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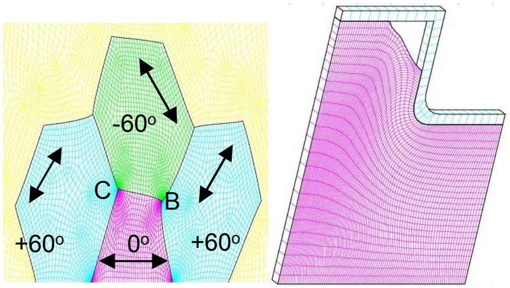

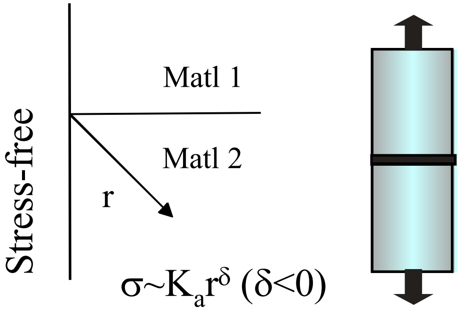

A Simple Mode-mixity Dependent Cohesive Zone Model with Application to Illustrative Interfacial Fracture Problems

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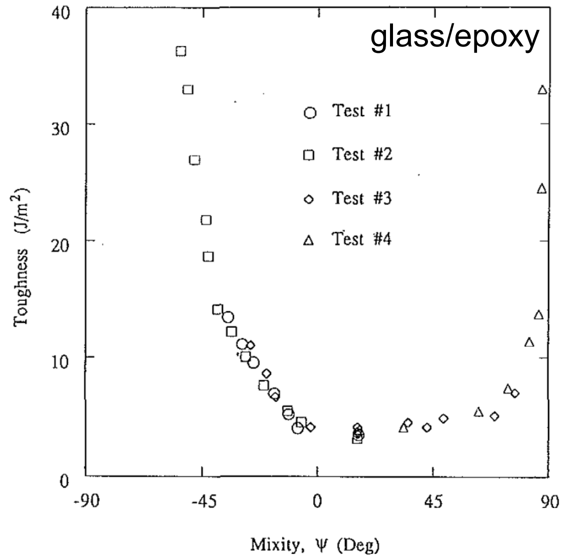
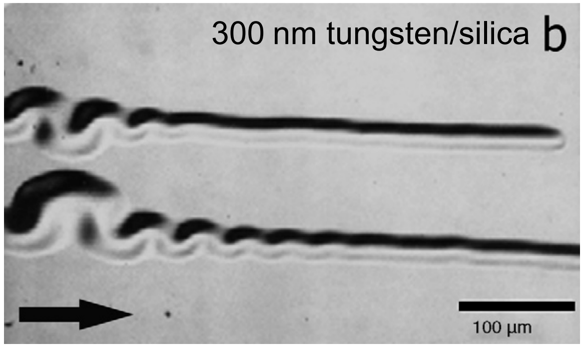
Background: mode-mixity dependent toughness

- The toughness of a polymer/solid interface increases with increasing mode-mixity.
- Behavior important in many problems.
- Widely used cohesive zone models do not include a mode-mixity dependent toughness (Tvergaard and Hutchinson, 1993, Xu and Needleman, 1994).
- Extension to include a mode-mixity dependent toughness has proved difficult.
  - A polynomial-based potential formulation defined by eight fracture parameters (Park and Paulino, 2011).
  - A nonpotential-based method that defines Mode I and Mode II response independently coupled by a mixed-mode failure condition (Yang and Thouless, 2001).



Liechti, K.M. and Chai, Y.S., Journal of Applied Mechanics, 1992. 59: p. 295-304.

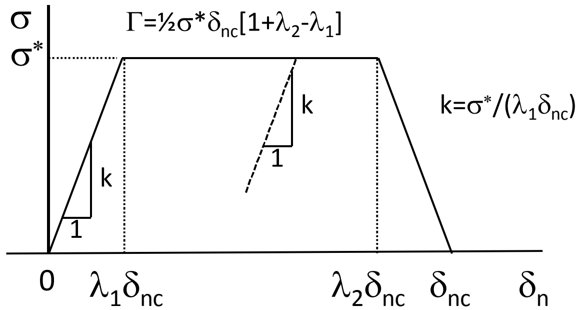


Cordill, M.J., et al., Materials Science and Engineering a-Structural Materials Properties Microstructure and Processing, 2007. 443(1-2): p. 150-155.

Simple CZ models that include mode-dependent toughness

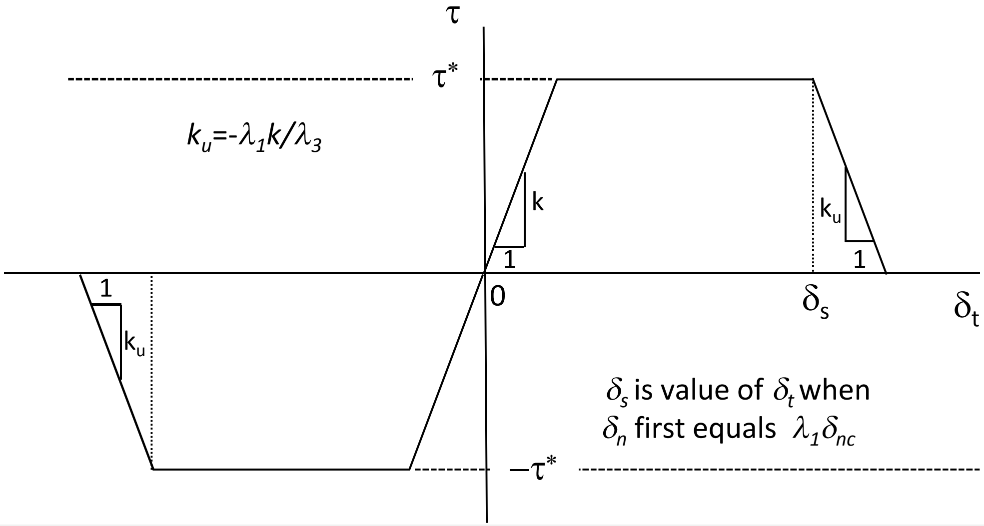
- Motivated by the recent development of the Adhesion/Atomistic Friction (Ad/AF) surface interaction model (Reedy, IJSS 2013).
  - Cohesive zone model includes all crack-tip dissipation generated within the process zone where:
    - Mode I dissipation is defined by a simple traction-separation law relating normal stress and opening.
    - Mode II (III) dissipation is associated with interfacial shear yielding (slip) that occurs in front of the region where Mode I softening occurs.
      - the amount of shear dissipation is not defined by a traction-separation law --- the extent of the slip zone is determined by the level of interfacial shear in front of the Mode I cohesive zone.

Mode-dependent toughness cohesive zone model (MDGc CZM)



Plane strain cohesive zone model for mode I opening (when δ_n <0; can define a multiple of k to penalize penetration)

Key parameters: interfacial strength σ^* and the intrinsic work of separation/unit area of interface Γ.



Plane strain cohesive zone model for mode II shear yielding prior to mode I softening (i.e., when $\delta_n < \lambda_1 \delta_{nc}$).

Key parameter: shear yield strength τ^*.

To generalize to 3D with anti-plane mode III slip δ_s in addition to the in-plane mode II slip δ_t, define effective shear stress τ_e and effective slip rate, $\dot{\delta}_e$.

$$\tau_e = \left(\tau_t^2 + \tau_s^2\right)^{1/2} \text{ and } \dot{\delta}_e = \left(\dot{\delta}_t^2 + \dot{\delta}_s^2\right)^{1/2}$$

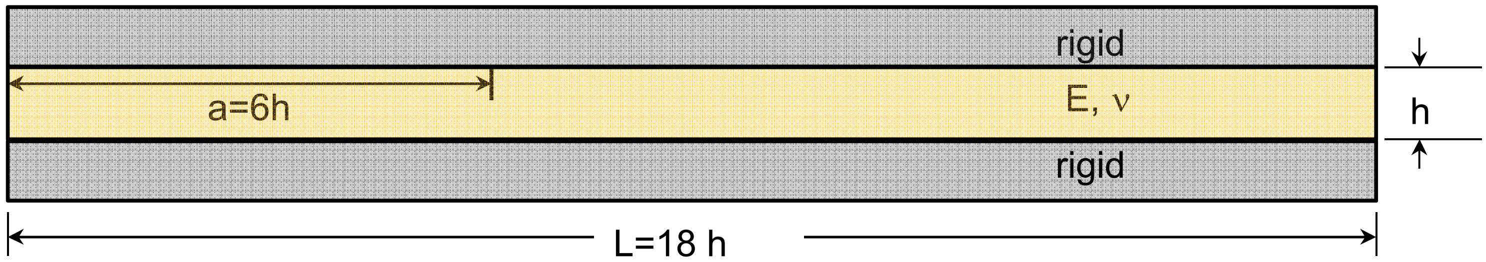
when $|\tau_e| < \tau^*$

$$\dot{\tau}_t = k\dot{\delta}_t \text{ and } \dot{\tau}_s = k\dot{\delta}_s$$

when $|\tau_e| = \tau^*$

$$\tau_t = \frac{\dot{\delta}_t}{\dot{\delta}_e}\tau^* \text{ and } \tau_s = \frac{\dot{\delta}_s}{\dot{\delta}_e}\tau^*$$

Simple problem that illustrates nature of MDG_c predictions



- Edge-cracked elastic layer that is sandwiched between rigid adherends.
- Apply uniform edge normal and tangential displacements (plane strain).
- Strip sufficiently long so that large region in central portion of ligament is uniformly stressed.
- Highly refined mesh in the region surrounding the initial crack tip (Δ/h=0.0025).
- Geometry similar to that used by Swadener and Liechti to measure mode-mixity dependent interfacial toughness (JAM 1998).

Definition of effective toughness and applied mode-mixity

- Energy release rate/unit area as the interfacial crack begins to propagate.

$$\Gamma_e = \frac{h}{2E_u}(\bar{\sigma}^c{}_{yy})^2 + \frac{h}{2G}(\bar{\sigma}^c{}_{xy})^2$$

  - $\bar{\sigma}^c{}_{yy}$ and $\bar{\sigma}^c{}_{xy}$ are critical stresses in uniformly stressed ligament.
  - $E_u=(1\text{-}v)\,E\,/((1+v)(1\text{-}2v))$) and G is the shear modulus.

- Applied mode-mixity as

$$\psi_a \equiv \tan^{-1}(2\bar{\sigma}^c{}_{xy} / \bar{\sigma}^c{}_{yy})$$

  - Crack tip mode-mixity at a distance l_o for same problem (Hutchinson and Suo,1992) is

$$\psi_{r=l_o} = \gamma + \omega + \varepsilon \ln(l_o / h)$$

when α=1.0 and β=0.25 (i.e., v=1/3), ω=-17^o and $\gamma=\psi_a$.

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Edge-cracked elastic layer sandwiched between rigid grips

- Nondimensional dependencies
  - Bulk properties: E, ν
  - MDGc properties: σ^*, τ^*, Γ, λ_1, λ_2, λ_3
  - Geometry: h
  - Loading: $\bar{\sigma}^c{}_{yy}, \bar{\sigma}^c{}_{xy}$ (Γ_e and ψ_a are defined in terms of these stresses)

- Choose primary dimensions Γ, and σ^*

$$\Gamma_e / \Gamma = f\left(\frac{\bar{\sigma}^c{}_{xy}}{\bar{\sigma}^c{}_{yy}}, \frac{\sigma^*}{E}, \frac{\tau^*}{\sigma^*}, \frac{\Gamma}{\sigma^* h}, \nu, \lambda_1, \lambda_2, \lambda_3 \right)$$

- In the base line calculation
  - σ^*/E=0.01, τ^*/σ^*=1.0
  - $\Gamma/(\sigma^* h)$=1e-4, ν=1/3
  - λ_1=0.1, λ_2=0.9, and λ_3 was typically set to 0.01
  - Use mesh with Δ~10-20δ_{nc} (as used by Tvergaard and Hutchinson,1993; relatively coarse since not trying to resolve stress within the process zone).

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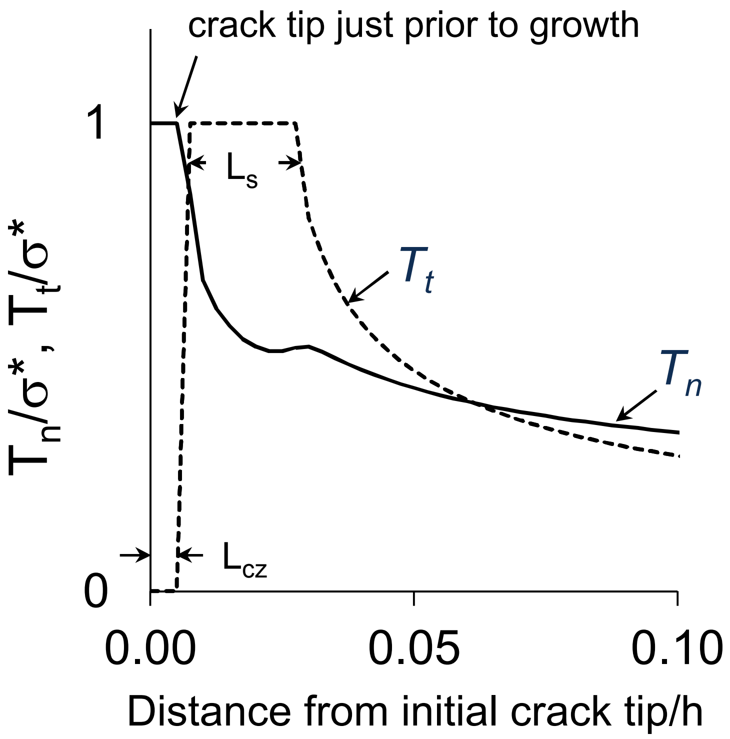
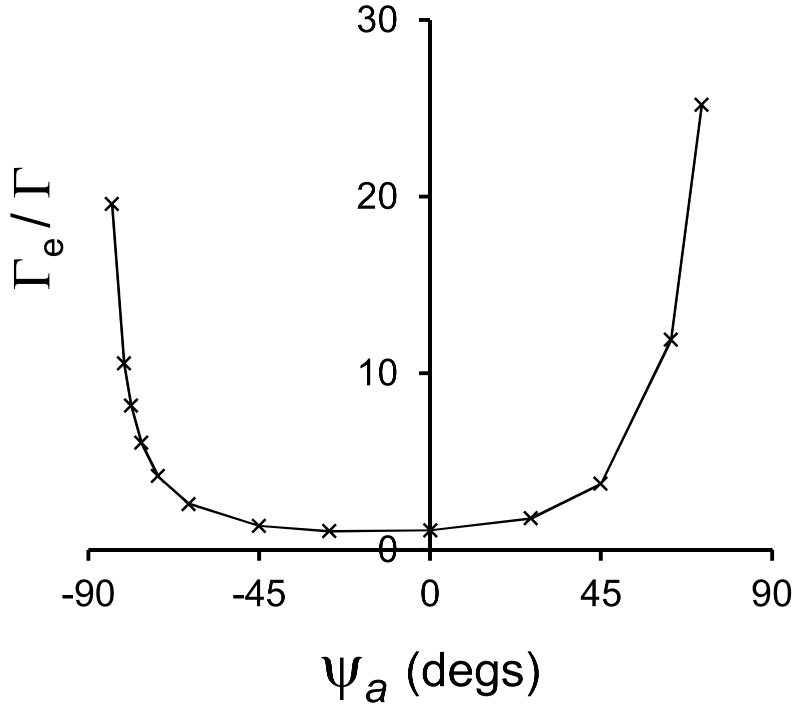
MDG$_c$ interfacial normal T_n and tangential T_t tractions

- Example of tractions in front of the initial crack tip as generated by MDG$_c$ CZ model.
  - plotted results for ψ_a=45^o at time just prior to crack propagation
  - Crack tip => point where the T_n first equals σ*.
  - Length of the cohesive zone L_{cz} => region where T_n=σ*.
  - Length of the plastic slip zone L_s => region where T_t=τ*.
  - Note that T_t=0 inside the cohesive zone since $\delta_n > \lambda_1 \delta_{nc}$.



Effective toughness vs. applied mode-mixity $\psi_a \equiv \tan^{-1}(2\sigma^c{}_{xy} / \sigma^c{}_{yy})$

- The MDG_c CZ model generates an effective toughness that increases rapidly with $|\psi_a|$.
  - $\Gamma_e/\Gamma=25$ when $\psi_a=+72^o$
  - $\Gamma_e/\Gamma=20$ when $\psi_a=-84^o$
- Calculated Γ_e/Γ displays asymmetry wrt ψ_a
  - $\Gamma_e/\Gamma=11.9$ when $\psi_a=+63^o$
  - $\Gamma_e/\Gamma=2.6$ when $\psi_a=-63^o$

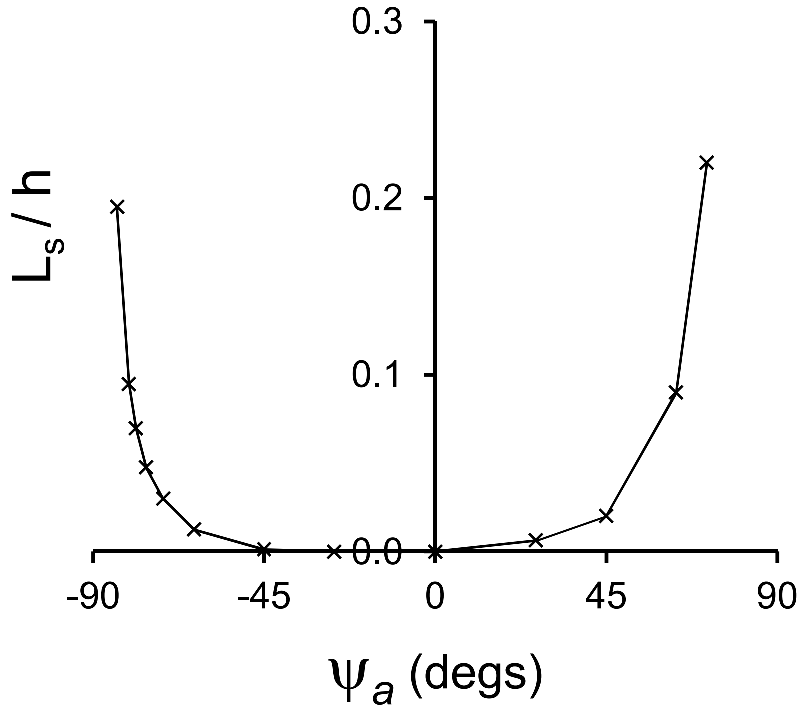


Length of slip zone at propagation *vs.* ψ_a

- L_s/h when crack begins to propagate.
  - similar dependence on ψ_a as Γ_e/Γ
  - ~ 0 when $-27<\psi_a<0$.
  - increases rapidly with $|\psi_a|$.
  - a sizable fraction *h* as $|\psi_a|$ increases.

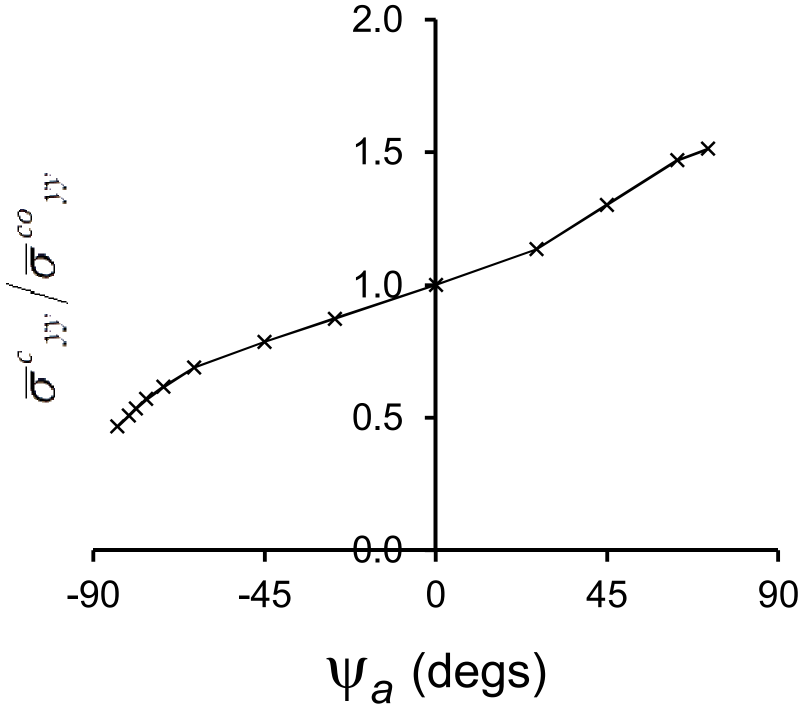
- L_s/h and Γ_e/Γ both display an asymmetric dependence on ψ_a.
  - $L_s/h=0.22$ $\psi_a=72^o$.
  - $L_s/h=0.03$ $\psi_a=-72^o$.

- Note: length of the cohesive zone L_{cz} is relatively insensitive to ψ_a.
  - $0.005<L_{cz}/h<0.009$ for $-84<\psi_a<72$



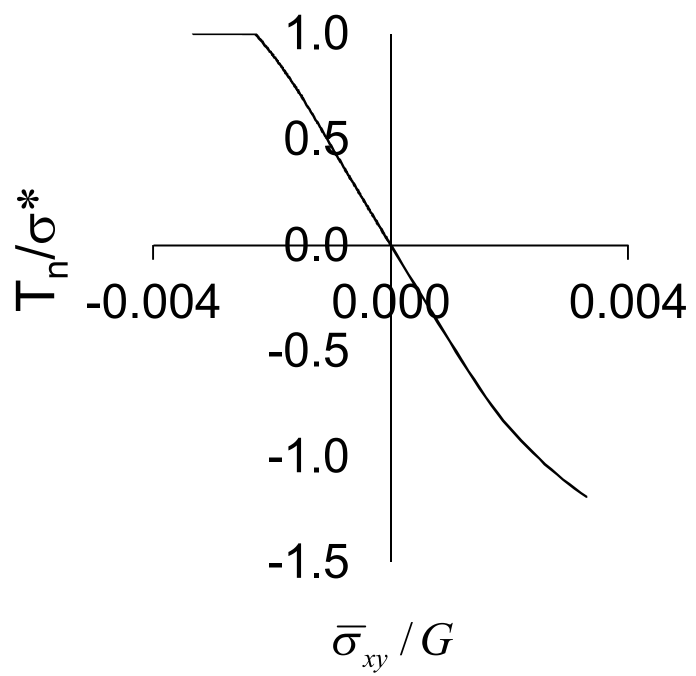
Discussion: Asymmetry in Γ_e/Γ vs. ψ_a

- Re-plotted results show how $\bar{\sigma}^c{}_{yy}$ depends on the Ψ_a.
  - $\bar{\sigma}^c{}_{yy}$ is strongly dependent on Ψ_a
  - significant asymmetry wrt ψ_a.
  - when ψ_a=-72°, $\bar{\sigma}^c{}_{yy} / \bar{\sigma}^{co}{}_{yy} = 0.62$
  - when ψ_a=+72°, $\bar{\sigma}^c{}_{yy} / \bar{\sigma}^{co}{}_{yy} = 1.51$

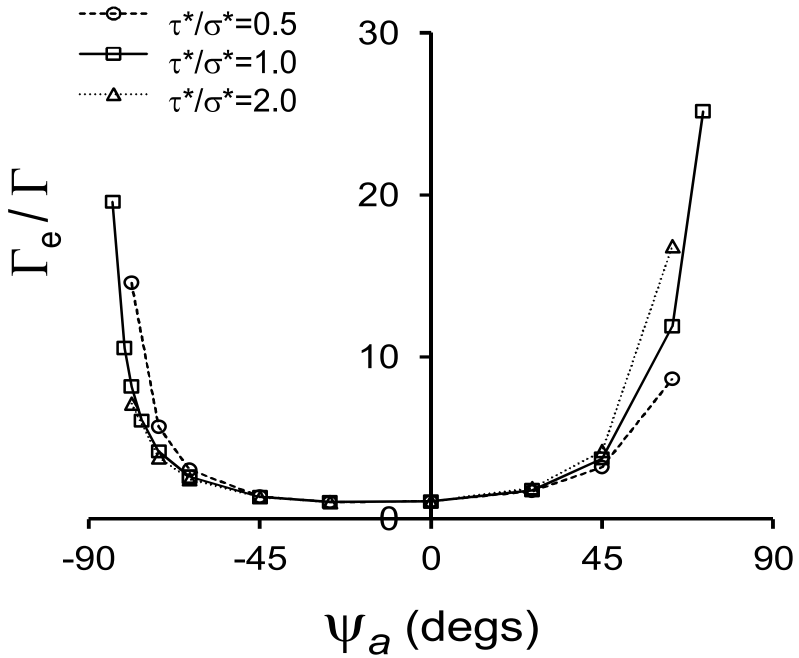


Discussion: Asymmetry in Γ_e/Γ vs. ψ_a

- Consider case where only +/- tangential edge displacement ($\Psi_a = \pm\infty$).
  - plot normal traction T_n at tip of initial crack tip vs. applied $\bar{\sigma}_{xy}$.
  - when $\Psi_a = +\infty$, $T_n < 0$
    - must overcome compression before normal edge displacement can open the interface.
  - when $\Psi_a = -\infty$, $T_n > 0$
    - only need to augment tension already induced by $\bar{\sigma}_{xy}$ to open the interface.
  - Similar response observed in experiments, Liechti and Chai (1992).



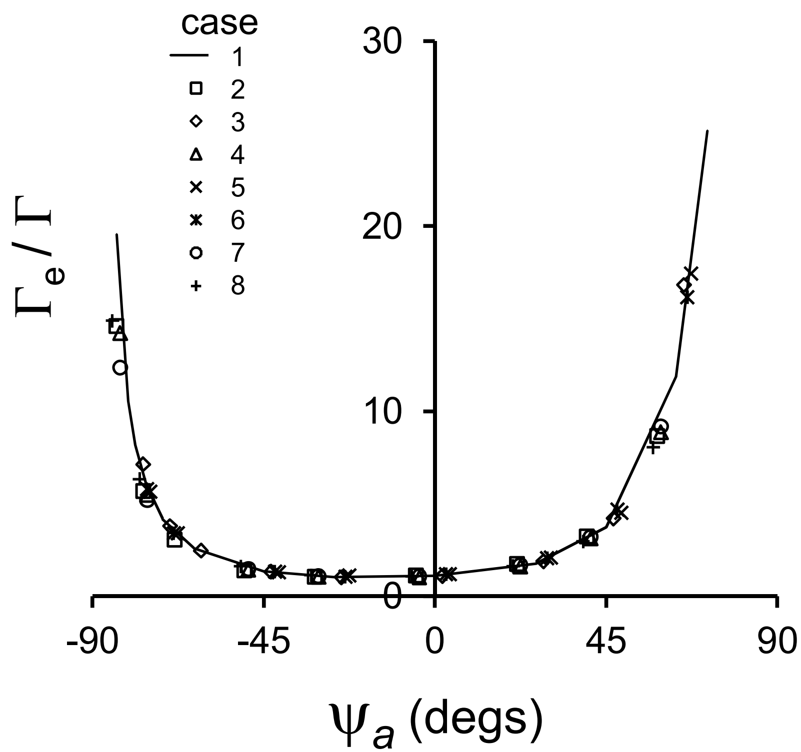
Dependence of Γ_e/Γ *vs.* ψ_a relationship on τ^*/σ^*



- Baseline ($\Gamma/(h\sigma^*)$=1e-4, σ^*/E=0.010) but vary τ^*/σ^*.
- Doubling or cutting τ^*/σ^* in half does not alter the basic shape.
- The τ^*/σ^*=*0.5 and* τ^*/σ^*=2.0 curves can be aligned with the τ^*/σ^*=1.0 curve by shifting the corresponding curve by -5^o (+2^o).

Discussion: A constant Ψ_a offset ~aligns calculated Γ_e/Γ vs. ψ_a

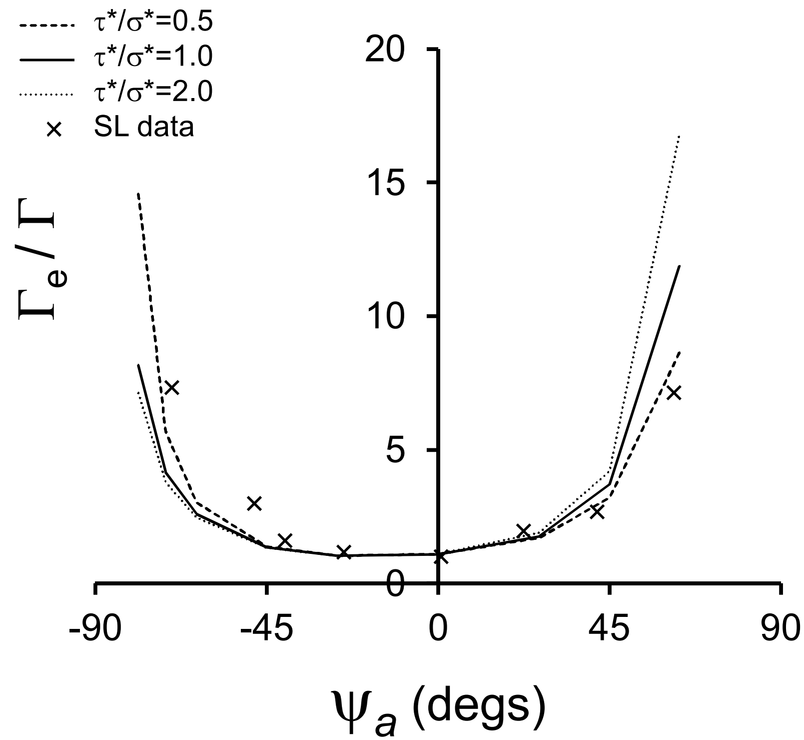
Case	τ^*/σ^*	σ^*/E	$\Gamma/(\sigma^* h)$	$\Delta\Psi_a$ (deg)
1	1.0	0.010	1e-4	-
2	0.5	0.010	1e-4	-5
3	2.0	0.010	1e-4	+2
4	1.0	0.005	1e-4	-4
5	1.0	0.020	1e-4	+4
6	1.0	0.010	5e-5	+3
7	1.0	0.010	2e-4	-4
8	1.0	0.025	1e-3	-6



- Apply a separate mode-mixity offset $\Delta\Psi_a$ to each of the calculated Γ_e/Γ vs. ψ_a relationships.
  - this is simply an observation

Discussion: comparison with experimental observation

- Compared calculated results to Swadener and Liechti (SL) data
- test geometry similar to that analyzed.
  - glass/epoxy interface with an ~ 0.25 mm thick epoxy bond.
- Data in good agreement with calculated baseline results (τ^*/σ^*=0.5).
  - MDGc CZM generates the Γ_e/Γ *vs.* ψ_a similar to that measured.
  - displays similar asymmetric response.
- Shape of calculated Γ_e/Γ *vs.* ψ_a relationship is not predefined.
  - shape of the relationship could differ if one analyzes a different type of specimen (not SCY).



Swadener, J. G. and Liechti, K. M., "Asymmetric Shielding Mechanisms in the Mixed-Mode Fracture of a Glass/Epoxy Interface", Journal of Applied Mechanics 65(1998), pp. 25-29.

Normalize data, convert to applied mixity with "rigid" glass as "material 1".

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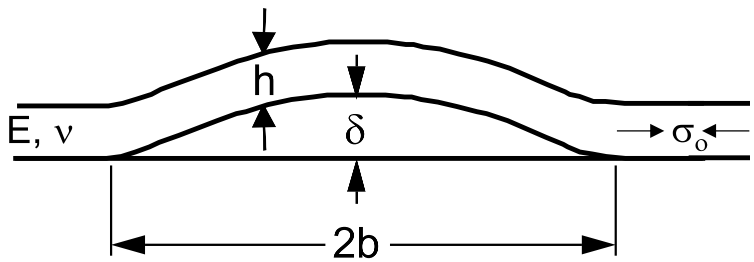
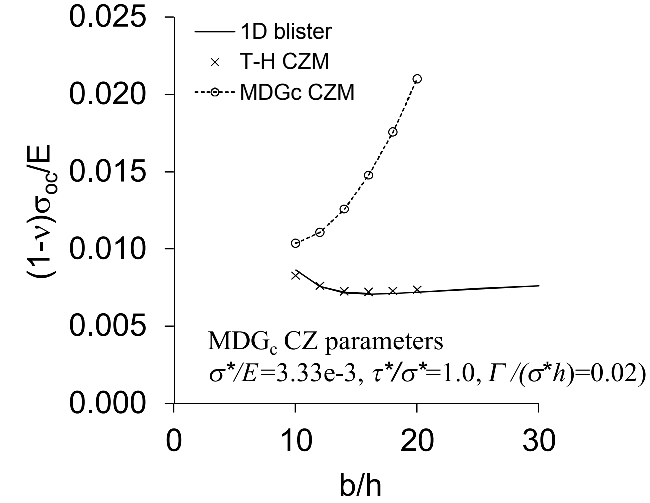
2^{nd} example: buckle-driven growth of 1-D blister on rigid substrate

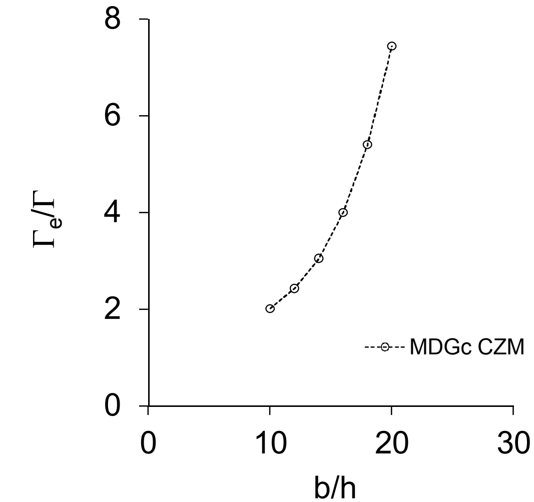


- Film subjected to increasing compressive biaxial stress reaches until reaches critical value σ_{oc} and initial buckled delamination extends.

- Results for mode-mixity independent T-H CZM compared to mode-mixity dependent MDG_c CZM.

- MDGc CZM predictions consistent with observation, sidewalls arrest.





Summary

- Conceptually simple MDGc CZM is composed of two elements.
  - Mode I dissipation is defined by a simple traction-separation law relating normal stress and opening.
  - Mode II (III) dissipation is associated with interfacial shear yielding (slip) that occurs in front of the region where Mode I softening occurs.
- Analyzed an edge-cracked elastic layer sandwiched between rigid grips
  - MDG_c CZ model generates an Γ_e/Γ *vs.* ψ_a relationship that increases rapidly with | ψ_a |.
  - Calculated results for Γ_e/Γ *vs.* ψ_a are in good agreement with published data and display a similar asymmetric response.