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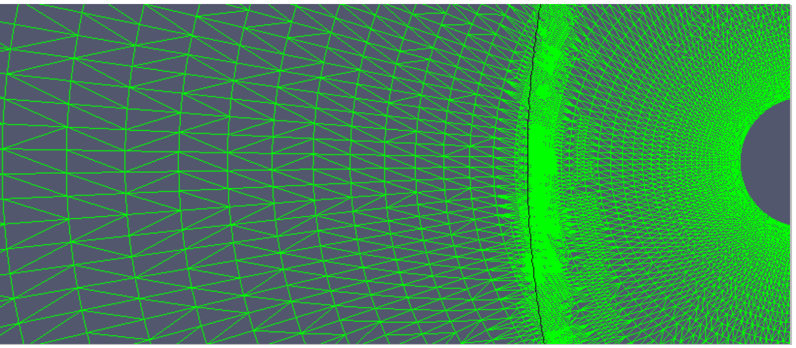

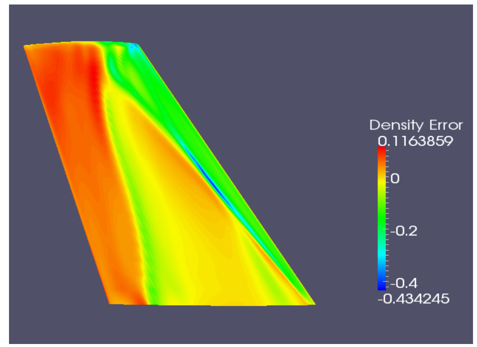

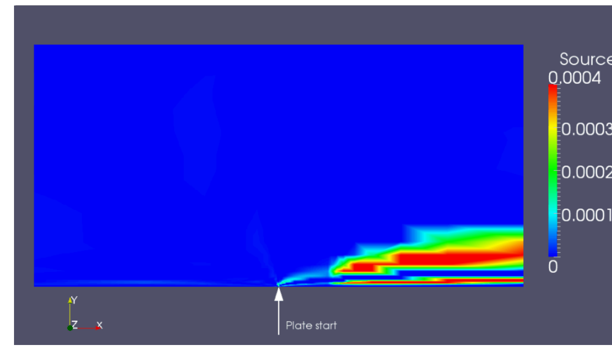

A Discrete Error Transport Equation Source Model for Mesh Adaptation

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52nd Aerospace Sciences Meeting

Overview

- Introduction
- Governing equations
- Error Transport Equations
- Error Source Models
- Comparison of Error Source Models
- Error for Mesh Adaptation
- Mesh Adaptation Study
- Conclusion

Introduction

- CFD is increasingly relied upon for its predictive capabilities.
- One of the primary shortcomings of CFD is the inability to robustly predict error in simulation results.
- Of particular interest is spatial discretization error (controllable) to guide mesh refinement.
- The discrete error transport equations offer a method to estimate local discretization error for targeted *in situ* refinement.
- DETE suffers from a requirement for accurate and well-behaved error source models. A method of approximating this error source for general fluxes is presented here.

Governing Equations

Sandia National Laboratories

- 3D Euler equations

$$\frac{\partial}{\partial t}\int_{\Omega} Q \, d\mathcal{V} + \int_{\partial\Omega} F(Q)\cdot\hat{\vec{n}}\, dA = 0$$

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e_t \end{bmatrix} \qquad F(Q)\cdot\hat{\vec{n}} = \begin{bmatrix} \rho\theta \\ \rho u\theta + P\hat{n}_x \\ \rho v\theta + P\hat{n}_y \\ \rho w\theta + P\hat{n}_z \\ \rho h_t\theta \end{bmatrix}$$

$$\theta = \hat{n_x}u + \hat{n_y}v + \hat{n_z}w$$

- Discretized for unstructured, general element meshes within the Sandia National Labs' SIERRA Gas Dynamics Module: Conchas.

Discrete Error Transport Equations

- 3D error transport equations are formulated by subtracting a discrete solution from the "exact" solution.

$$\frac{\partial}{\partial t}\int_{\Omega}(Q - Q_H)\, d\mathcal{V} + \int_{\partial\Omega} F(Q) - F(Q_H)\cdot \hat{\vec{n}}\, dA = -\int_{\Omega} R(Q_H)\, d\mathcal{V}$$

- The error term is defined as the difference between the "exact" solution vector and the discrete version.

$$\frac{\partial}{\partial t}\int_{\Omega}\epsilon\, d\mathcal{V} + \int_{\partial\Omega} A(Q_H)\epsilon\cdot \hat{\vec{n}}\, dA = -\int_{\Omega} R(Q_H)\, d\mathcal{V}$$

- The term on the right hand side is referred to as the error source (residual) and can be derived exactly via Richardson's extrapolation. However, here we assume that expense is too great or impossible with current computing resources.

Error Source Models – Dissipation

- Zhang *et al* suggest utilizing the leading terms of a standard 1D truncation error analysis (Roe's flux) for computing the error source term.

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} = -\frac{1}{2\Delta x}\left(|\tilde{A}_{i+1/2}|(Q_{i+1} - Q_i) - |\tilde{A}_{i-1/2}|(Q_i - Q_{i-1}) \right) - \frac{\Delta t}{2}\frac{\partial^2 Q}{\partial t^2} - \frac{\Delta x^2}{6}\frac{\partial^3 F}{\partial x^3} - \frac{\Delta t^2}{6}\frac{\partial^3 Q}{\partial t^3} + O(\Delta x^3, \Delta t^3)$$

- Cavallo extended this to 3D unstructured grids

$$R_i(Q_H) = -\frac{1}{2V_i}\sum_{j=1}^{N faces} (|\tilde{A}_j|\,\Delta Q_j)\cdot \vec{n}$$

Error Source Models - MUSCL

- Utilizing a MUSCL type extrapolation in 3D we can write the conservative variable values at a face location.

$$Q^{HO} = Q + \nabla Q \cdot \vec{r}$$

- Taking the difference between these extrapolated values and the averaged values (1st and 2nd order spatially) results in an approximate error source.

$$R_i(Q_H) = \frac{1}{V_i} \int_{\partial\Omega} (F(Q_H)^{HO} - F(Q_H)^{LO}) \cdot \hat{\vec{n}}\, dA$$

- This error source is generally applicable to any flux formulation and does not have to be re-derived when changing fluxes (HLLC, HLLE, Roe, AUSM, etc.)

Error Source Models – MUSCL

- Another benefit of using the MUSCL flux model is its applicability to viscous fluxes.

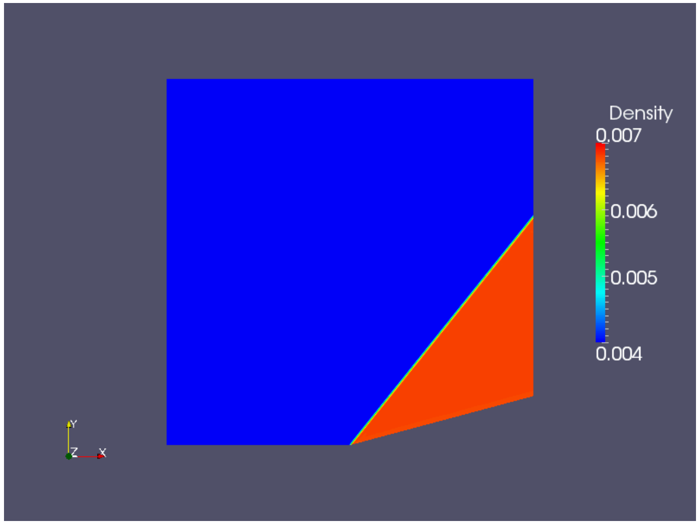
$$(\nabla q)^{HO} = \frac{1}{2}(\nabla q_L + \nabla q_R) + [q_R - q_L - \frac{1}{2}(\nabla q_L + \nabla q_R) \cdot \vec{\Delta s}]\frac{\vec{\Delta s}}{|\vec{\Delta s}|^2}$$

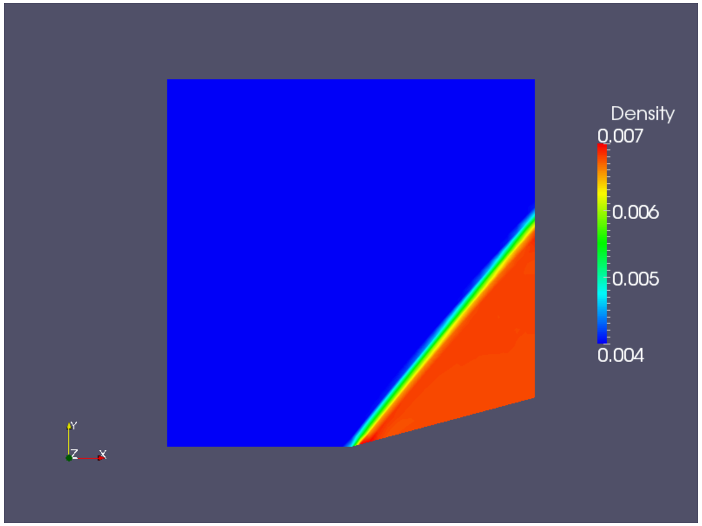
$$(\nabla q)^{LO} = \frac{1}{2}(\nabla q_L + \nabla q_R)$$

- This difference in fluxes can be surface integrated in the same way as the convective source.
- Allows for an approximation of error introduced into the viscous fluxes based on spatial discretization.

Comparison of Error Source Models

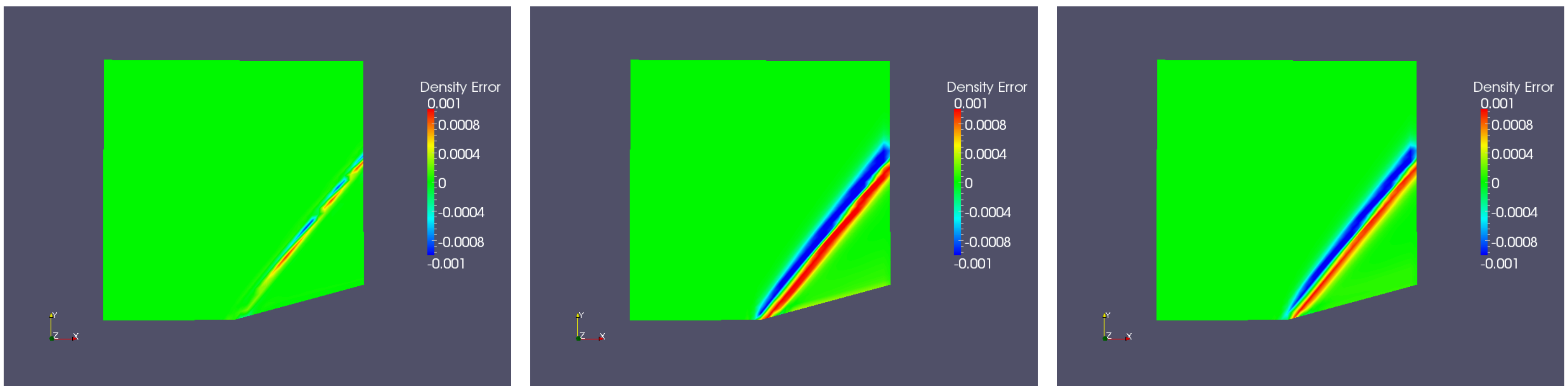
- 15 degree supersonic ramp. Mach 1.9 flow.
- Fine mesh 250 x250 nodes
- Coarse mesh 40 x 40 nodes





Comparison of Error Source Models

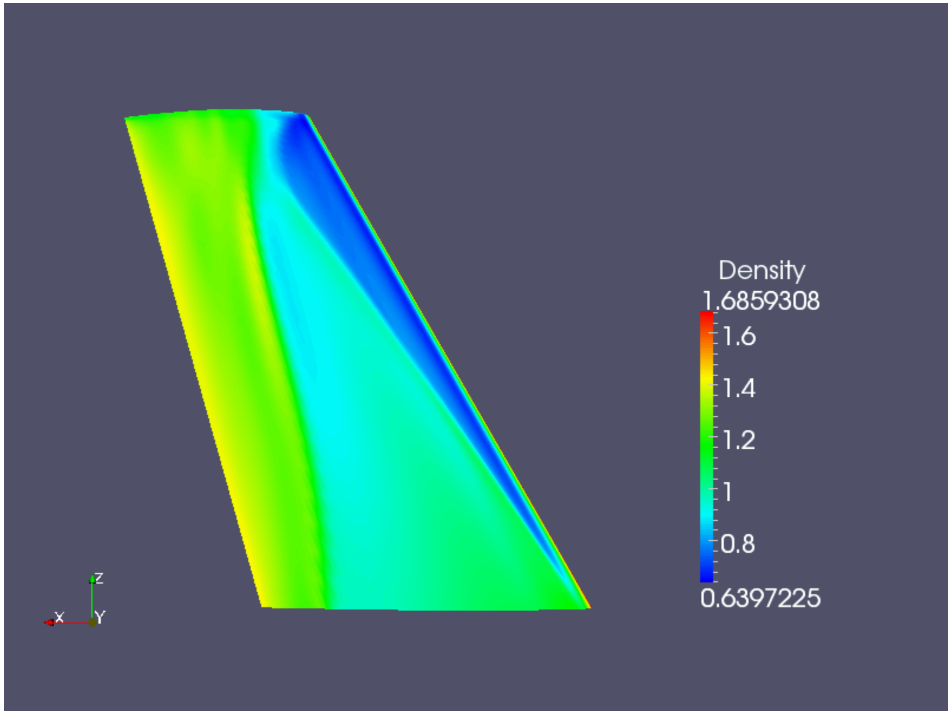
- Maximum error in dissipation model (Center) is nearly twice actual error. MUSCL based error model (Right) predicts half the maximum error in comparison and shows only ~17% relative difference when compared to the actual error (Left).



Density error shown clipped for comparison purposes

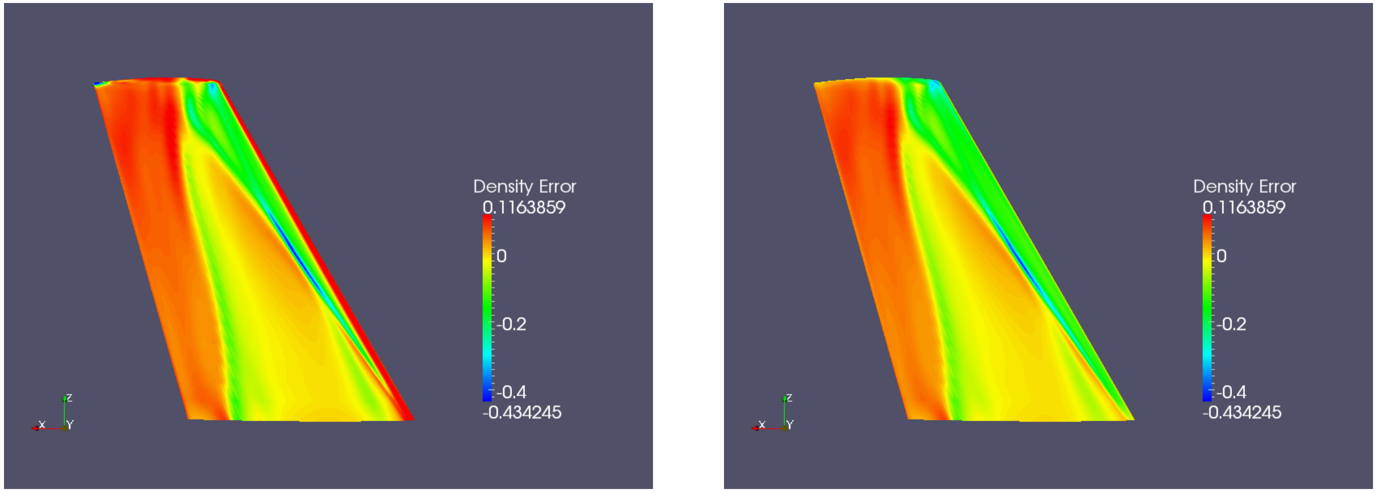
Comparison of Error Source Models

- Onera M6 wing at Mach 0.8395 with an angle of attack of 3.06 degrees.



Comparison of Error Source Models

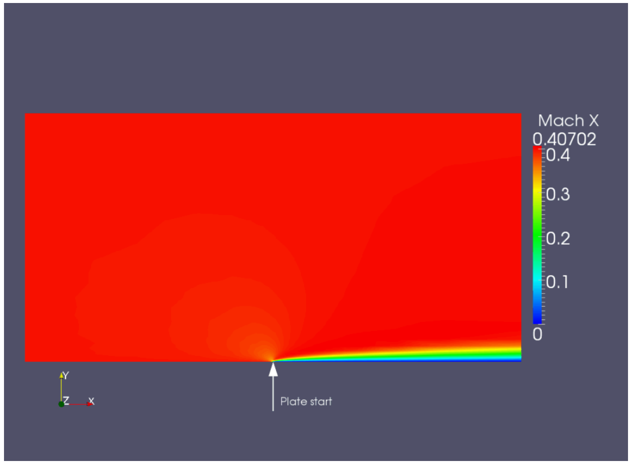
- Computed error with dissipation model is three times the maximum value of density (Left). Noted in literature that the dissipation model has difficulty with overshoots in the vicinity of sharp gradients. DETE is non-diffusive.
- Computed error with MUSCL model is more well-behaved (Right).

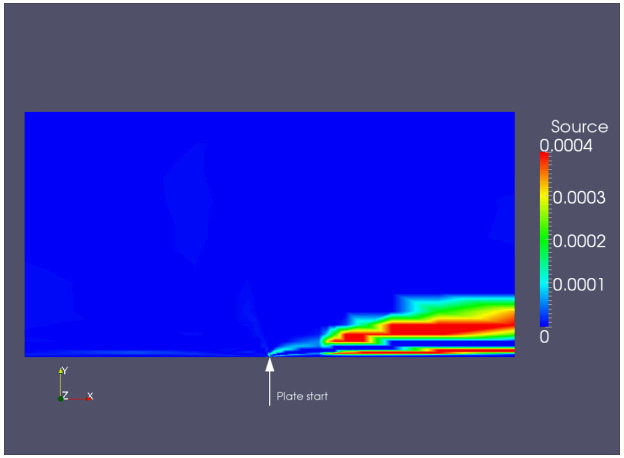


Values clipped for comparison purposes

Comparison of Error Source Models

- Laminar flat plate Reynold's number of 500. Mach 0.4.
- Shown is the viscous error source for the x-momentum equation utilizing the MUSCL model.
- It is unclear at this time whether this model will allow for appropriate grid refinement in difficult problems (shear dominated).





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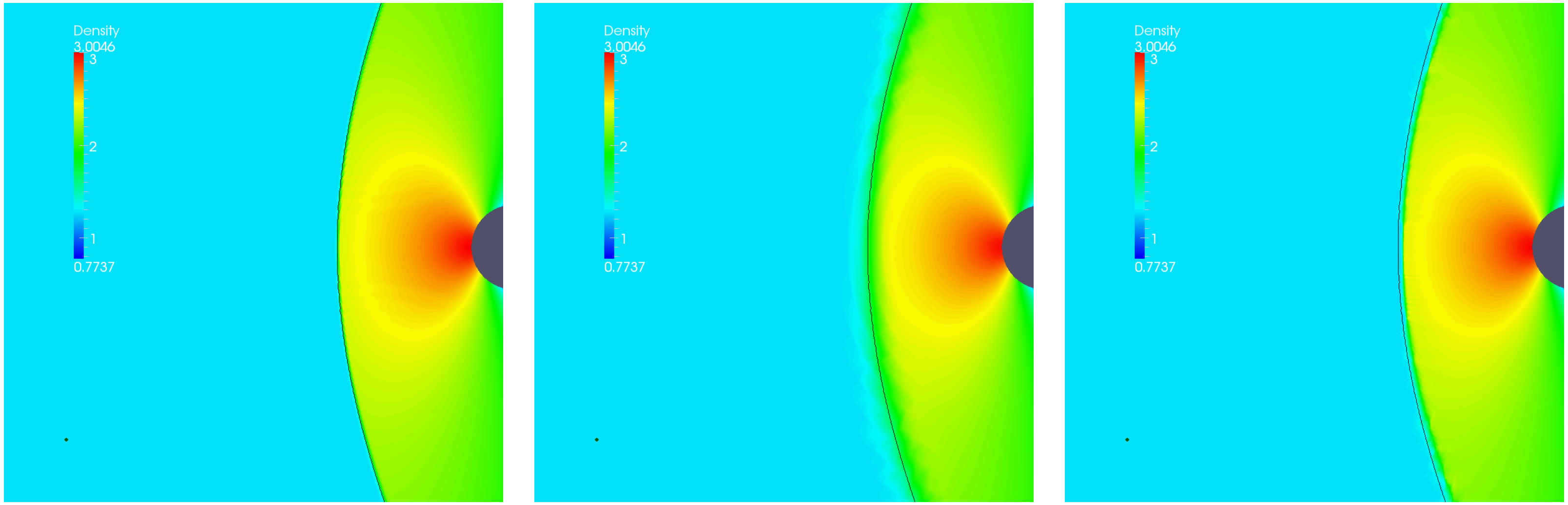
Error For Mesh Adaptation

- There are three basic requirements if we expect error estimators and automated refinement to be useful in solving engineering problems.
  - The error indicator must be computed at relatively low expense. Currently many large scale simulations cannot be uniformly refined due to computational cost.
  - The error indicator must be at least as robust as the core flow solver.
  - The error indicator must be capable of directing refinement in regions which are non-obvious to the end-user. The end goal of refinement is increased accuracy in engineering quantities (Cd, Cl, etc.)
- If these three requirements can be met, we will have a useful tool for *in situ* mesh adaptation which would greatly relieve the human cost in generating "good" meshes.

Mesh Adaptation Study

- Comparison of “truth” mesh, coarse mesh, and refined mesh based on ETE solution.



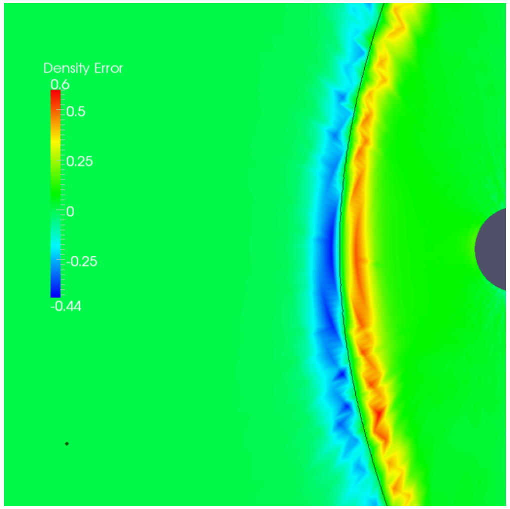
- Refinement reduces normed error by over 60%

	Number of nodes	$\|\text{Actual Error}\|_2$
Fine Mesh	81,929	–
Coarse Mesh	5,000	.07217
Refined Mesh	21,932	.02607

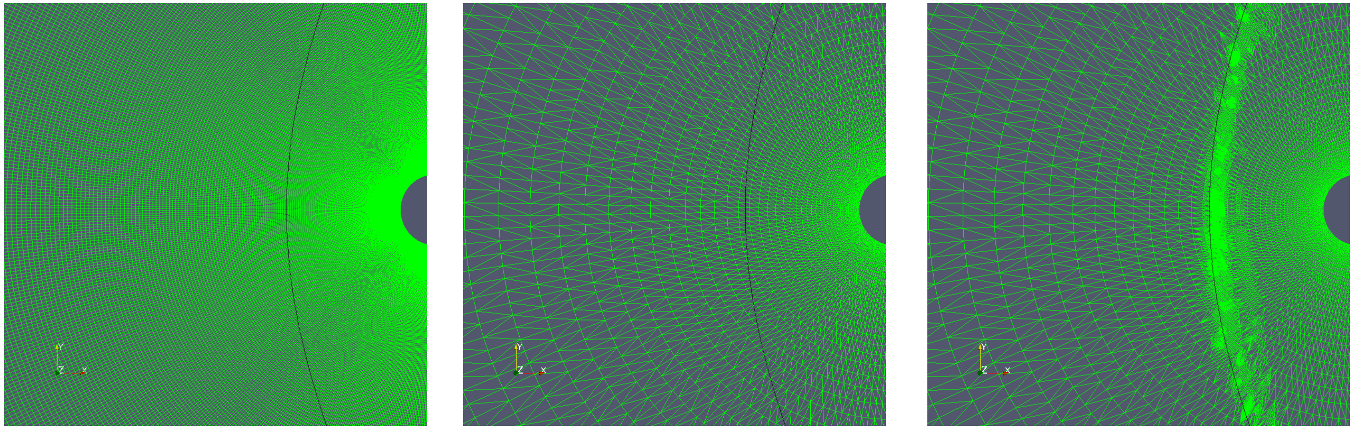
Mesh Adaptation Study

- Computed density error spans the

Shock location computed on "truth" mesh



- Refinement sequence meshes



Conclusion

- A new MUSCL extrapolation based error source model is introduced within the Sandia National Labs' SIERRA Gas Dynamics Module: Conchas.
- The model compares favorably with existing models published in literature, namely, dissipation based models.
- MUSCL model allows for reliable prediction and refinement of under-resolved regions in supersonic problems.