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SAND2013-5738C

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Estimation of the Critical Time Step for Peridynamic Models

12th U.S. National Congress on Computational Mechanics

22 July 2013

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SAND2013-XXXXX

U.S. DEPARTMENT OF ENERGY | NNSA

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Advancing Computational Simulation with Peridynamics

PERIDYNAMICS IS UNIQUE IN ITS ABILITY TO CAPTURE PERVASIVE MATERIAL FAILURE

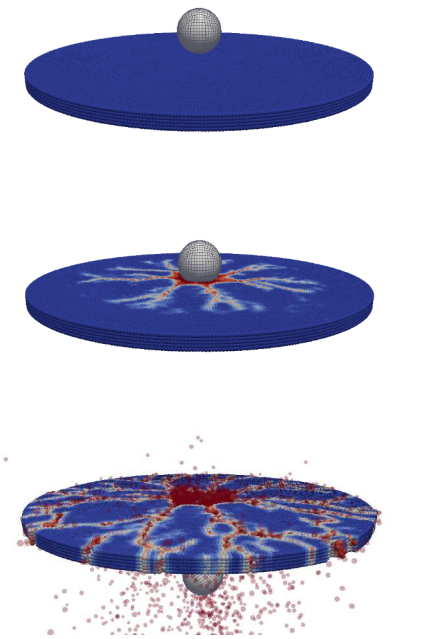
- Potential to enable rigorous simulation of failure and fracture
- Directly applicable to Sandia's national security missions

SIERRA CODE SUITE

- Engineering mechanics simulation code suite supporting the nation's nuclear weapons mission as well as other customers
- Advanced Simulation and Computing (ASC) code
- Peridynamic modeling for explicit transient dynamics (ETD)

IMPORTANCE OF THE ETD CRITICAL TIME STEP ESTIMATE

- Necessary condition for stable simulation
- Directly impacts computational expense



Sierra/SolidMechanics
Simulation of brittle fracture

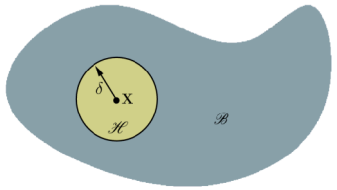
→ Successful application of peridynamics for engineering analyses requires an accurate estimate of the critical time step

Peridynamic Theory of Solid Mechanics

Peridynamics is a mathematical theory that unifies the mechanics of continuous media, cracks, and discrete particles

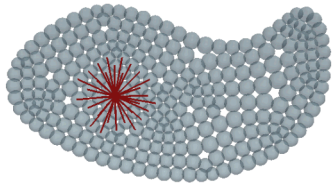
- Peridynamics is a nonlocal extension of continuum mechanics
- Remains valid in presence of discontinuities, including cracks
- Balance of linear momentum is based on an integral equation

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x},t) = \underbrace{\int_{\mathcal{B}} \left\{\underline{\mathbf{T}}[\mathbf{x},t]\,\langle \mathbf{x}'-\mathbf{x}\rangle - \underline{\mathbf{T}}'[\mathbf{x}',t]\,\langle \mathbf{x}-\mathbf{x}'\rangle\right\} \mathrm{d}V_{\mathbf{x}'}}_{\text{Divergence of stress replaced with integral of nonlocal forces.}} + \mathbf{b}(\mathbf{x},t)$$



- Peridynamic bonds connect any two material points that interact directly
- Peridynamic forces are determined by force states acting on bonds
- A peridynamic body may be discretized by a finite number of elements

$$\rho(\mathbf{x})\ddot{\mathbf{u}}_h(\mathbf{x},t) = \sum_{i=0}^{N} \left\{\underline{\mathbf{T}}[\mathbf{x},t]\,\langle \mathbf{x}'_i-\mathbf{x}\rangle - \underline{\mathbf{T}}'[\mathbf{x}'_i,t]\,\langle \mathbf{x}-\mathbf{x}'_i\rangle\right\} \Delta V_{\mathbf{x}'_i} + \mathbf{b}(\mathbf{x},t)$$



S.A. Silling. Reformulation of elasticity theory for discontinuities and long-range forces. *Journal of the Mechanics and Physics of Solids*, 48:175-209, 2000.

Silling, S.A. and Lehoucq, R. B. Peridynamic Theory of Solid Mechanics. *Advances in Applied Mechanics* 44:73-168, 2010.

Constitutive Models for Peridynamics

MATERIAL MODEL FORMULATION STRONGLY AFFECTS CRITICAL TIME STEP

- Presence of multiple length scales differs from the classical (local) approach
- Complex deformation modes possible within a nonlocal neighborhood
- Material failure through the breaking of bonds may alter the stable time step

Microelastic Material [1]

- Bond-based constitutive model
- Pairwise forces are a function of bond stretch

$$s = \frac{y - x}{x}$$

- Magnitude of pairwise force density given by

$$\underline{t} = \frac{18k}{\pi\delta^4} s$$

Linear Peridynamic Solid [2]

- State-based constitutive model
- Deformation decomposed into deviatoric and dilatational components

$$\theta = \frac{3}{m}\int_{\mathcal{H}} (\underline{\omega}\,\underline{x}) \cdot \underline{e}\, dV \qquad \underline{e}^d = \underline{e} - \frac{\theta\,\underline{x}}{3}$$

- Magnitude of pairwise force density given by

$$\underline{t} = \frac{3k\theta}{m}\,\underline{\omega}\,\underline{\mathrm{x}} + \frac{15\mu}{m}\,\underline{\omega}\,\underline{e}^{\mathrm{d}}$$

Definitions

$\underline{x}$	bond vector
x	initial bond length
y	deformed bond length
s	bond stretch
$\underline{e}$	bond extension
$\underline{e}^d$	deviatoric bond extension
$\underline{\omega}$	influence function
V	volume
$\mathcal{H}$	neighborhood
m	weighted volume
θ	dilatation
δ	horizon
k	bulk modulus
μ	shear modulus
$\underline{t}$	pairwise force density

1. S.A. Silling. Reformulation of elasticity theory for discontinuities and long-range forces. *Journal of the Mechanics and Physics of Solids*, 48:175-209, 2000.
2. S.A. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari, Peridynamic states and constitutive modeling, *Journal of Elasticity*, 88, 2007.

Classical Material Models Can Be Applied in Peridynamics

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WRAPPER APPROACH RESULTS IN A NON-ORDINARY STATE-BASED MATERIAL MODEL [1]

- Approximate deformation gradient based on initial and current locations of material points in family

Approximate Deformation Gradient

$$\bar{\mathbf{F}} = (\underline{\mathbf{Y}} * \underline{\mathbf{X}})\,\mathbf{K}^{-1}$$

Shape Tensor

$$\mathbf{K} = \underline{\mathbf{X}} * \underline{\mathbf{X}}$$

Definitions

$\mathbf{X}$	reference position vector state
$\mathbf{Y}$	deformation vector state
$\mathbf{K}$	shape tensor
$\bar{\mathbf{F}}$	approximate deformation gradient
ξ	bond
$\underline{\omega}$	influence function
σ	Piola stress

- Kinematic data passed to classical material model
- Classical material model computes stress
- Stress converted to pairwise force density

$$\underline{\mathbf{T}}\langle\xi\rangle = \underline{\omega}\langle\xi\rangle\ \sigma\mathbf{K}^{-1}\xi$$

- Suppression of zero-energy modes (optional) [2]

1. S.A. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari, Peridynamic states and constitutive modeling, *Journal of Elasticity*, 88, 2007.
2. Littlewood, D. A Nonlocal Approach to Modeling Crack Nucleation in AA 7075-T651. Proceedings of the ASME 2011 International Mechanical Engineering Congress and Exposition, Denver, Colorado, 2011.

Candidate Approaches for Estimating the Critical Time Step

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INVESTIGATE MULTIPLE APPROACHES

- Courant-Friedrichs-Lewy (CFL) condition [1]
- Approach of Silling and Askari for microelastic materials (von Neumann analysis) [2]
- Generalized Silling and Askari approach incorporating bond angles
- Global estimate using the Lanczos method [1,3]
- Largest eigenvalue of 3x3 nodal stiffness matrix

MEASURES OF SUCCESS

- Accuracy of estimate
- Computational expense

STRATEGY FOR ASSESSING CRITICAL TIME STEP ESTIMATES

- Evaluate via test simulations
- Compare against empirical result
  - Stable time step determined by numerical experiment

1. Hughes, T.J.R. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*. Prentice-Hall, Inc., Englewood Cliffs, NJ, 1987.
2. Silling, S.A. and Askari, E. A meshfree method based on the peridynamic model of solid mechanics. *Computers and Structures* 83:1526-1535, 2005.
3. Koteras, J.R. and Lehoucq, R.B. Estimating the critical time-step in explicit dynamics using the Lanczos method. International Journal for Numerical Methods in Engineering 69:2780-2788, 2007.

Courant-Friedrichs-Lewy (CFL) condition

- Common stability condition for explicit time integration schemes
- Heuristic interpretation: size of time step must be less than the time for a wave to pass to an adjacent grid point

$$\frac{c\,\Delta t}{\Delta x} \leq 1$$

where c is the wave speed, Δt is the time step, and Δx is a characteristic length of the discretization

- Wave speed computed from bulk modulus and density

$$c = \sqrt{\frac{k}{\rho}}$$

- CFL limit is given by

$$\Delta t \leq \frac{\Delta x}{c}$$

- For peridynamic models, what is the characteristic length scale?
  - Investigate both the mesh spacing and the horizon

Lanczos Global Critical Time Step Estimate

- For numerical solutions (e.g., classical FEM, peridynamics) after spatial discretization, linearization, and global assembly

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}$$

- **Explicit transient dynamics**: central difference time integration; lumped diagonal mass matrix **M**; real, symmetric, and positive-definite tangent stiffness matrix **K**
- Global critical time step

$$\Delta t_c = \frac{2}{\sqrt{\lambda}}$$

where λ is the maximum eigenvalue, and **x** the associated eigenvector, of the generalized eigenvalue problem

$$(\mathbf{K} - \lambda\mathbf{M})\,\mathbf{x} = 0$$

- This requires an efficient algorithm to find the maximum global eigenvalue.

Lanczos Global Critical Time Step Estimate (ct'd)

- **Lanczos Iteration** [1] : Define $\mathbf{A} = \mathbf{M}^{-1}\mathbf{K}$, $\beta_0 = 0$, $\mathbf{q}_0 = 0$, $\mathbf{b}$ = arbitrary, and $\mathbf{q}_1 = \mathbf{b}/||\mathbf{b}||$. For n = 1, 2, 3, …

$$\mathbf{v} = \mathbf{A}\mathbf{q}_n$$
$$\alpha_n = \mathbf{q}_n^T \mathbf{v}$$
$$\mathbf{v} = \mathbf{v} - \beta_{n-1}\mathbf{q}_{n-1} - \alpha_n \mathbf{q}_n$$
$$\beta_n = \|\mathbf{v}\|$$
$$\mathbf{q}_{n+1} = \frac{\mathbf{v}}{\beta_n}$$

- The largest eigenvalue of the tri-diagonal symmetric matrix with α_n on the diagonal and β_n on the off-diagonals converges to the largest eigenvalue of **A** as $n \to \infty$.
- **Implementation Efficiency**: Each iteration requires $\mathbf{A}\mathbf{q}_n = (\mathbf{M}^{-1}\mathbf{K})\mathbf{q}_n$. **M** is diagonal, and $\mathbf{K}\mathbf{q}_n$ is simply the (linearized) internal force. Thus for component *i*:

$$[\mathbf{A}\mathbf{q}_n]_i = \frac{1}{m_i} F_i^{\text{int}}(\mathbf{q}_n)$$

1. Trefethen, L.N. and Bau, D., III. *Numerical Linear Algebra*. SIAM 1997.

Estimate of Critical Time Step for Microelastic Material

- One-dimensional analysis of Silling and Askari for microelastic material [1]

$$\Delta t_c = \sqrt{\frac{2\rho}{\sum_p V_p C_{ip}}}$$

where ρ is the density, p iterates over all bonds at node i, V_p is the volume associated with each neighbor, and C_{ip} is the micromodulus between nodes i and p

- Multiple dimensions and nonlinear materials [1]

$$C_{ip} = |\mathbf{C}(x_p - x_i)| = \left| \frac{\partial \mathbf{f}}{\partial \boldsymbol{\eta}} \right|$$

where the **f** is the force function of each bond and $\boldsymbol{\eta}$ is the relative displacement of the nodes in the bond

- The suggested multi-dimensional measure represents the worst case scenario of all bonds being aligned.

1. Silling, S.A. and Askari, E. A meshfree method based on the peridynamic model of solid mechanics. *Computers and Structures* 83:1526-1535, 2005.

Multidimensional Estimate of Critical Time Step for Microelastic Material

- We investigate using the assembled bond structure in the critical time step estimate

$$\Delta t_c = \sqrt{\frac{2\rho}{\left|\sum_p V_p \mathbf{C}_{ip}\right|}}$$

where the for the *jk* component of $\mathbf{C}_{ip}$ we have

$$[C_{ip}]_{jk} = C_{ip} a_j a_k$$

where C_{ip} is the scalar linearized bond stiffness and a_j are the direction cosines associated with the bond *p* orientation

- The norm we use is the maximum principal stiffness, *i.e.*, in three dimensions the maximum eigenvalue of the assembled 3x3 nodal stiffness

Critical Time Step Estimate Based on Probed Nodal Stiffness Matrix

- Linearized bond-based and state-based peridynamic equation of motion [1,2]

$$\rho\ddot{\mathbf{u}} = \int_{\mathscr{B}} \mathbf{C}\left(\mathbf{x},\mathbf{q}\right)\left(\mathbf{u}(\mathbf{q},t)-\mathbf{u}(\mathbf{x},t)\right)\mathrm{d}V_q + \mathbf{b}(\mathbf{x},t)$$

  where the two theories involve different mathematical objects but similar formulations

- Rewriting the equation of motion

$$\rho\ddot{\mathbf{u}} = \int_{\mathscr{B}} \mathbf{C}\left(\mathbf{x},\mathbf{q}\right)\mathbf{u}(\mathbf{q},t)\mathrm{d}V_q - \mathbf{P}(\mathbf{x})\mathbf{u}(\mathbf{x},t) + \mathbf{b}(\mathbf{x},t)$$

  where

$$\mathbf{P}(\mathbf{x}) = \int_{\mathscr{B}} \mathbf{C}\left(\mathbf{x},\mathbf{q}\right)\mathrm{d}V_q$$

1. S.A. Silling. Reformulation of elasticity theory for discontinuities and long-range forces. *Journal of the Mechanics and Physics of Solids*, 48:175-209, 2000.
2. S.A. Silling. Linearized Theory of Peridynamic States, *Journal of Elasticity*, 99:85-111, 2010.

Critical Time Step Estimate Based on Probed Nodal Stiffness Matrix (ct'd)

- **P**(**x**) is seen to be, after discretization, the denominator of the multi-dimensional bond-based time step estimate

$$\mathbf{P}(\mathbf{x}) = \int_{\mathscr{B}} \mathbf{C}\left(\mathbf{x}, \mathbf{q}\right) \mathrm{d}V_q \approx \sum_p V_p \mathbf{C}_{ip}$$

- Given a displacement field such that at $\mathbf{x}_0$ the displacement is **e** and everywhere else it is 0, we have the following interpretation [1]

> *The value of the vector* $\boldsymbol{P(x_0)e}$ *is therefore the force density (per unit volume) at* $\boldsymbol{x_0}$ *required to displace* $\boldsymbol{x_0}$ *by the vector* ***e****, holding all other points fixed.*

- *I.e.* the internal force from a nodal probe at $\mathbf{x}_0$ will be $\mathbf{P}(\mathbf{x}_0)\mathbf{e}$.
- Probing in all dimensions will allow recovery of $\mathbf{P}(\mathbf{x}_0)$, giving a bond- or state-based version of the denominator in our multi-dimensional critical time step estimate.
- We investigate the applicability of probing to obtain the nodal 3x3 stiffness and subsequent time step estimate for both bond-based and state-based analyses.

1. S.A. Silling. Linearized Theory of Peridynamic States, *Journal of Elasticity*, 99:85-111, 2010.

Test Case: Elastic Wave Propagation

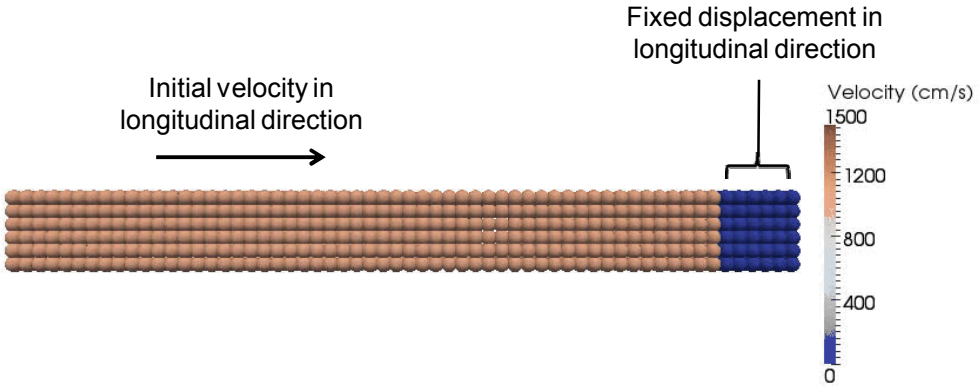
- Investigate material models
  - Microelastic bond-based
  - Linear peridynamic solid state-based
  - Wrapped classical elastic model
- Investigate critical time step estimates
  - Empirical (numerical experiment)
  - 1D approach of Silling and Askari
  - Generalized Silling and Askari
  - Element time step (3x3 stiffness probe)
  - Lanczos global estimate

Material Parameters

Density	7.8 g/cm^3
Young's Modulus	300.0 GPa
Poisson's Ratio	0.25
Horizon	0.5075 cm

Simulation

Bar Length	10.0 cm
Bar Width	1.0 cm
Initial Velocity	10.0 m/s
Time Step	0.48 μs



Microelastic Bond-Based Material Model

Time Step	Kinetic Energy
0.1 μs	3.51 J
0.2 μs	3.51 J
0.3 μs	3.51 J
0.4 μs	3.51 J
0.5 μs	14.1 J
0.6 μs	NaN
0.7 μs	NaN
0.8 μs	NaN
0.9 μs	NaN
1.0 μs	1.75e+299 J

Silling and Askari
max. time step = 0.241 μs
max. kinetic energy = 3.51 J

CFL Limit (element size)
max. time step = 0.329 μs
max. kinetic energy = 3.51 J

Generalized Silling and Askari
max. time step = 0.414 μs
max. kinetic energy = 3.51 J

Nodal Stiffness Matrix
max. time step = 0.414 μs
max. kinetic energy = 3.51 J

Empirical Observation
max. time step = 0.499 μs
max. kinetic energy = 3.51 J

Global Lanczos
max. time step = 0.500 μs
max. kinetic energy = 3.51 J

CFL Limit (horizon)
max. time step = 1.00 μs
max. kinetic energy = unstable

Note: Lanczos estimate of critical time step updated continuously throughout simulation

Linear Peridynamic Solid State-Based Material Model

Time Step	Kinetic Energy
0.1 μs	3.51 J
0.2 μs	3.51 J
0.3 μs	3.51 J
0.4 μs	NaN
0.5 μs	NaN
0.6 μs	NaN
0.7 μs	NaN
0.8 μs	NaN
0.9 μs	NaN
1.0 μs	NaN

Nodal Stiffness Matrix
max. time step = 0.314 μs
max. kinetic energy = 3.51 J

CFL Limit (element size)
max. time step = 0.329 μs
max. kinetic energy = 3.51 J

Empirical Observation
max. time step = 0.381 μs
max. kinetic energy = 3.51 J

Global Lanczos
max. time step = 0.381 μs
max. kinetic energy = 3.51 J

CFL Limit (horizon)
max. time step = 1.00 μs
max. kinetic energy = unstable

Wrapped Classical Material Model

Time Step	Kinetic Energy
0.1 μs	3.51 J
0.2 μs	3.51 J
0.3 μs	3.51 J
0.4 μs	3.51 J
0.5 μs	1.33e+135 J
0.6 μs	3.08e+151 J
0.7 μs	1.73e+109 J
0.8 μs	2.17e+124 J
0.9 μs	2.66e+116 J
1.0 μs	4.86e+208 J

CFL Limit (element size)
max. time step = 0.329 μs
max. kinetic energy = 3.51 J

Empirical Observation
max. time step = 0.490 μs
max. kinetic energy = 3.51 J

Global Lanczos
max. time step = 0.490 μs
max. kinetic energy = 3.51 J

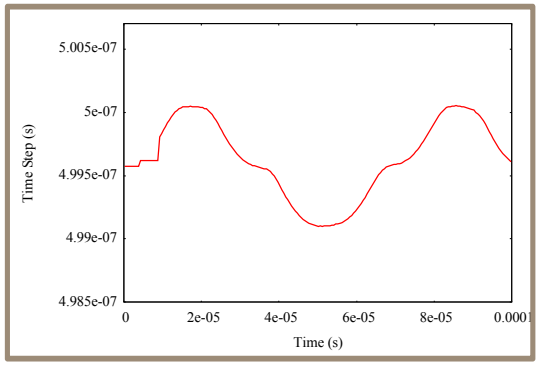
Nodal Stiffness Matrix
max. time step = 0.498 μs
max. kinetic energy = unstable

CFL Limit (horizon)
max. time step = 1.00 μs
max. kinetic energy = unstable

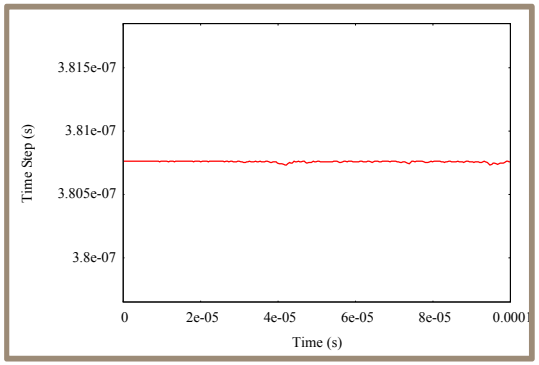
The Lanczos Estimate is a Function of Material Deformation

- Estimate of the critical time step varies over the course of the simulation
- Choice of material model affects behavior of critical time step estimate
  - Microelastic material model exhibits greatest degree of variation

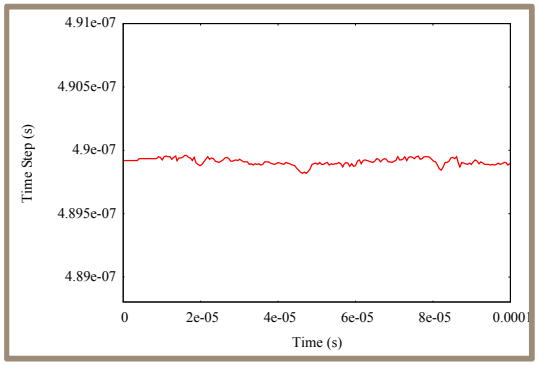
Microelastic Material



Linear Peridynamic Solid

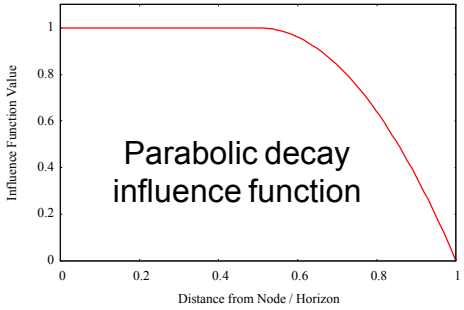


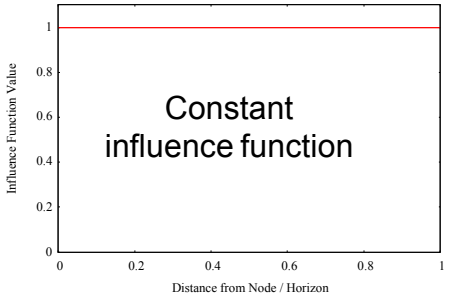
Wrapped Classical Model



The Influence Function Affects the Critical Time Step

- Choice of influence function affects critical time step
- Lanczos algorithm successfully detects changes in critical time step
- Observation: Influence function that decays with increasing bond length results in reduced critical time step





Peridynamic Linear Solid

	Parabolic decay influence function	Constant influence function
Max. Lanczos time step	0.381 μs	0.434 μs
Empirical result	0.381 μs	0.434 μs

14% Increase

Wrapped Classical Material Model

	Parabolic decay influence function	Constant influence function
Max. Lanczos time step	0.490 μs	0.549 μs
Empirical result	0.490 μs	0.549 μs

12% Increase

Test Case: Fragmenting Ring

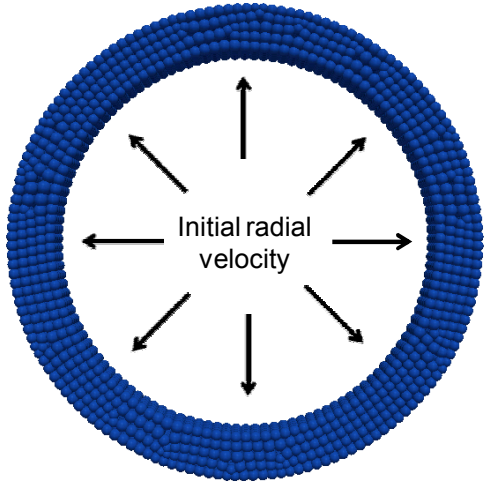
- Investigate material models
  - Microelastic bond-based
  - Linear peridynamic solid state-based
  - Wrapped classical elastic model (nosb)
- Investigate critical time step estimates
  - Empirical
  - 1D approach of Silling and Askari
  - Generalized Silling and Askari
  - Element time step (3x3 stiffness probe)
  - Lanczos global estimate

Material Parameters

Density	7.8 g/cm^3
Young's Modulus	300.0 GPa
Poisson's Ratio	0.25
Critical Stretch	0.01 cm/cm
Horizon	0.603 cm

Simulation

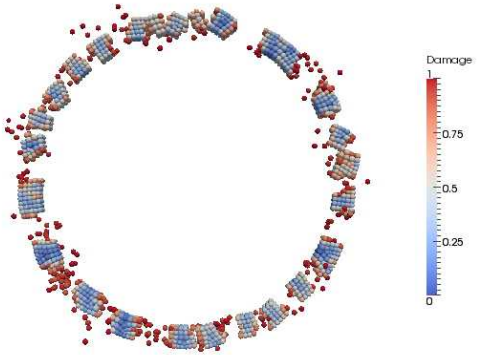
Ring Diameter	4.5 cm
Ring Width	1 cm
Initial Radial Velocity	200.0 m/s



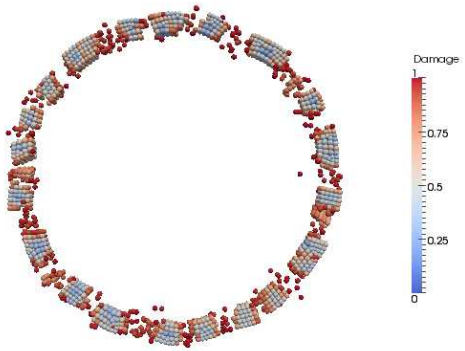
Unstable Time Step Manifests as Increased Bond Failure

Simulation results for microelastic material

Time Step	Percentage of Broken Bonds	Maximum Kinetic Energy (t > 10 μs)
0.01 μs	44.3 %	3.83 kJ
0.1 μs	44.5 %	3.82 kJ
0.2 μs	44.7 %	3.82 kJ
0.3 μs	45.3 %	3.82 kJ
0.4 μs	45.3 %	3.82 kJ
0.5 μs	45.4 %	3.82 kJ
0.6 μs	46.7 %	3.81 kJ
0.7 μs	49.1 %	3.83 kJ
0.8 μs	73.5 %	3.82 kJ
0.9 μs	95.3 %	4.39 kJ
1.0 μs	99.1 %	6.40 kJ



Time step = 5.0 μs
46.7% of bonds broken



Time step = 7.5 μs
62.7 % of bonds broken

Microelastic Bond-Based Material Model

Time Step	Percentage of Broken Bonds	Maximum Kinetic Energy ($t > 10\ \mu s$)
0.01 μs	44.3 %	3.83 kJ
0.1 μs	44.5 %	3.82 kJ
0.2 μs	44.7 %	3.82 kJ
0.3 μs	45.3 %	3.82 kJ
0.4 μs	45.3 %	3.82 kJ
0.5 μs	45.4 %	3.82 kJ
0.6 μs	46.7 %	3.81 kJ
0.7 μs	49.1 %	3.83 kJ
0.8 μs	73.5 %	3.82 kJ
0.9 μs	95.3 %	4.39 kJ
1.0 μs	99.1 %	6.40 kJ

Note: Empirical observation corresponds to the largest constant time step that results in no more than 50% bond failure

Silling and Askari
max. time step = 0.290 μs
percentage of broken bonds = 44.7 %
max. kinetic energy = 3.82 kJ

CFL Limit (element size)
max. time step = 0.395 μs
percentage of broken bonds = 45.3 %
max. kinetic energy = 3.51 J

Nodal Stiffness Matrix
max. time step = 0.485 μs
percentage of broken bonds = 45.4 %
max. kinetic energy = 3.82 J

Generalized Silling and Askari
max. time step = 0.486 μs
percentage of broken bonds = 45.4 %
max. kinetic energy = 3.82 kJ

Global Lanczos
max. time step = 0.682 μs
percentage of broken bonds = 46.0 %
max. kinetic energy = 3.83 kJ

Empirical Observation
max. time step = 0.707 μs
percentage of broken bonds = 50.0 %
max. kinetic energy = 3.83 kJ

CFL Limit (horizon)
max. time step = 1.19 μs
percentage of broken bonds = 99.1 %
max. kinetic energy = unstable

Linear Peridynamic Solid State-Based Material Model

Time Step	Percentage of Broken Bonds	Maximum Kinetic Energy (t > 10 μs)
0.01 μs	40.3 %	3.43 kJ
0.1 μs	40.2 %	3.43 kJ
0.2 μs	40.4 %	3.43 kJ
0.3 μs	41.6 %	3.42 kJ
0.4 μs	42.0 %	3.44 kJ
0.5 μs	44.7 %	3.45 kJ
0.6 μs	95.6 %	4.33 kJ
0.7 μs	97.3 %	5.54 kJ
0.8 μs	98.6 %	7.14 kJ
0.9 μs	99.4 %	19.8 kJ
1.0 μs	99.8 %	62.8 kJ

CFL Limit (element size)
max. time step = 0.395 μs
percentage of broken bonds = 41.3 %
max. kinetic energy = 3.51 J

Global Lanczos
max. time step = 0.494 μs
percentage of broken bonds = 42.8 %
max. kinetic energy = 3.43 kJ

Nodal Stiffness Matrix
max. time step = 0.505 μs
percentage of broken bonds = 44.8 %
max. kinetic energy = 3.82 kJ

Empirical Observation
max. time step = 0.509 μs
percentage of broken bonds = 50.0 %
max. kinetic energy = 3.46 kJ

CFL Limit (horizon)
max. time step = 1.19 μs
percentage of broken bonds = 99.1 %
max. kinetic energy = unstable

Conclusions

- There are a number of potentially useful ways to estimate the Explicit Transient Dynamics critical time step with peridynamics
- Any estimate should be material-type dependent (or else excessively conservative)
- The most accurate and reliable is the Lanczos method
  - Efficient implementation
  - Extremely accurate for all material types
  - Operates on the global linear system and is independent of underlying theory
  - May be applicable to mixed FEM-peridynamic analyses
- Bond-based nodal estimates (1D or multi-dimensional) are accurate for bond-based materials
- Probed nodal estimate is accurate for bond-based and ordinary state-based materials but extremely expensive
- Element size CFL limit is conservative but fails to account for material type
- Horizon size CFL limit is non-conservative and fails to account for material type

1. S.A. Silling. Linearized Theory of Peridynamic States, *Journal of Elasticity*, 99:85-111, 2010.

Questions?

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