SAND2011-5881C

Metamaterials Science & Technology

Subcell Models with Application to Split-Ring Resonators in the Infrared

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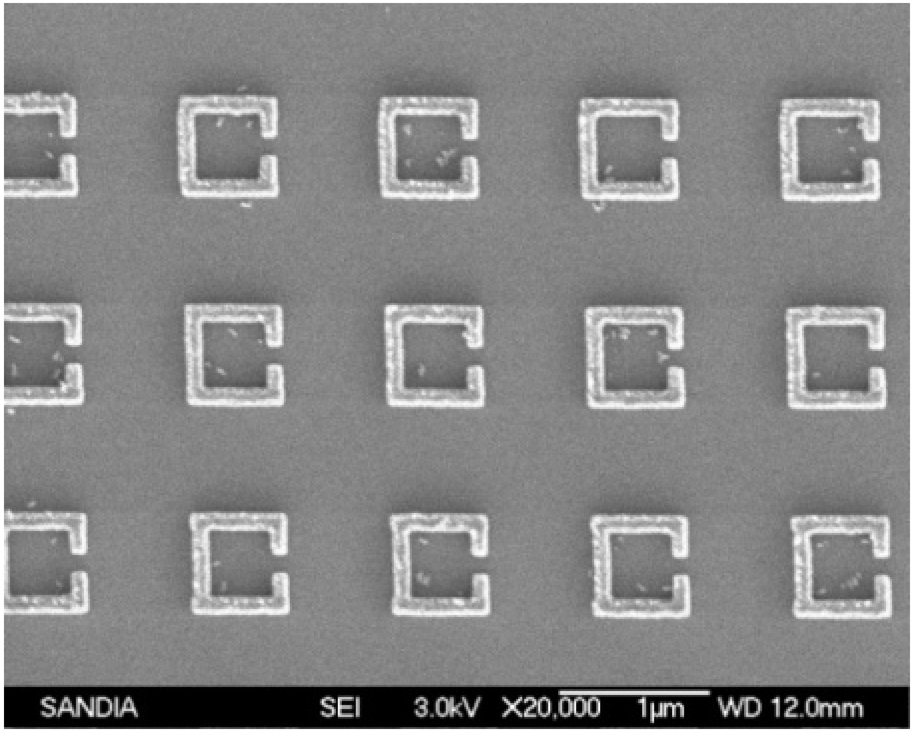
A rectangular lattice of gold split-ring resonators on top of a GaAs half-space



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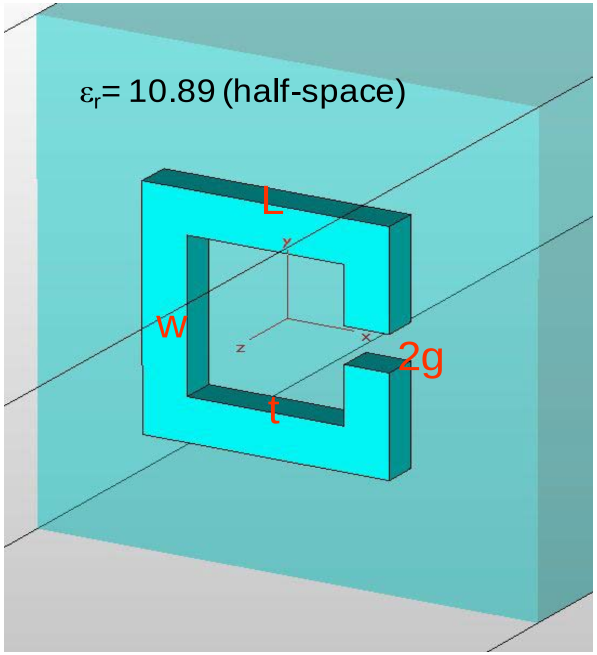
A single gold split-ring resonator on top of a thick GaAs half-space



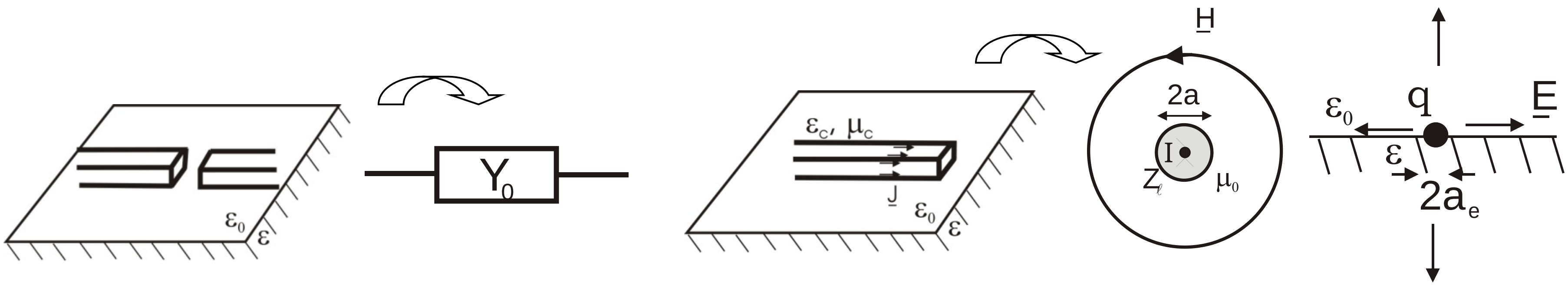
The dimensions of the SRR are 2g=0.10 μm, L=0.66 μm, t=0.1 μm, w=0.12 μm.
The relative permittivity of the substrate is ε_r= 10.89

Outline

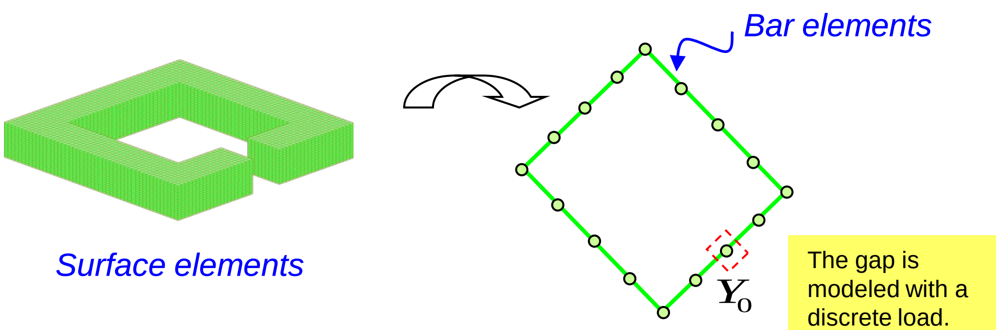
- Subcell Models
  - capacitive corrections
  - internal impedance correction
- Comparison of Subcell Models and Explicit-Mesh Simulations
- Applications and HPC Subcell Calculations

Subcell Models

- **Local descriptions**
  - Equivalent radius (Hallen)
  - Equivalent radius on dielectric half space
  - Impedance per unit length expansion (Cockcroft, Vainshstein, Zhurav, Warne, Holloway)
  - Internal admittance per unit length (negligible effect for metals)
  - Gap, fringe, and transition capacitance



Rectangular SRR Example



Subcell: Gap and Fringe Capacitance

Capacitance Correction of thin wire:

$$\Delta C \approx \frac{1}{\pi}(\varepsilon + \varepsilon_0)(w+t)\left[\ln(2a_e / g) - .5772 - \frac{2}{15})\right]$$

$$\mathrm{C}_{\mathrm{pp}} = \varepsilon_0 \frac{wt}{2g}$$

$a_e \approx$ 0.036μm (GaAs half-space)

$a_e = a \approx$ 0.065 μm (air)

$\Delta C + C_{pp} = \left(-2.5206+1.0625\right)\ \mathrm{aF} =$ -1.4581 aF (GaAs half-space)

$\Delta C + C_{pp} = \left(0.3011+1.0625\right)\ \mathrm{aF} =$ 1.3636 aF (air)

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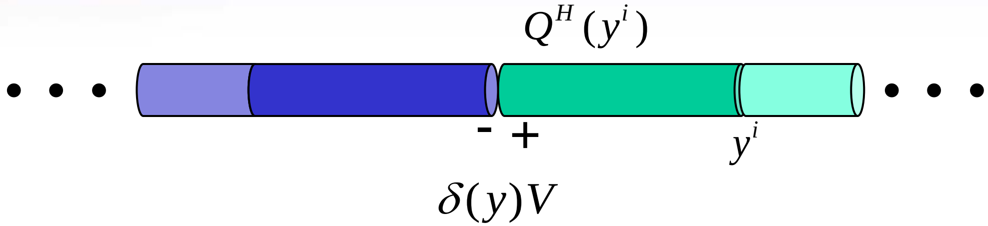
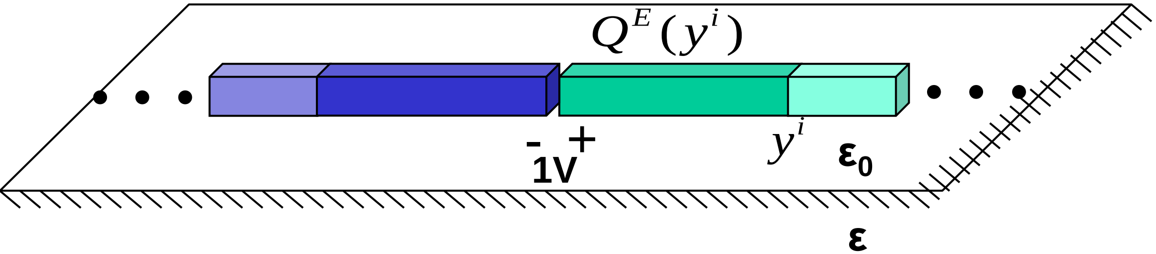
Subcell Capacitance Corrections: Hallen and EIGER facet 4





$$C_t = C_E^i - C^H - \left(\Delta C + C_{pp}\right)$$

$$\Delta C_f = 2a_e\left(\varepsilon + \varepsilon_0\right)\left[\ln(4(3.4)a_e / s_0) - \gamma - \frac{1}{9}\right]$$

$$C_{Load} = \Delta C + C_{pp} + C_t - \Delta C_f$$

$$C^H(y^i) = \frac{\varepsilon + \varepsilon_0}{2\varepsilon_0} Q^H(y^i)$$

$C^H(y^i) =$ 63.727 aF, 12.960 aF

$$C_E^i = Q^E(y^i) = \sum_{l=1}^{i} q_E^l$$

$Q^E(y^i) =$ 61.321 a F, 13.960 aF

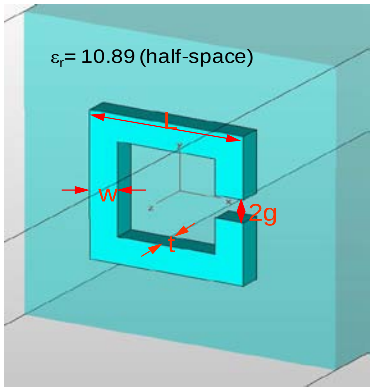
$C_t =$ -0.947214 aF , -0.36345 aF

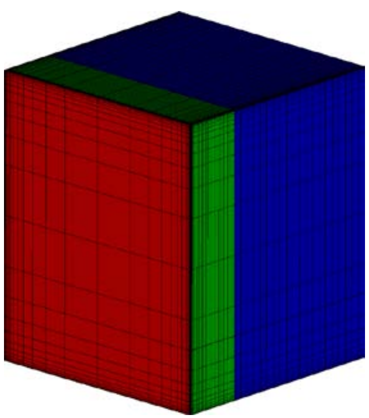
$\Delta C_f =$ 9.875 aF, 2.7308 aF

$C_{Load}^{magneta} =$ -12.3 aF, -1.73 aF

Electrostatic simulations for improvement of the analytical capacitances

Charge calculations for 1V across the gap and the SRRs arms extended





Index	Color	Top y µm	Length µm	GaAs q/ε_0 nCm/F	Air q/ε_0 nCm/F
1	red	0.050	0.000	186.6973	168.0576
2	green	0.075	0.025	643.5120	139.8804
3	blue	0.150	0.075	844.6347	191.9064
4	white	0.375	0.225	1663.647	355.1039
5	magenta	1.050	0.675	3587.186	721.7294
6	orange	3.075	2.025	8260.145	1593.920
7	cyan	9.150	6.075	20000.92	3754.312
8	pink	27.375	18.225	50244.16	9255.789
9	light blue	82.050	54.675	129717.3	23578.98
10	black	246.075	164.025	342439.5	61636.23
11	gold	738.150	492.075	925501.6	165389.4
12	grey blue	2214.37	1476.225	2672127.	476384.1

Subcell: impedance per unit length

$$a \approx \frac{1.18(w+t)}{4} \approx .065\mu m$$

$$Z_s = \frac{\omega\mu_0}{k} \quad (w,t > 4\delta)$$

f (THz)	ε_r	k (μm^{-1})	δ (nm)	Z ($\Omega/\mu m$)
31.6	-3753.8 -j1242.7	6.6197 -j41.059	24.3	3.667 +j19.40
63.1	-1051.5 -j185.33	3.7648 -j43.04	23.2	3.829 +j37.62

Gold properties at the first resonances of the SRRs above the GaAs half-space and in air

Near square cross-section:

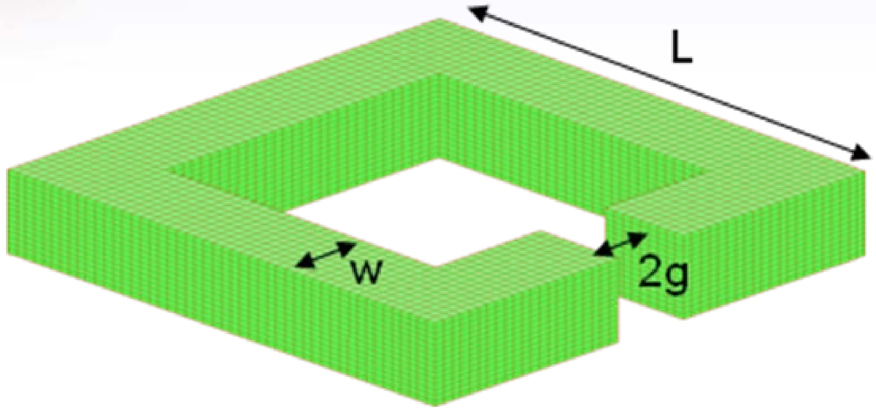
$$Z \approx \frac{2Z_s}{\pi(w+t)}\left[1-0.102\left(\frac{-4j}{k\left(w+t\right)}\right)-\frac{2jZ_S}{\omega\mu_0\left(w+t\right)}\right]$$

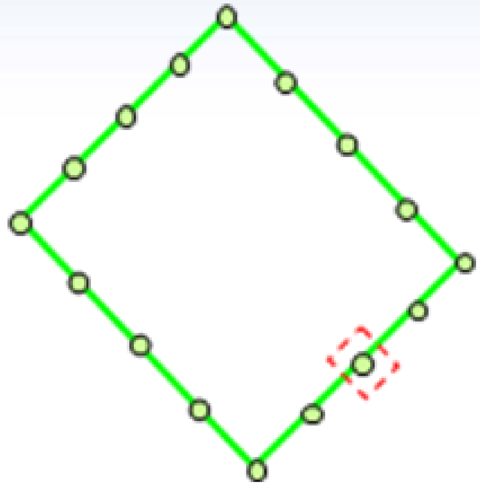
$$\Delta Z_L = Z + \frac{j\omega\mu_0}{2\pi}\ln(a_e/a)$$

$$\Delta Z_C = 0$$

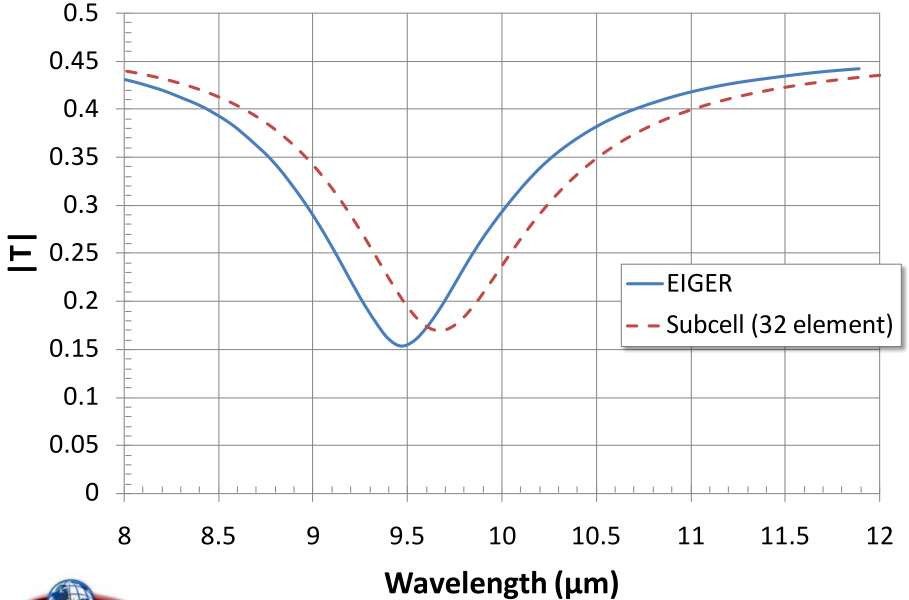
$$\mathrm{E}_s(a_e,s) + \Delta Z_c \frac{d^2}{ds^2} I_s(s) - \Delta Z_L I_s(s) = -E_s^{inc}(s)$$

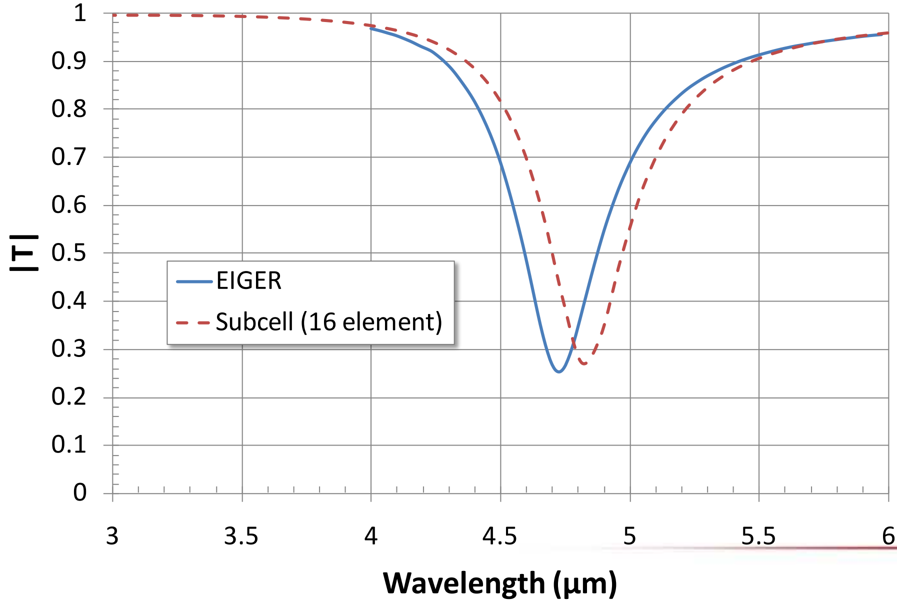
Comparison of EIGER and Subcell Simulations





1000 to 1 reduction in unknowns and runtime!





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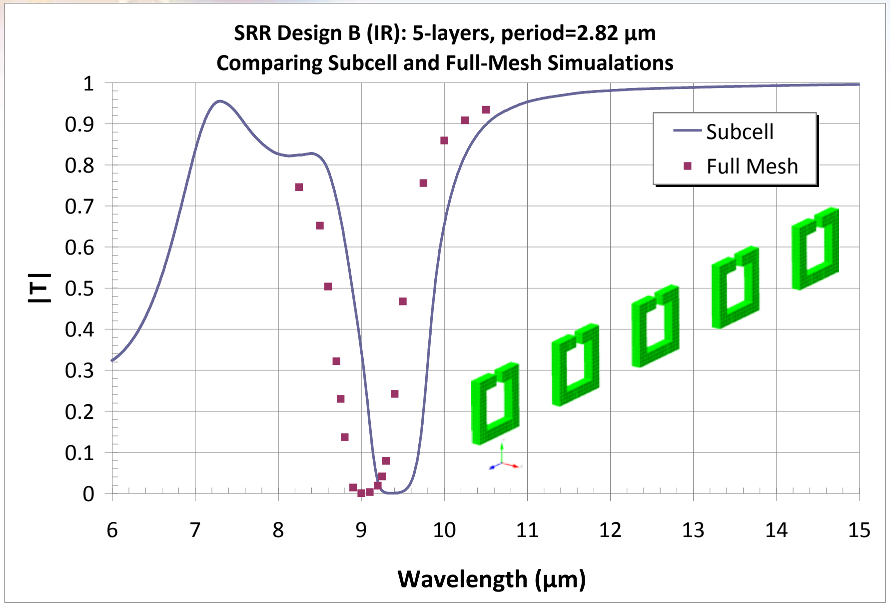
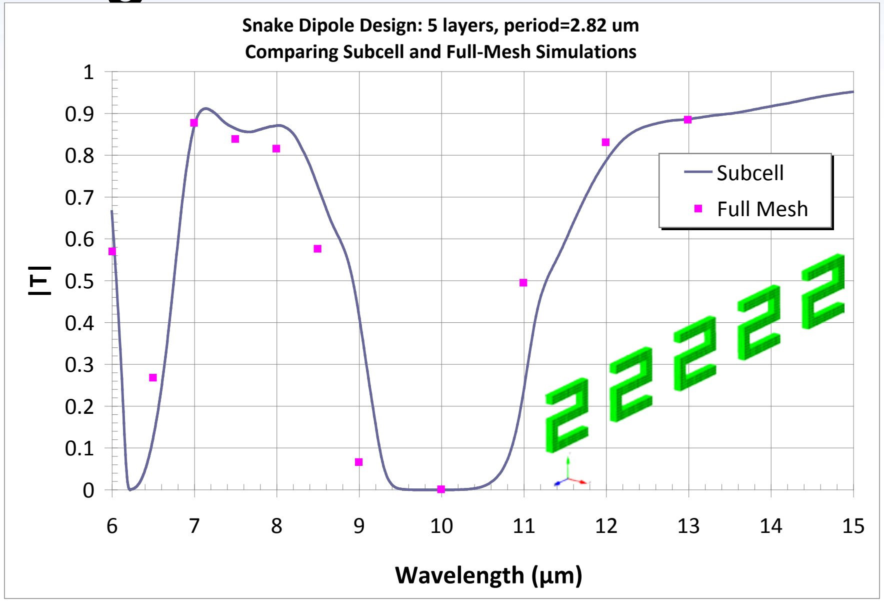
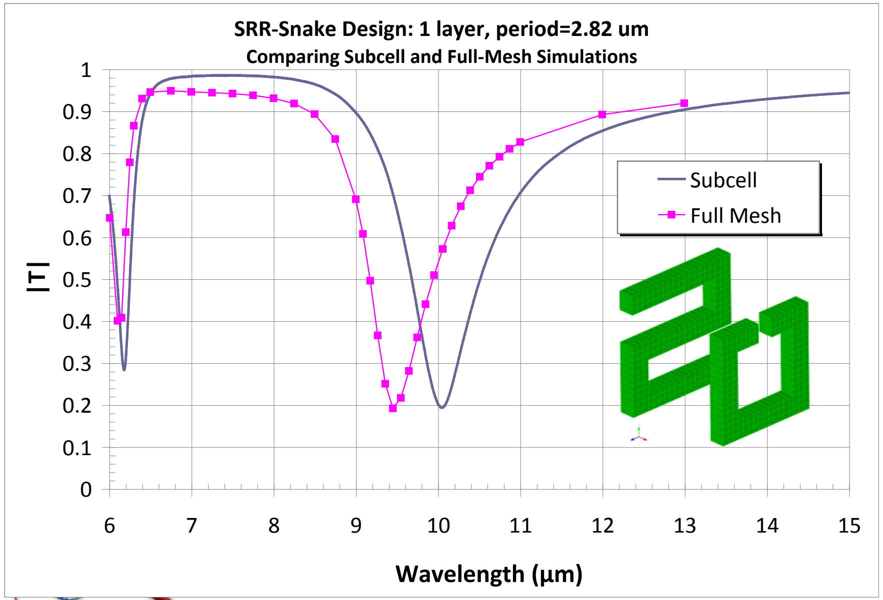
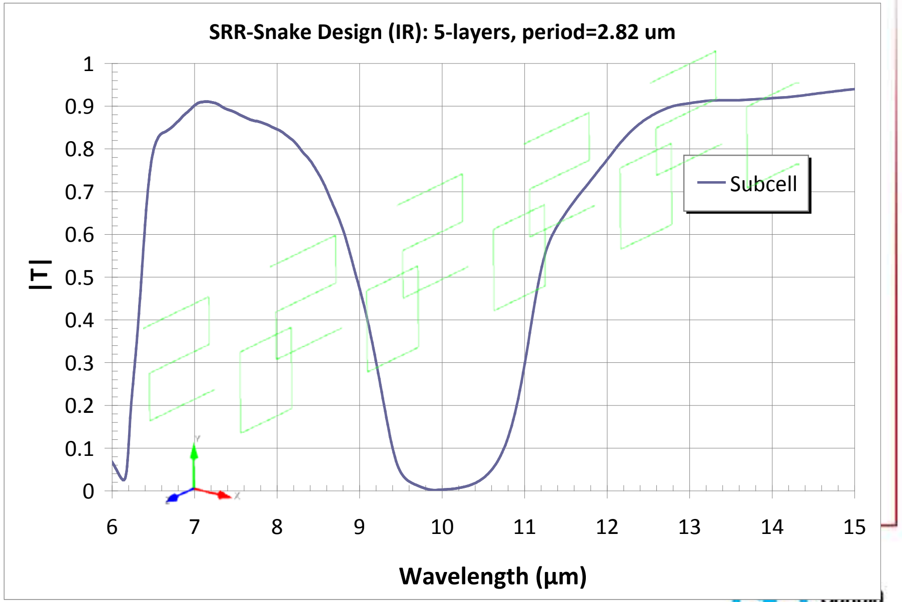
HPC Simulations Subcell Simulations

Subcell Models Can be Used For Various Metal Designs









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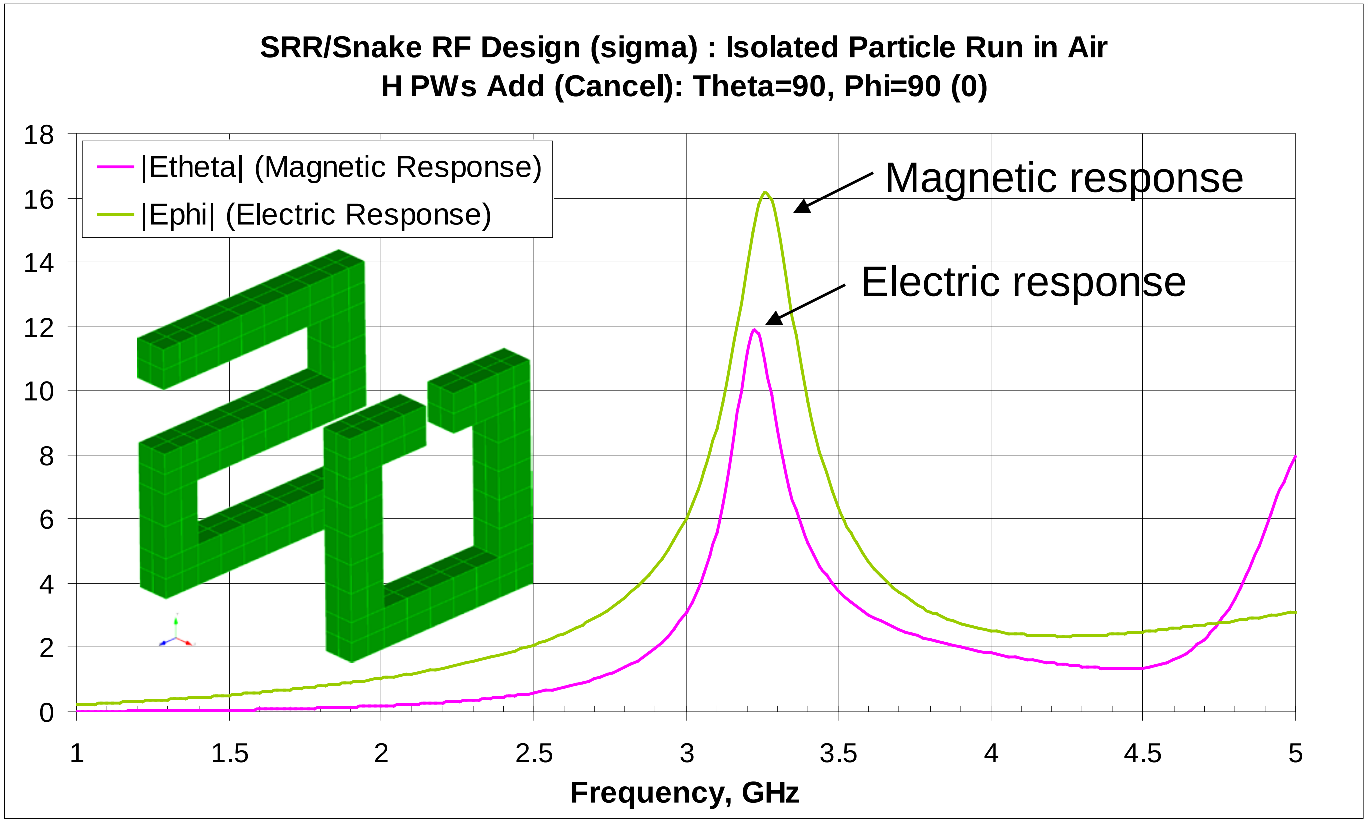
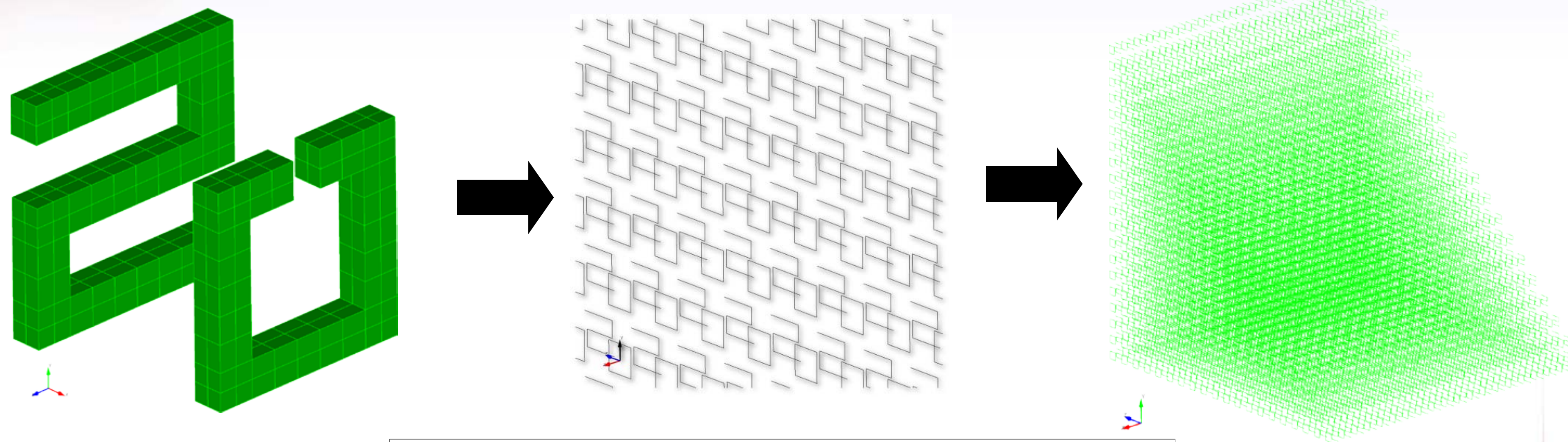
Subcell Models Can Be Used to Model Negative-Index Prisms (3D)

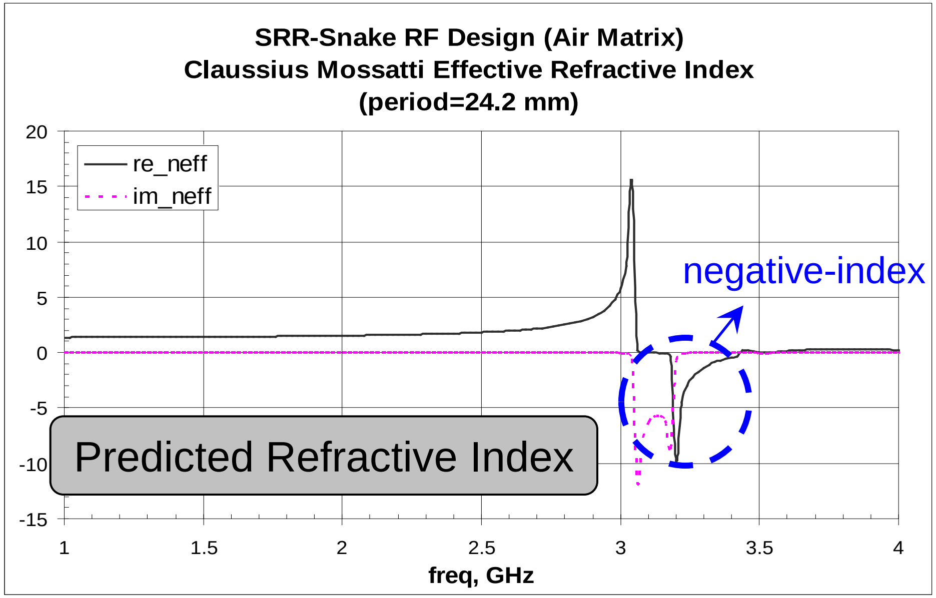


SRR generating magnetic response around 3.2 GHz

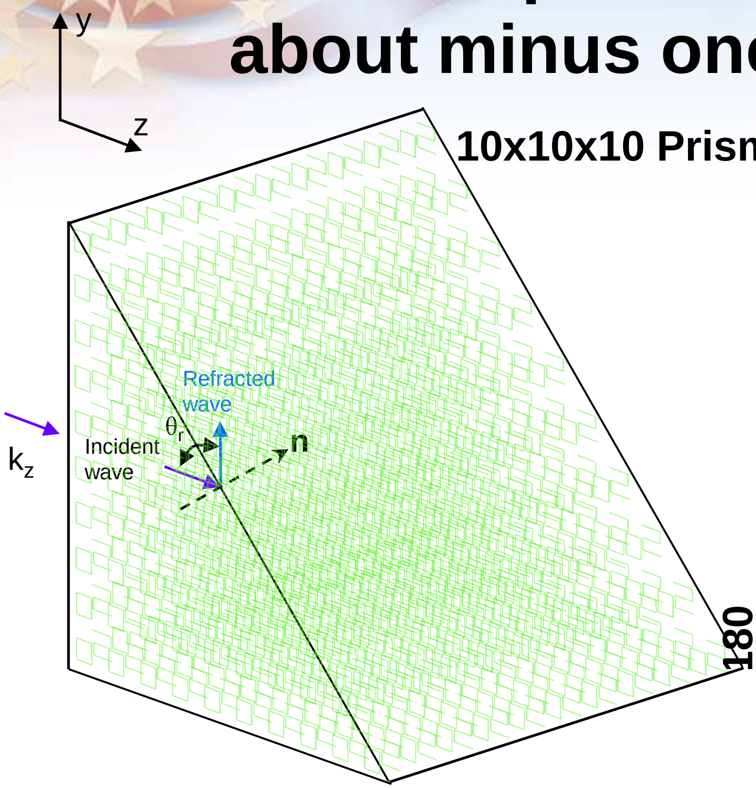
Z dipole generating electric response around 3.2 GHz

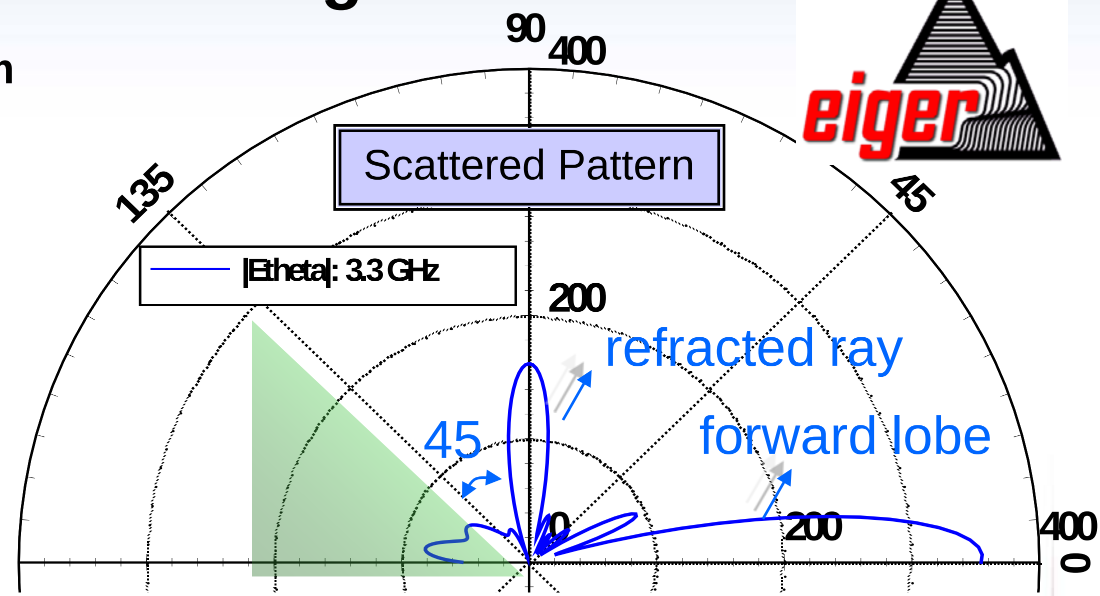
We assemble a 3D prism of subcells to realize a negative-refractive index

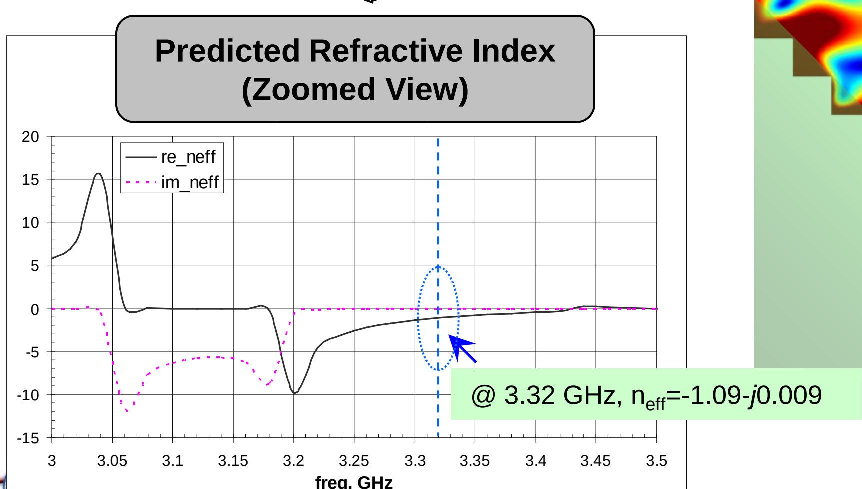


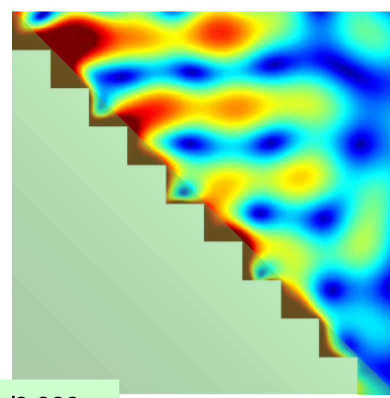


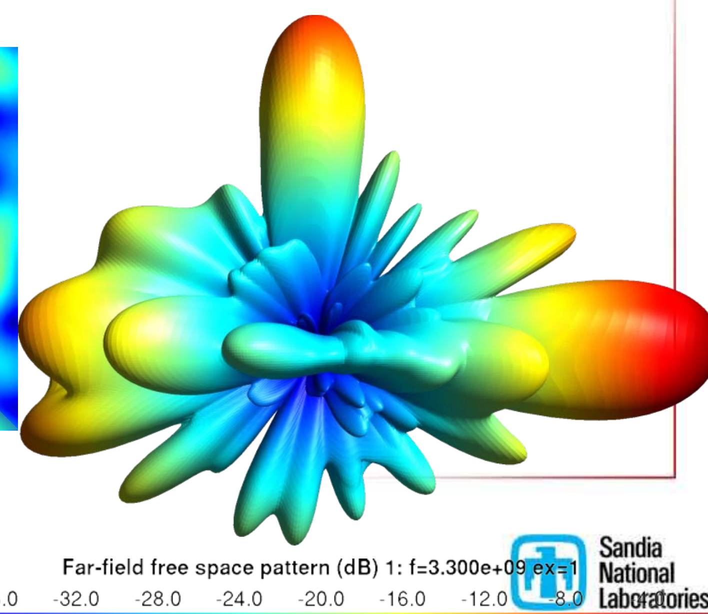
The RF 3D prism run shows a negative index of about minus one occurring around 3.3 GHz



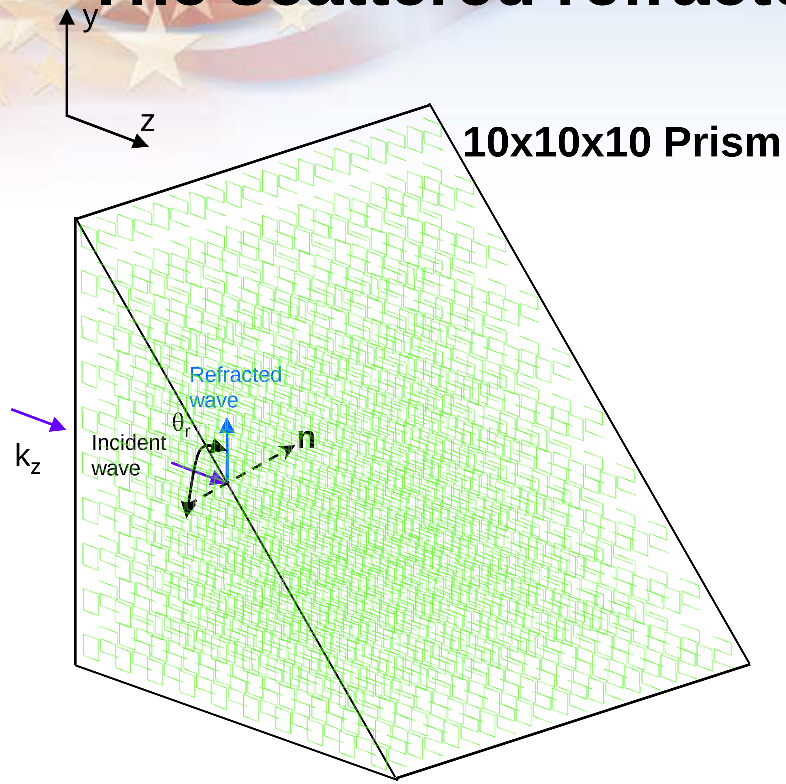


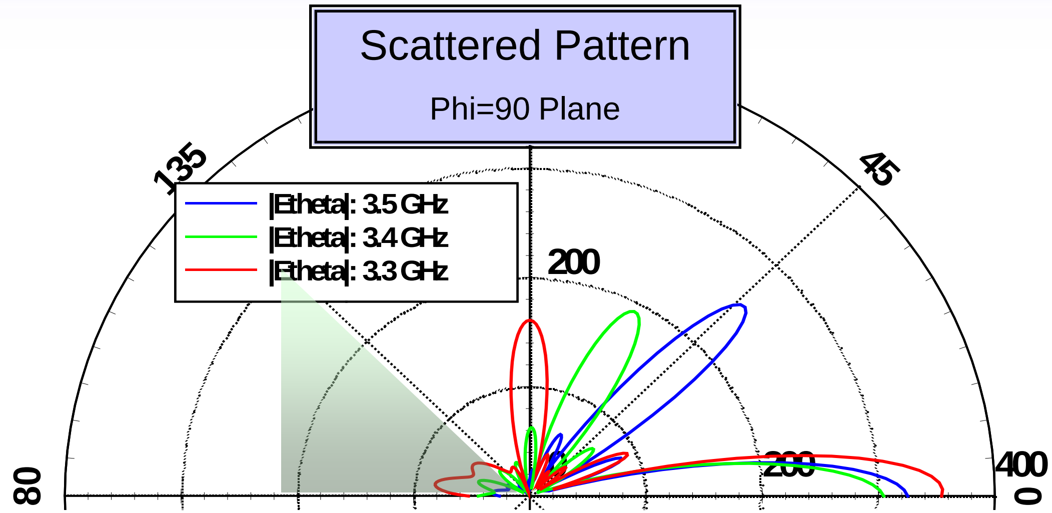


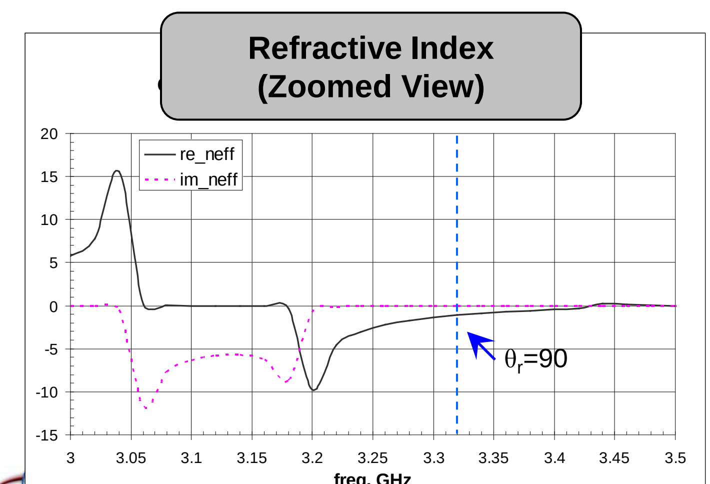


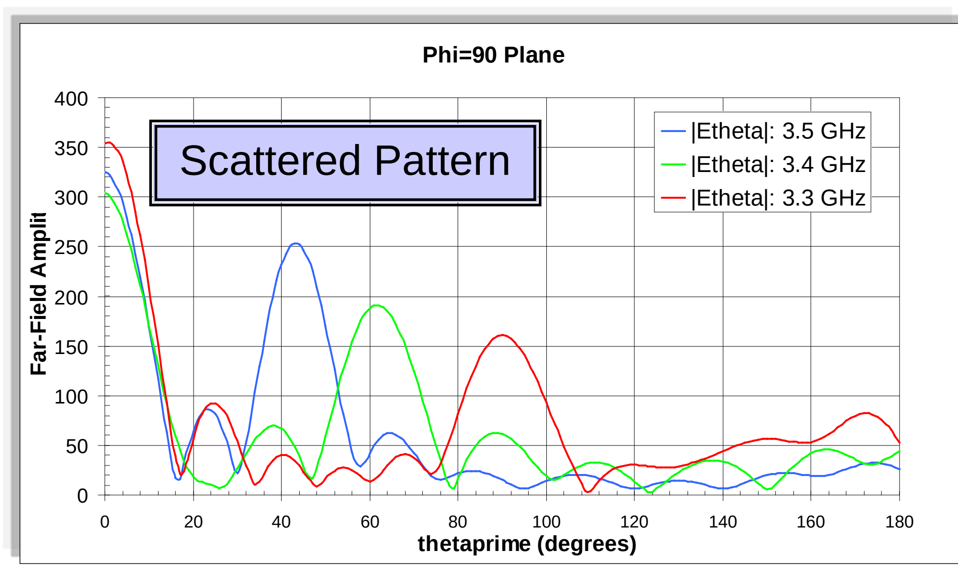


The scattered refracted beam scans with frequency









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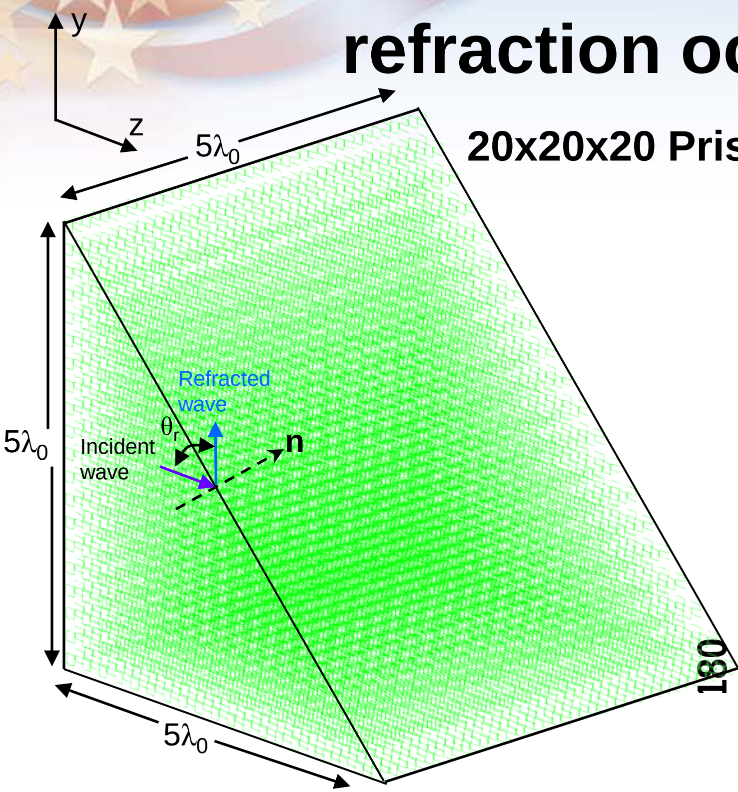
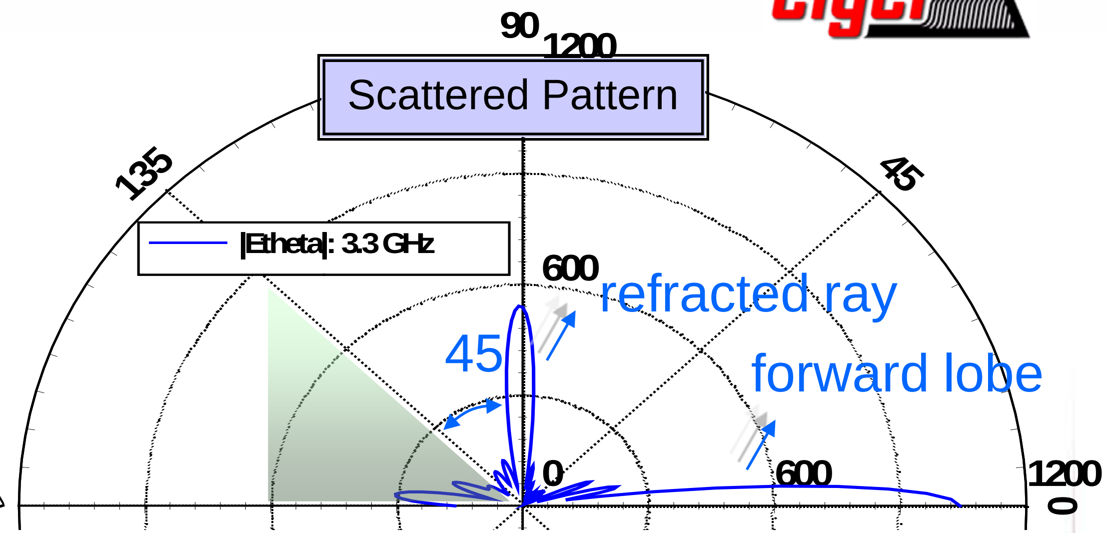
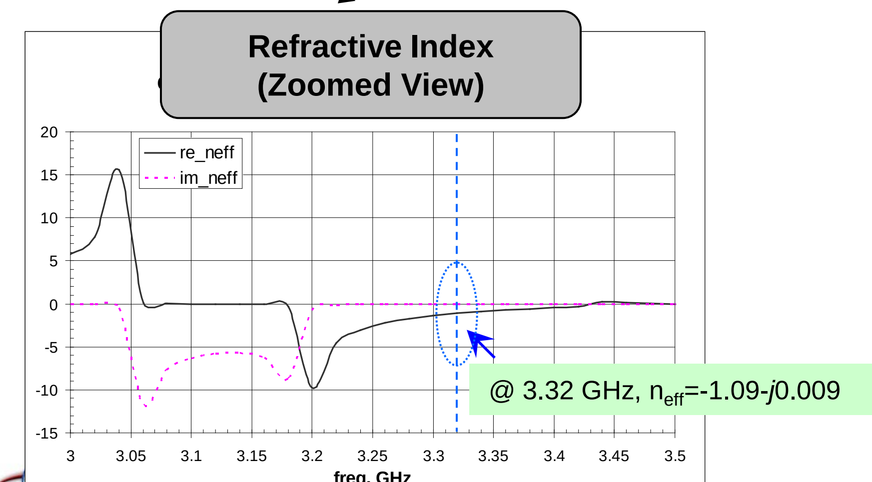
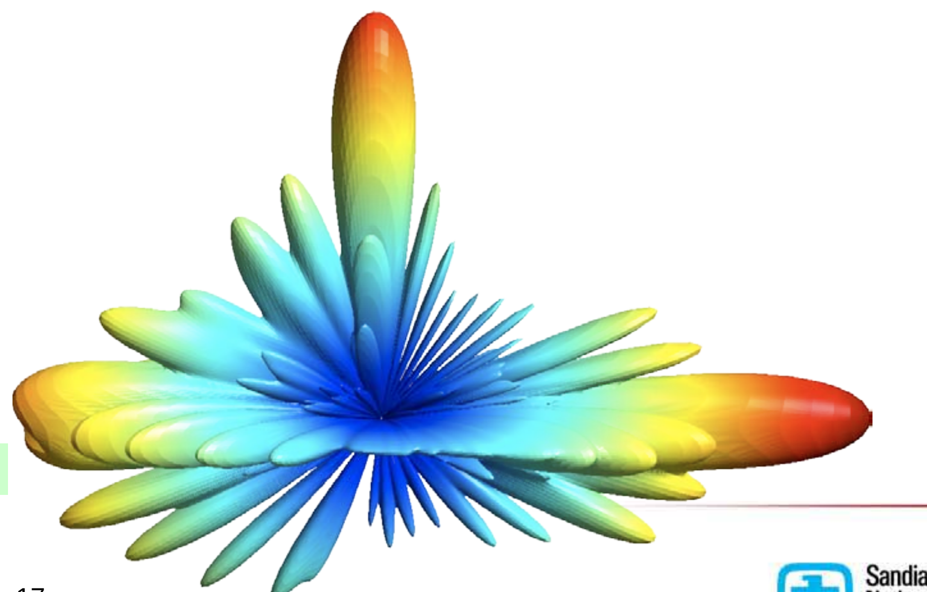
The results of the RF 3D prism run show negative refraction occurring around 3.3 GHz







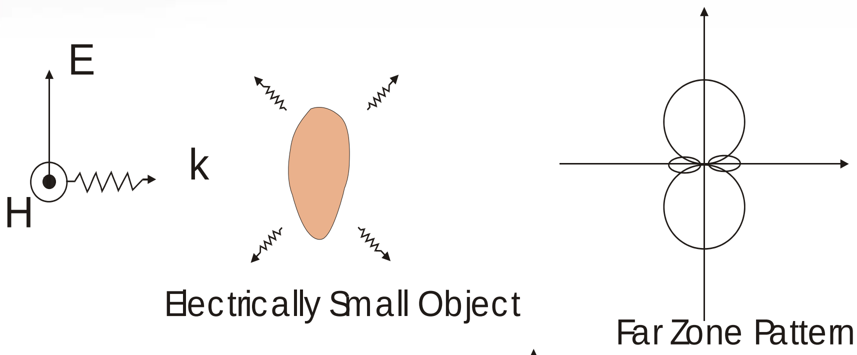


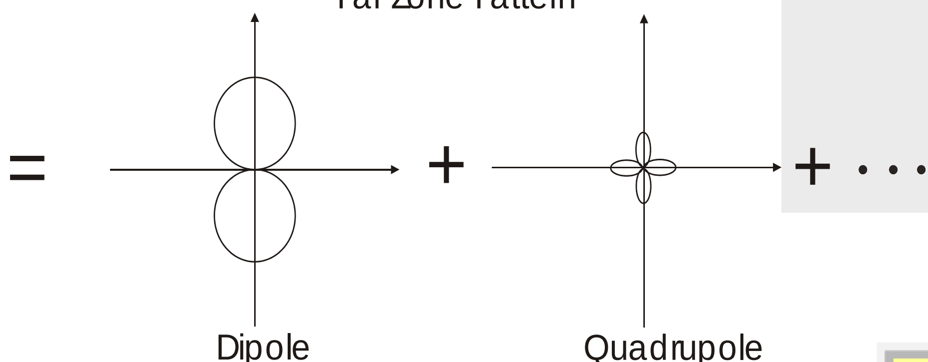
Acknowledgements

is being jointly developed by Sandia and the University of Houston (Donald Wilton, David Jackson, and their co-workers) with contributions from former students now at Johnson-Space center, NASA.

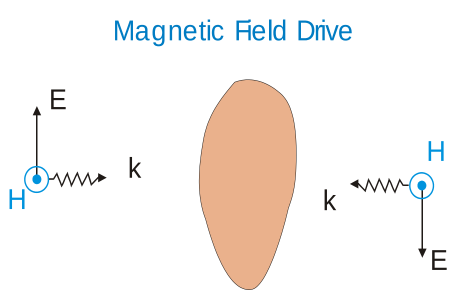
This tool has been very valuable in carrying out this work.

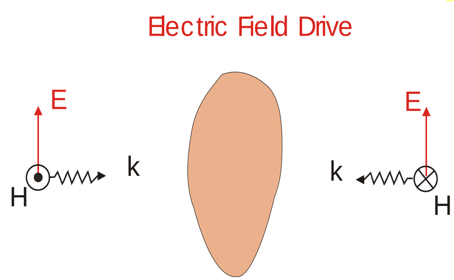
A dual-plane wave excitation of a single resonator simplifies interpretation of effective-media results





- ❑ separate electric and magnetic responses
- ❑ far-field amplitudes yield *spectral positions of resonances*
- ❑ far-field patterns yield polarizabilites and *type of response*
  - ❑ electric vs. magnetic
  - ❑ dipole vs. quadrapole
  - ❑ effective media parameters (with removal of radiation damping)





Design Procedure: Isolated Particle Simulation and Moments

IPSaM

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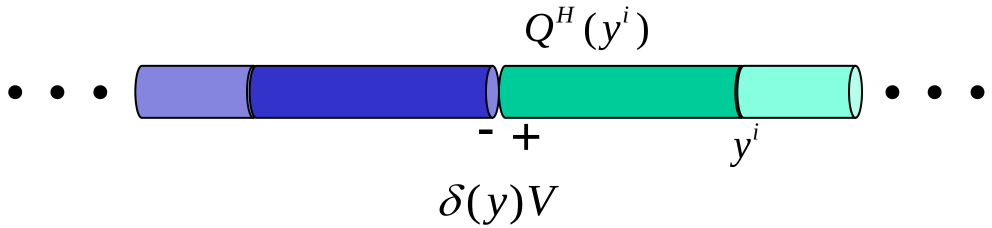
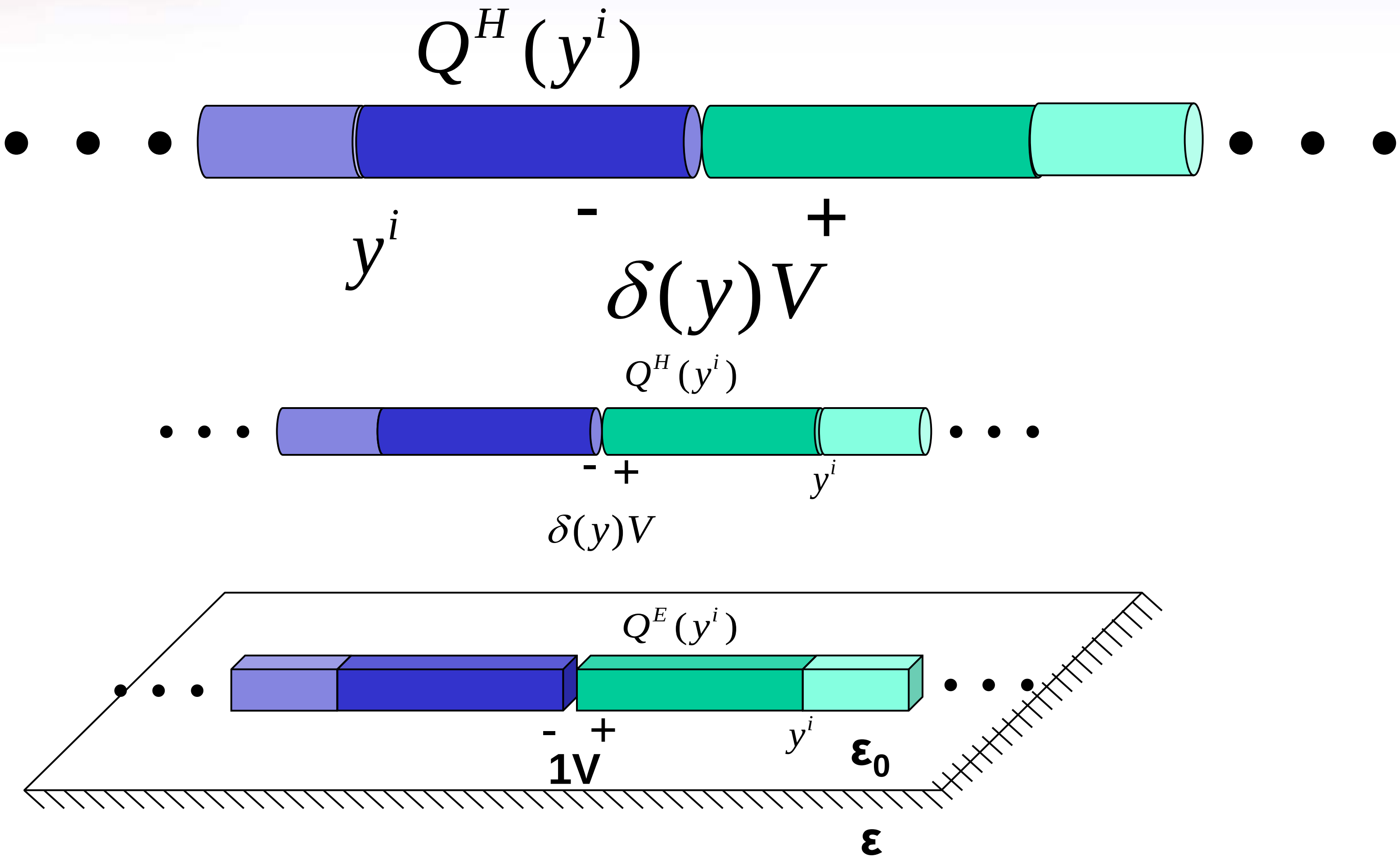

$Q^H(y^i)$
- +
y^i
$\delta(y)V$



$Q^H(y^i)$
y^i
-
+
$\delta(y)V$
$Q^H(y^i)$
- +
y^i
$\delta(y)V$
$Q^E(y^i)$
- +
y^i
1V
ε_0
ε