

# Using Ring Crack Initiation to Measure Surface Flaw Densities in Glasses

## Influence of Humidity, Friction, and Searched Areas

**Rajan Tandon and Cory Gibson**

Sandia National Laboratories, Albuquerque, NM

**Bhasker Paliwal**

Georgia Institute of Technology, Atlanta, GA 30332, and Unite Mixte Internationale Georgia Tech  
Lorraine/CNRS, UMI-2958, 57070 Metz, France

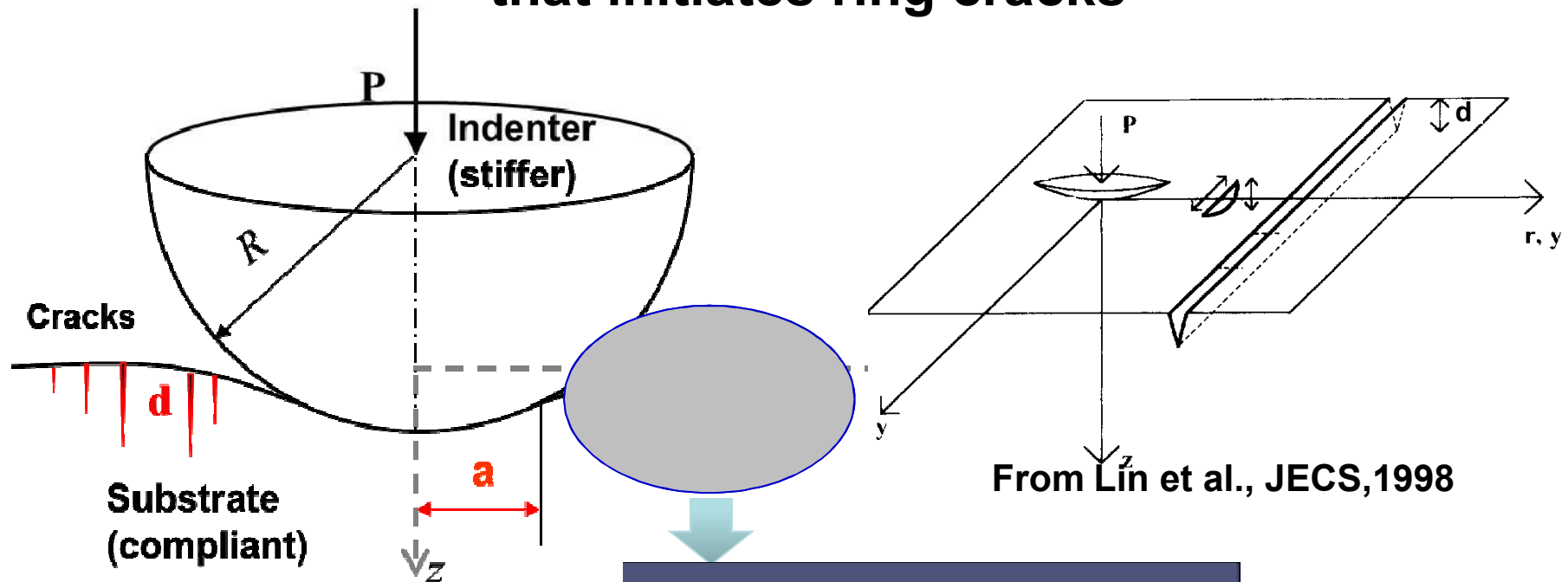
Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.



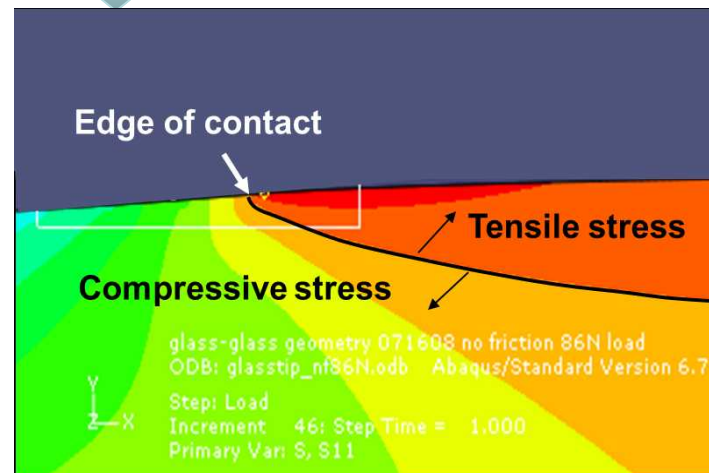
Sandia  
National  
Laboratories

# Ring Crack Initiation

Sphere loaded onto a flat surface generates radial tension that initiates ring cracks



From Lin et al., JECS, 1998





# Motivation

---

- Conventional strength tests usually measure the largest flaw in a sample.
- Here it is possible to sample the smaller flaws, and maybe even the entire distribution.

**Smaller flaws: Crack initiation under intense local loads, assessing surface quality, laser-damage thresholds, models for damage initiation under impact**

- |                         |  |
|-------------------------|--|
| - Role of friction:     | ? on how significant-Wang et. al- JMS-2003<br>-how to use- Jadaan et. al- IJACT-2011 |
| - Role of Humidity:     | Largely ignored (Argon, JACS, 1960,<br>Langitan, Lawn, JAP 1971)                     |
| - Surface flaw density- | # of flaws in a size range/area-<br>(Wilshaw-JAP-1971)                               |



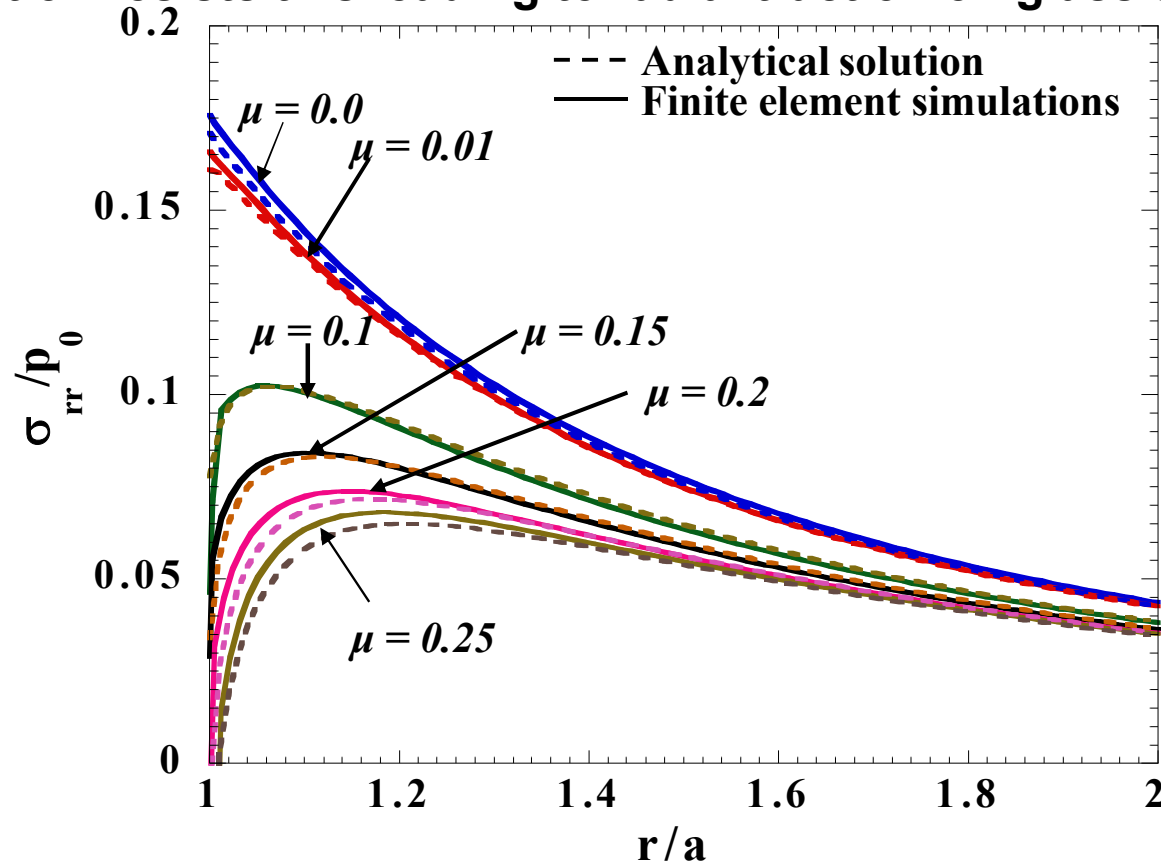
# Outline

---

- **Describe stress, stress intensity factors for dissimilar contact (WC-Glass)**
- **Using experimental and analytical results:**
  1. **Demonstrate the importance of controlling humidity**
  2. **Demonstrate the effect of friction on stress distributions under dissimilar elastic contact**
  3. **Flaw size densities on brittle material surface under similar/dissimilar contacts**

# Dissimilar Contact with Stiffer Indenter: Surface Stresses

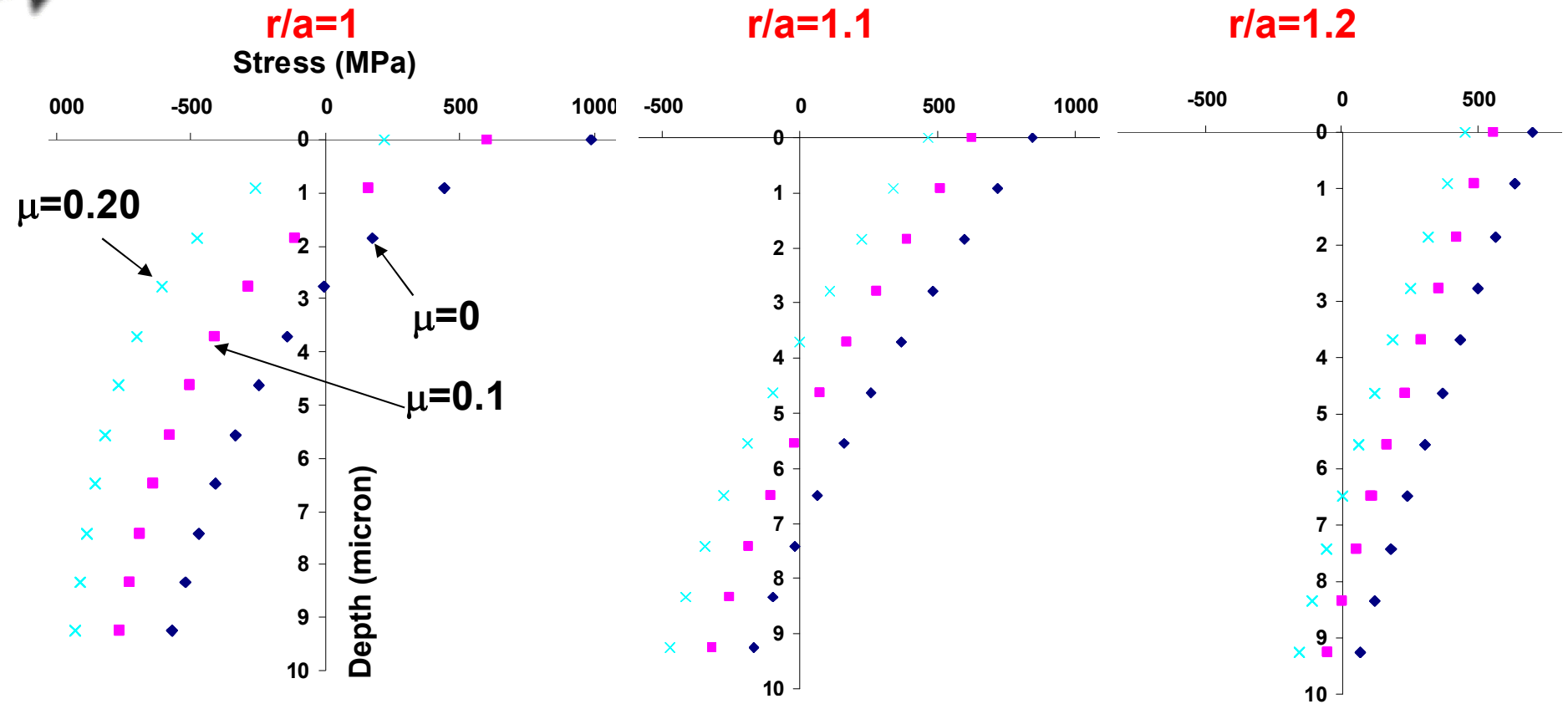
Elastic mismatch creates differential slip  
Friction resists this leading to radial traction on glass surface



Friction shifts the maximum in the stress farther from contact edge  
Friction also leads to reduction in normalized stress

Paliwal B , R Tandon et al., Assessing the Hertzian Indentation Approach for Measuring Fracture Toughness, J. Amer. Ceram. Soc., 2011

# Calculated sub-surface stress distributions



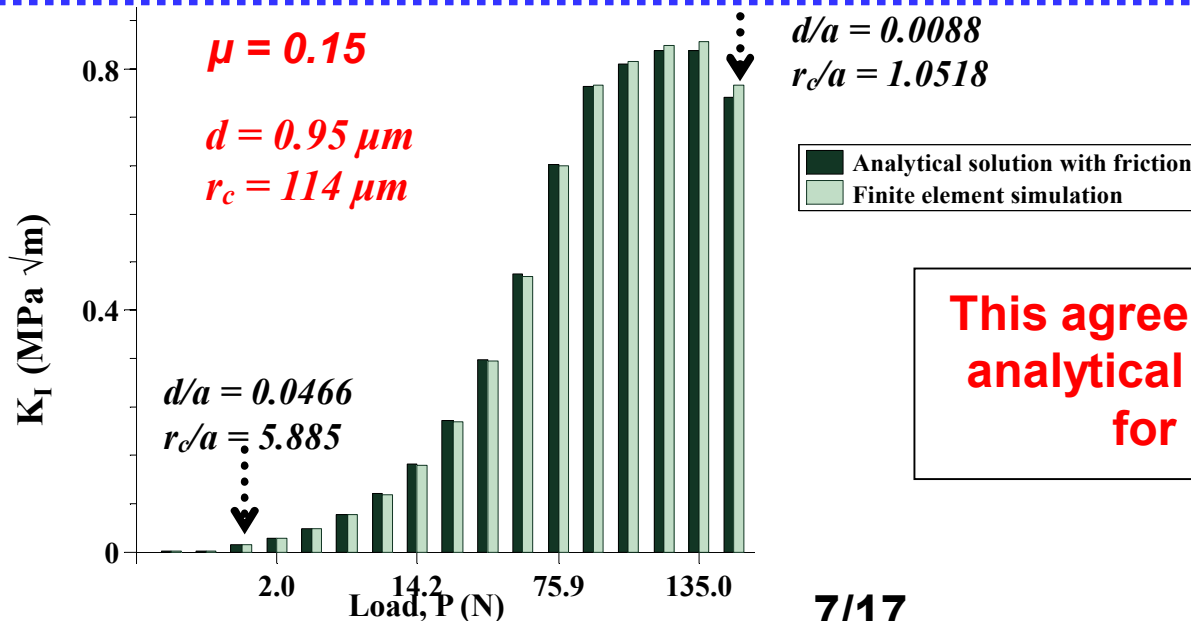
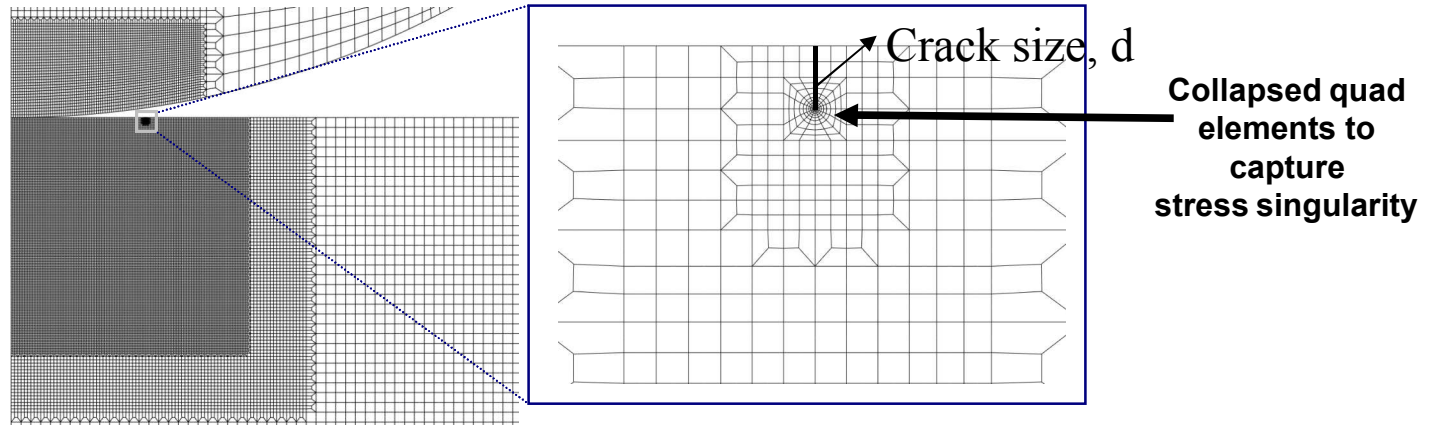
Subsurface distribution are parallel to each other for various  $r/a$  values.  
Therefore, the effect of friction can be captured by the lowered surface stress and the Hertzian sub-surface drop-off in stress (no slope change)  
The drop-offs are  $\sim$ linear for  $r/a \geq 1.1$



# Comparison of Analytical and FE Approach to K

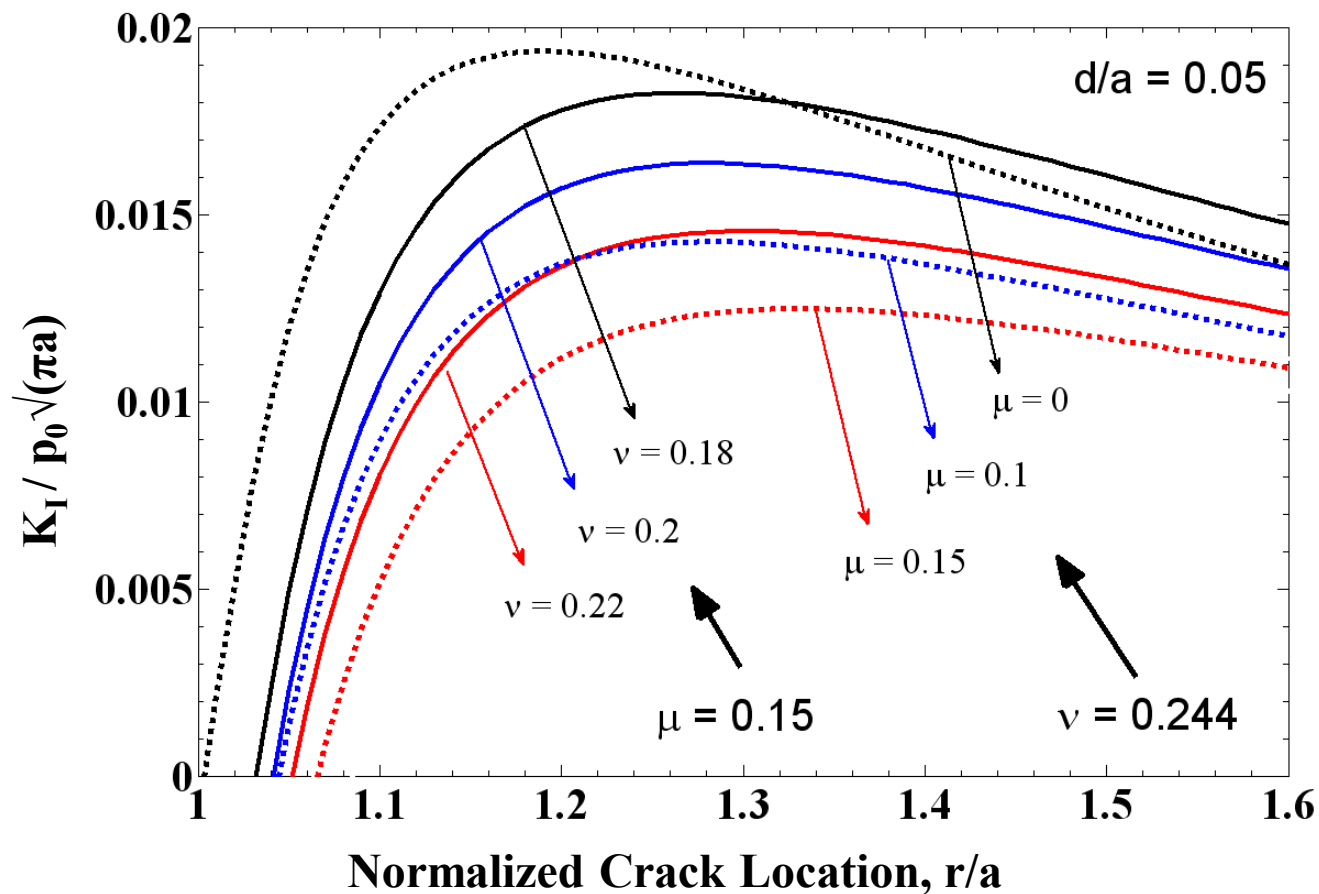
Knowing the radial stress distribution in the presence of friction, mode I stress intensity factor can be written as

$$\frac{K_I}{p_0 \sqrt{\pi a}} = \frac{2}{\pi} \sqrt{\frac{d}{a}} \int_0^{d/a} \frac{\sigma_{rr}(r_c, z)/p_0}{\sqrt{(d/a)^2 - (z/a)^2}} d(z/a)$$



**This agreement allows us to use our analytical formulation, with friction, for parametric studies**

# $K_I$ for a particular crack size, $d=0.05a$



Maximum in  $K$  is shifted well away from edge of contact  
Values of  $K$  are very sensitive to  $\nu$  and  $\mu$





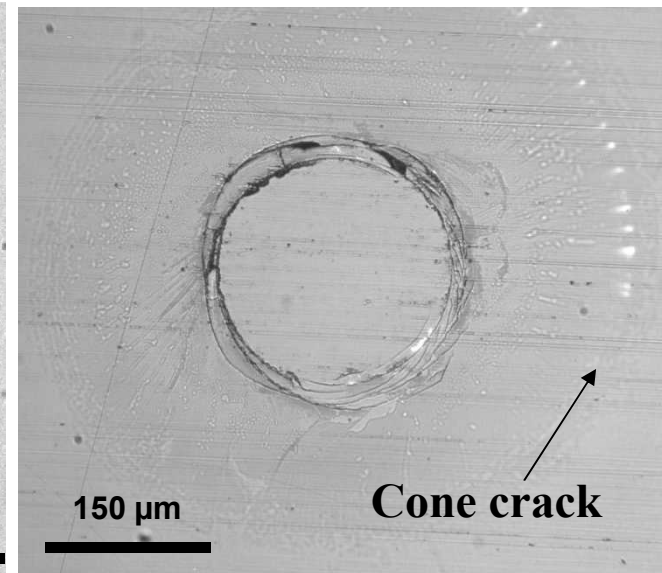
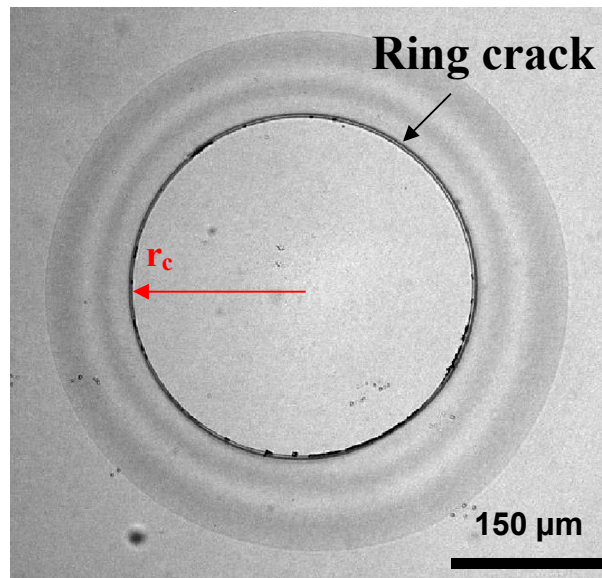
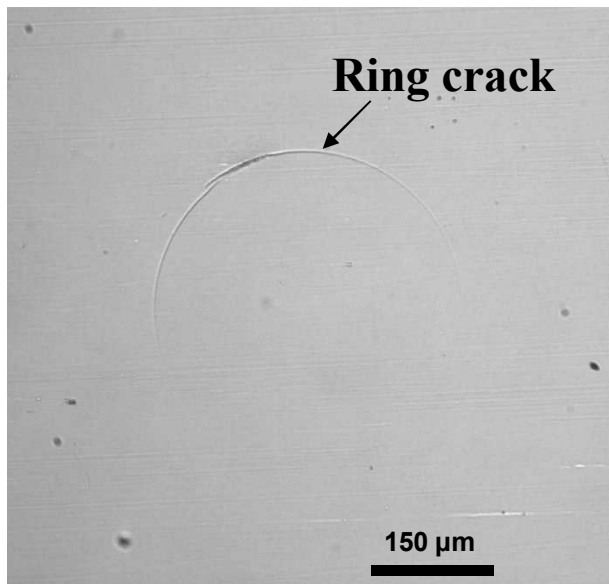
# Outline

---

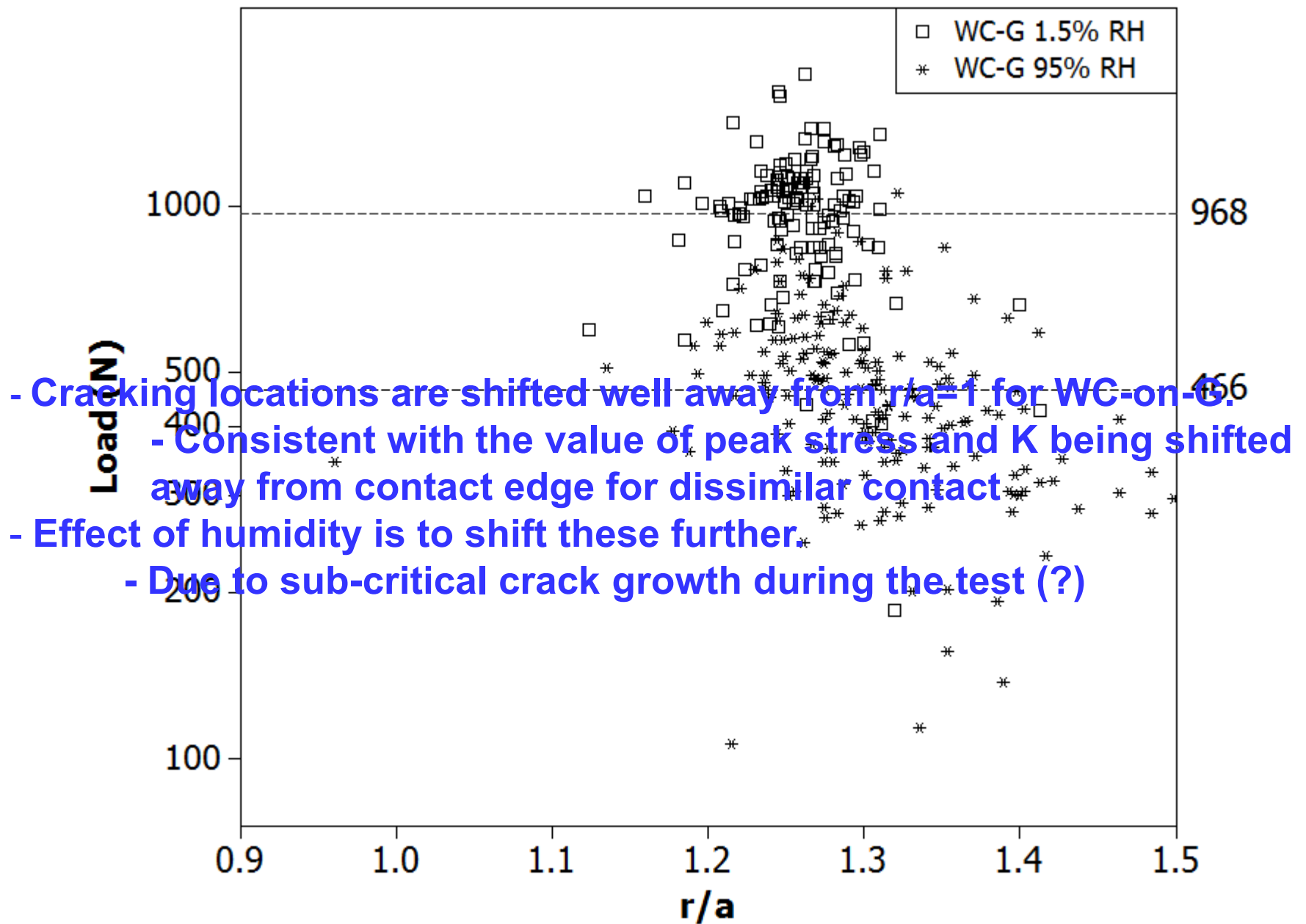
- Describe stress, stress intensity factors for dissimilar contact
- Using experimental and analytical results:
  1. Demonstrate the importance of controlling humidity
  2. Demonstrate the effect of friction on stress distributions under dissimilar elastic contact
  3. Outline procedure to calculate flaw size distribution on brittle material surface

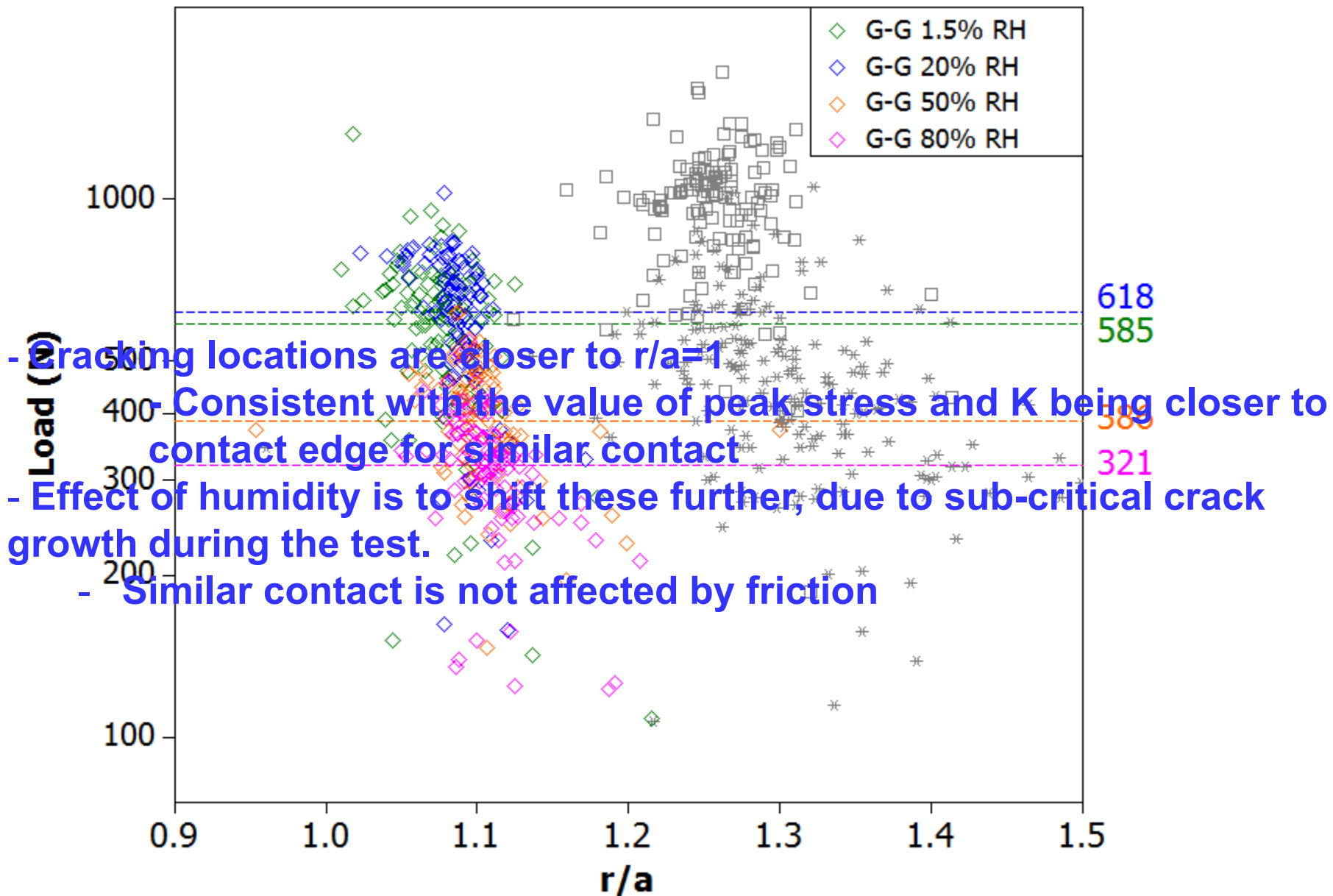
# Experimental Approach

- >500 tests with WC sphere on SLS glass (dissimilar contact),  $R=0.8$  mm
- >500 tests with glass sphere on SLS glass (similar contact),  $R=0.8$  mm
- Various controlled RH's, loading rate=0.2 mm/min, 10-15 sec per test
- Acoustic emission was used to measure the load at cracking
- Friction coefficient of glass/ WC was measured ( $\mu=0.15$ ) [scratch test]
- **Cracking locations were measured**

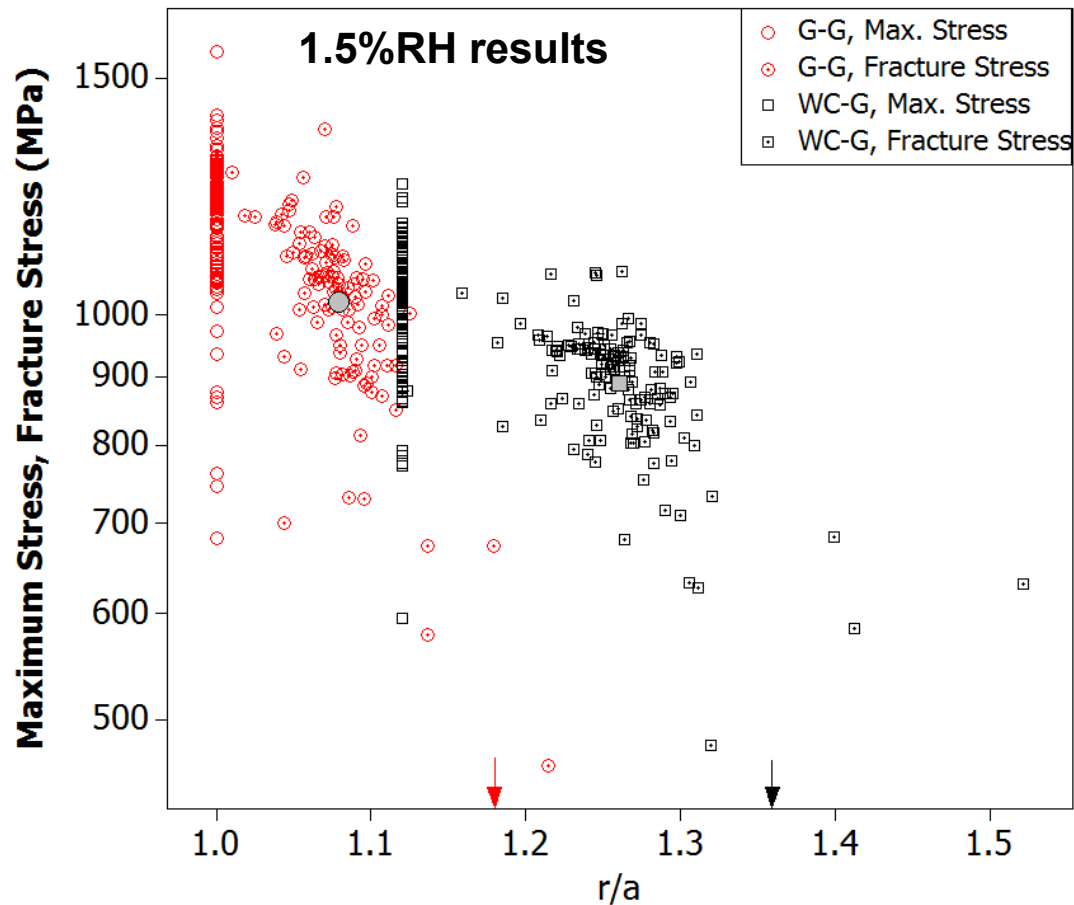


As-received Surface





# Fracture Stresses



Boundary lubrication friction value ( $\mu=0.15$ ) obtained from simple friction tests can be used to obtain reasonable estimates of stress.

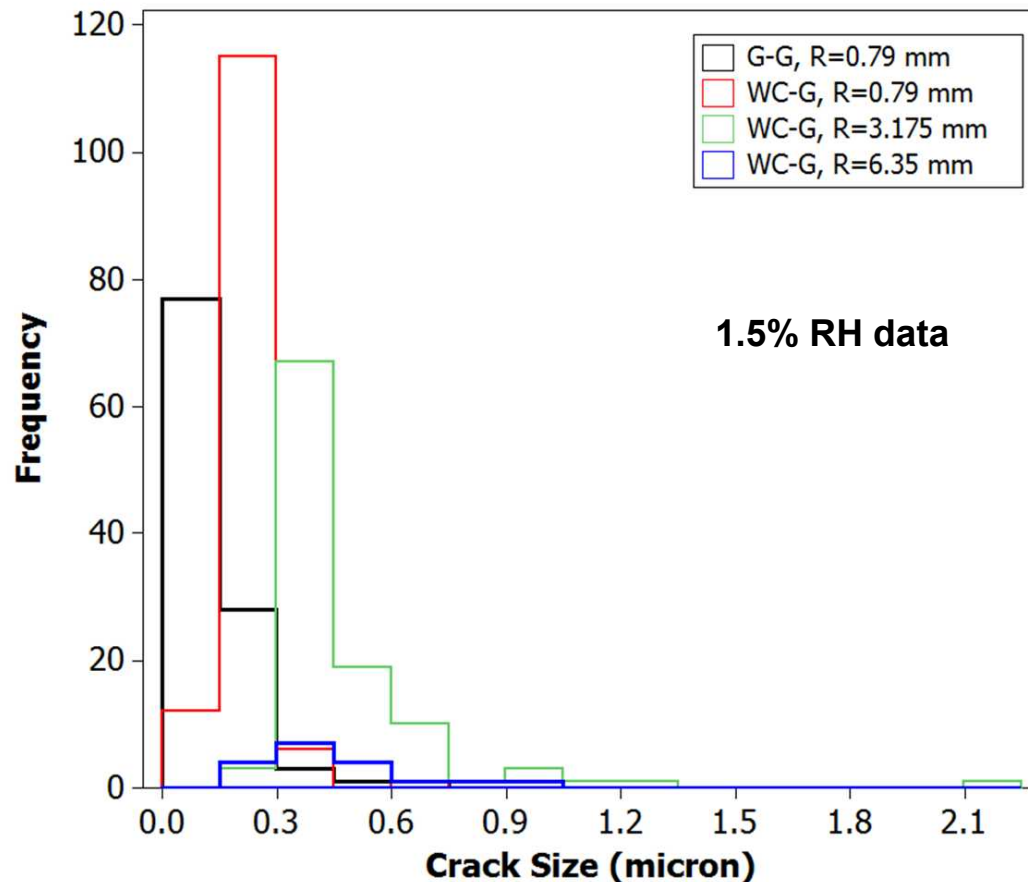
Fair agreement when the stresses at fracture for WC-G, G-G compared.

# Calculation of Crack Size Distribution

$$\sigma_{rr} = \sigma_{surf} \left( 1 - \alpha \left( \frac{z}{a} \right) \frac{3 \left( \frac{r}{a} \right)^2}{(1-2\nu)} \right)$$

$$\frac{K_I}{\sqrt{\pi a}} = \sigma_{surf} \left[ 1.12 - \frac{2\alpha}{\pi} \left( \frac{d}{a} \right) \frac{3 \left( \frac{r}{a} \right)^2}{(1-2\nu)} \right] \left( \frac{d}{a} \right)^{\frac{1}{2}}$$

(Warren, JECS, 1995)



**r** = radial coord.  
**a**=contact rad.  
**d**=crack size  
**z**=depth coord.

**Cracks are  
Small**

**(d/a@0.03)**

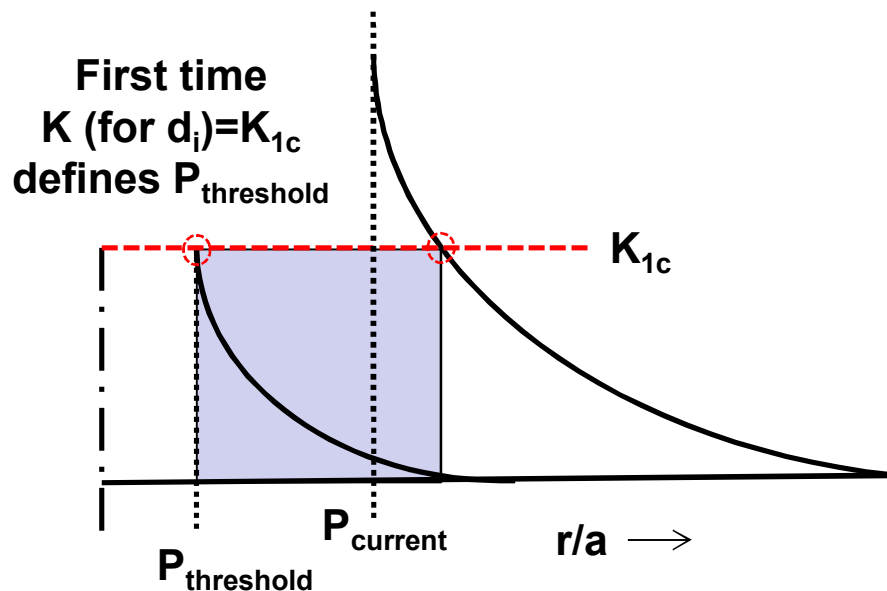
**Larger indenters find larger flaws**  
**These crack sizes are << than those in routine strength tests**

# “Searched Area” for small cracks ( $d/a < 0.03$ )

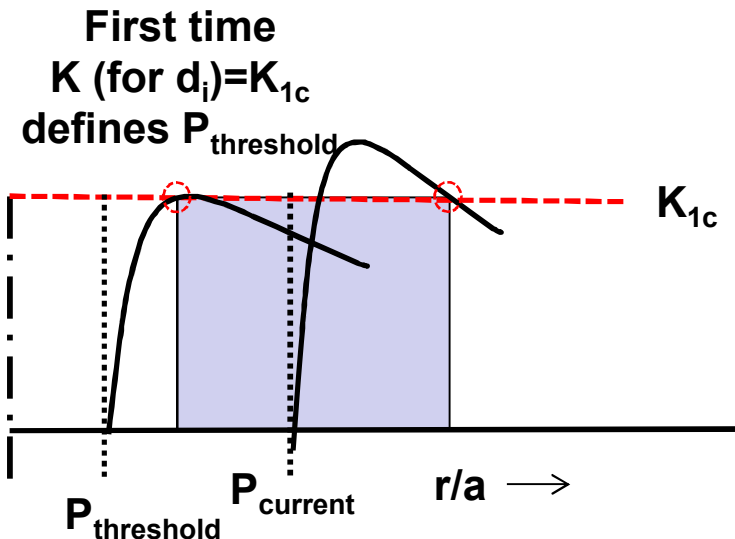
For any particular test, the “area searched” for a crack of depth  $d_i$  by loading up to a force  $P$  is that area of the surface within which, had there existed a crack of depth  $d_i$ , it would have caused fracture under some force  $\leq P$ .

$$\text{For small cracks, } K_1 = 1.12\sigma_{\text{surf}} (\pi d_i)^{1/2}$$

## Frictionless Case



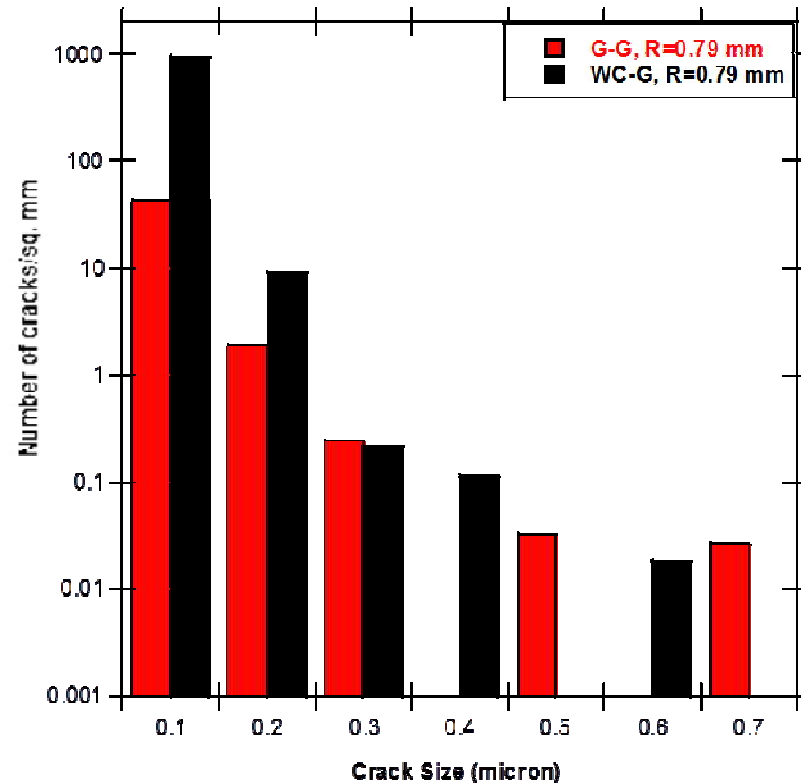
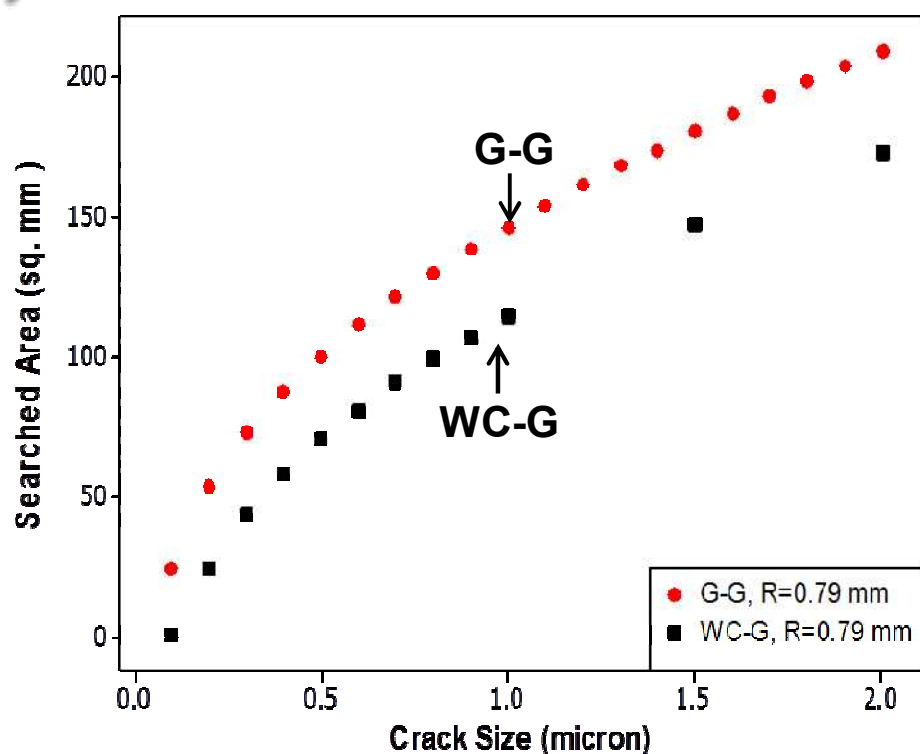
## Case with Friction



Sum up the area searched from each test for different flaw sizes



# “Searched Area” and Crack Size Distributions



CSD on this glass surface:  $n(c)=\eta c^{-\rho}$ , with  $\rho\sim 3.8$  for G-G contact

If exponential CSD is followed,  $m=2\rho-2$

$m\sim 5.6$  which is a reasonable value



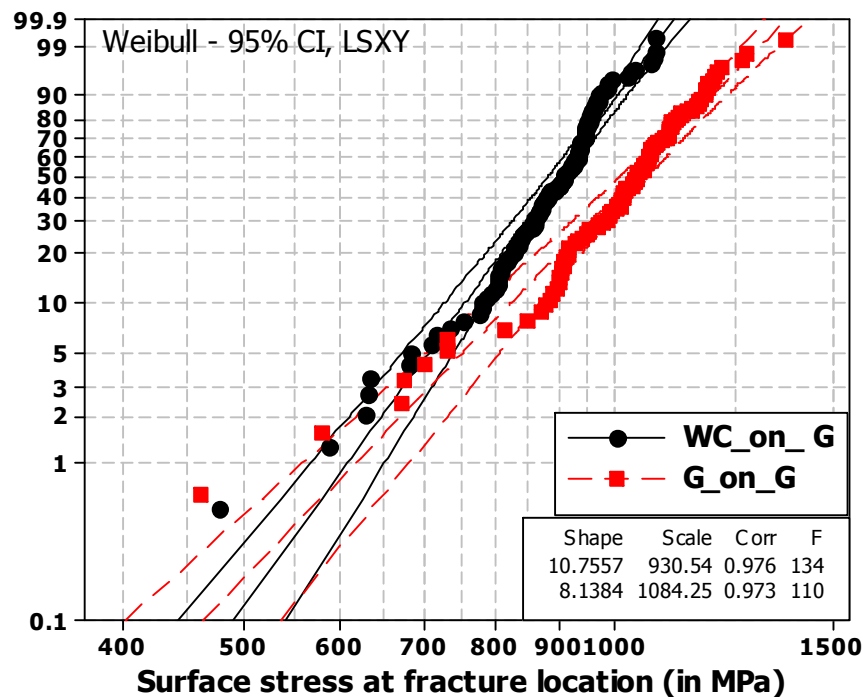
# Conclusions

---

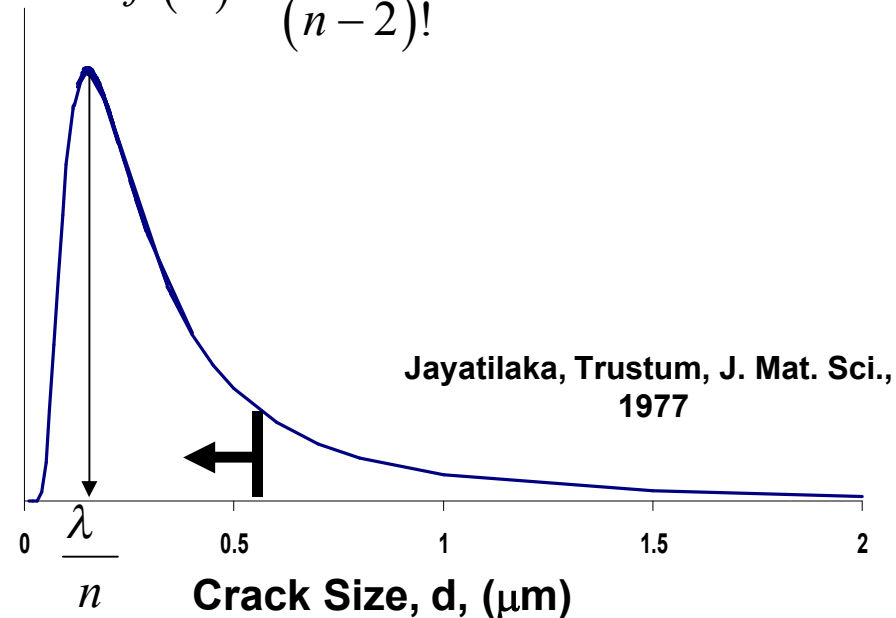
- Humidity control in Hertzian tests on brittle materials is critical
- r/a locations, and higher loads for WC-on-G support the calculated shape, and magnitude of the stress reduction due to friction
  - This effect must be accounted for in meaningful data analyses
- Friction shifts the stress distribution lower w/o slope change
  - Sub-surface stress can be obtained using the Hertzian slope
- Results from two contacts for crack size distribution are close
  - The approach detects very small flaws/high strengths
- The searched area concept provides reasonable fits to exponential crack size distribution (CSD) on glass surfaces
- CSD can be related to Weibull moduli measured in bulk strength tests

**Backup Slides**

# Surface Stress Distributions at Crack Initiations



$$PDF = f(d) = \frac{\lambda^{n-1}}{(n-2)!} d^{-n} e^{-d/\lambda}$$



Shape factors (m) are higher than obtained in routine strength tests (4-6)  
 Stresses needed for crack initiation range from ~450 MPa to ~1500 MPa

Higher strengths and higher m values may be more relevant for failures in locally stressed regions

# Analytical Approach for Radial Stress in Dissimilar Elastic Contact

Radial stress in non-friction Hertzian solution is:

$$\sigma_{rr}^p(r_a, z_a) = \left[ \frac{(1-2\nu)}{3r_a^2} \left\{ 1 - \left( \frac{z_a}{\sqrt{u}} \right)^3 \right\} + \frac{z_a}{\sqrt{u}} \left\{ (1+\nu)\sqrt{u} \tan^{-1} \frac{1}{\sqrt{u}} + (1-\nu) \frac{u}{1+u} + \frac{z_a^2}{u^2 + z_a^2} - 2 \right\} \right] p_0$$

It is modified due to friction <sup>7, 8</sup>

- (a) Calculating stick-slip boundary in terms of elastic parameters & friction
- (b) Calculating the surface shear tractions via the function

$$q(r_a) = \mu p_0 \operatorname{sgn}(\beta) \left[ \sqrt{1-r_a^2} - r_a \int_{r_a}^{c_a} \frac{\chi(x, c_a)}{x^2 \sqrt{1-x^2}} dx \right]$$

- (c) The modifying stress in terms of the potential function,  $\psi$ , is

$$\sigma_{rr}^q = 2\psi_{rr} + 2\nu \frac{\psi_r}{r} + z_a \psi_{rrz}$$

and can be calculated using the boundary values for the problem.

7. Spence DA. The Hertz contact problem with finite friction. **J. Elasticity** 1975; 5:297.

8. Hills DA, Sackfield A. The stress field induced by normal contact between dissimilar spheres. **J. Appl. Mech.** 1987;54:8.

**Practical Aspects of Using Hertzian Ring Crack Initiation to Measure Surface Flaw Densities in Glasses: Influence of Humidity, Friction, and Searched Areas**

**Rajan Tandon<sup>1</sup>, Bhasker Paliwal<sup>2</sup> and Cory Gibson<sup>1</sup>**

**<sup>1</sup>Materials Science and Engineering Center, Sandia National Laboratories, Albuquerque, NM 87185  
USA**

**<sup>2</sup>George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0405, USA and Unite Mixte Internationale Georgia Tech Lorraine/CNRS, UMI-2958, 57070 Metz, France**

**Hertzian ring crack initiation loads on glass, using spherical WC and glass (G) indenters, are measured and analyzed. Our measurements demonstrate that humidity plays a key role in determining load to fracture; experiments conducted without controlling this variable cannot be used to obtain material properties. The role of friction is explicitly considered for dissimilar (WC-G) elastic contacts. For this material pair, the stresses at fracture are well described by a boundary lubrication value of friction coefficient. The fracture loads are used in a fracture-mechanics formulation to calculate crack sizes on glass surfaces. The “searched-area” concept for dissimilar contacts is described, and used to estimate crack density values for these surfaces.**

**Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.**

# Damage Propagation Models need Crack Density

