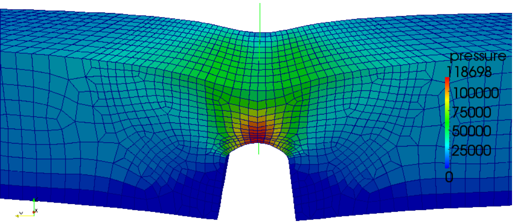

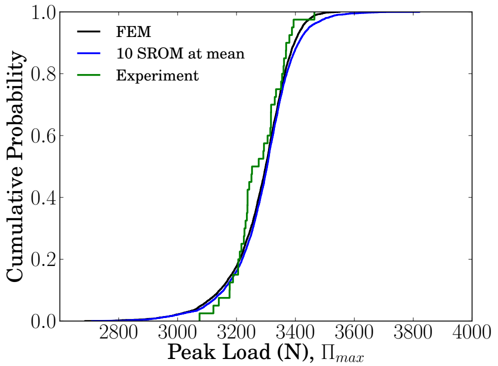

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SAND2014-1132C

Reliability calculations for ductile laser welds with stochastic reduced-order models

TMS 2014 Annual Meeting

February 19, 2014

J. Emery[1], R. Field[1], M. Grigoriu[2], K. Karlson[1], J. Foulk[1]

[1] Sandia National Laboratories

[2] Cornell University

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Laser welding – very briefly

what? / why?

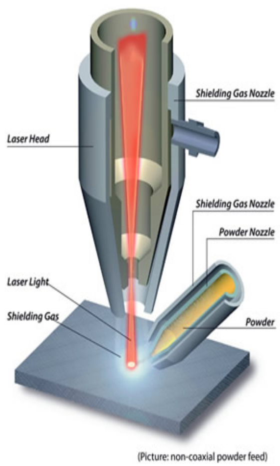
- a variety of tools/methods: fiber lasers; pulsed waves; etc.
- low heat input (a benefit for nearby heat-sensitive, parts)
- high automation
- high quality

but:

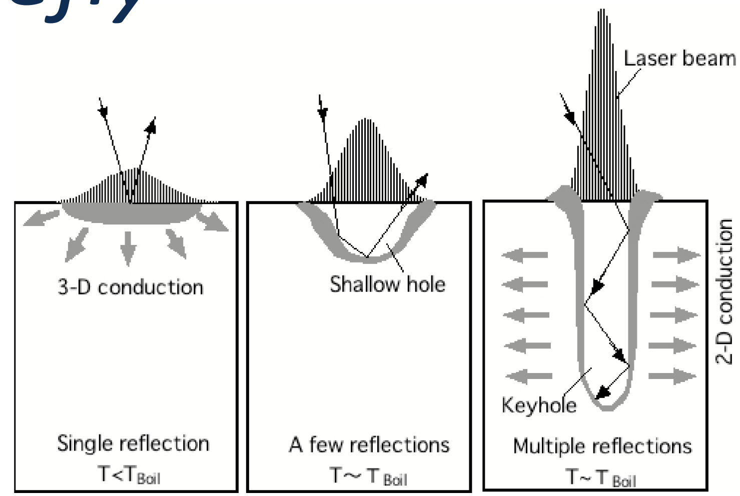
- beam power & quality affect solidification (upper right)
- can develop porosity due to off-gassing or key-hole collapse

consequences:

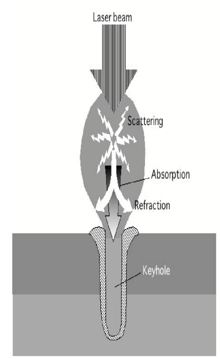
- wide range of variability in geometry
- wide range in response under mechanical loading (peak load / “strain” at peak load)



(stellite.co.uk)



(Miyamoto & Knorovsky 2008)



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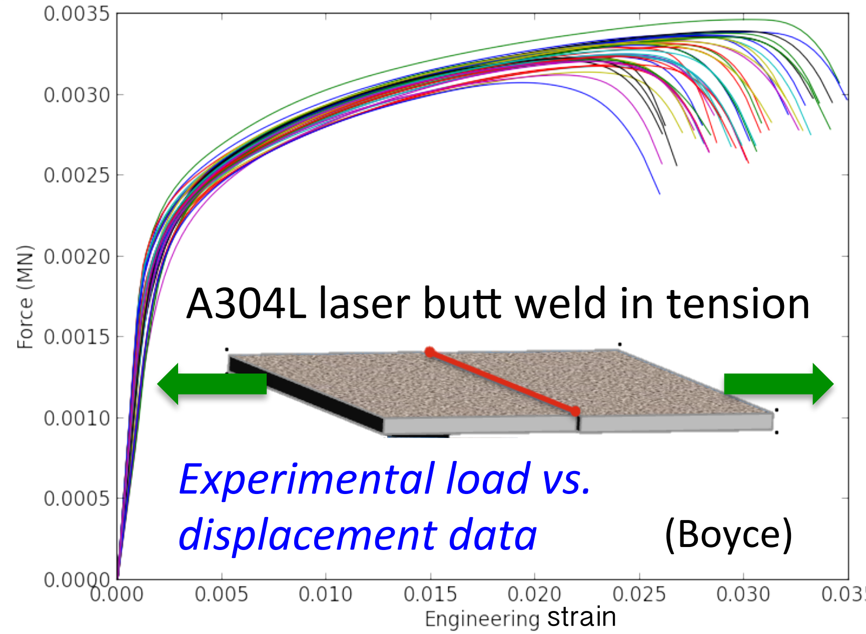
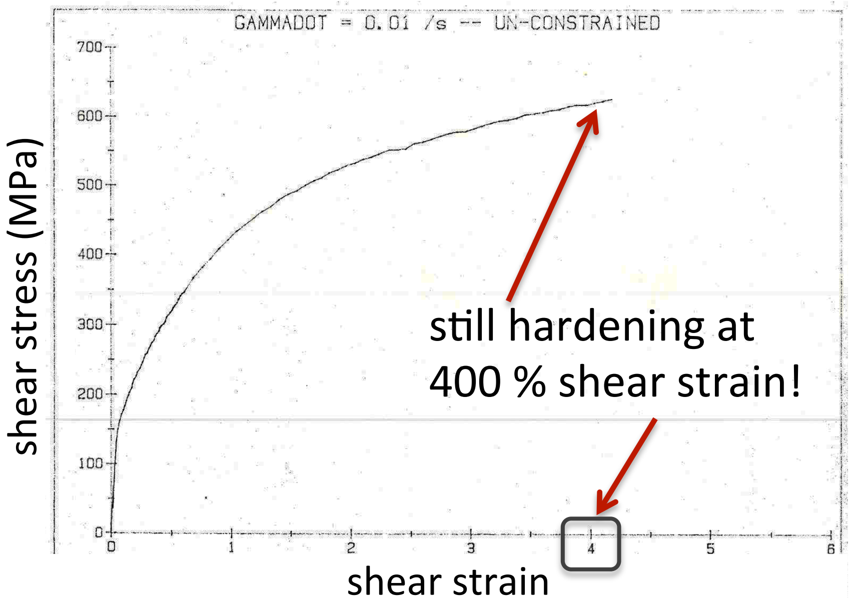


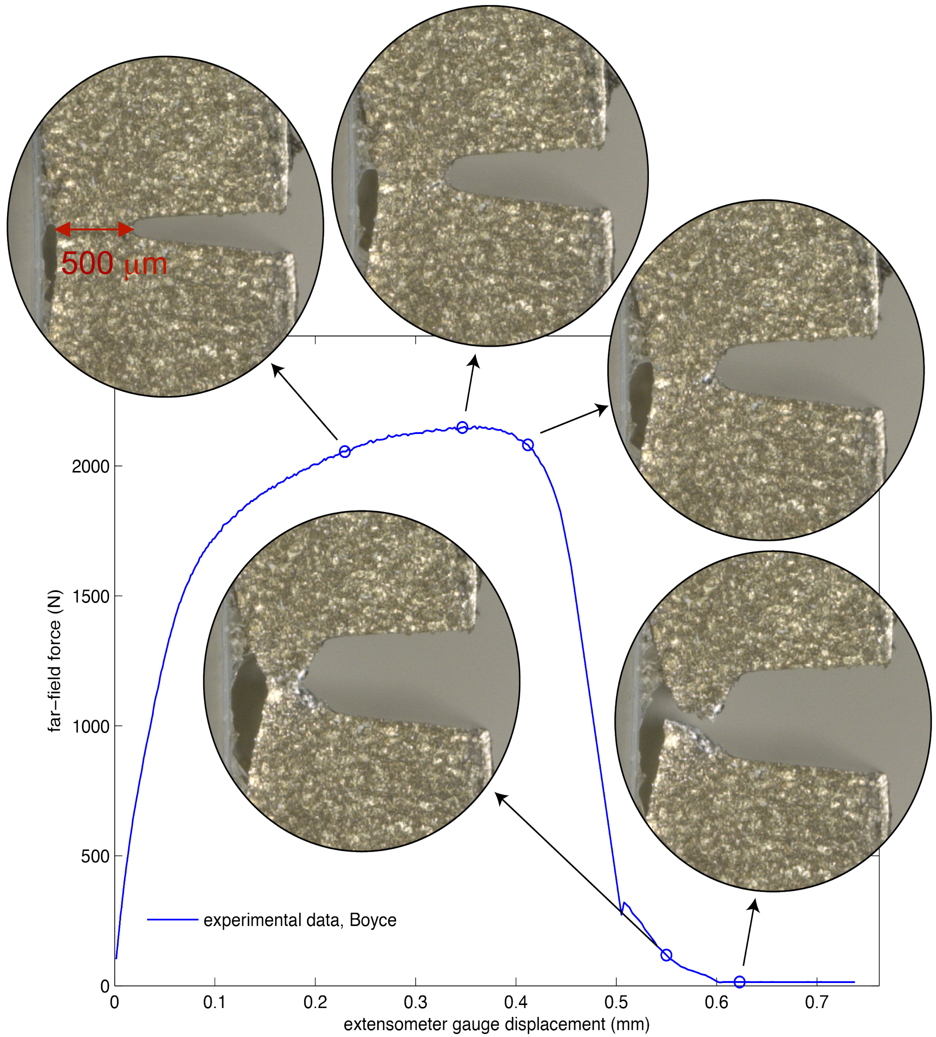
Motivation – mechanics

- A304L hardens to high plastic strains

Thin tubes in torsion (W. Kawahara, ~1980)

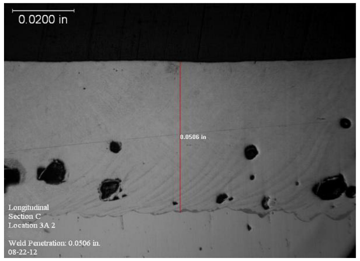


- We observe geometric instability (necking) prior to crack nucleation and propagation

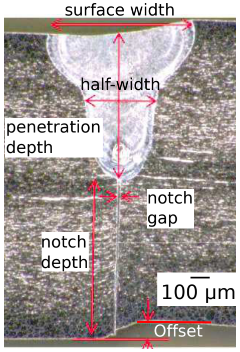


A304L SS laser-welded tensile coupon

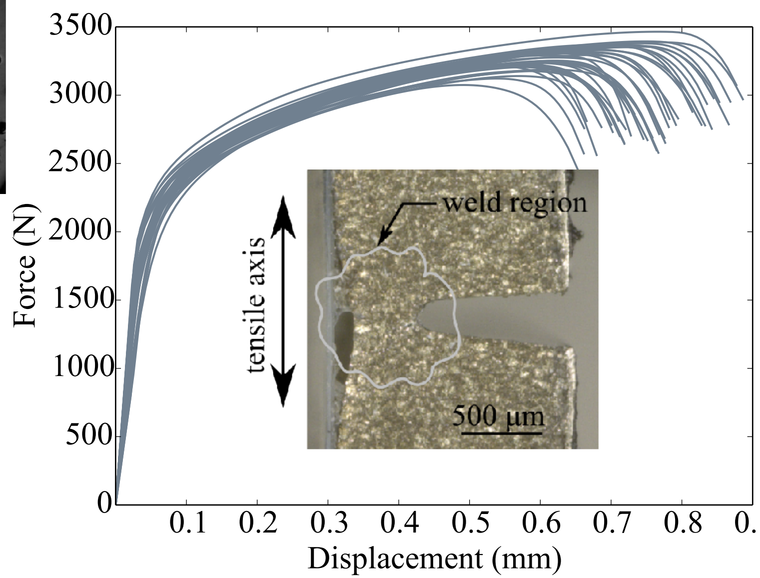
Motivation – variability

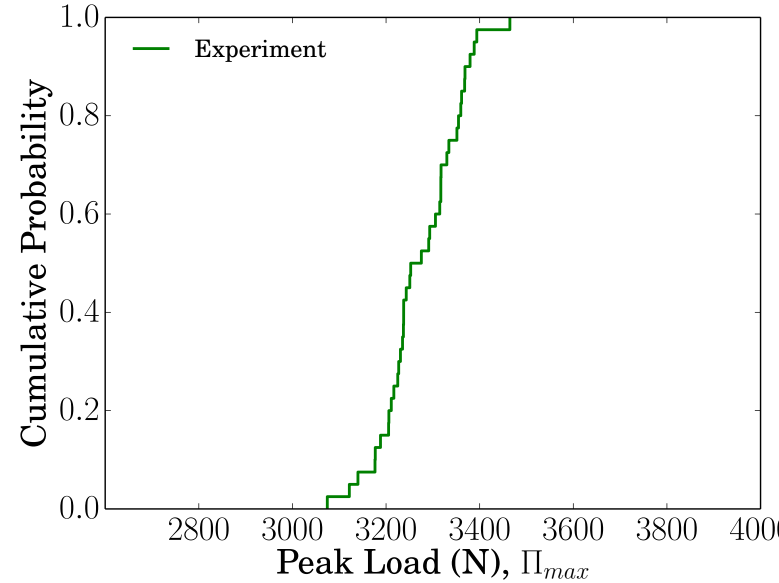


porosity



geometry

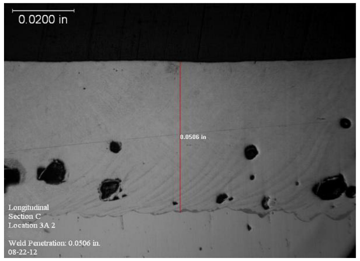




40 laser-welded tensile coupons

- Randomness in weld microstructure manifests itself as randomness in weld response.
- Our goal is to propagate this uncertainty through simulation of welded components to make reliability predictions.

Motivation – variability



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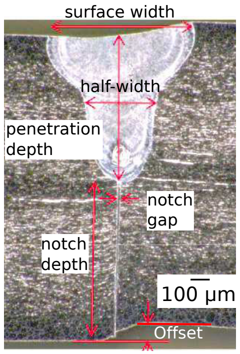

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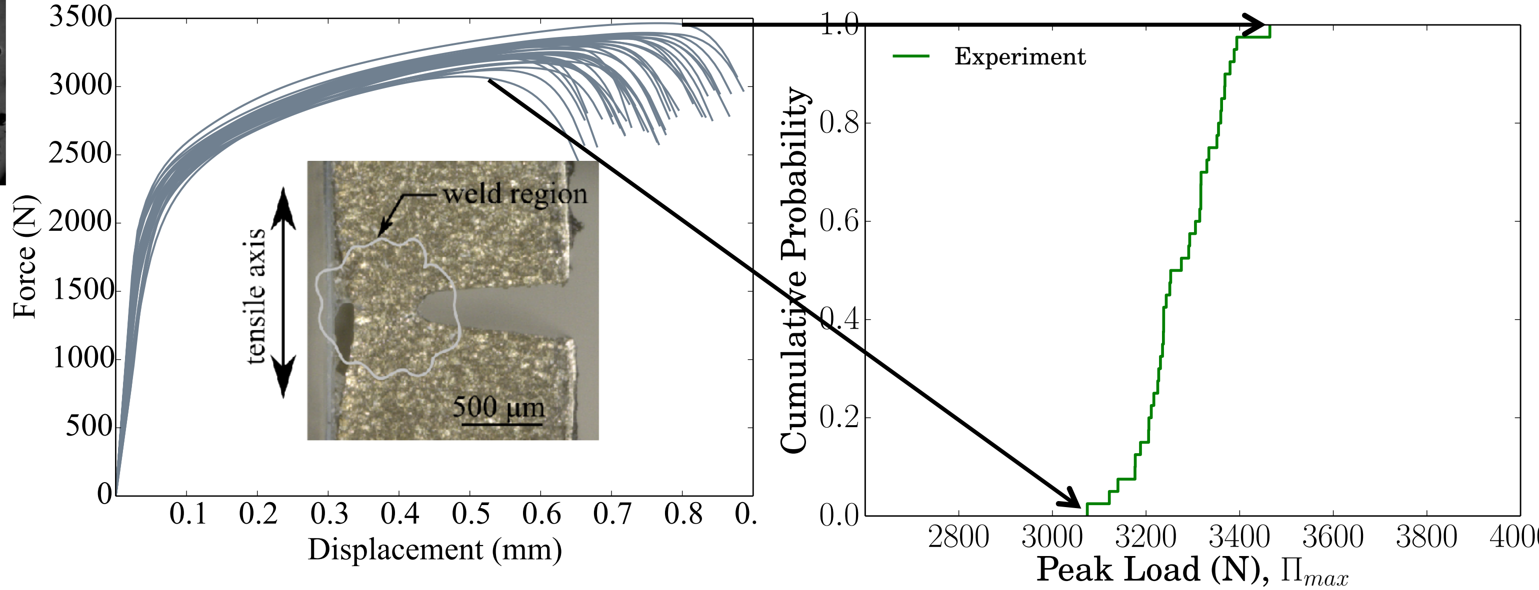

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- Randomness in weld microstructure manifests itself as randomness in weld response.
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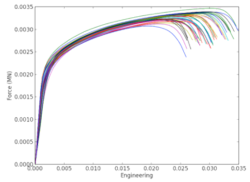
Outline for the rest of the talk

How to propagate the observed uncertainty through component analysis?

Input → **Calibration**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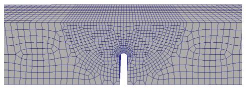

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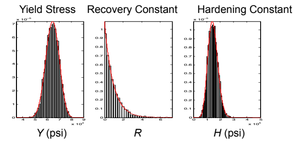
Enrichment



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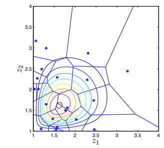
Stochastic Reduced-order Model



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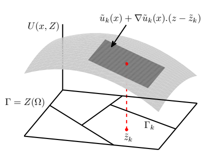
FE Analysis ↔ **SROM-based Surrogate Model**



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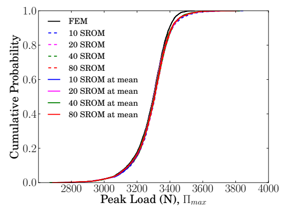
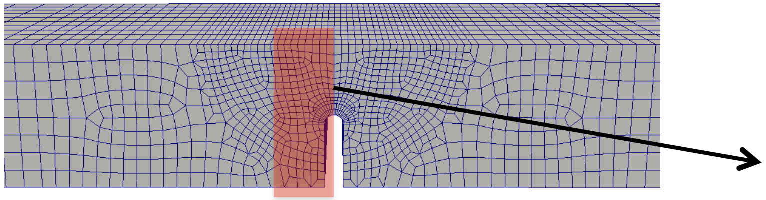
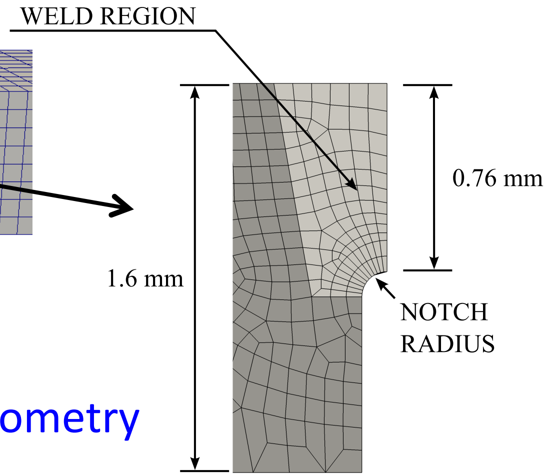
Output



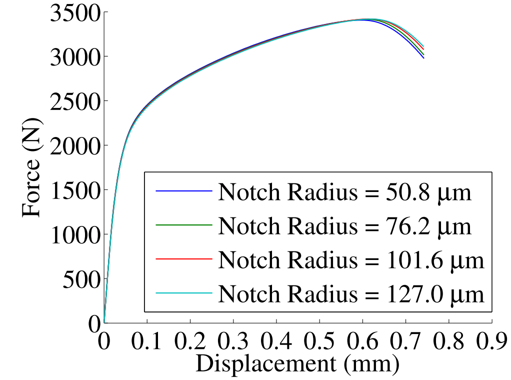
Calibration – stochastic dimension & FE model



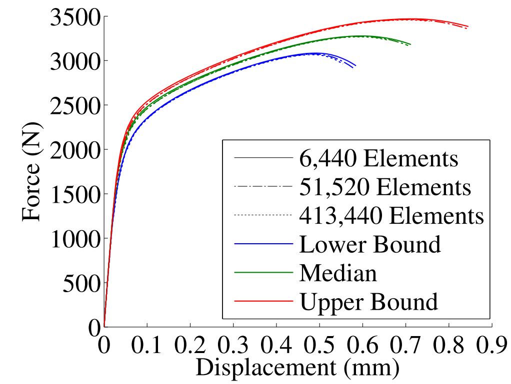


base sheet: deterministic
weld region: stochastic

3D FE model – idealized weld geometry



notch sensitivity



mesh convergence

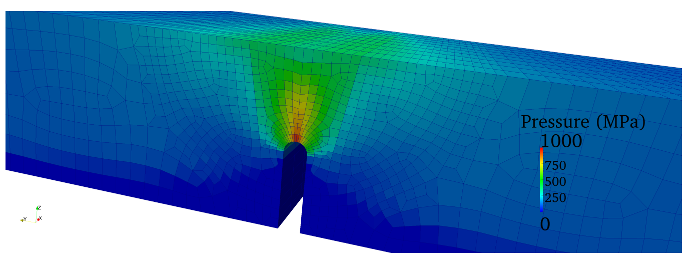
Model:

1. *Idealize weld geometry for smooth evolution of fields through very large plastic strains*
2. *Assume weld is rate/temperature independent*
3. *Assume the weld is homogeneous*
4. *Assume elastic properties are deterministic*
5. *Use BCJ_MEM material model w/ 3 uncertain parameters*

$$\sigma_y = Y + \kappa \qquad \dot{\kappa} = \left[H - R\kappa\right]\dot{\epsilon}_p$$

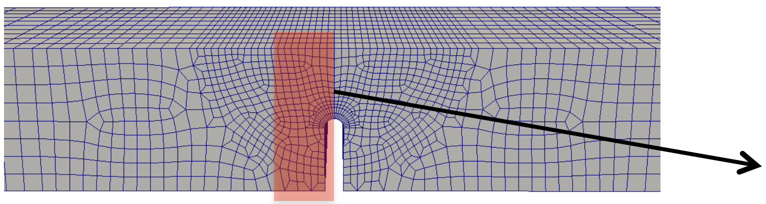
$$\kappa\left(\epsilon_p\right) = \frac{H}{R}\left[1 - \exp\left(-R\epsilon_p\right)\right]$$

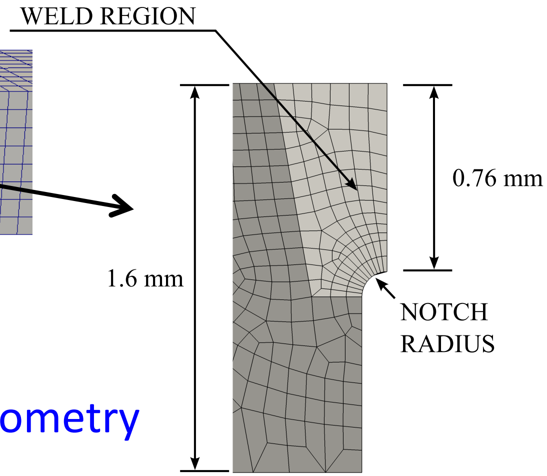
- Y initial yield stress
- H hardening (linear)
- R recovery coefficient



smooth pressure – no locking

Calibration – stochastic dimension & FE model





base sheet: deterministic
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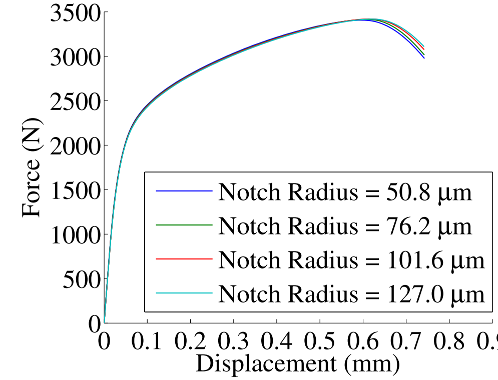

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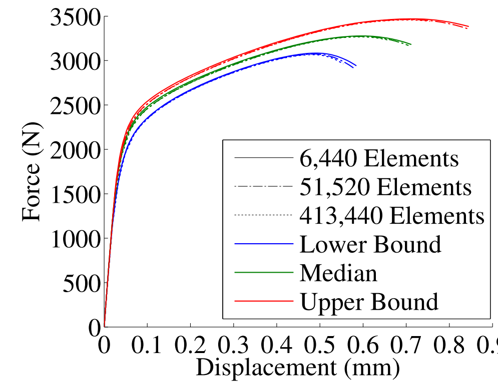

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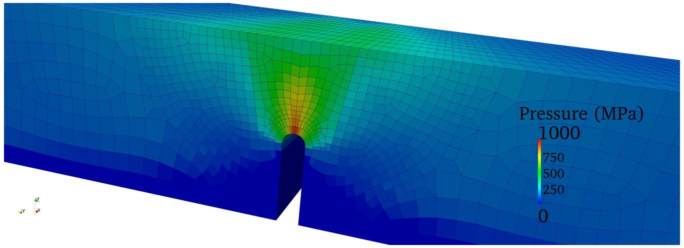
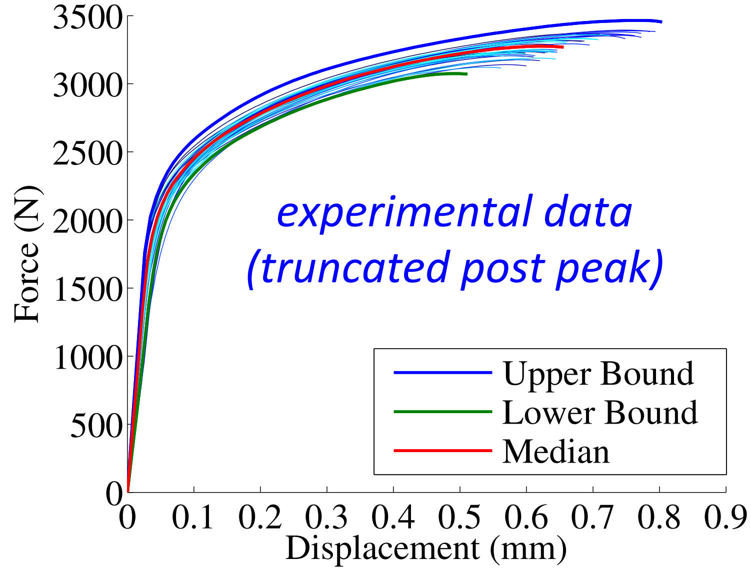
stochastic dimension = 3 $\Theta = \begin{bmatrix} Y \\ H \\ R \end{bmatrix}$ initial yield stress, hardening (linear), recovery coefficient

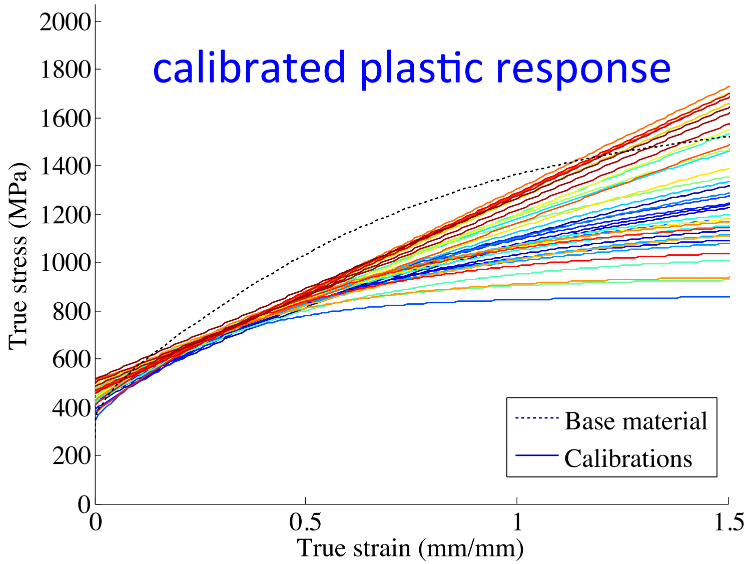


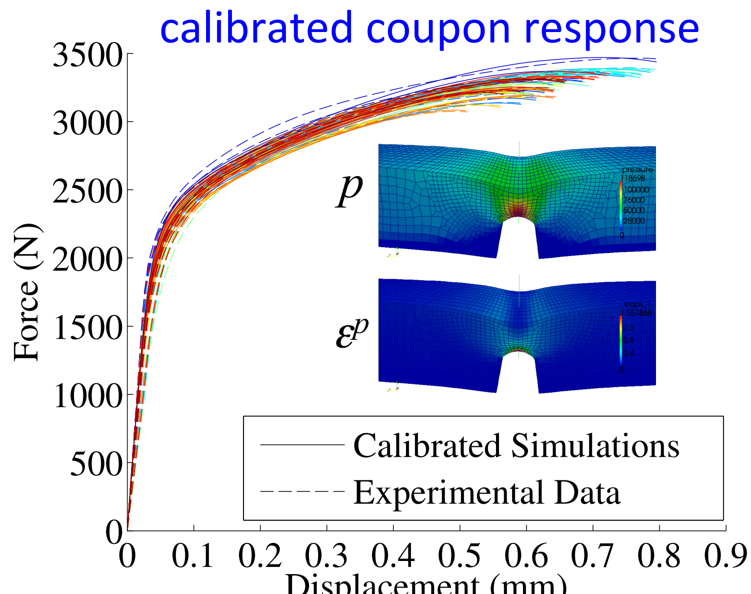
smooth pressure – no locking

Calibration through optimization – MatCal wraps DAKOTA

- First calibrated to the upper/lower bound & median of the experimental data using the global optimization routine *ncsu_direct* from DAKOTA.
  - weighted to favor data points between the elastic region and peak load.
  - large parameter bounds chosen for the optimizations.
- With parameters bounds from the global optimizations, the remaining 37 data were calibrated with the least-squares algorithm *nl2sol*.



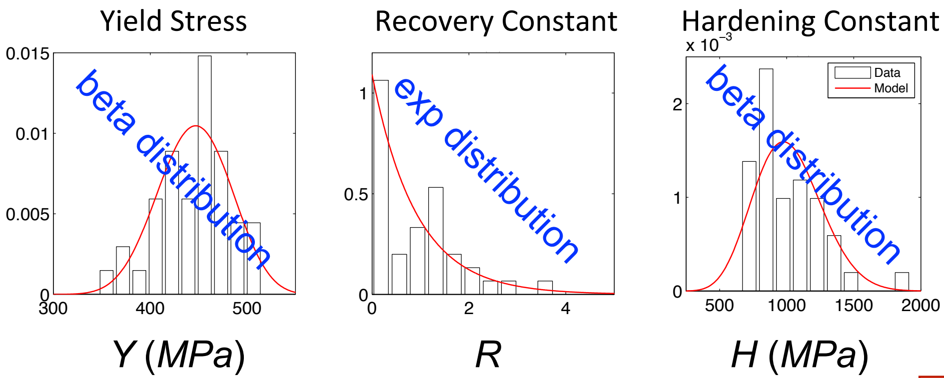


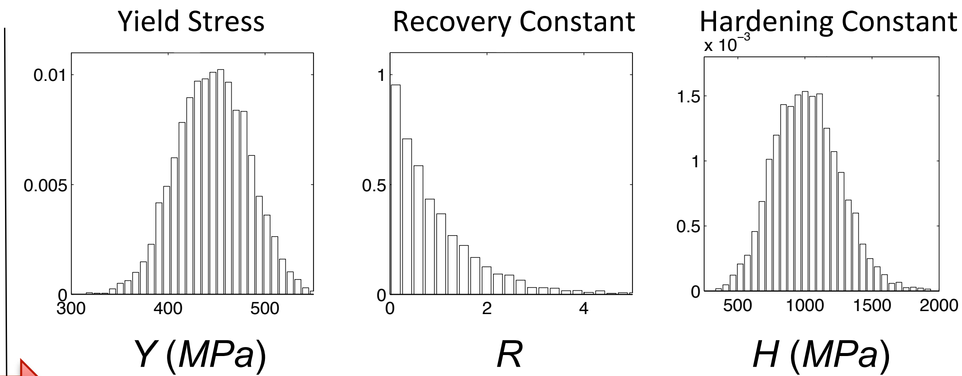


Enrichment

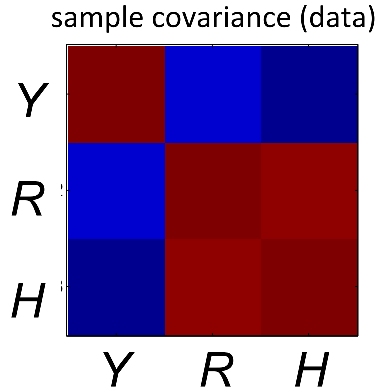
- Translation random vectors (S. Arwade. *Probabilistic Engineering Mechanics* 2005)
  - A probabilistic model with functional form based on physical arguments, calibrated to available data
  - Able to match second-moment properties (mean, covariance) and marginal distributions; these are quantities we can easily estimate from data
- Available information
  - 40 sets of model parameters calibrated to experimental measurements
    - Yield stress (Y), recovery constant (R), hardening constant (H)
  - Lower and upper bounds on each model parameter
    - Expert judgment, FE analysis to determine onset of unrealistic material behavior
  - Estimate covariance matrix based on 40 samples of model parameters
- Modeling assumptions
  - Yield stress and hardening constant follow a beta distribution
  - Recovery constant follows an exponential distribution
  - Consistent with bound information and some literature
  - Alternative distributions can be studied at a later date

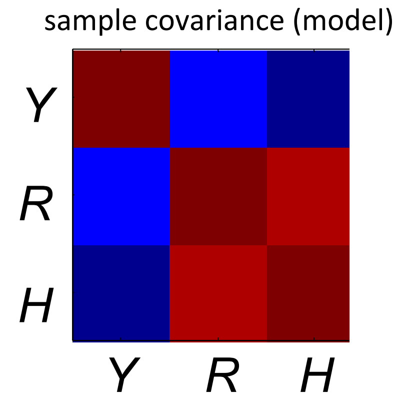
Enrichment results





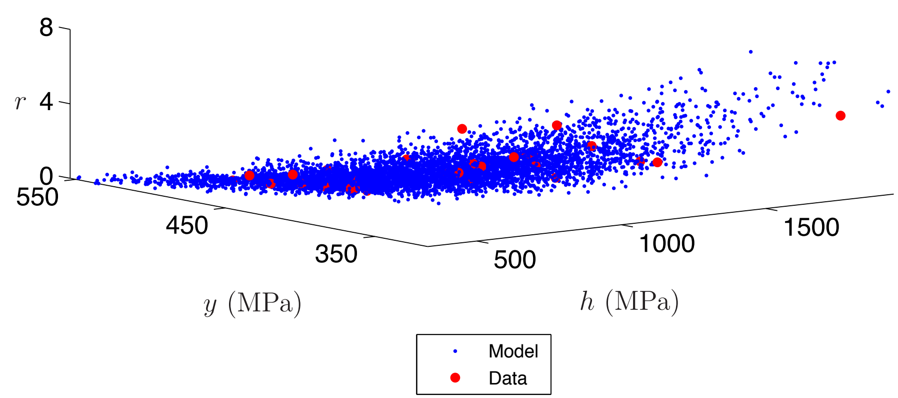
Experimental Data





Translation Model

3D scatter plot of the data (red dots) and 5,000 samples from the translation model (blue dots)



Stochastic reduced-order model (SROM)

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To develop a model that optimally represents the uncertainty in the input we choose a discrete random variable $\tilde{\boldsymbol{\Theta}}$. The SROM is then defined by the collection $\left(\tilde{\boldsymbol{\theta}}_k, \tilde{p}_k\right)$ k = 1, …, m that minimizes an objective function of the form:

$$\underbrace{\max_{1\le r\le \bar{r}} \max_{1\le s\le d} \alpha_{s,r} |\tilde{\mu}_s(r) - \hat{\mu}_s(r)|}_{\text{moments}} + \underbrace{\max_{1\le s\le d} \beta_s |\tilde{F}_s(x) - \hat{F}_s(x)|}_{\text{cumulative distribution}} + \underbrace{\zeta_{s,t} \max_{s,t} |\tilde{c}(s,t) - \hat{c}(s,t)|}_{\text{correlation}}$$

SROM (solve for $\tilde{p}_k$ given a set of m randomly chosen samples from V)

$$\tilde{\mu}_s(r) = \mathrm{E}[\tilde{\Theta}_s^r] = \sum_{k=1}^{m} p_k \, (\tilde{\theta}_{k,s})^r$$

$$\tilde{F}_s(x) = \Pr(\tilde{\Theta}_s \le x) = \sum_{k=1}^{m} p_k \, 1(\tilde{\theta}_{k,s} \le x)$$

$$\tilde{c}(s,t) = \mathrm{E}[\tilde{\Theta}_s \, \tilde{\Theta}_t] = \sum_{k=1}^{m} p_k \, \tilde{\theta}_{k,s} \, \tilde{\theta}_{k,t}$$

Estimates of uncertainty

$$\hat{\mu}_s(r) = \sum_{i=1}^{n} (1/n) \, (\theta_{i,s})^r,$$

$$\hat{F}_s(x) = \sum_{i=1}^{n} (1/n) \, 1(\theta_{i,s} \le x)$$

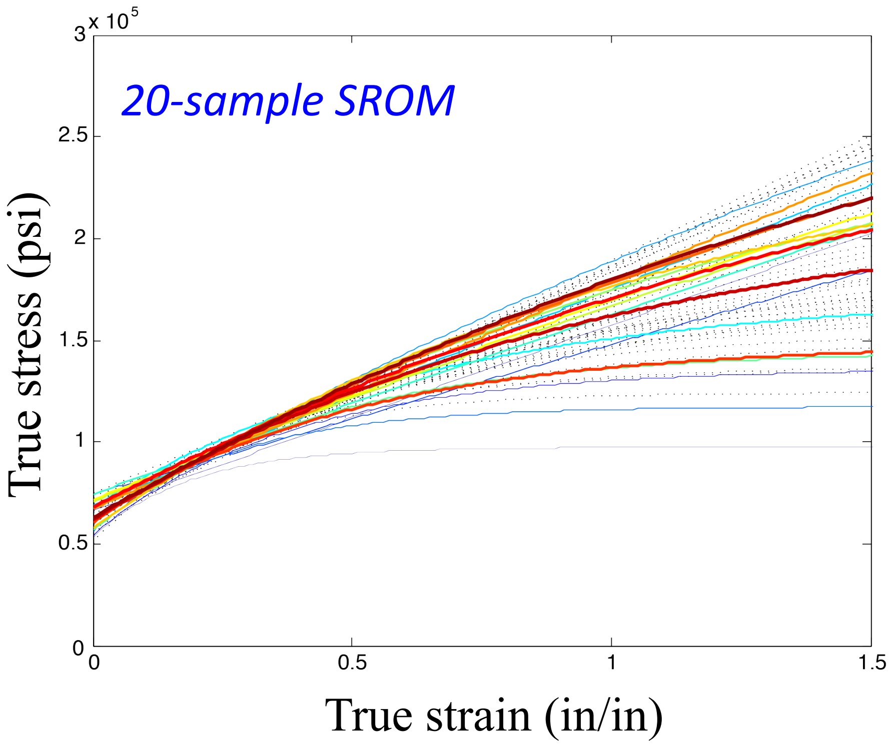
$$\hat{c}(s,t) = \sum_{i=1}^{n} (1/n) \, \theta_{i,s} \, \theta_{i,t}$$

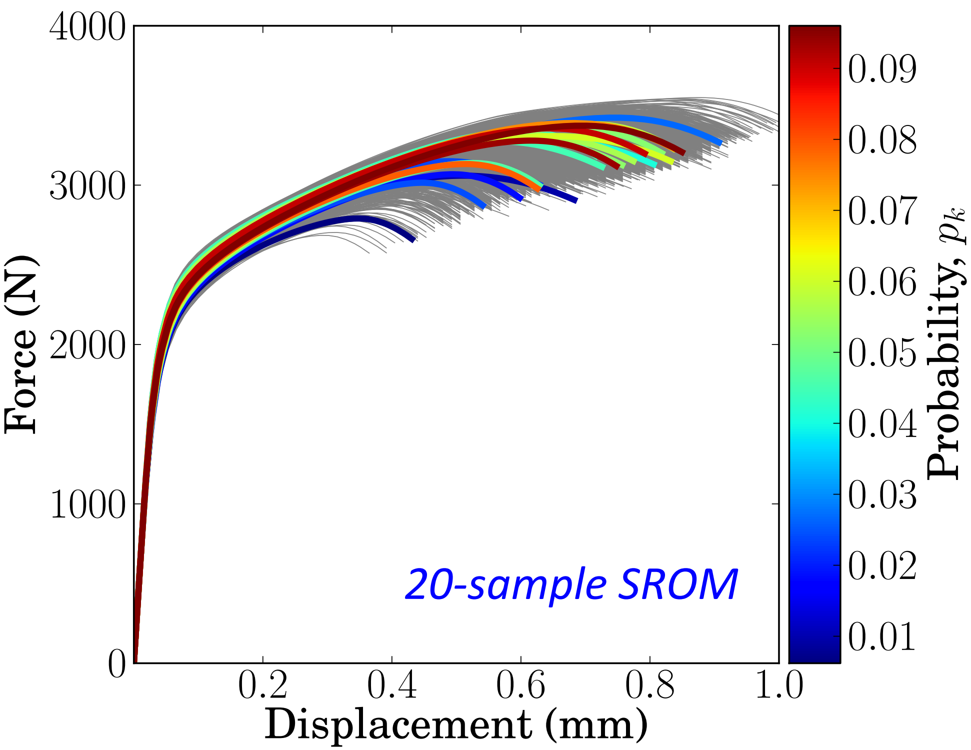
with m << n and $\alpha, \ \beta, \ \zeta > 0$ are weights and subject to probabilities $\tilde{p}_k \ge 0$ and $\sum_k \tilde{p}_k = 1$.

Graphical representation of SROMs

This figure compares the SROMs (solid lines) to the calibrated fits (fine dotted lines). The SROMS are derived from the enriched samples. The SROMs are painted from least probable (blue, thin) to most probable (red, thick).

This figure compares the load versus displacement curves. The colored lines are the FE-computed results using the 20-sample SROMs (color indicates probability). The fine gray lines are FE-computed results using 5,000 samples from the enrichment process.

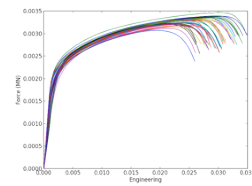




Outline for the rest of the talk

How to propagate the observed uncertainty through component analysis?

Input → **Calibration**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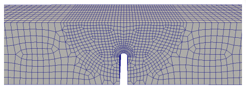
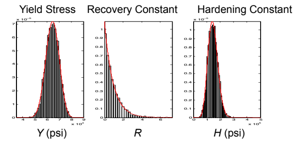
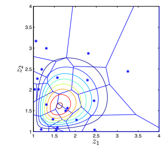


Up to now, we have only discussed models for uncertain inputs

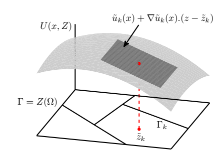
Enrichment



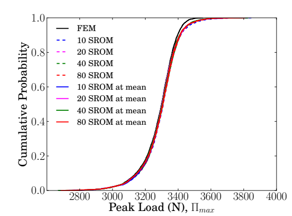
Stochastic Reduced-order Model



FE Analysis ↔ **SROM-based Surrogate Model**



Output



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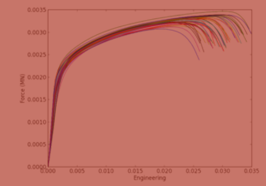

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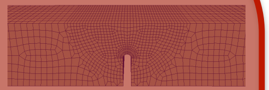

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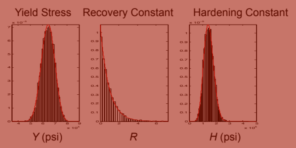
Enrichment



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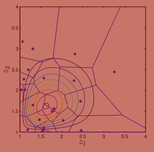
Stochastic Reduced-order Model



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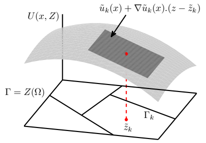
FE Analysis ↔ **SROM-based Surrogate Model**



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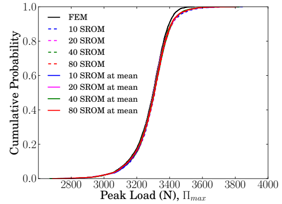
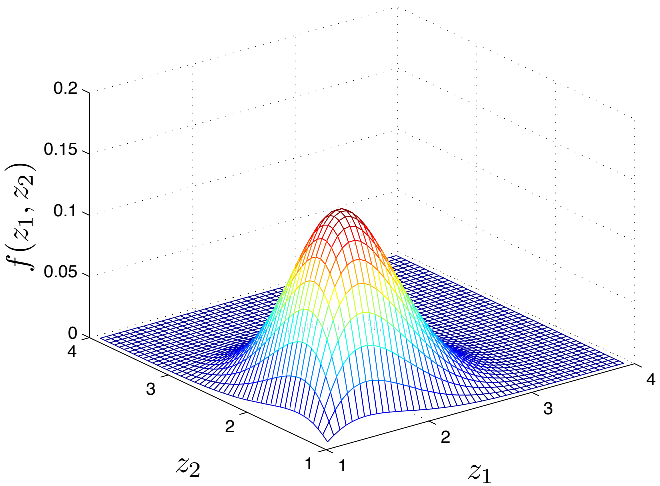
Output



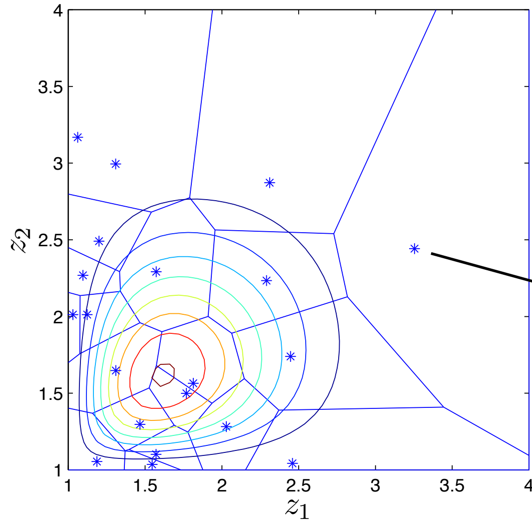
Now, propagation through "component" analysis

Construction of SROM-based surrogate

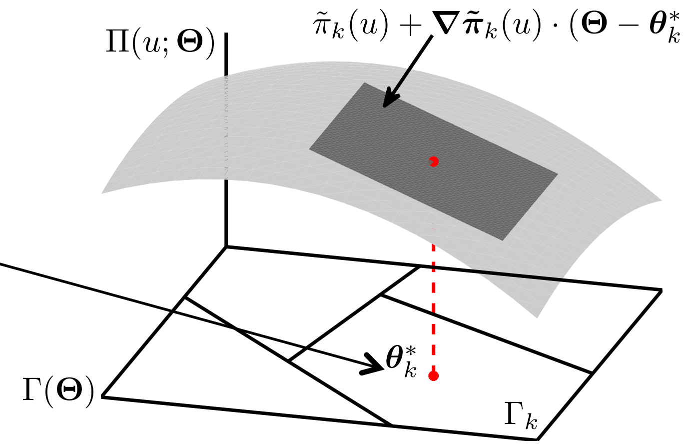
Example 2D probability density



* SROM points



Response surface



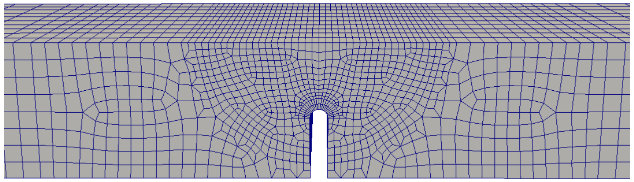
- A response surface is constructed for the structural response of the component, $\Pi(u;\Theta)$
- The surface is a series of hyper-planes described with a first-order Taylor approximate of the structural response

$$\tilde{\Pi}_L(u;\boldsymbol{\Theta}) = \sum_{k=1}^{m} 1(\boldsymbol{\Theta} \in \Gamma_k)\ [\tilde{\pi}_k(u) + \boldsymbol{\nabla}\tilde{\boldsymbol{\pi}}_k(u) \cdot (\boldsymbol{\Theta} - \boldsymbol{\theta}_k^*)]$$

- The SROM samples are used as the expansion points $\theta_k{}^*$ and the domain Γ_k are determined by the Voronoi tessellation of the uncertain parameters

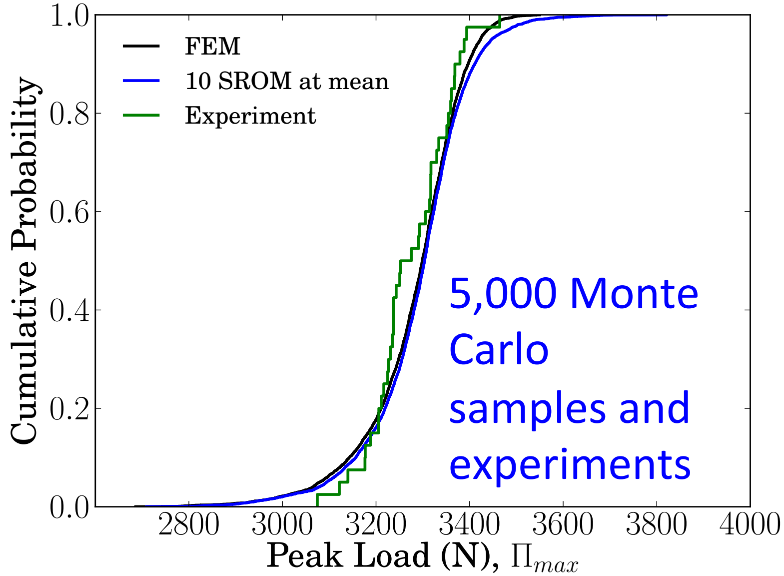
Assumption: The structural response is differentiable

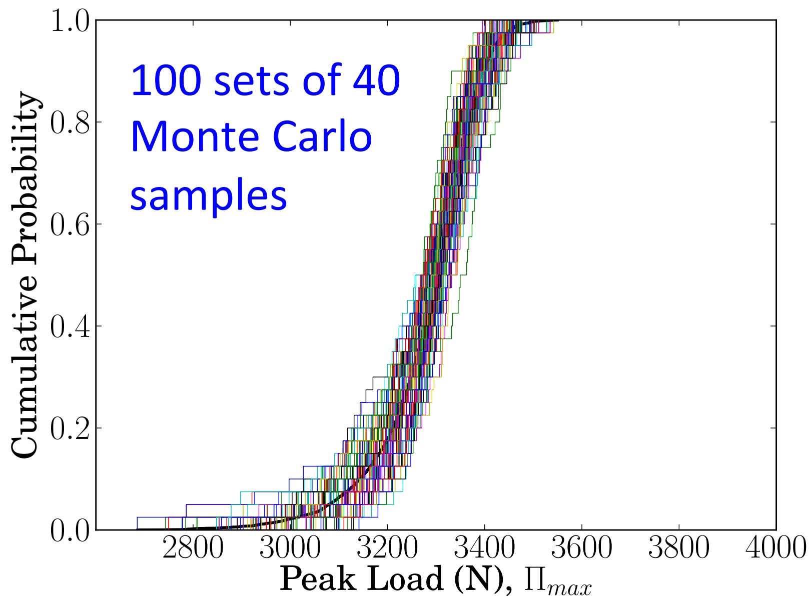
Monte Carlo simulation for the weld coupon



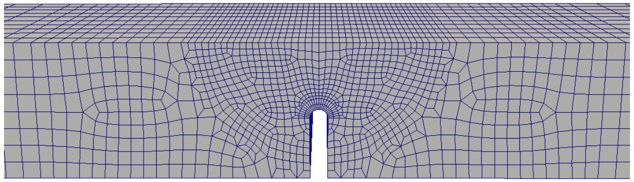
Our "component"

- For verification, we perform Monte Carlo simulation for the response of the laser welded tensile coupon.
- We generate 5,000 samples of **Θ**. We do 5,000 FE calculations for the response of the coupon (solid black line in plots).
- We compare to the SROM-based surrogate (top).
- The 10 sample SROM-based surrogate model requires 40 FE calculations to construct (10 for each sample, 30 for the gradients).
- The CDF on the bottom was constructed with 100 sets of 40 FE calculations, no surrogate. It shows the wide confidence and large error…





Monte Carlo simulation for the weld coupon



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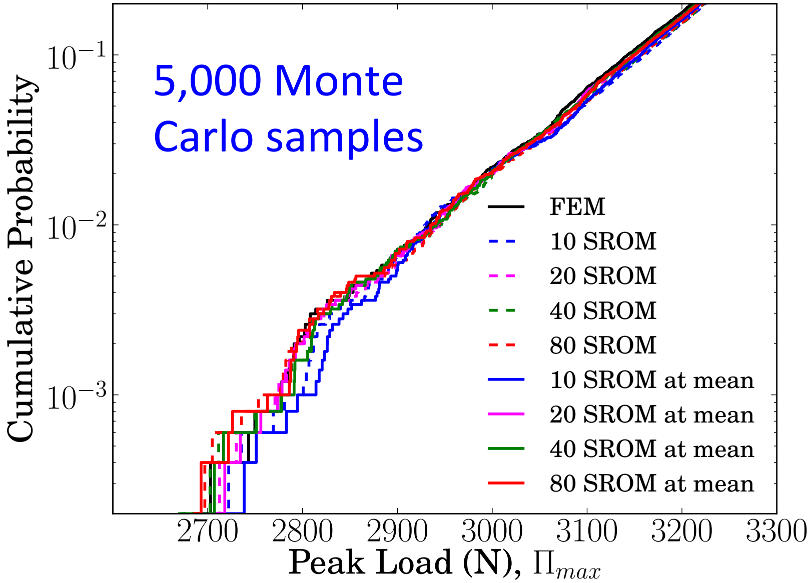
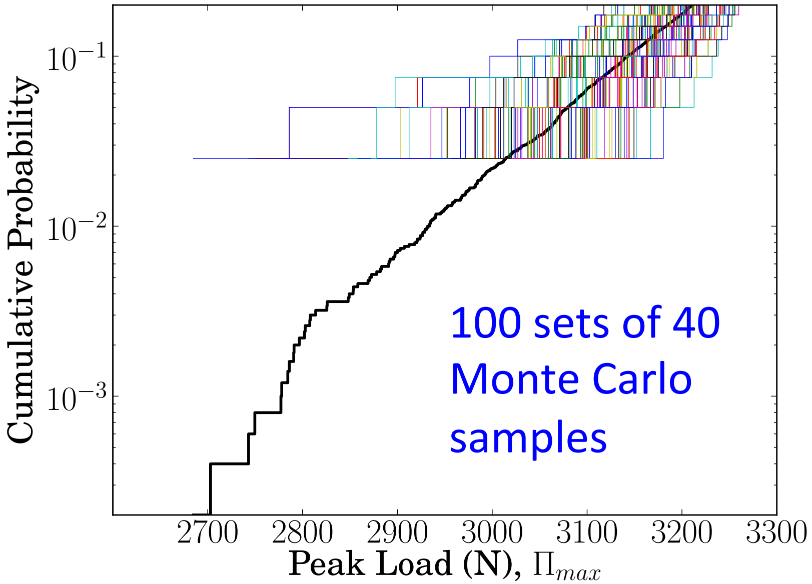
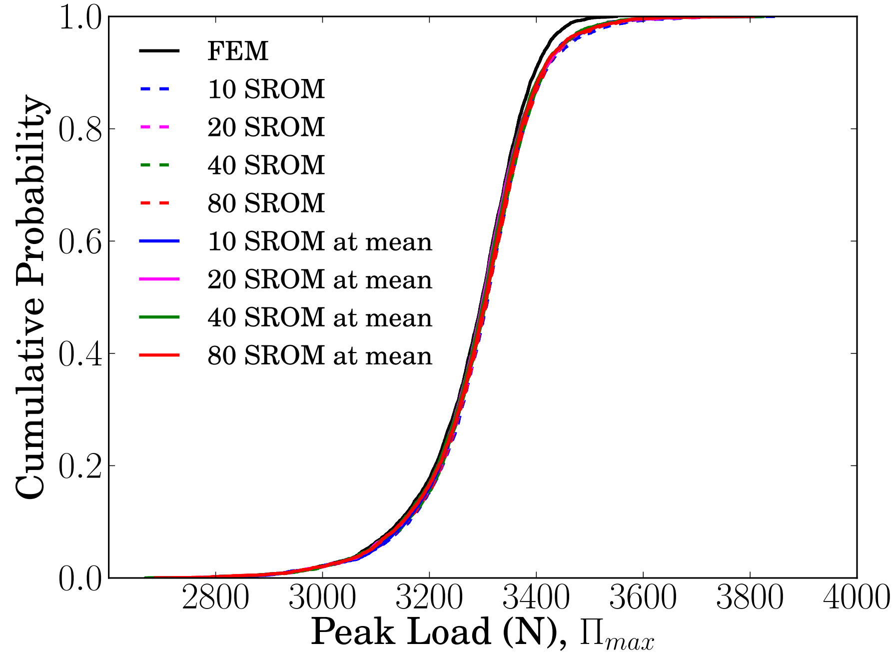
- … and when we zoom in, it can't capture probability below 1/40.





Cost savings with SROM-based surrogates

- SROM-based surrogate models replace component level FE models expediting Monte Carlo simulation while providing comparable accuracy.
- In practice, component level FE models cannot be run thousands of times. The SROM-based surrogate can.
- CPU time results are for the example shown here and compared with 5,000 FE calculations.



Computational expense in CPU seconds.

	Construct SROM*	FE calculations **	Evaluate surrogate*	Total
Brute force MCS	n.a.	33,400,000 (5,000 FE calculations)	n.a.	33,400,000
10 SROM at mean	948	511,000 (40 FE calculations)	6.69	512,000

* Intel ® Xeon ® x5675 CPU @ 3.07 GHz w/ 48GiB RAM

** Intel ® Nehalem ® x5570 CPU @ 2.93 GHz w/ 1.5GiB RAM

Conclusions

- Issues involving nuclear safety require high confidence
- We cannot afford "brute force" MCS. SROMs provide a path forward.
- Developed tools for calibration, enrichment, and the construction of SROMs
- Applied "brute force" Monte Carlo with 5000 finite element (FE) calculations to obtain the "truth"
- SROM-based surrogate accurately computed the cumulative distribution function, capturing the lower tail, at 0.8% of the computational cost (40 vs 5000 FEA)
- On an equal computational footing, the SROM-based surrogate is far more accurate.