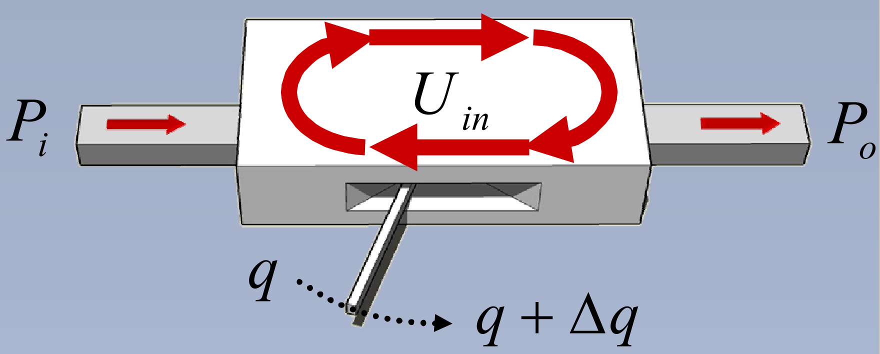


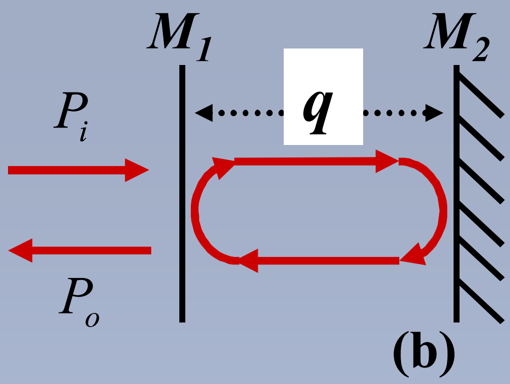






Response theory of optical forces (RTOF)





Assumptions:

1) Photons conserved

2) Energy conserved

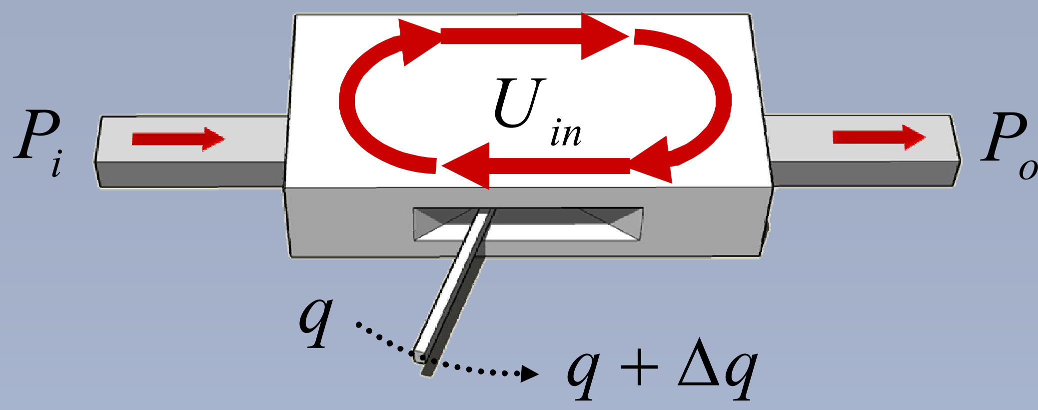
➔ Lossless

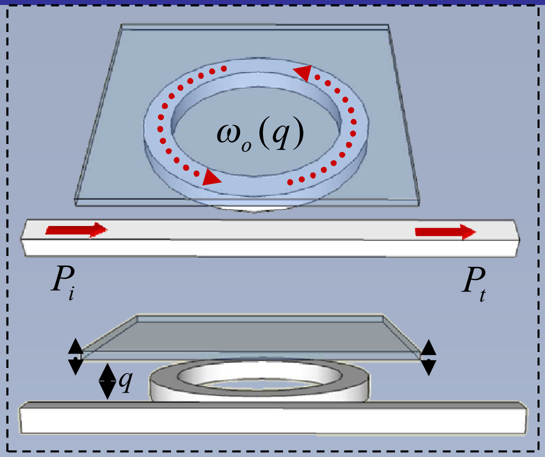
S-matrix

Single port eqn Here

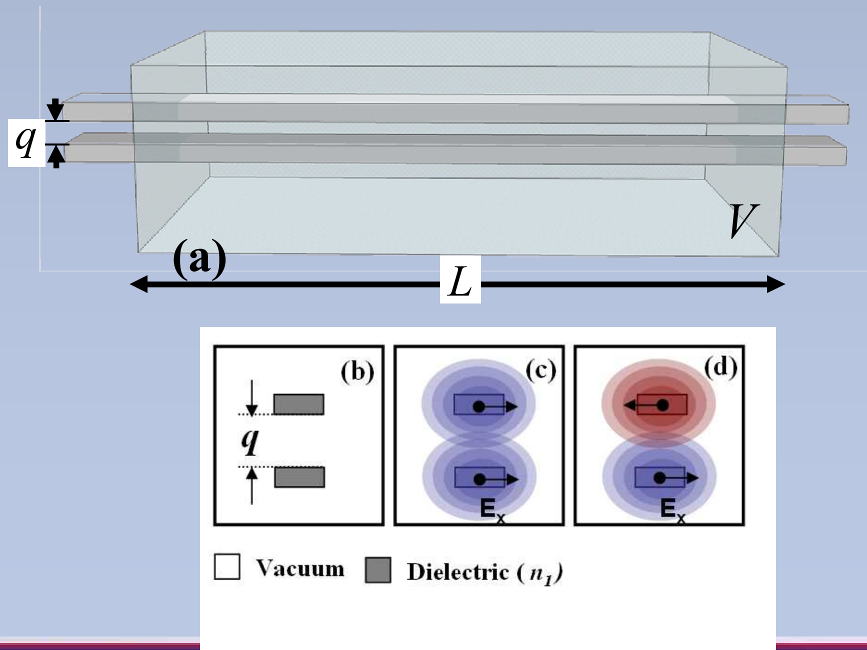
Multi-port eqn Here

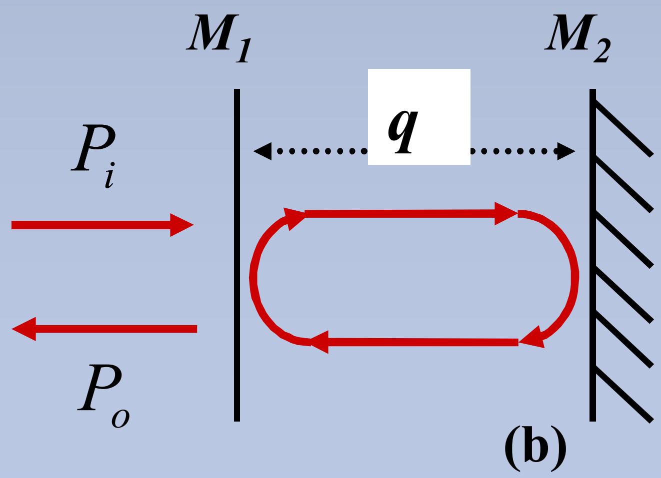
Can we extend to arbitrary optomechanically variable systems?



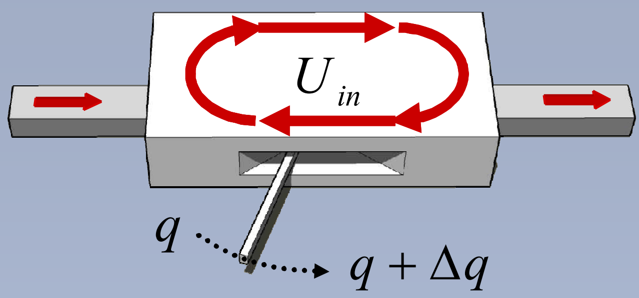


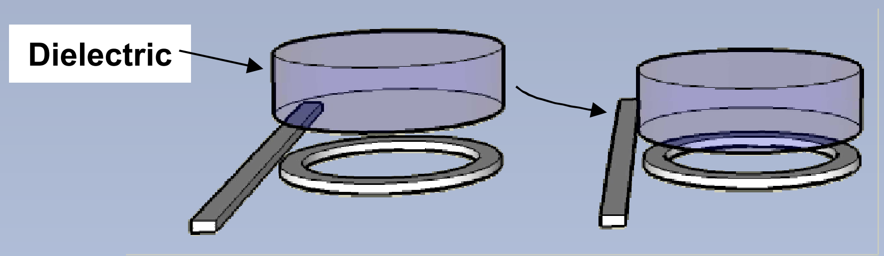
Rakich et. al. Opt. Lett. 31, 1241-1243 (2006).

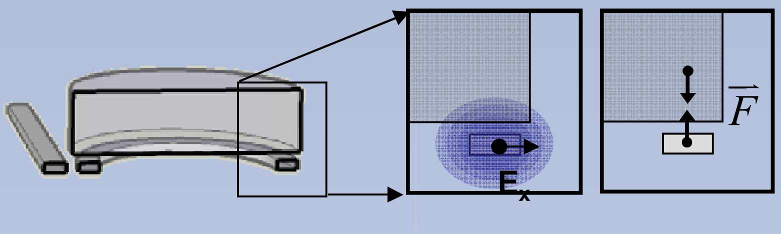




Energy in optomechanically variable systems:



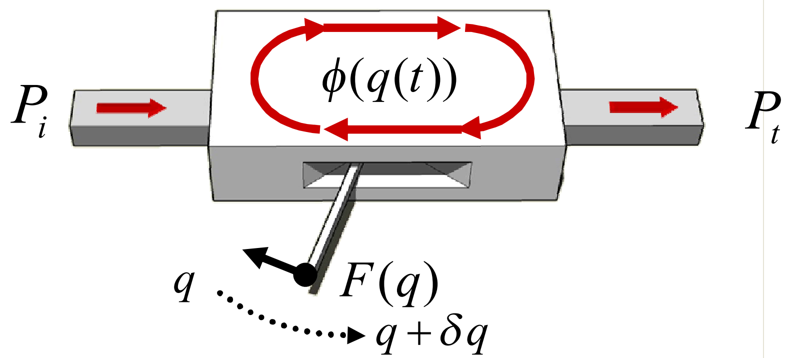




Constant flow of photons passes through an OM variable device.
Work done against the optical forces. Q: Where does energy go?
A: imparted to the transmitted optical wave.

Conservation of energy in optomechanically variable systems:

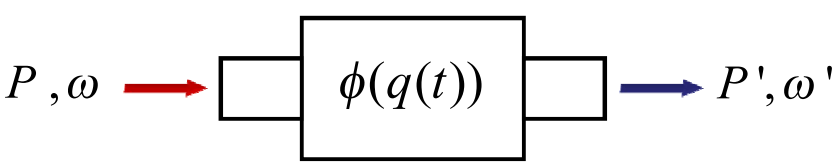
Smoothly time-varying optomechanical system:



For simplicity, we assume:

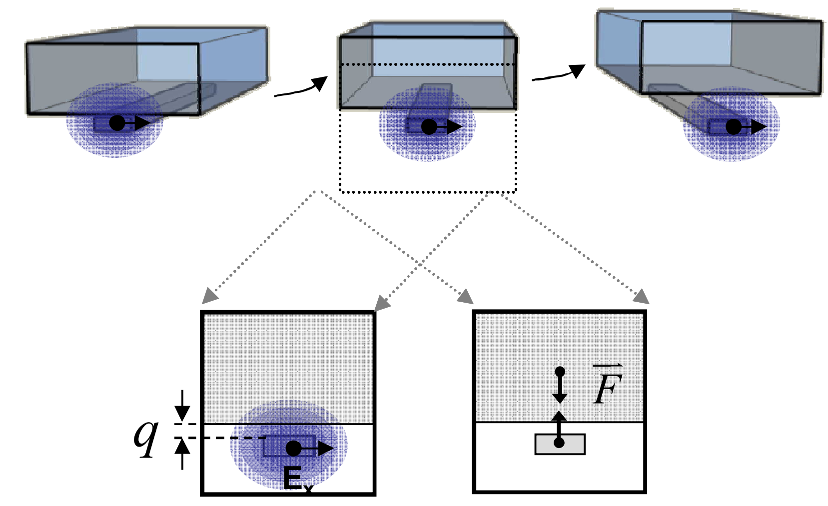
- **Device = Reflectionless one-port**
- **No Losses ➔ Coordinate, *q*, effects phase.**
  (System =optomechanical phase modulator)

$$exp[-i(\omega t)] \rightarrow exp[-i(\omega t - \phi(q))]$$



For time-varying *q*, frequency & power modified.

Simple example of reflectionless one-port:



$$n_{eff}(q)$$

$$W = \int F^{opt}(q) \cdot dq$$

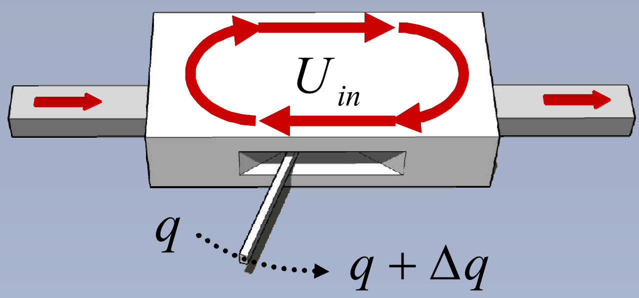
Energy conservation:

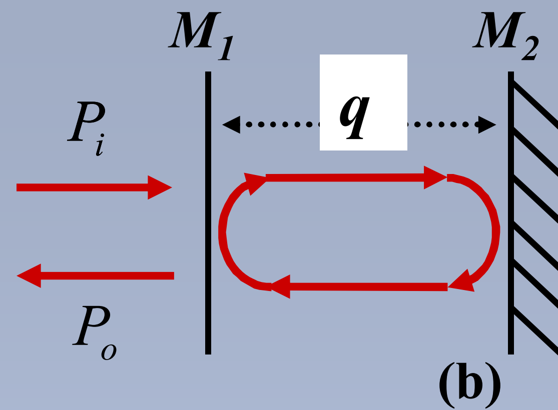
$$\Delta U_{EM} = \int (P' - P) \cdot dt$$

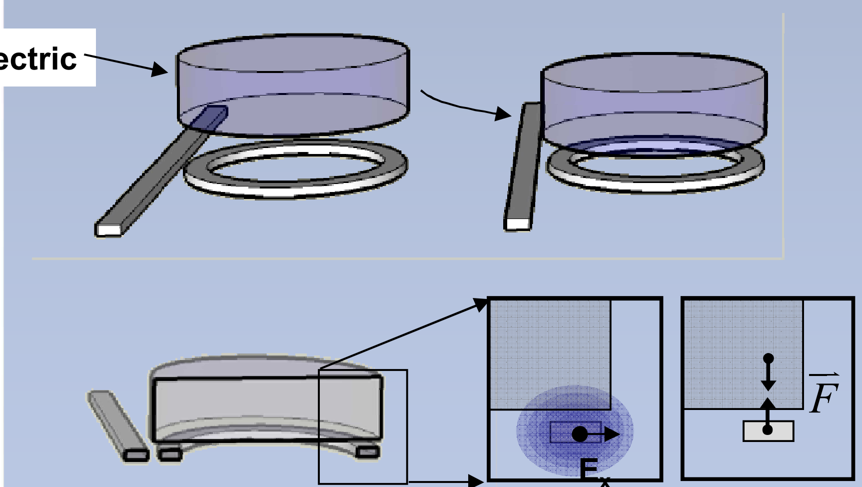
Work done must equal the change in electromagnetic energy!

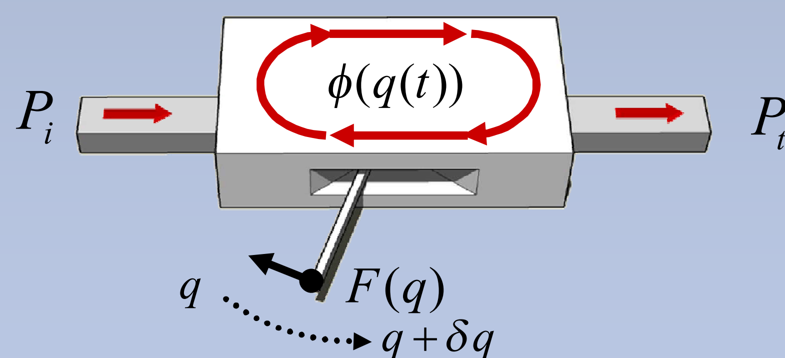
$$\Delta U_{EM} = W$$

Various lossless reflectionless one-port systems to consider:



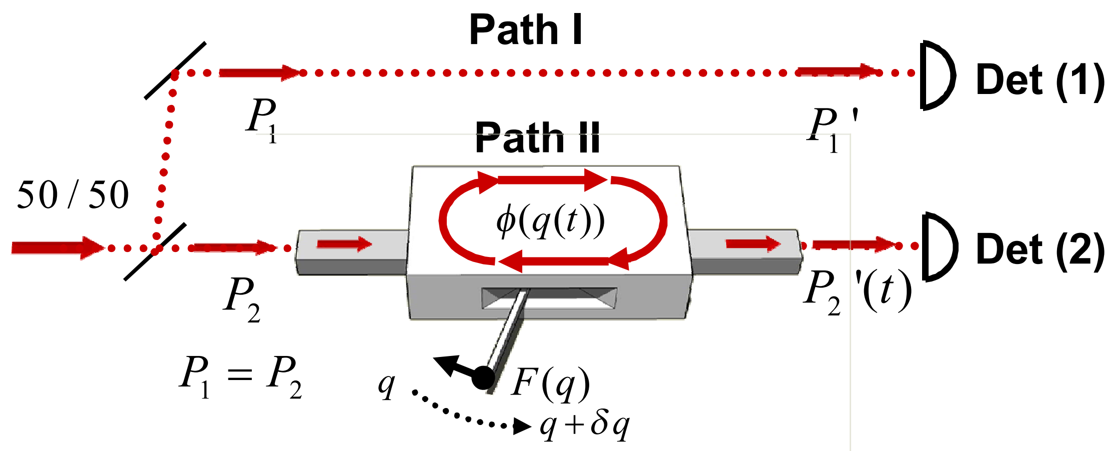






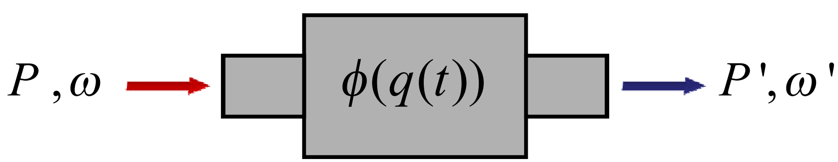
A closer look at energetics of time-varying system:

Thought experiment Illustrating Energetics



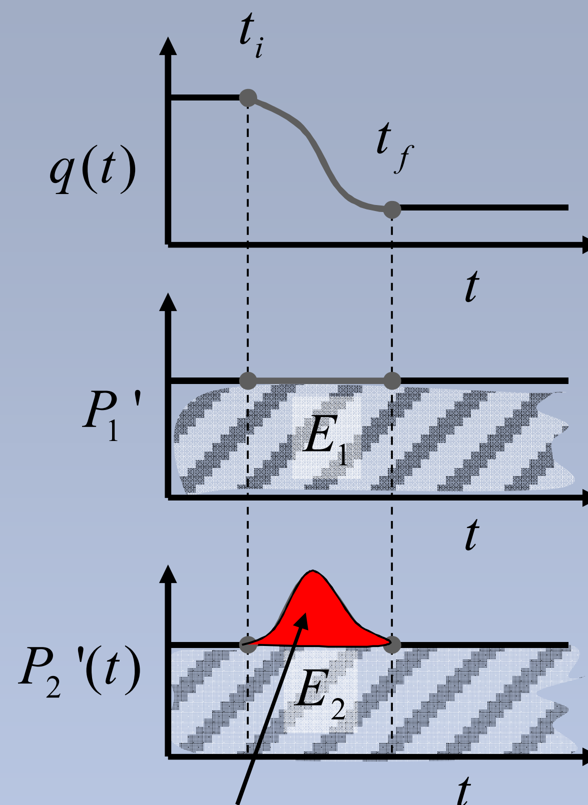
$$exp[-i(\omega t)] \rightarrow exp[-i(\omega t - \phi(q))]$$

(Lossless one port) = (Optomechanical phase modulator)



Change in energy carried by EM wave:

$$\Delta U_{EM} = \int (P' - P) \cdot dt$$



Work done against optical forces:

$$W = \int F^{opt}(q) \cdot dq$$

How to compute the change in EM energy?

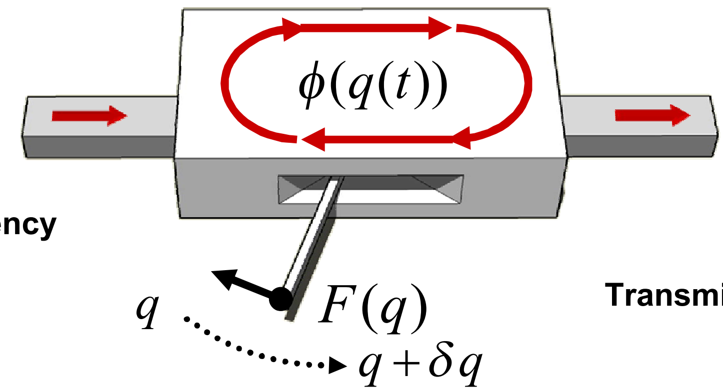
Great simplification through use of the photon construct:

(1) Relate the incident power to photon energy and photon flux.
(2) Assume that *q(t)* evolves slowly.
(3) Assume that photons are conserved.

$$P_i = \Phi_i \cdot \hbar\omega$$

Incident power (P_i), **Incident photon flux** (Φ_i), **Incident frequency** (ω)

P_i



P_t

Transmitted power (P_t)

$$P_t = \Phi_t \cdot \hbar\omega'$$

Trans. photon flux (Φ_t), **Trans. frequency** (ω')

If *q(t)* evolves slowly, it is reasonable to assume:

$$\Phi_i = \Phi_t = \Phi$$ **(Photon flux, conserved)**

$$\omega' = \omega + \delta\omega(t) = \omega - \dot{\phi}(q(t))$$

$$\Delta U_{EM} = \int (P' - P) \cdot dt = \Phi\hbar \cdot \int \delta\omega(t) \cdot dt = -\Phi\hbar \cdot (\phi_f - \phi_i)$$

How to compute the change in EM energy?

Great simplification through use of the photon construct:

$$\Delta U_{EM} = \int (P' - P)\cdot dt = \Phi\hbar \cdot \int \delta\omega(t)\cdot dt = -\Phi\hbar\cdot(\phi_f - \phi_i)$$

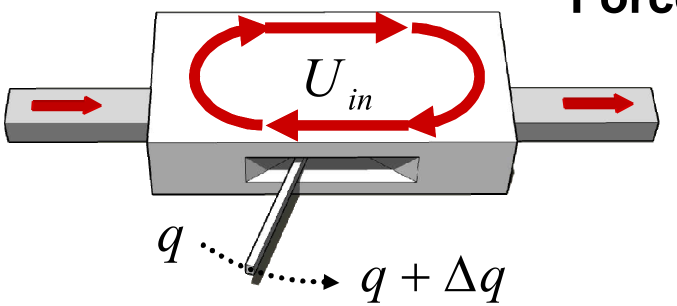
Since $\Delta U_{EM} = W$ **Potential:**

$$U^{eff}(q) = -\Phi\hbar\cdot\phi(q)$$

Force:

$$F(q) = \Phi\hbar\cdot\left(\frac{\partial\phi}{\partial q}\right)$$

$$F(q) = -\left(\frac{\partial U_{eff}}{\partial q}\right)$$



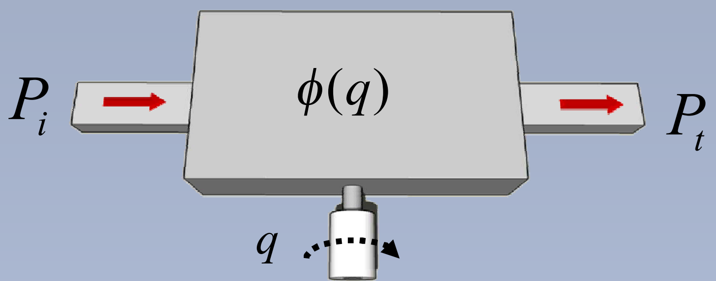
Result:

Given $\phi(q)$ we can compute the optical force on coordinate, q.

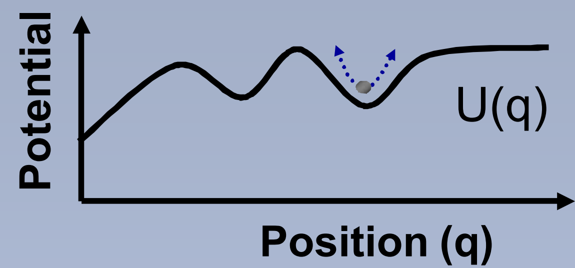
Extension to multi-port systems.

Lossless single-port, with a single mechanical degree of freedom, q.
(Input flux (Φ) and frequencies (ω) are assumed to be fixed).

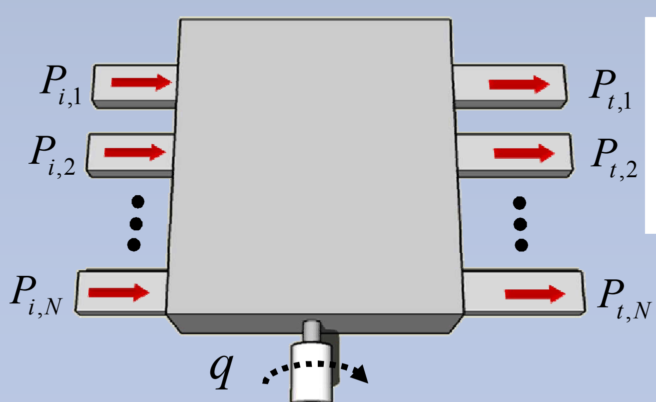
$$U_{eff}(q) = -\Phi \cdot \hbar\phi(q)$$



- Allows direct potential synthesis!
- Phase synthesis already known [1].



Lossless multi-port, with a single mechanical degree of freedom, q.
(Input fluxes (Φ_i's) and frequencies (ω_i's) are assumed to be fixed).



$$U_{eff}(q) = -\hbar \cdot \int \left[\sum_k \Phi_{t,k}(q) \cdot \frac{d\phi_{t,k}(q)}{dq} \right] \cdot dq$$

Phase response

Photon flux (k[th] output port)

[1] Madsen, C. K. & Zhao, J. H. *Optical flter design and analysis* (Wiley, New York, 1999).

Background: Optical forces

- How do we define an optical force?

  -Forces that are a direct consequence of an applied electromagnetic field.

- Many different forms:
  - Gradient forces,
  - Trapping,
  - Radiation pressure
- All can be understood from energetics.

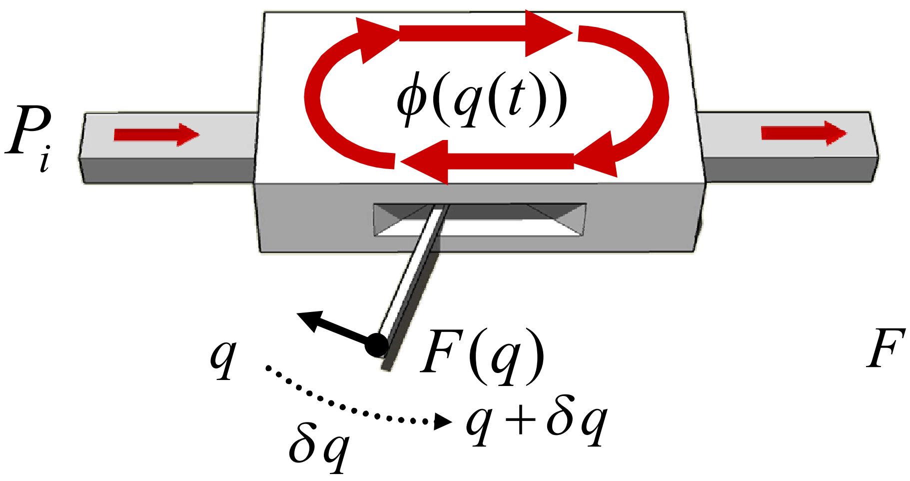
How to compute the change in EM energy?

Great simplification through use of the photon construct:

$$\Delta U_{EM} = \int (P' - P) \cdot dt = \Phi\hbar \cdot \int \delta\omega(t) \cdot dt = -\Phi\hbar \cdot (\phi_f - \phi_i)$$

Since $\Delta U_{EM} = W$ **Since** $U^{eff}(q) = -\Phi\hbar \cdot \phi(q)$

$$F(q) = \Phi\hbar \cdot \left(\frac{\partial \phi}{\partial q}\right)$$



$$F(q) \cdot \delta q = \delta W$$

How to compute the change in EM energy?

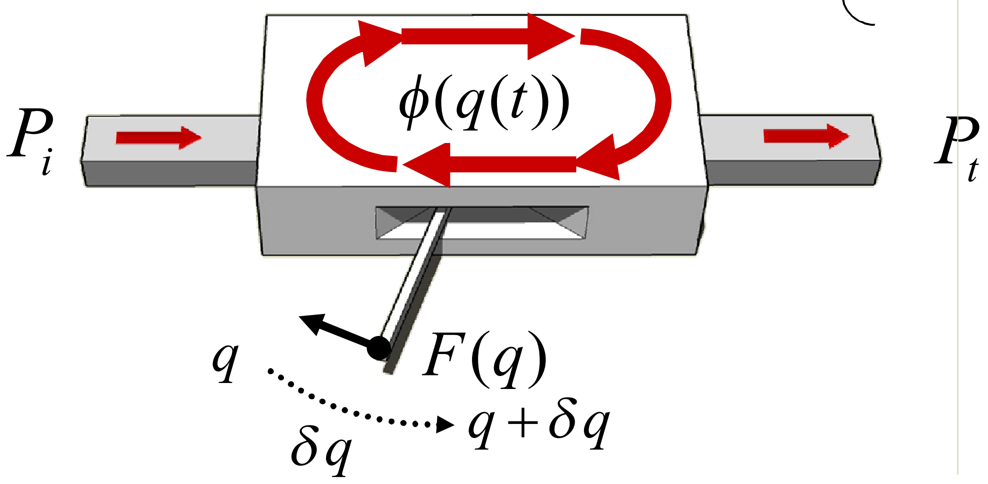
Great simplification through use of the photon construct:

$$\Delta U_{EM} = \int \left(P' - P\right) \cdot dt = \Phi\hbar \cdot \int \delta\omega(t) \cdot dt = -\Phi\hbar \cdot (\phi_f - \phi_i)$$

Since $\Delta U_{EM} = W$

$$U^{eff}(q) = -\Phi\hbar \cdot \phi(q)$$

$$F(q) = \Phi\hbar \cdot \left(\frac{\partial\phi}{\partial q}\right)$$



Result:

Given $\phi(q)$ we can compute the optical force on coordinate, q.

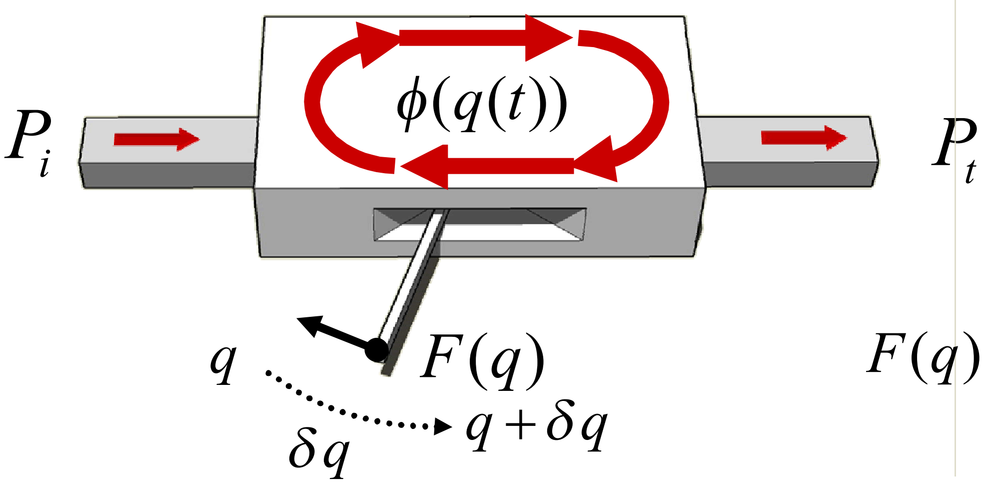
How to compute the change in EM energy?

Great simplification through use of the photon construct:

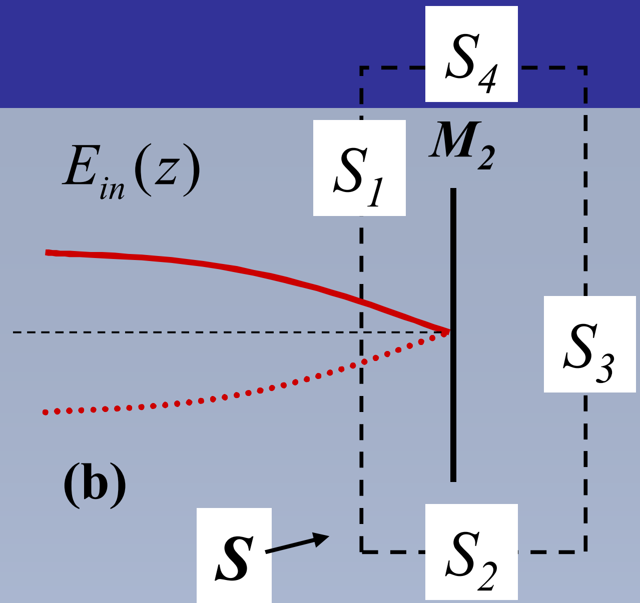
$$\Delta U_{EM} = \int (P' - P) \cdot dt = \Phi\hbar \cdot \int \delta\omega(t) \cdot dt = -\Phi\hbar \cdot (\phi_f - \phi_i)$$

Since $\Delta U_{EM} = W$ **Since** $U^{eff}(q) = -\Phi\hbar \cdot \phi(q)$

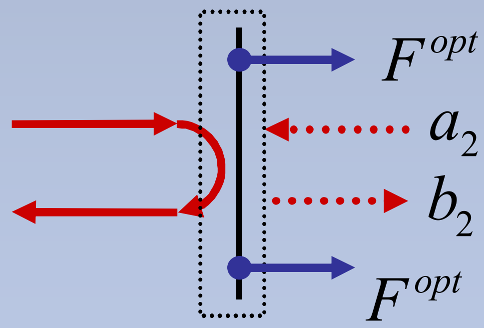
$$F(q) = \Phi\hbar \cdot \left(\frac{\partial\phi}{\partial q}\right)$$



$$F(q) \cdot \delta q = \delta W$$



S_4
S_1
M_2
$E_{in}(z)$
S_3
(b)
$\boldsymbol{S}$
S_2

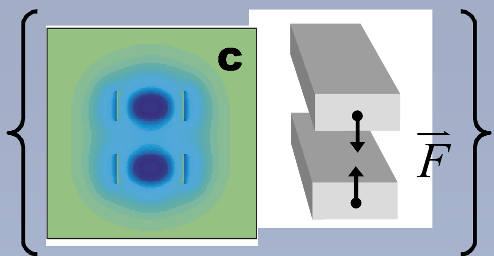


F^{opt}
a_2
b_2
F^{opt}

Enhanced optical forces via guided modes

Nanometer-scale optical waveguides.

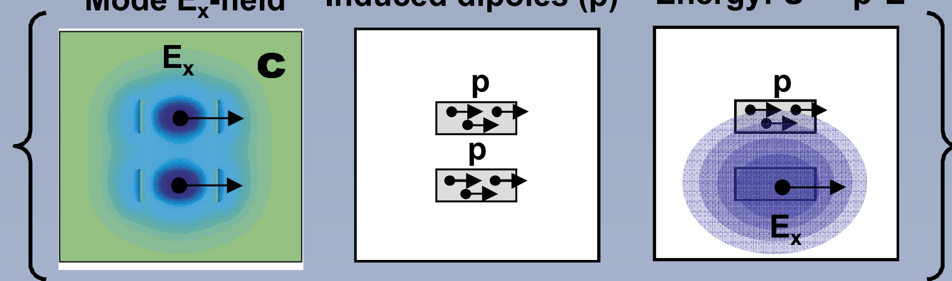
***Symmetric* (+)**



Attractive Force

<u>Coupled optical waveguides</u>

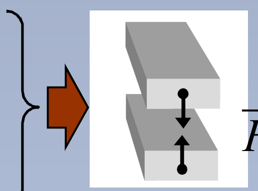
How are forces produced between waveguides?



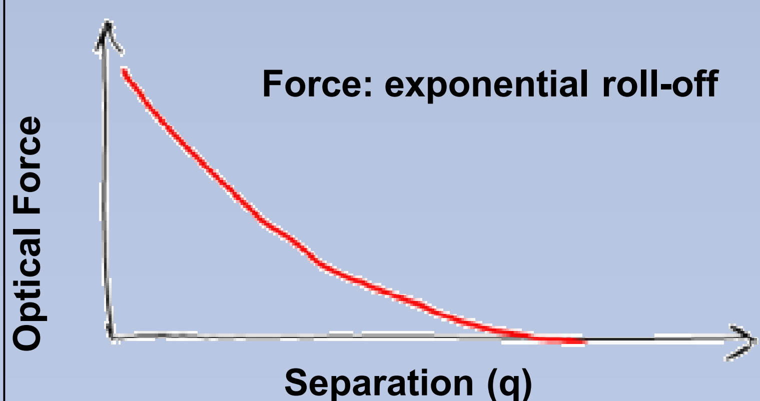
Energy of interaction: U = -p*E

For symmetric mode:

- **Since p || E, U = low energy state**
- **Attractive forces produced.**



Attractive Force



Effect of strong confinement:

- **Large Field Enhancements**
- **Large Field Gradients**

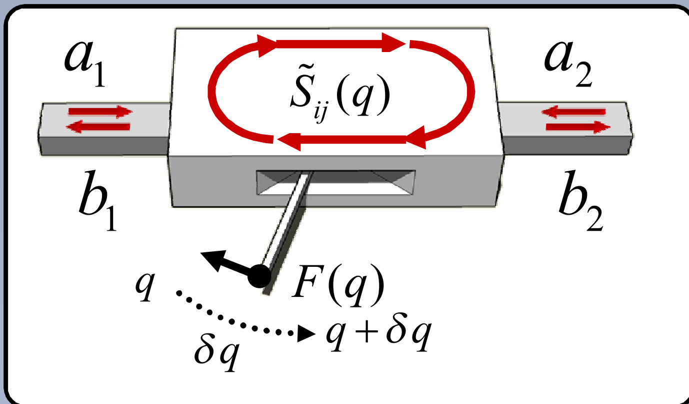
Result:

- **Optical forces Greatly enhanced!!**

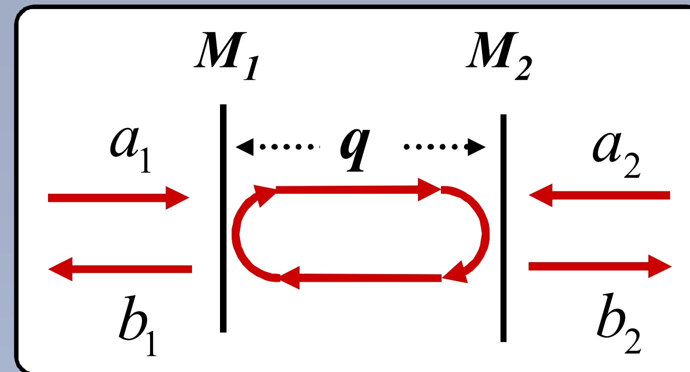
Forces ~ 1 uN (P = 1 mW)

Results: The response theory of optical forces (RTOF)

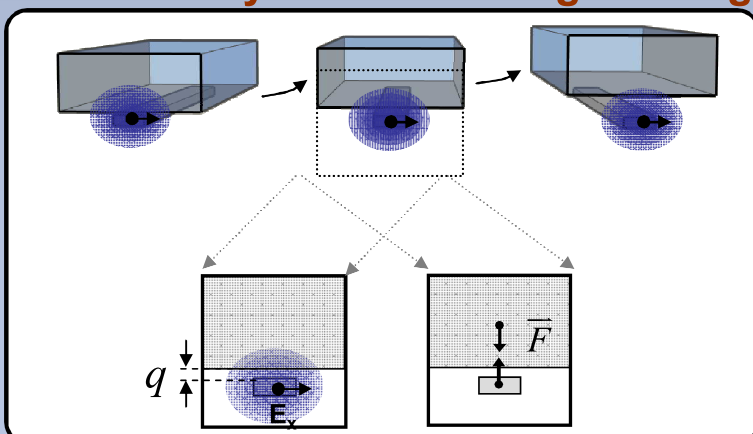
Generalized Mech. variable Two-port system



Fabry-Perot: Two-port system



Optomechanically variable waveguide segment:



Fabry-Perot: Two-port system

$$\begin{bmatrix} b_1(q) \\ b_2(q) \end{bmatrix} = \left[\tilde{S}(q)\right] \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\varphi_{o,k}(q) = \arg\left[\tilde{b}_k(q)\right]$$

Optical force expressed on mechanical degree of freedom, q, given by:

$$F_q(q) = \frac{1}{\omega}\sum_k \left|b_k(q)\right|^2 \cdot \frac{d\phi_{o,k}(q)}{dq}$$

Trajectory of optomechanics

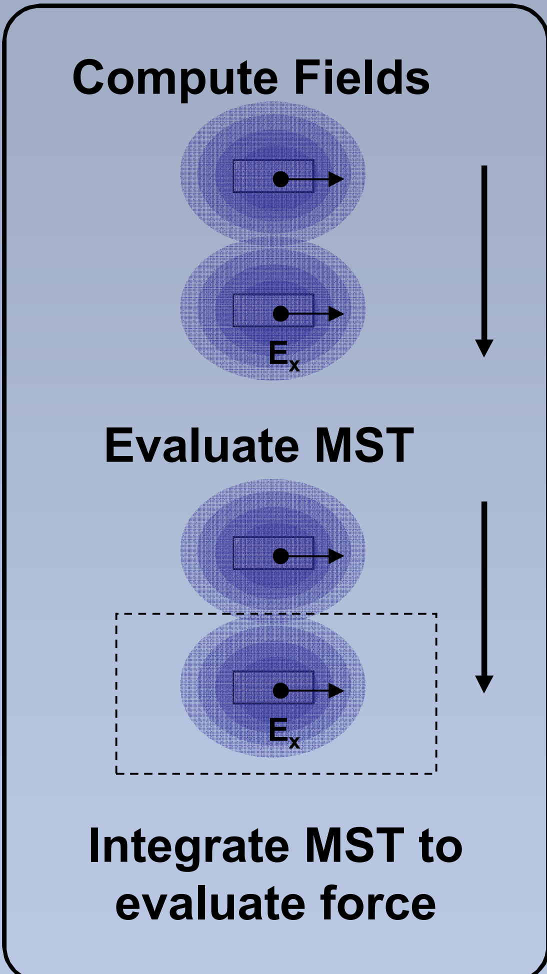
- How to Synthesize forces?
- How to deal with complexity?
- MST methods→ Very complex and offer little physical insight

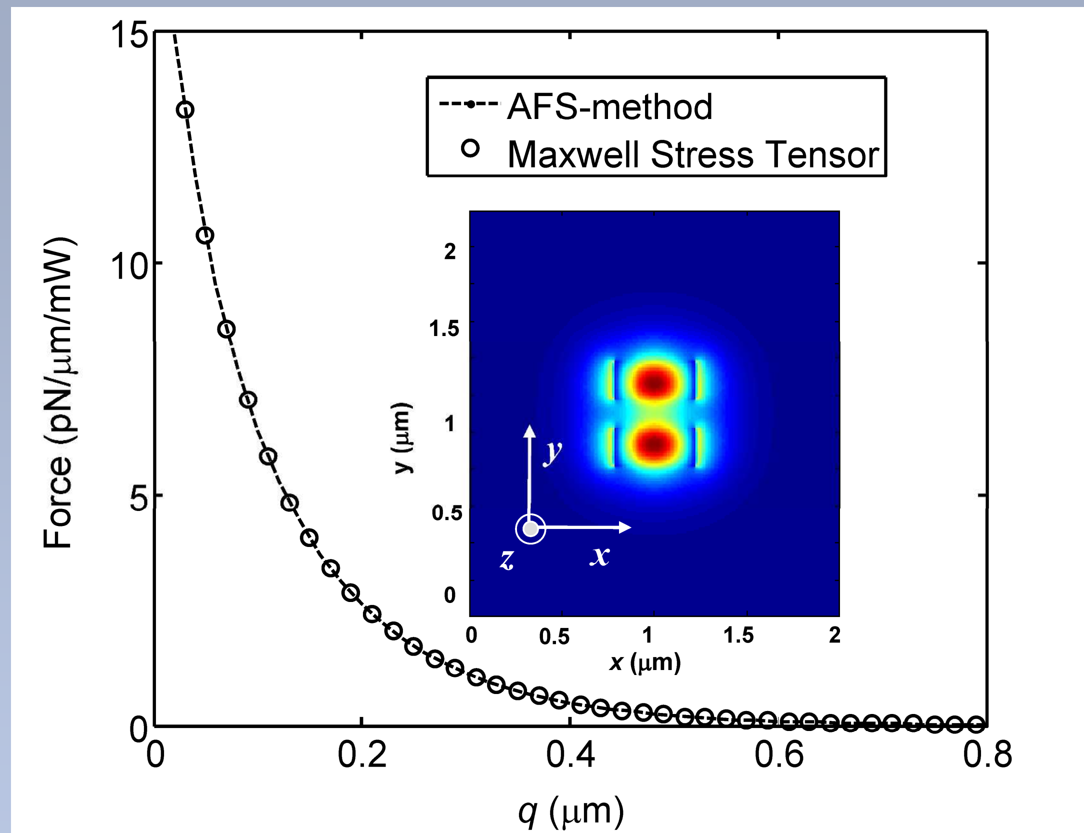
Conventional means of computing forces: Maxwell Stress Tensor

- Requires full field computation.
- Small variations in design require complete re-computation. Show systems of identical optical response… will show that they generate identical forces. (no need to compute again)
- Offers Little intuition

Conventional means of computing forces: Maxwell Stress Tensor (MST)

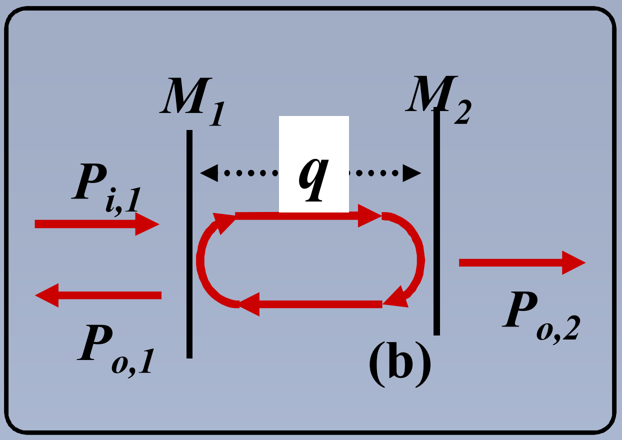
Procedure for MST

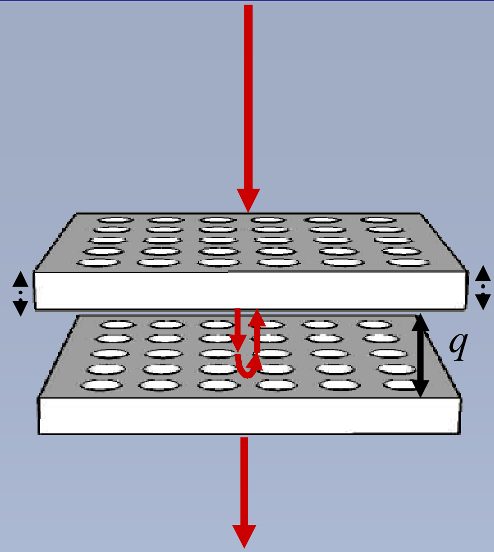


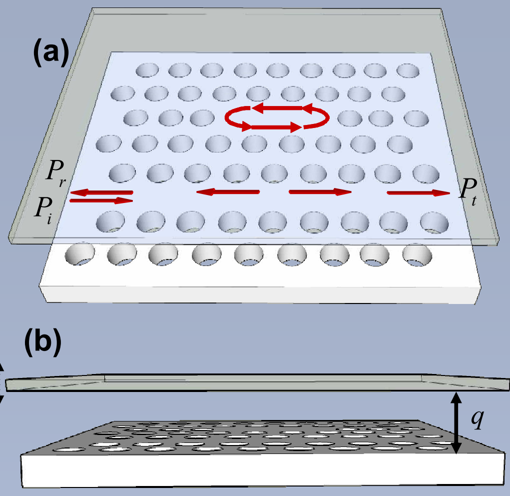


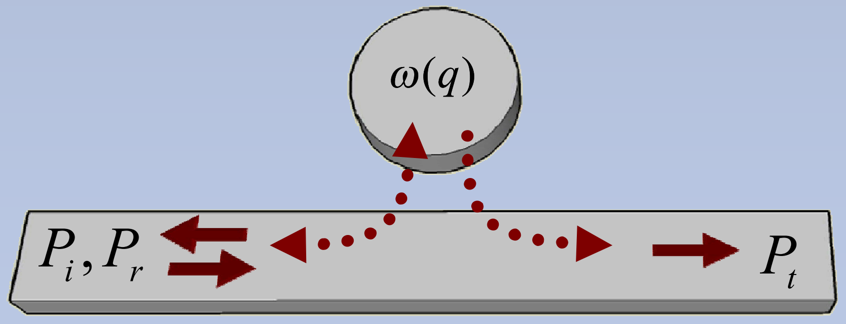
- Requires full field computation.
- Small variations in design require complete re-computation. Show systems of identical optical response… will show that they generate identical forces. (no need to compute again)
- Offers Little intuition

Similarity of optical systems

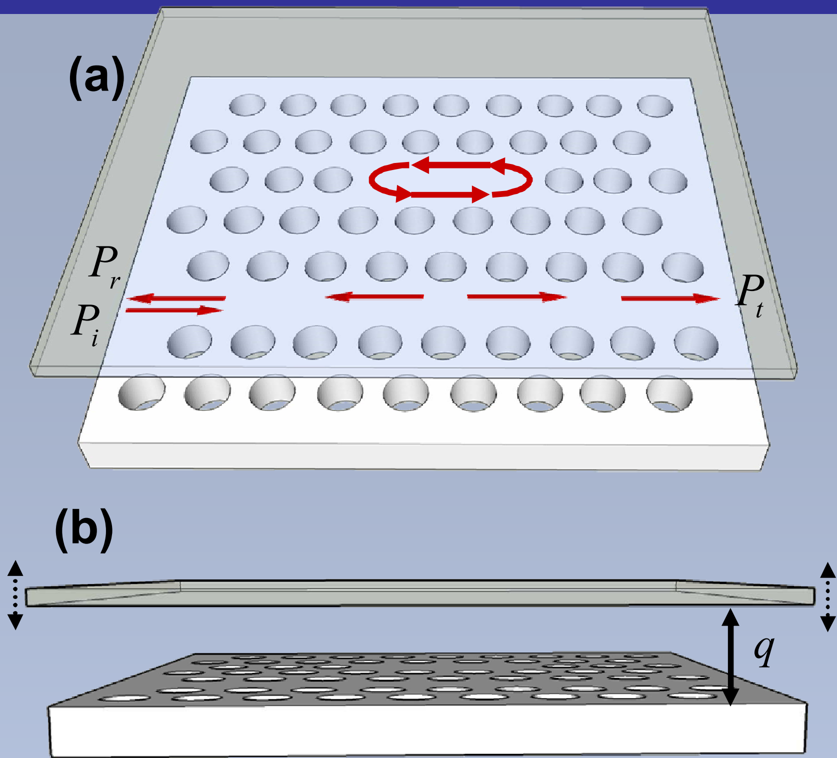






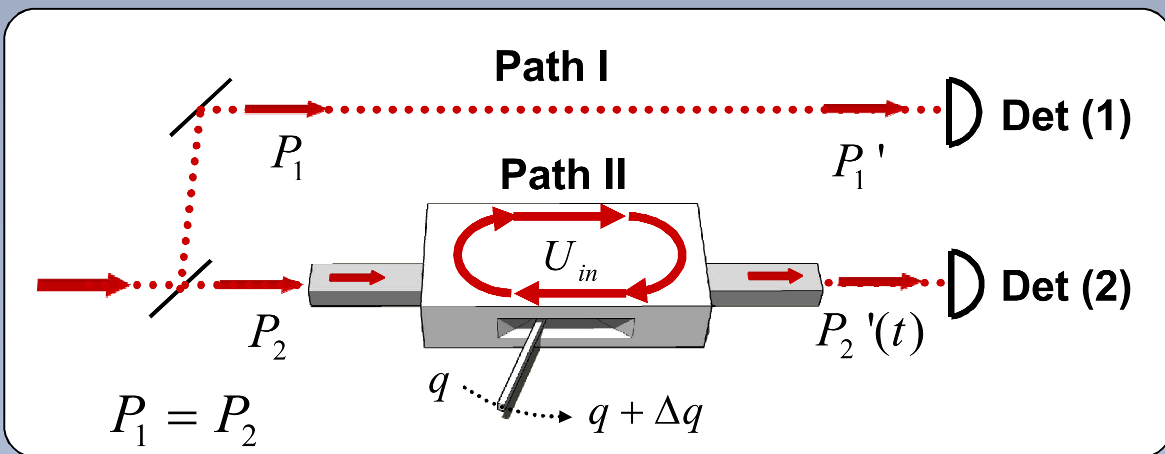


- Systems look quite different, but their response can be identical.
  - All are two-port resonant systems (ω_o)
  - $\omega_o(q)$
  - $H(\omega_o(q))$
- Despite similarities, Maxwell stress tensor requires complete reformulation of problem electromagnetically.

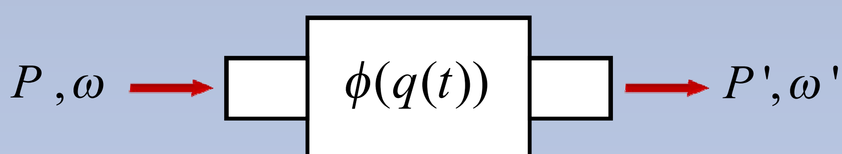

(a)
P_r
P_i
P_t
(b)
q

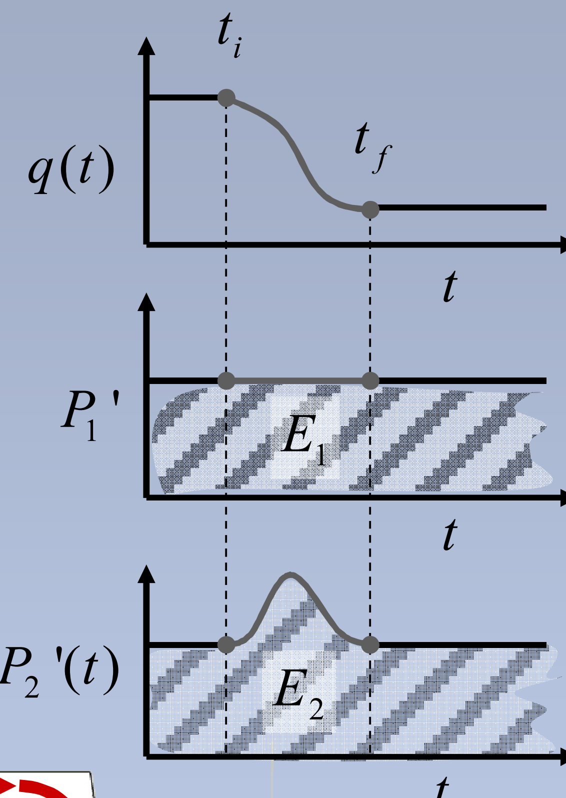
Conceptual Basis for RTOF

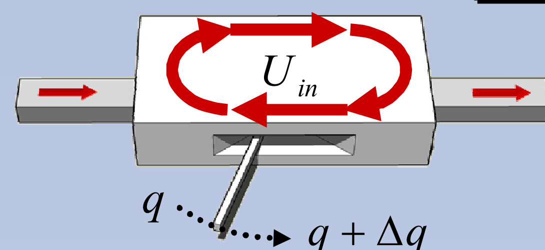
Thought experiment Illustrating Energetics



Since lossless: $exp[-i(\omega t)] \rightarrow exp[-i(\omega t - \phi(q))]$







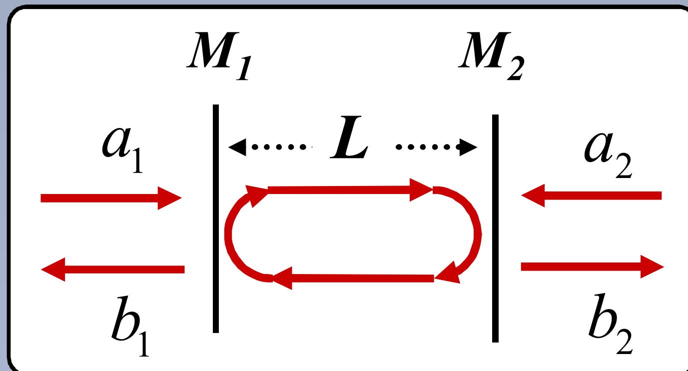
P_i

P_o

Results: The response theory of optical forces (RTOF)

Response of a static two port system

Fabry-Perot: Two-port system

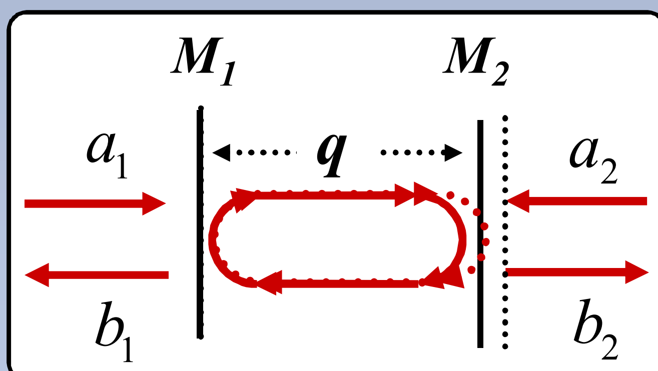


Fabry-Perot: Two-port system

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \left[\tilde{S}\right] \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Response of an Optomechanically variable two-port system

Optomechanically variable Two-port system



Fabry-Perot: Two-port system

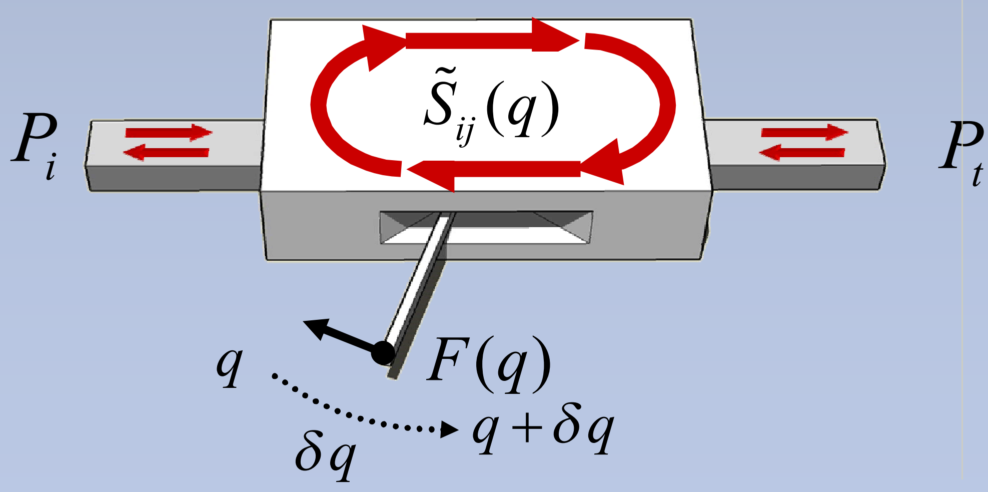
$$\begin{bmatrix} b_1(q) \\ b_2(q) \end{bmatrix} = \left[\tilde{S}(q)\right] \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\varphi_{o,k}(q) = \arg\left[\tilde{b}_k(q)\right]$$

Optical force expressed on mechanical degree of freedom, q, given by:

$$F_q(q) = \frac{1}{\omega} \sum_k \left|b_k(q)\right|^2 \cdot \frac{d\phi_{o,k}(q)}{dq}$$

Conventional Force Calculations: Maxwell Stress Tensor (MST) Method



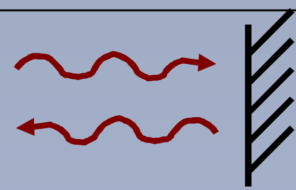
Relation between optical forces & Energy

- If we can compute the dE for a given dq, we can generally compute the optical forces.
- Capacitor + plates (electrostatics)
- Change in length of waveguide/cavity.
- How to treat an Open system and all of degrees of freedom associated with it?
- Will just give a conceptual outline

Basis for a formulation of forces which treats forces in open systems: Thought experiment

Photon momentum:

$$|p| = \hbar k$$



Imparted momentum per photon:

$$|\Delta p| = 2\hbar \cdot k$$

Force on Mirror:

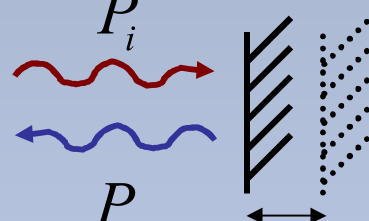
$$F = \frac{|\Delta p|}{\Delta t} = 2\hbar \cdot k \cdot \Phi$$

Virtual work:

For: Δq

$$\Delta U_{EM} = ?$$

P_i



P_r

Δq

While mirror static:

$$P_i = P_r$$

While mirror is moving:

$$P_i \neq P_r$$

$$\Delta U_{EM} = \int P_r - P_i \cdot dt \Rightarrow \sum_k N_k \Delta\omega_k$$

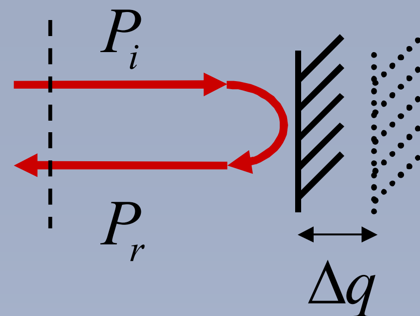
$$F = -\frac{\Delta U_{EM}}{\Delta q} = 2\hbar \cdot k \cdot \Phi$$

Basis for a formulation of forces which treats forces in open systems:

Virtual work:

For: Δq

$\Delta U_{EM} = ?$



While mirror static:

$$P_i = P_r = \Phi\hbar\omega$$

While mirror is moving:

$$P_i \neq P_r = \Phi\hbar\omega'$$

$$\phi \equiv 2(\omega/c)\cdot q$$

$$\Delta U_{EM} = \int P_r - P_i \cdot dt \;\Rightarrow \sum_k N_k \Delta\omega_k$$

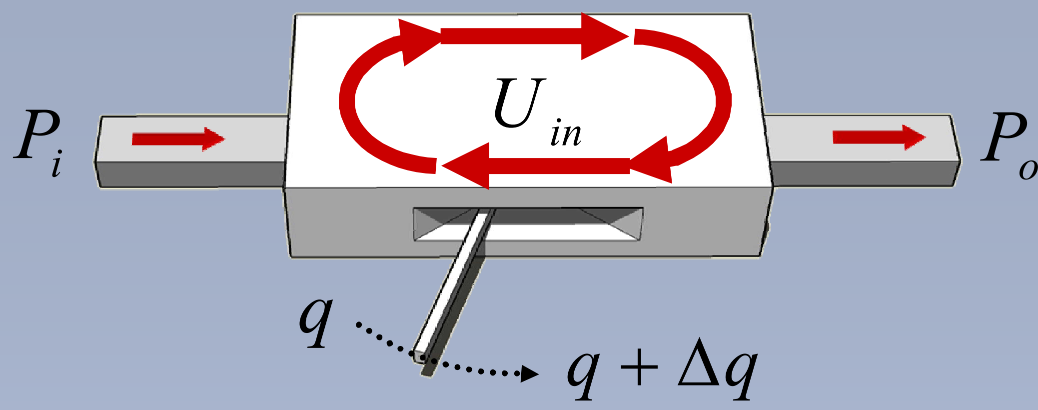
$$\Delta\omega_k \cong -\dot{\phi} = 2(\dot{q}/c)\cdot\omega$$

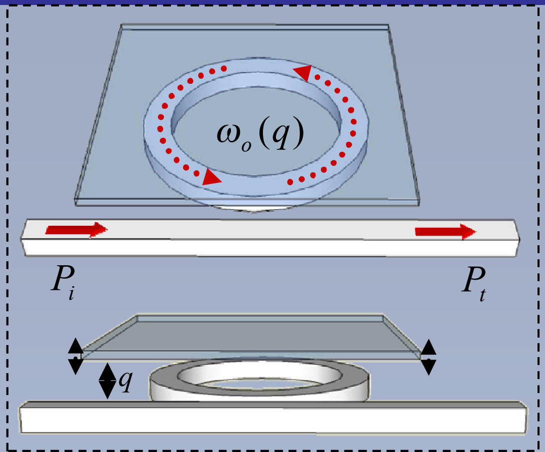
$$(P_r - P_i) = -\Phi\hbar\dot{\phi} \qquad \Delta U_{EM} \approx -\Phi\hbar\Delta\phi$$

$$F = -\frac{\Delta U_{EM}}{\Delta q} = 2\hbar\cdot k\cdot\Phi$$

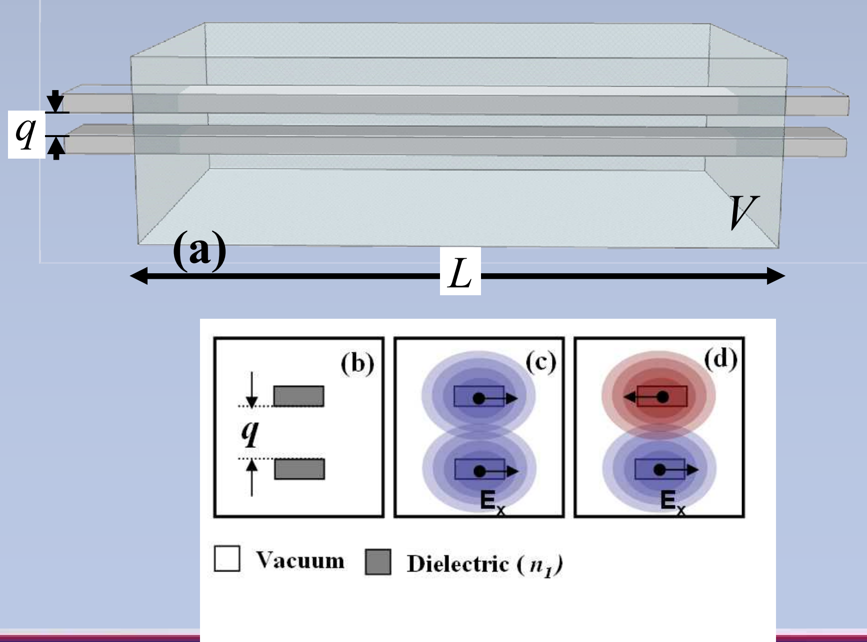
Correct: Is there a more fundamental relationship between the phase and the Energy of the system? Can we extend this concept into a well developed theory?

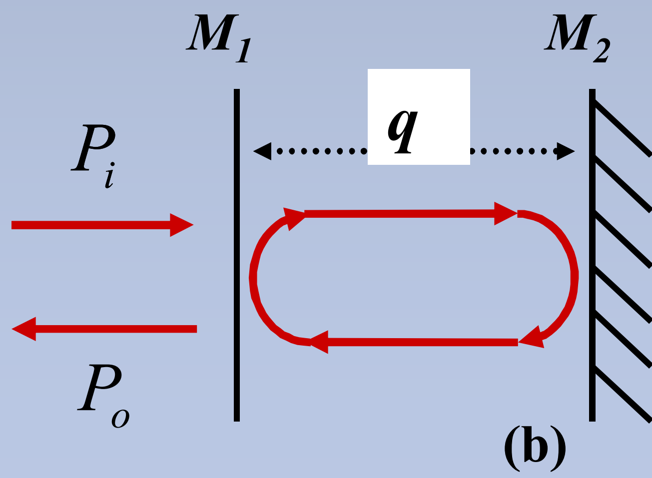
Can we extend to arbitrary optomechanically variable systems?





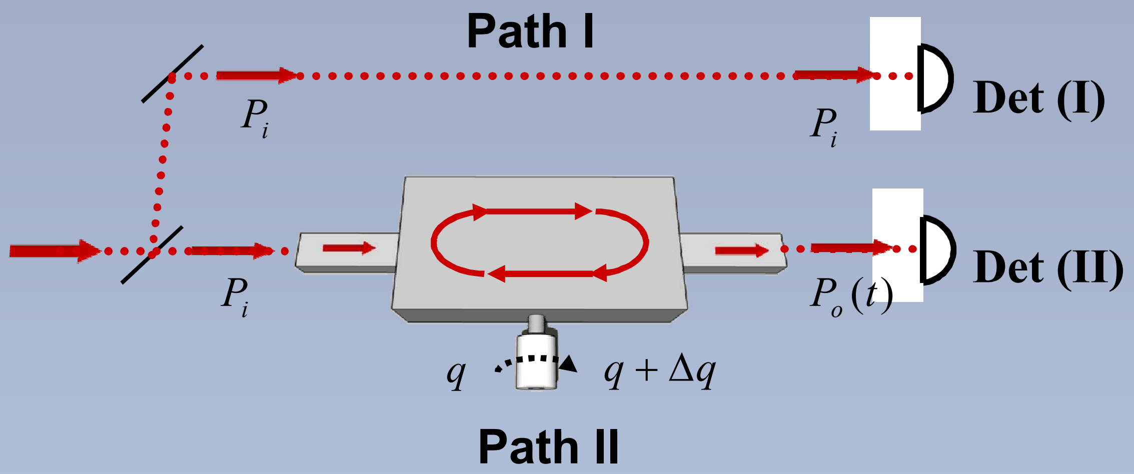
Rakich et. al. Opt. Lett. 31, 1241-1243 (2006).

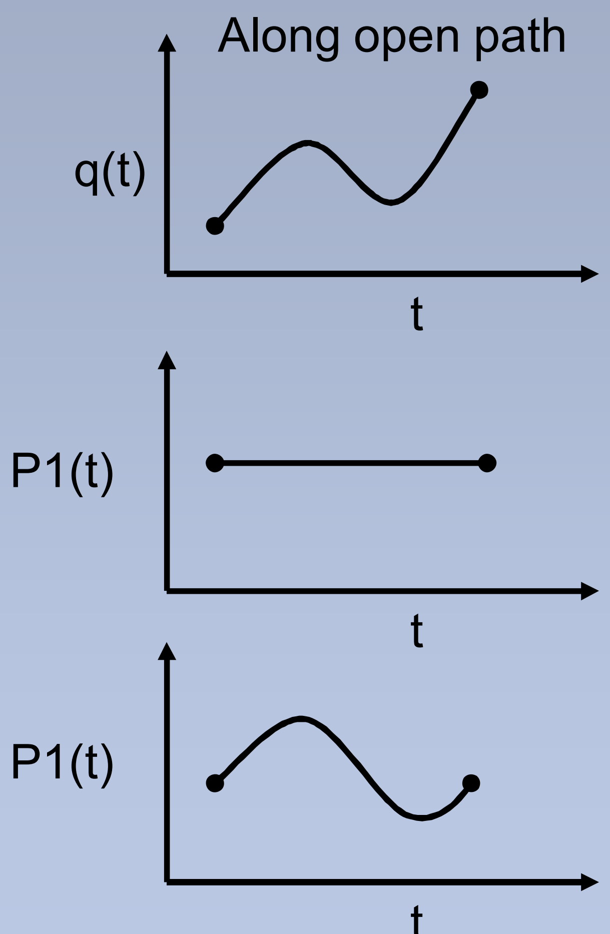




Can we (more generally) formulate forces in a simpler manner?

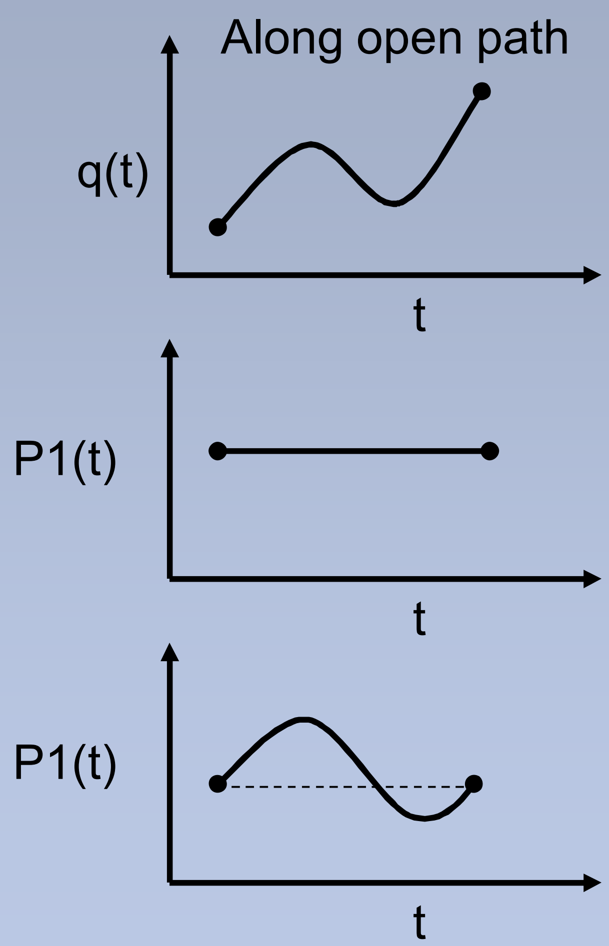
Thought experiment





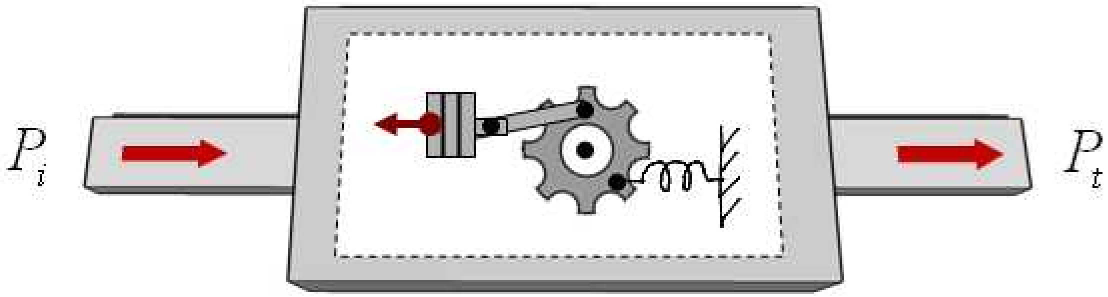
Can we (more generally) formulate forces in a simpler manner?

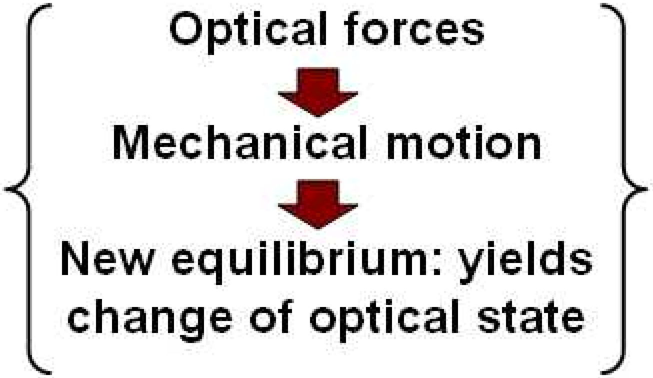
Thought experiment

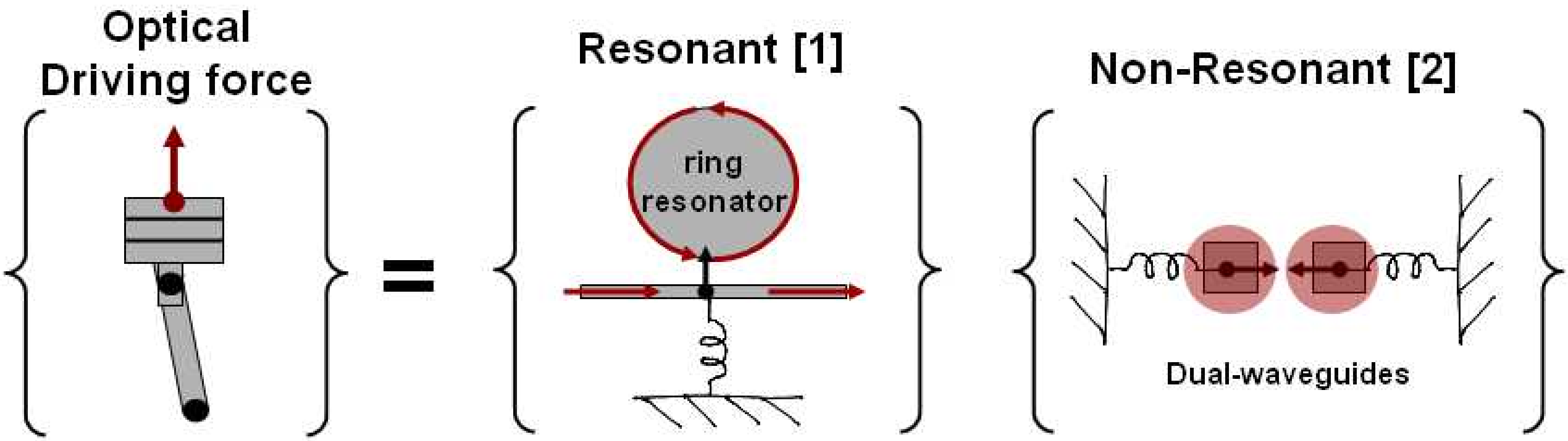


State of optomechanics: so far.. simple

Simple optomechanical system:
Single driving force & spring





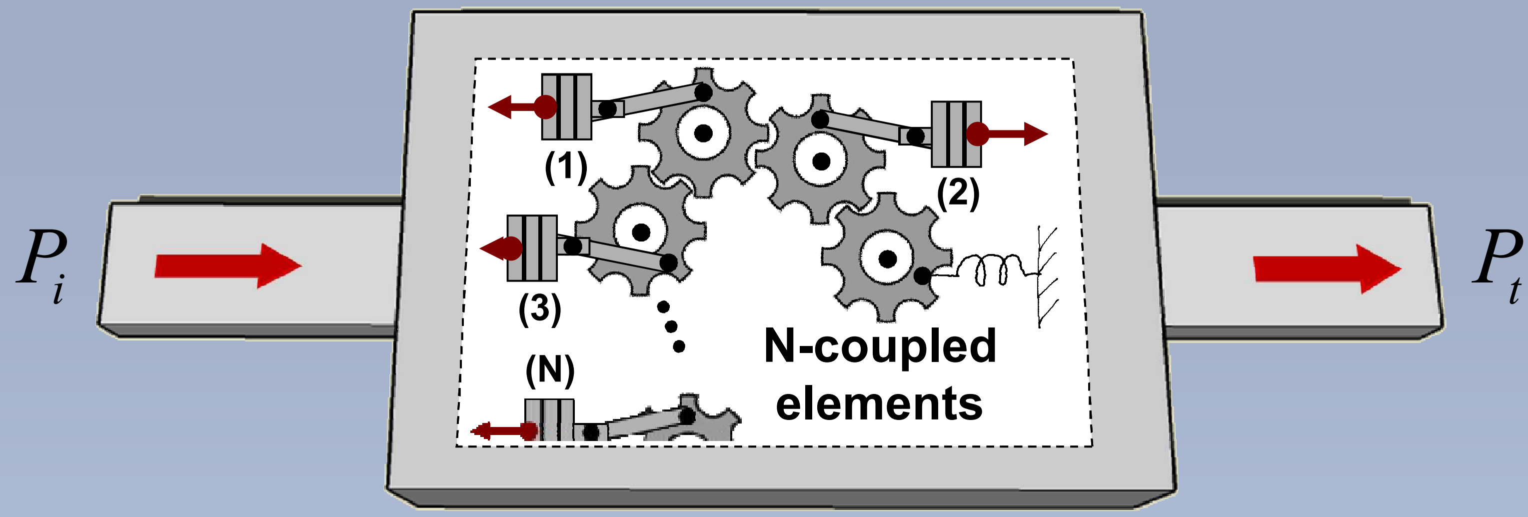


- Simple optomechanical systems require only force-based calculations.
- Problem: with increasing complexity, force-based treatments aren't practical!

[1] Eichenfield, M. *et al.* Nature Photon. 1, 416–422 (2007).
[2] Povinelli, et. al. Opt. Lett. 30,3042 (2005).

How to treat more complex systems?

Complex optomechanical system:



Newtonian mechanics:

$$m\ddot{\vec{x}} = \sum_{k=1}^{N} \vec{F}_k$$

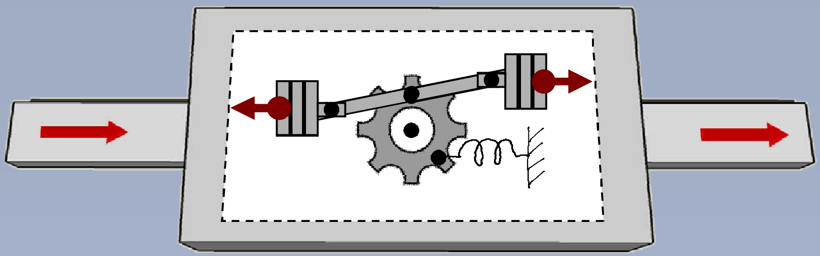
Lagrangian formulation:

$$L(q,\dot{q}) = \frac{\dot{q}^2}{2m} + U(q)$$

- **Lagrangian approach is far more general (and better with complexity).**
  - **Analyze stability & equilibrium states of system**
- **Problem: no general way to formulate optically-induced potential *U(q)*.**

Potentials useful for new applications:

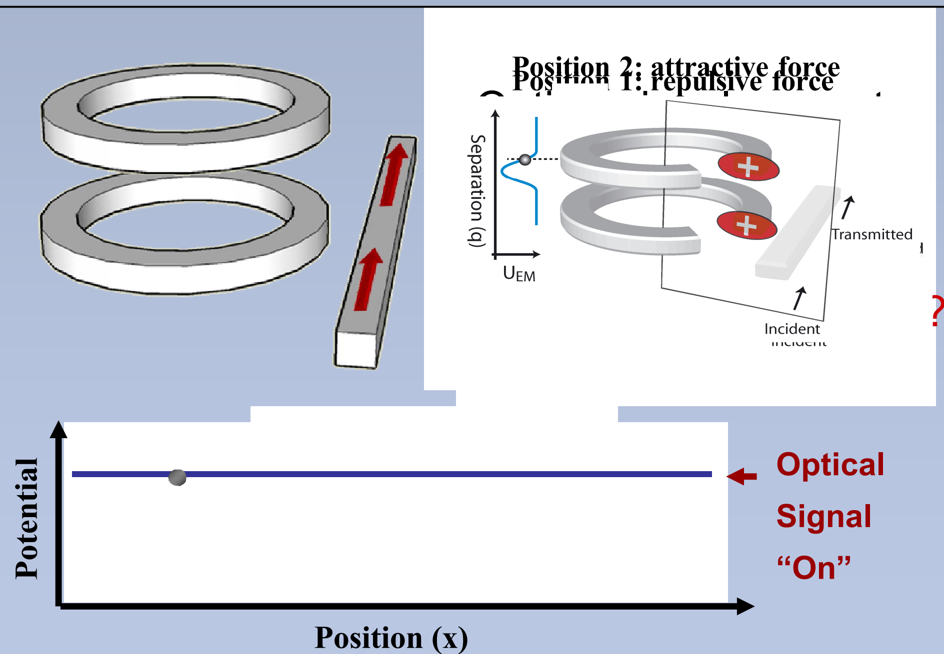
All-optical potential-well



All-optical potential-well:

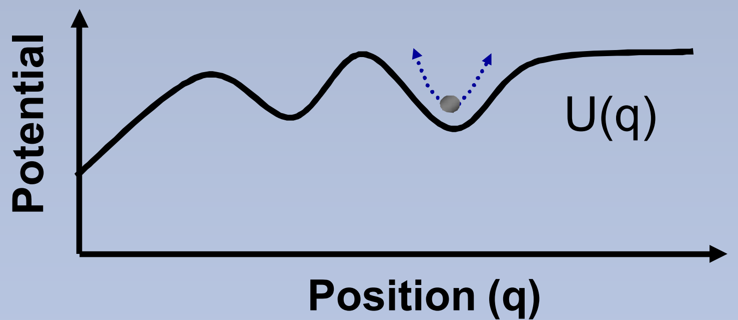
- Manipulation of state possible by simply tuning laser-frequency.
- Larger range & higher stability.
- Intuition was our guide here.

Mech. equilibrium dictated by light:



Q: How to *design* arbitrary potential?

- Design states of stable equilibrium

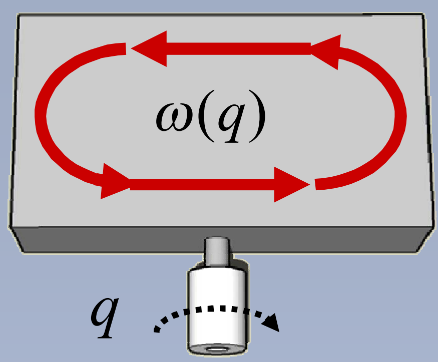


- To do this, we need to look at things in a very different manner…

Peter T. Rakich, Milos A. Popovic, Marin Soljacic, Erich P. Ippen, *Nature Photonics* 1, 658-65 (2007)

Open system and closed system energetics:

Closed System:



- Number of photons (*N*) is fixed.
- Only frequency (ω) can change.
- Mech. variable (*q*) changes state.

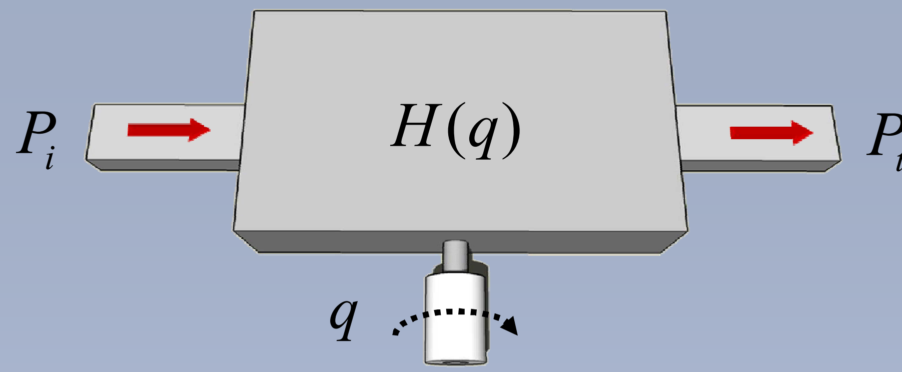
$$U_{EM}(q) = N\hbar \cdot \omega(q)$$

Number of photons (N); **cavity frequency** ($\omega(q)$)

$$\boxed{F_{opt} = -dU_{EM}/dq}$$

- Can yield useful exact model (e.g. [1])

Open System:



- Photons constantly entering and exiting.
- Response (*H*) is a function of (*q*)

Q: How to compute U_{EM} of open-system?

- Only option: assume time variation and apply power conservation

$$dU_{EM}/dt = (P_t - P_i) + dU_{in}/dt$$

Transmitted power (P_t); **Incident power** (P_i); **Stored energy** (dU_{in}/dt)

[1] Povinelli, *et. al.* Opt. Lett. 30,3042 (2005).

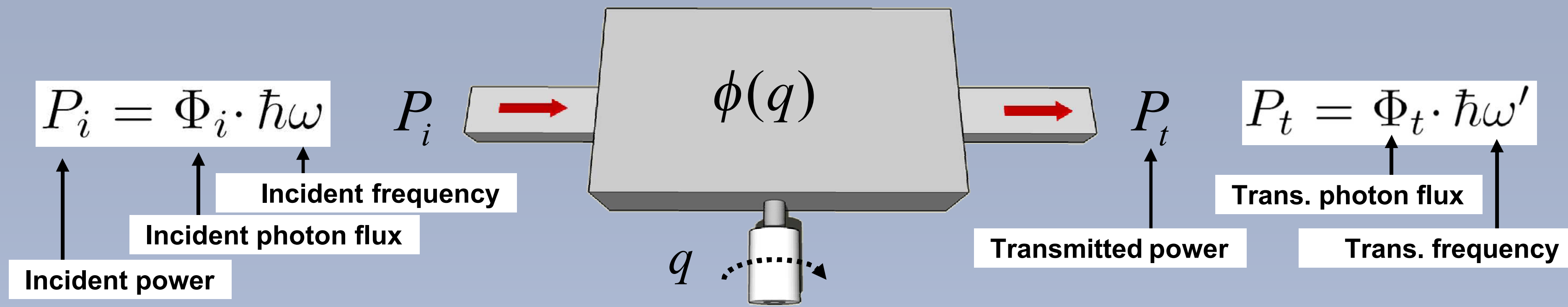
Open-system energetics: Static case

Assumptions: **(1) system is lossless (2) spatial coordinate, *q*, effects response.**

Since lossless: $exp[-i(\omega t)] \rightarrow exp[-i(\omega t - \phi(q))]$

$P_i = \Phi_i \cdot \hbar\omega$

$P_t = \Phi_t \cdot \hbar\omega'$



Static case: system is at steady-state

$\Phi_i = \Phi_t = \Phi$ (Photon flux, conserved)

$\omega' = \omega$ (Photon frequency, unchanged)

$$\therefore dU_{EM}/dt = (P_t - P_i) + dU_{in}/dt = 0 \quad \text{(Static Case)}$$

Must examine energetics of open system in time-varying case to say anything.

Open-system energetics: Dynamic case

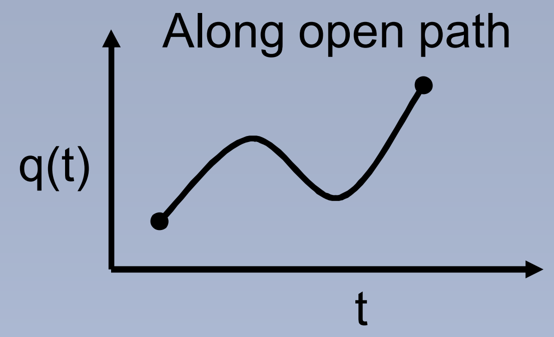
General approach:

- Express power (dU_{EM}/dt) in terms of optical response of system
- We assume: $q(t)$ varies in <u>quasi-static</u> manner (i.e. very slowly).

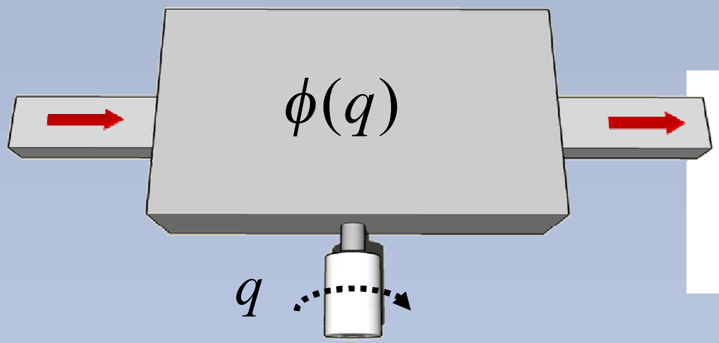
Quasi-static limit:

$$\phi(t) \cong \phi(q(t))$$

$$\omega' = \omega + \delta\omega = \omega - \dot{\phi}$$



$$P_i = \Phi_i \cdot \hbar\omega$$



$$P_t = \Phi_t \cdot \hbar\omega' \cong \Phi_t \cdot \hbar(\omega - \dot{\phi})$$

$$U_{in} = N\hbar\omega\text{"} = \left(\frac{\partial\phi}{\partial\omega} \cdot \Phi_i\right) \cdot \hbar\omega\text{"}$$

…Given this, what can we say about dU_{EM}/dt?

Open-system energetics: dynamic case

Expressing all quantities in terms of the phase-response of the system:

$$P_i = \Phi_i \cdot \hbar\omega$$
$$P_t \cong \Phi_t \cdot \hbar(\omega - \dot{\phi})$$

$$U_{in} = N\hbar\omega" = \left(\frac{\partial\phi}{\partial\omega} \cdot \Phi_i\right) \cdot \hbar\omega"$$

One can show: $dU_{EM}/dt = \left(P_t - P_i\right) + dU_{in}/dt$

$$= -\Phi \cdot \hbar\dot{\phi} + \mathrm{O}(\dot{\phi}^2) + \cdots$$

Here $\dot{\phi} = \frac{d\phi}{dq} \cdot \frac{dq}{dt}$. Thus, in the static limit, (i.e. as dq/dt → 0) we have:

Optically-induced force:

$$\frac{dU_{EM}}{dq} = -\Phi \cdot \hbar \frac{d\phi}{dq} = -F_{opt}$$

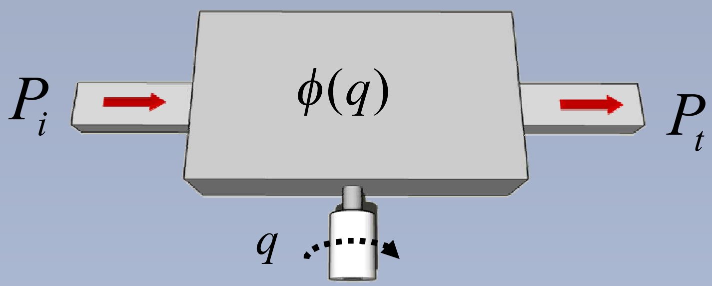
Optically-induced potential:

$$U_{eff}(q) = -\Phi \cdot \hbar\phi(q)$$

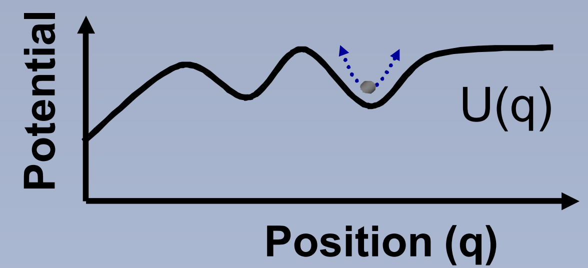
Extension to multi-port systems.

Lossless single-port, with a single mechanical degree of freedom, q.
(Input flux (Φ) and frequencies (ω) are assumed to be fixed).

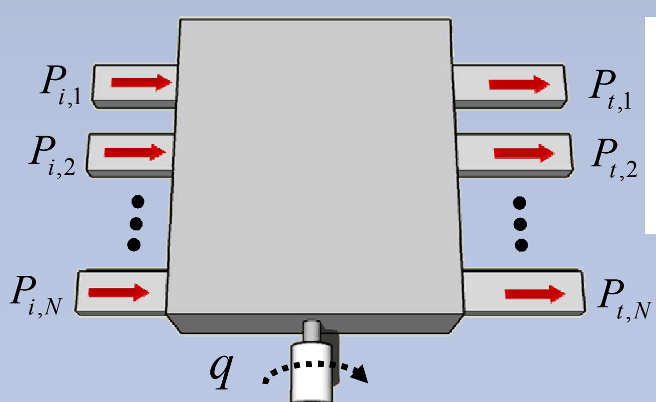
$$U_{eff}(q) = -\Phi \cdot \hbar \phi(q)$$



- Allows direct potential synthesis!
- Phase synthesis already known [1].



Lossless multi-port, with a single mechanical degree of freedom, q.
(Input fluxes (Φ_i's) and frequencies (ω_i's) are assumed to be fixed).



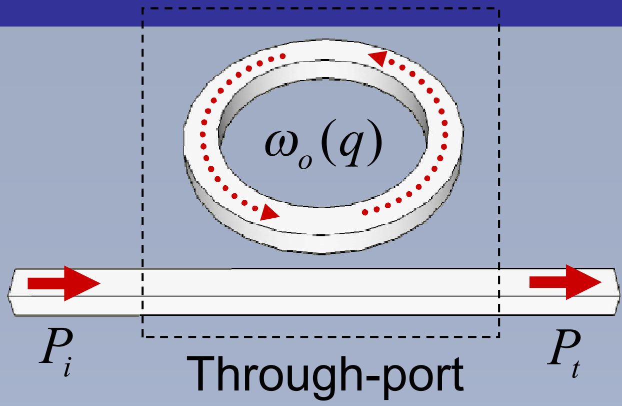
$$U_{eff}(q) = -\hbar \cdot \int \left[\sum_k \Phi_{t,k}(q) \cdot \frac{d\phi_{t,k}(q)}{dq} \right] \cdot dq$$

Phase response

Photon flux (k^{th} output port)

[1] Madsen, C. K. & Zhao, J. H. *Optical flter design and analysis* (Wiley, New York, 1999).

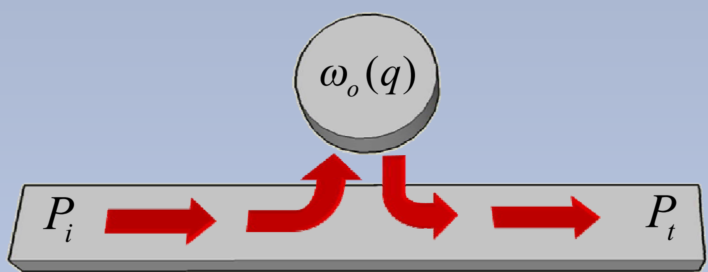
Example: All-pass filter.



$$\phi(q) = 2 \cdot \arctan[(\omega - \omega_o(q))\tau_t]$$

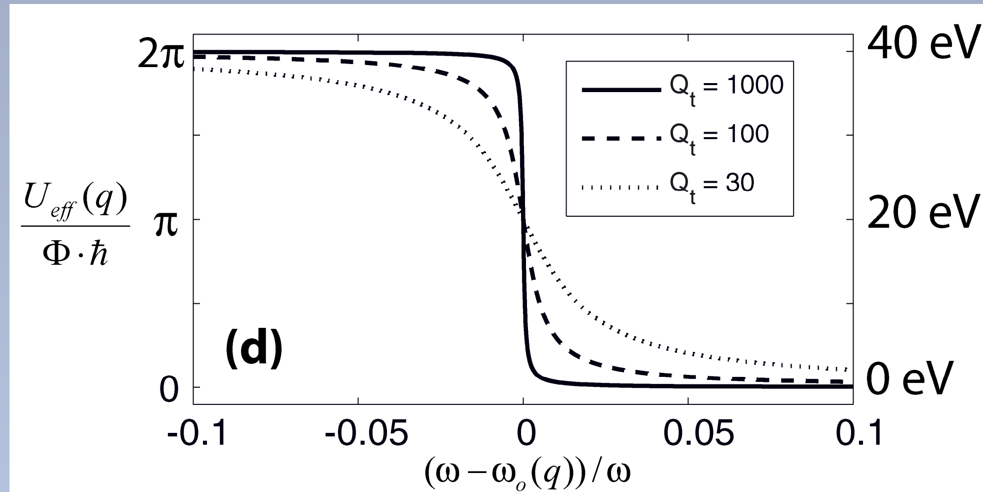
$$U_{eff}(q) = -\Phi \cdot \hbar\phi(q)$$

C.M.T. Model of microring:



$$\frac{d}{dt}a = i\omega_o a - \frac{1}{\tau}a + i\kappa s_i$$

$$\kappa = \sqrt{2/\tau_t}$$



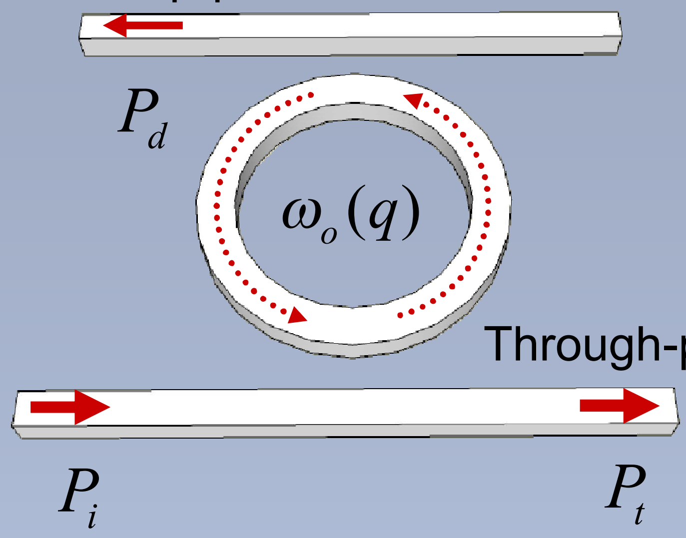
Potential makes "jump" of

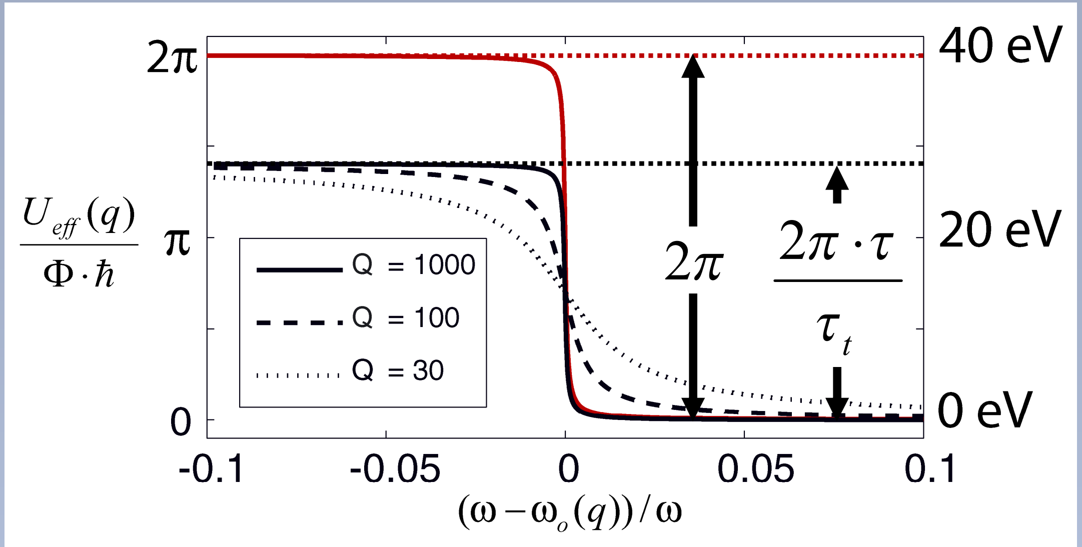
$$\Delta U_{eff} = \Phi \cdot \hbar \cdot 2\pi$$

Independent of cavity Q!

Example: Multi-port ring resonator system.

$$1/\tau = 1/\tau_t + 1/\tau_d$$



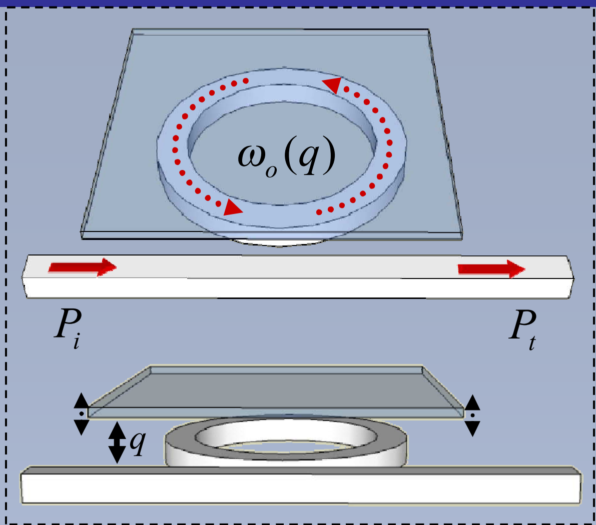


Single-port system: $U^o_{eff} = \hbar \cdot \Phi \cdot 2 \cdot \arctan[(\omega - \omega_o(q))\tau_t]$

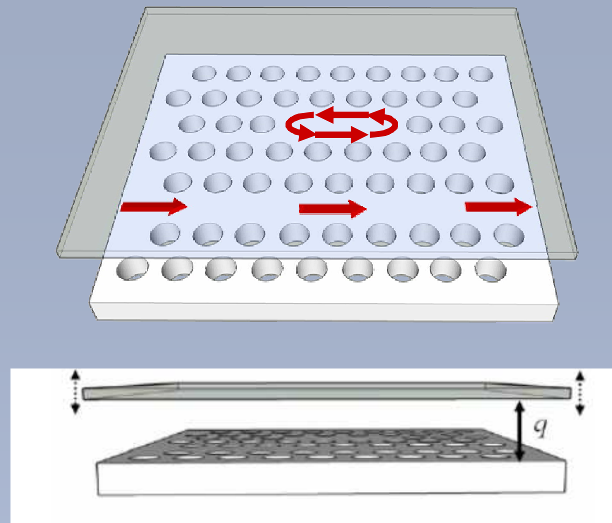
Multi-port system: $U^o_{eff} = \hbar \cdot \Phi \cdot \frac{2\tau}{\tau_t} \cdot \arctan[(\omega - \omega_o(q))\tau]$

Same relation for N-ports: $1/\tau = 1/\tau_t + 1/\tau_1 + 1/\tau_2 \ldots + 1/\tau_N$

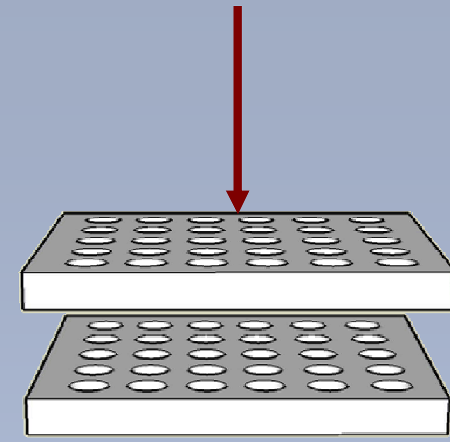
Analyze any system representable by CMT!



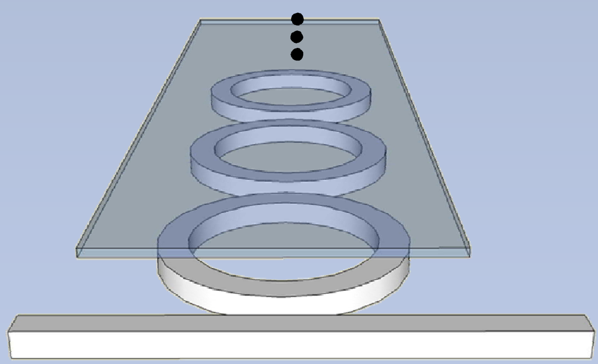
Rakich et. al. Opt. Lett. 31, 1241-1243 (2006).

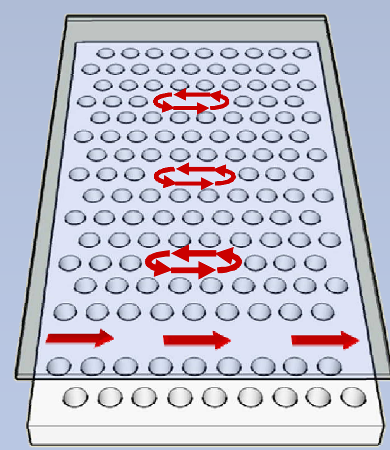


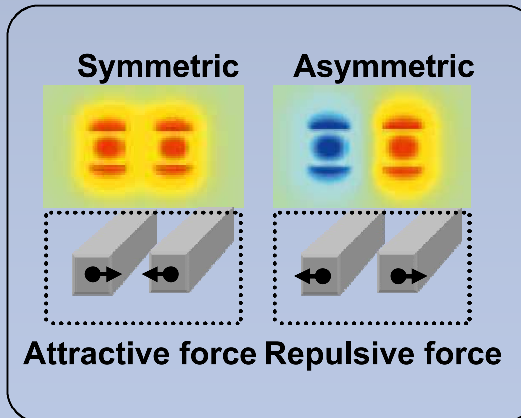
M. Notomi, et. al.PRL 97(2), 023903 (2006).



Suh et al, Opt. Lett., vol. 28, p1763 (2003)







Povinelli, et. al. Opt. Lett. 30,3042 (2005).

Conclusions:

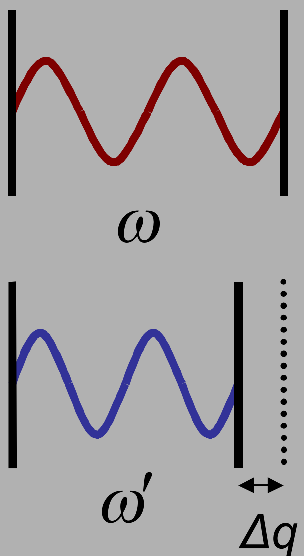
- **Developed unique and simple treatment of forces and potentials from elementary energetics considerations, showing:**
  - **Minimum information to compute potential: phase and amplitude response multi-port system.**
- **This theory produces "Exact" agreement with similar analyses using close-system energetics.**
- **This is an important step toward a synthesis theory for forces and potentials in the context of optomechanics**

Acknowledge Zheng Wang for helpful technical discussions.

Special thanks to Marin Soljacic, Erich P. Ippen and Yoel Fink for generous support and encouragement

Closed System Energetics:

Lossless Fabry-Perot Cavity



$$U_{EM} = N\hbar\omega$$

cavity frequency

Number of photons

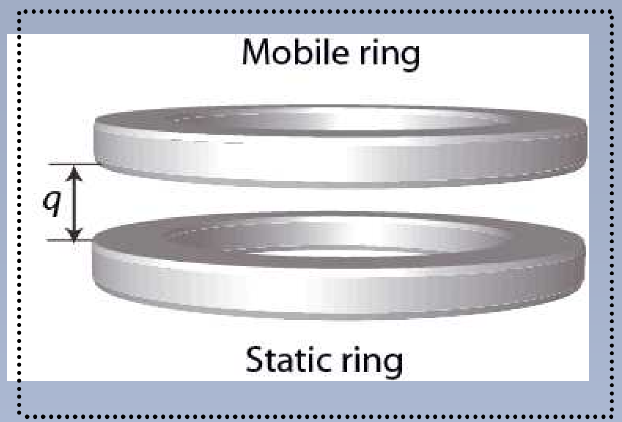
$$\delta\omega = \omega' - \omega$$

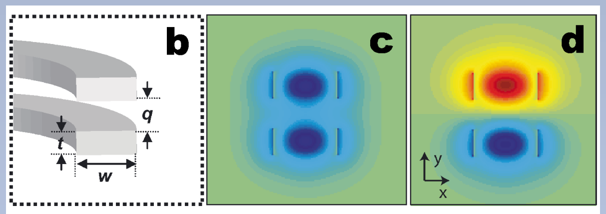
$$F_q = -\frac{dU_{EM}}{dq} = -N\hbar \cdot \frac{d\omega}{dq}$$

$$\left\{ N, \frac{d\omega}{dq} \right\}$$

- Cavity frequency is tuned.
- Photon energy adiabatically shifted.
- Virtual work reveals sign of force.

Dual-ring microcavity: How to analyze?





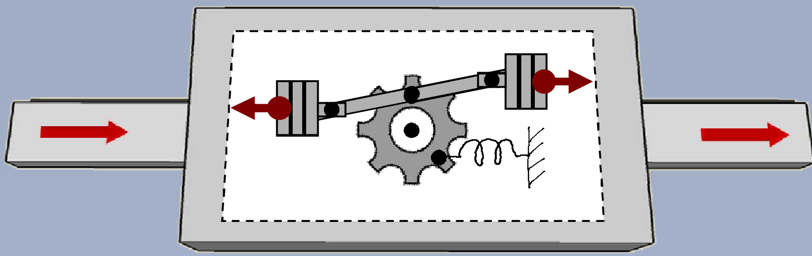
- Mechanical degree of freedom: ring-separation
- Dual-rings will have two cavity modes to consider
- How will photon-energy change as q is varied?

Peter T. Rakich, Milos A. Popovic, Marin Soljacic, Erich P. Ippen, *Nature Photonics* 1, 658-65 (2007)

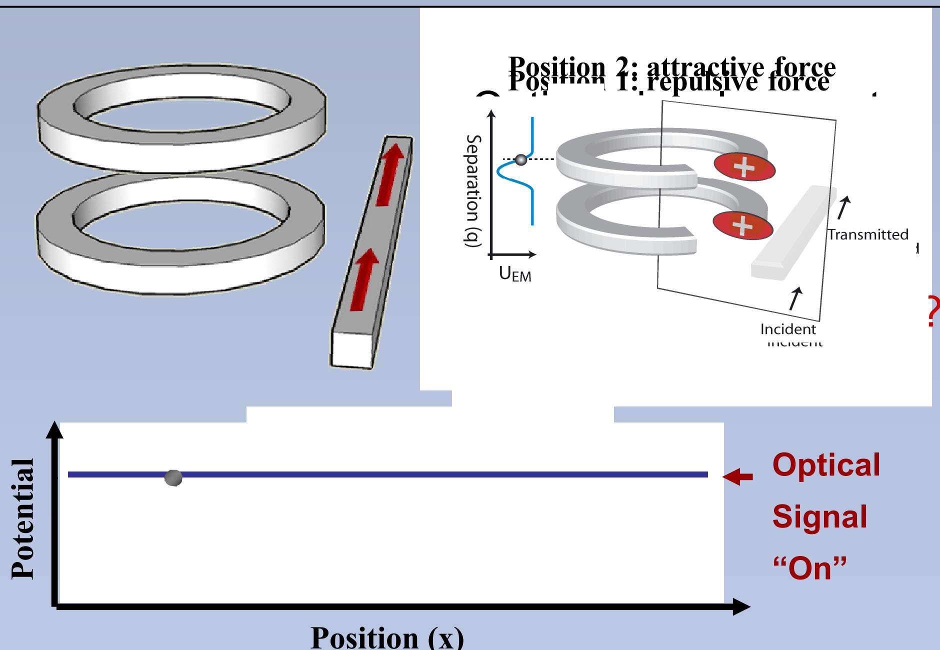
Junk Slides Follow…

Potentials useful for new applications:

All-optical potential-well



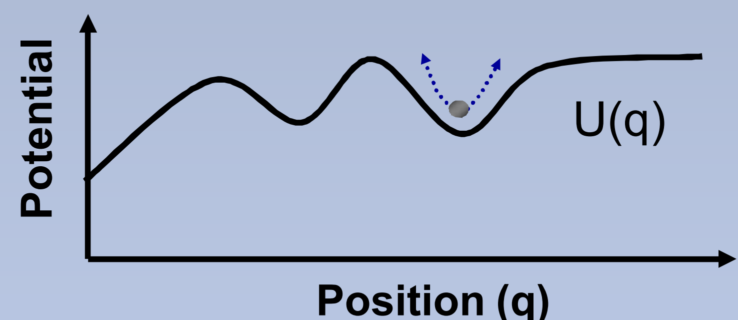
Mech. equilibrium dictated by light:



All-optical potential-well:

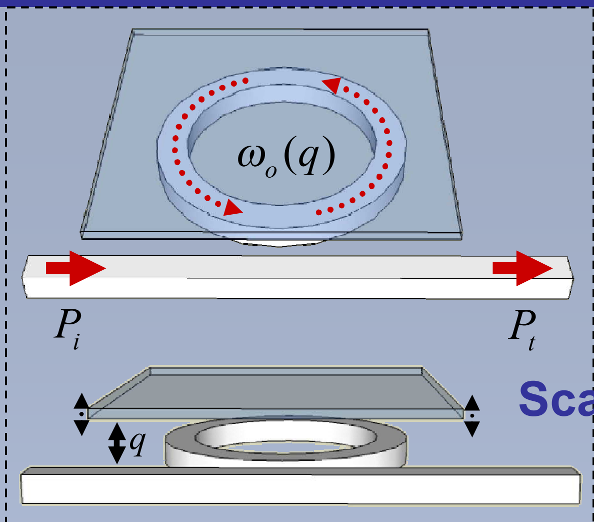
- Potential-well tailored by two optical resonances (attractive & repulsive).
- Larger range & higher stability.
- Yields (1) self-aligning cavity and (2) wavelength-to-position converter
- Intuition was our guide here.

Q: How to *design* arbitrary potential?



- To do this, we need to look at things in a very different manner…

Analyze any system representable by CMT!



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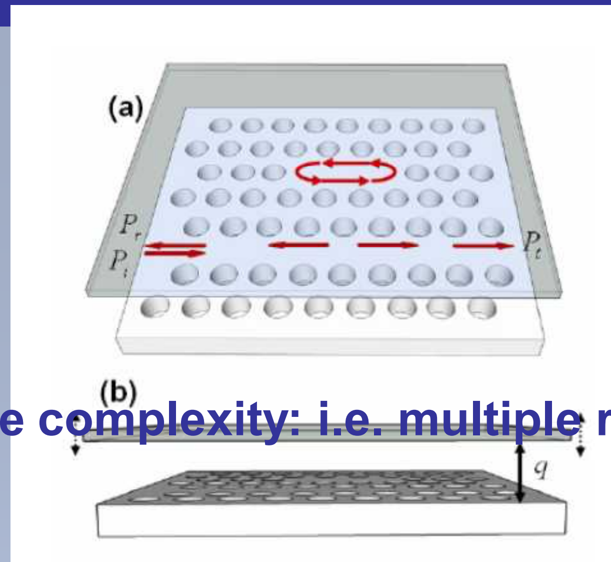
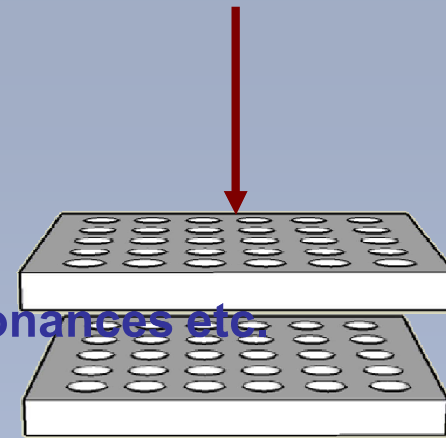
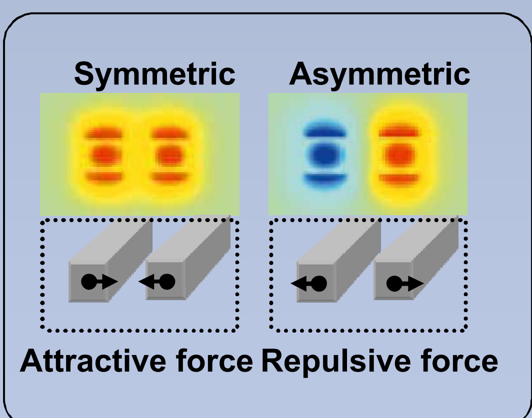


M. Notomi, et. al.PRL 97(2), 023903 (2006).



Suh et al, Opt. Lett., vol. 28, p1763 (2003)

Scale complexity: i.e. multiple resonances etc.



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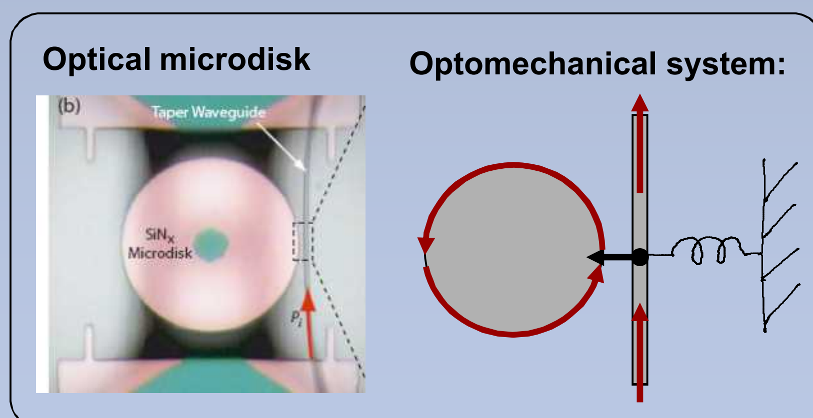
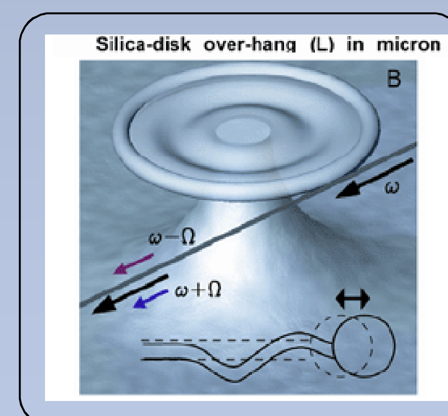


Eichenfield, M. *et al.* Nature Photon. 1, 416–422 (2007).



Vahala Group: Caltech

What's this talk about?

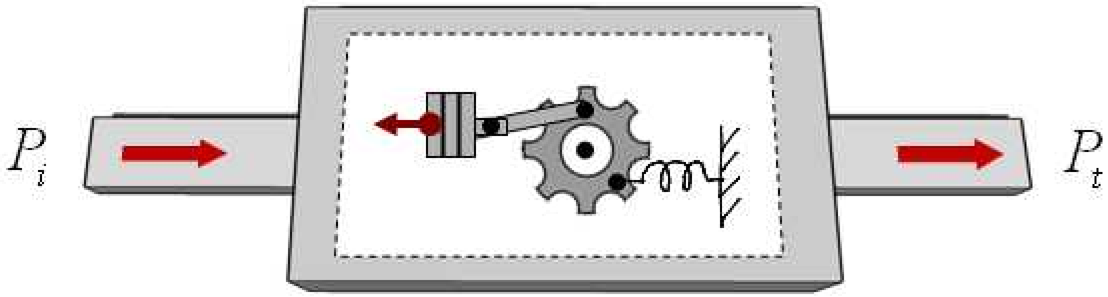
- **We present new treatment of optically-induced forces and potentials in optomechanically variable systems.**
- **Q: What does a new treatment of forces and potentials give us?**

  (1) Greatly simplified computation optically-induced optical forces & potentials based only on optical response of system.

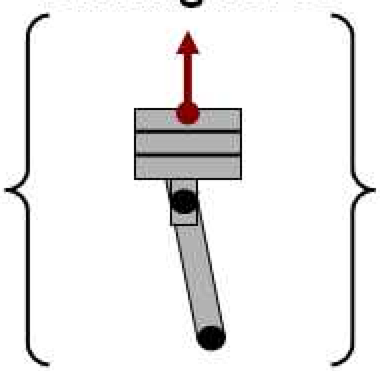
  (2) Since forces & potentials are computed from optical response alone → potential synthesis is possible.
- **<u>Illustrative examples:</u> resonant single-port and multi-port systems.**
- **Conclude**

State of optomechanics: so far.. simple

Simple optomechanical system:
Single driving force & spring



Optical forces
↓
Mechanical motion
↓
New equilibrium: yields change of optical state



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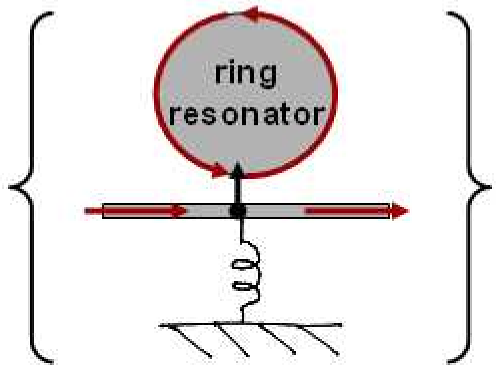
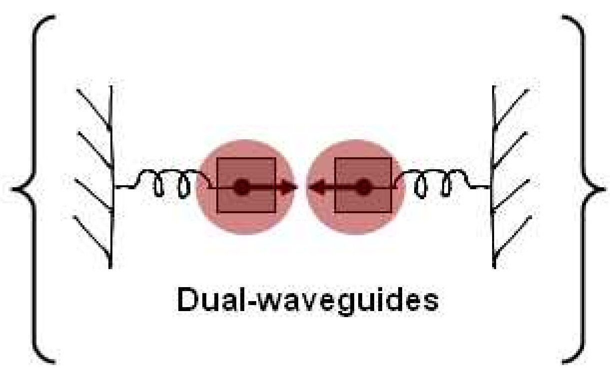
Resonant [1]



Non-Resonant [2]



- Simple optomechanical systems require only force-based calculations.
- Problem: with increasing complexity, force-based treatments aren't practical!

[1] Eichenfield, M. *et al.* Nature Photon. 1, 416–422 (2007).
[2] Povinelli, et. al. Opt. Lett. 30,3042 (2005).

State of optomechanics: so far.. simple

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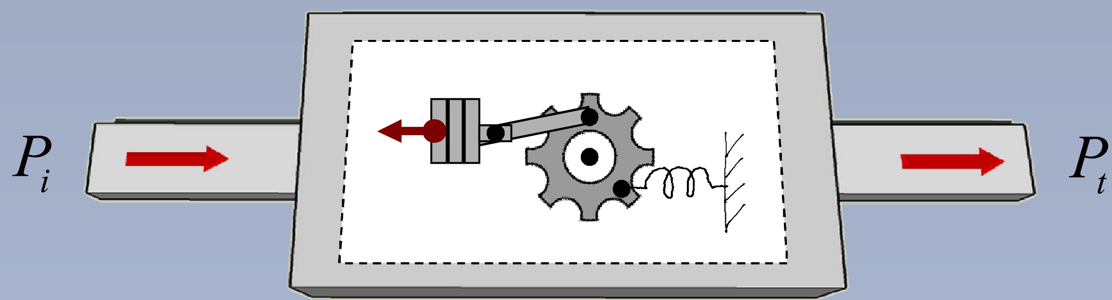


Optical forces

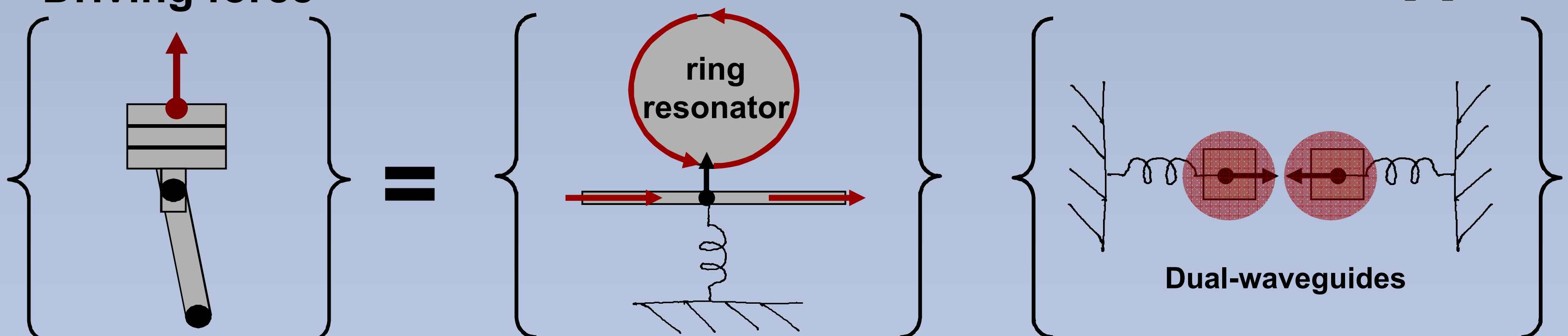
↓

Mechanical motion

↓

New equilibrium: yields change of optical state

Optical Driving force = **Resonant [1]** **Non-Resonant [2]**



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[2] Povinelli, et. al. Opt. Lett. 30,3042 (2005).

History of Radiation Pressure

Idea that light can induce motion has been around for a long time:

- **Timeline for Radiation Pressure:**

  (1619) Kepler: Speculated solar repulsion

  (1873) Maxwell: Theoretical basis for pressure

  (1901) Lebedew, Nichols: Experimental evidence

- **Optical Forces Very small:**
  (60 Watts → Force ≈ 400nN)

- **Thermal lamp, Torsion balance**

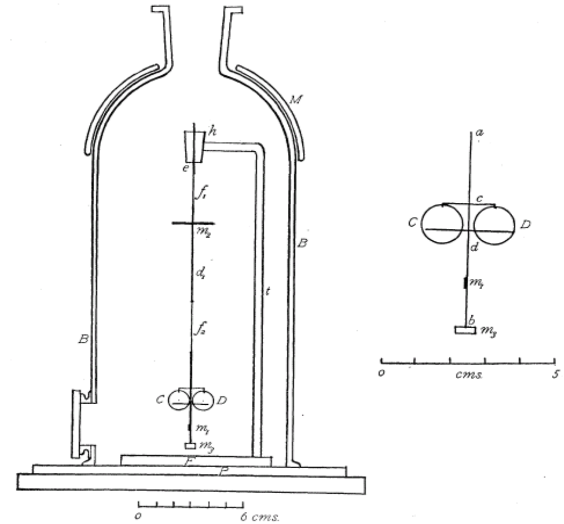
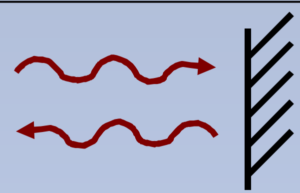


Fig. 2.

[1] P. Lebedew, Ann. Phys., VI., 457, 1901.

Photon momentum:

$$|p| = \hbar k$$



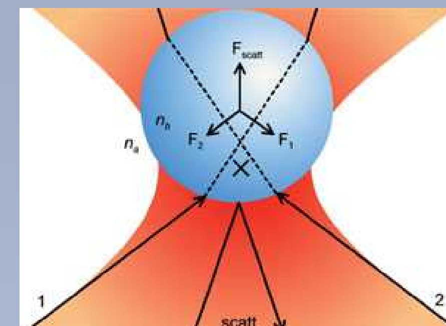
Imparted momentum:

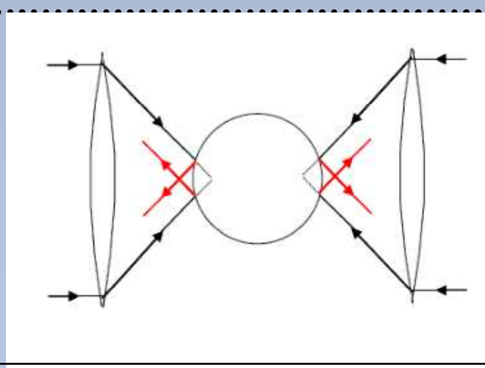
$$|\Delta p| = 2\hbar \cdot k$$

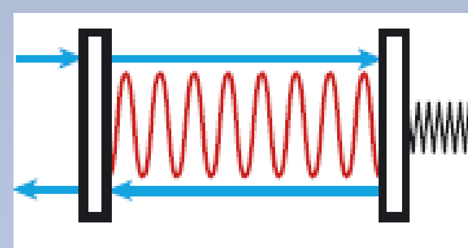
1. Nichols, E. F. & Hull, G. F. *Phys. Rev.* **13,** 307–320 (1901).
2. Maxwell, J. C. *A Treatise on Electricity and Magnetism* 1st edn, Vol. 2, 391 (Oxford Clarendon, USA, 1873).
3. Lebedev, P. *Ann. Phys. (Leipz.)* **6,** 433–458 (1901).

Useful Mechanical Motion from Radiation Pressure?

- Optical tweezers: Trapping small of particles (Power = 50 mW, Force ≈ 50 pN)
- Laser Trapping and Cooling: Atoms
- Free-space Interferometers: Optical Bistability (Power = 100 mW, Force ≈ 650 pN)
- Forces are small: difficult to create useful motion.



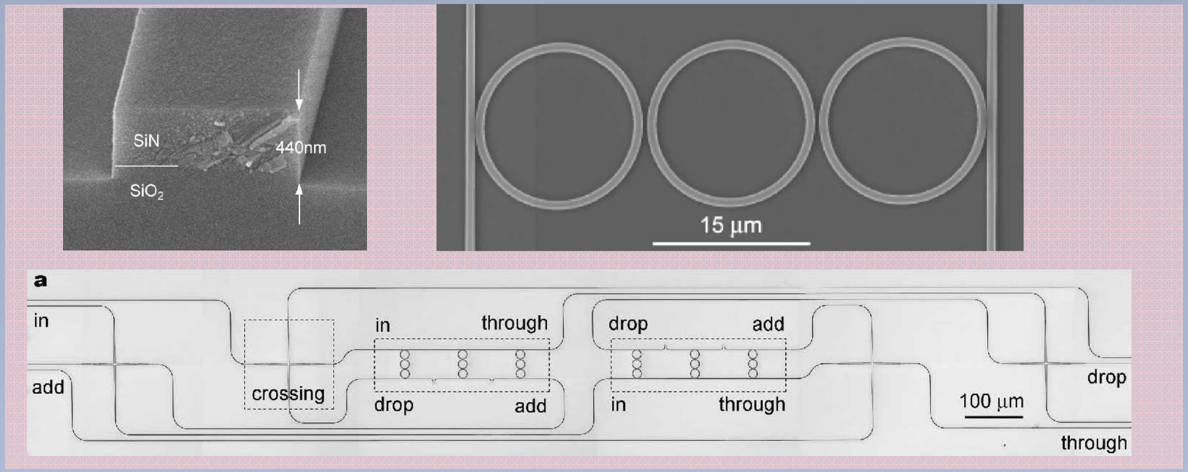




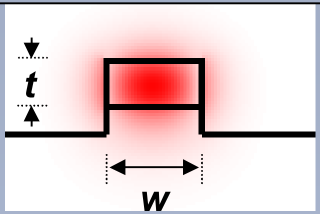
5. Ashkin, A. *Phys. Rev. Lett.* **24,** 156–159 (1970).

New Opportunities: Integrated photonics

Micron-scale optical components:



Computed Mode Profile



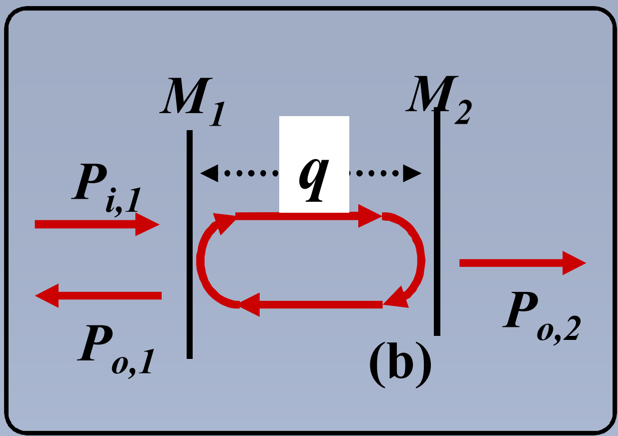
Core: Silicon
w = 450 nm
t = 200 nm

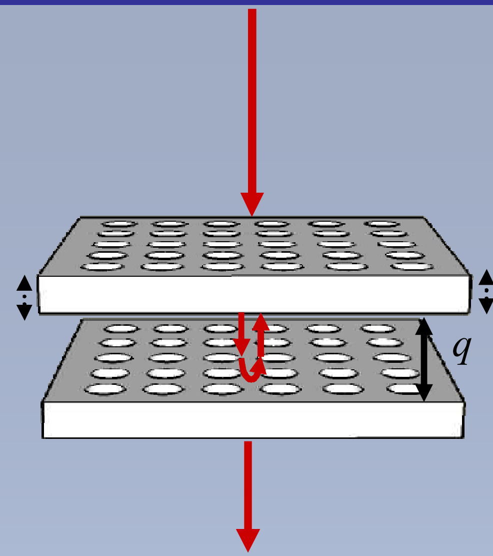
Compelling case for utility of optical forces in integrated photonics:

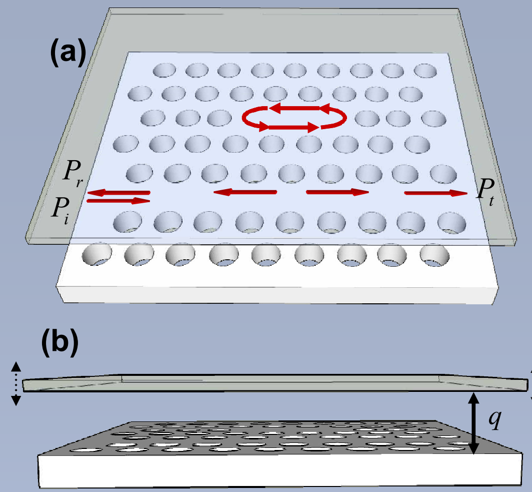
- **Nanometer scale modes ➔ forces scale to large values (*F* = 1-10 uN, *P* = 1mW)**
- **MEMS-like components can be tethered to surface**
  - **Enhanced forces strong at these scales**
  - **Small mass, little inertia**
  - **Rapid response to optical forces**

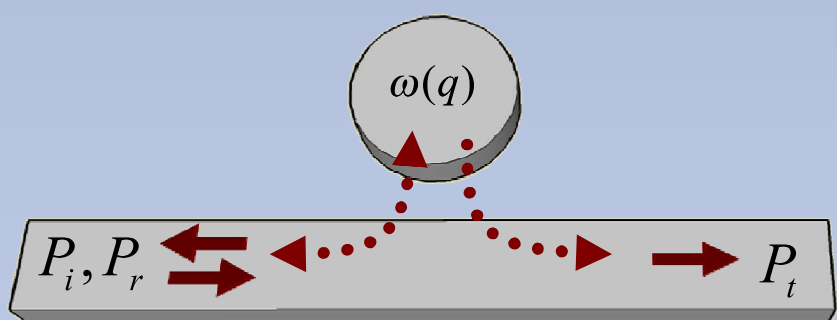
Similarity of optical systems

Radiation Pressure





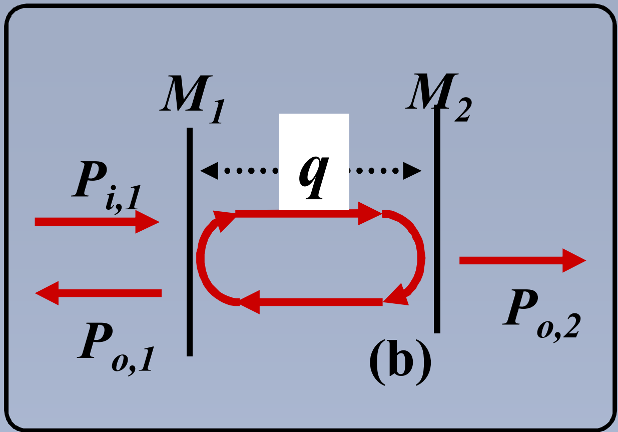


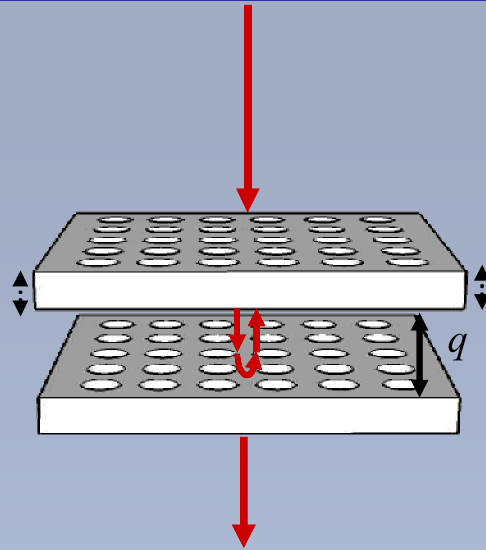


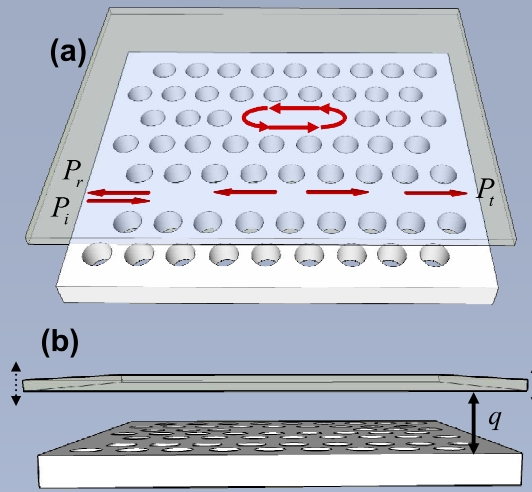
- Systems look quite different, but their response can be identical.
  - All are two-port resonant systems (ω_o)
  - $\omega_o(q)$
  - $H(\omega_o(q))$
- Despite similarities, Maxwell stress tensor requires complete reformulation of problem electromagnetically.

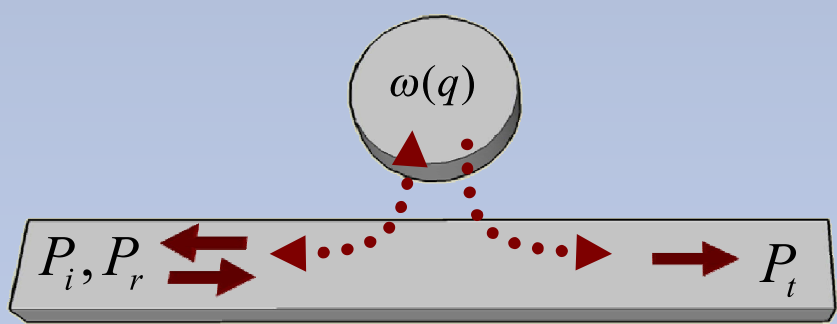
Similarity of optical systems

Radiation Pressure









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