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Concrete Fracture and Failure Modeling with Peridynamics

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Outline

- Theory
  - What makes this different?
- Single crack growth
- Concrete applications
  - Perforation
    - Single panel: effect of impact angle
    - Multiple panels
    - Effect of reinforcement
  - Damage accumulation due to multiple impacts
  - Fragmentation and fragment distribution
  - Blast loading
  - Deep penetration
    - Target size effect

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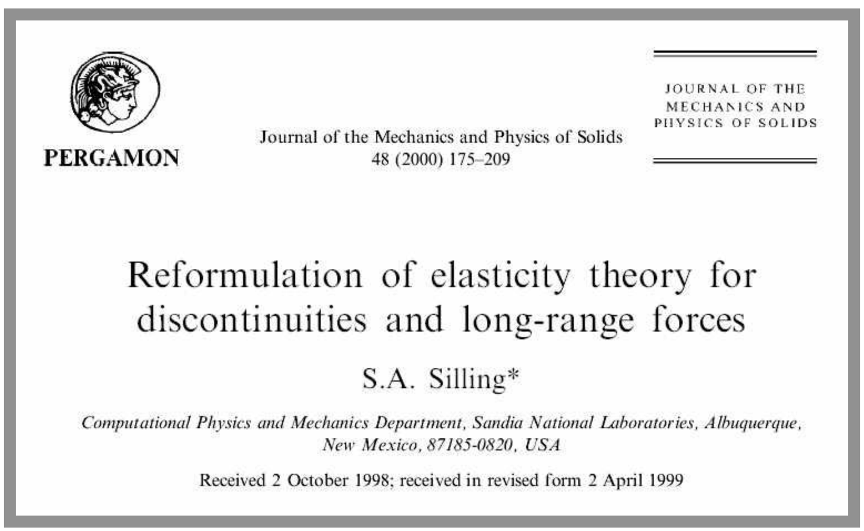
Peridynamic theory:
What makes this different

- Why is it so hard to model fracture with conventional finite element codes?
  - The fundamental PDEs do not apply on a crack.
- New approach: <u>peridynamic</u> theory uses integral rather than differential equations.
  - Reformulation of the fundamental equations.
  - Equations apply everywhere regardless of discontinuities.
  - No need for externally supplied “crack growth law”.
  - Cracks initiate, grow spontaneously.
  - Theory first published in 2000:



PERGAMON

Journal of the Mechanics and Physics of Solids
48 (2000) 175–209

JOURNAL OF THE MECHANICS AND PHYSICS OF SOLIDS

Reformulation of elasticity theory for discontinuities and long-range forces

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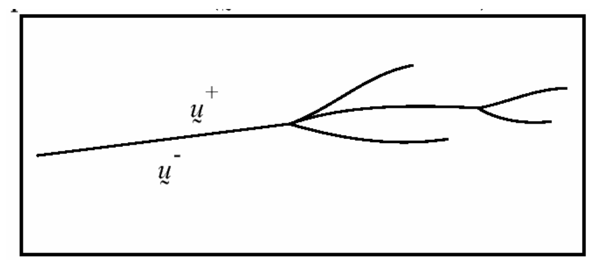

Real life:
Discontinuities can evolve in complex patterns not known in advance.

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Peridynamic theory: basic equations

Classical theory:

$$\rho \ddot{\underset{\sim}{u}}(\underset{\sim}{x}, t) = \nabla \bullet \underset{\sim}{\sigma}(\underset{\sim}{x}, t) + \underset{\sim}{b}(\underset{\sim}{x}, t)$$

where ρ = density, $\underset{\sim}{u}$ = displacement, $\underset{\sim}{\sigma}$ = stress tensor field, and $\underset{\sim}{b}$ = body force field.

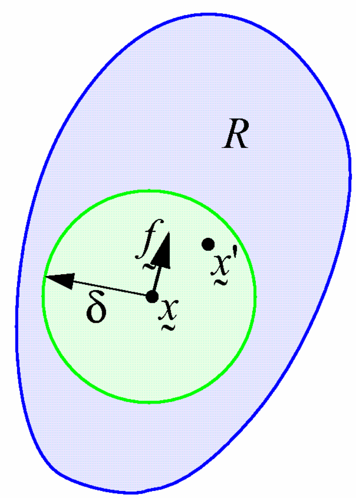
Peridynamic theory:

$$\rho \ddot{\underset{\sim}{u}}(\underset{\sim}{x}, t) = \underset{\sim}{L}_u(\underset{\sim}{x}, t) + \underset{\sim}{b}(\underset{\sim}{x}, t)$$

where

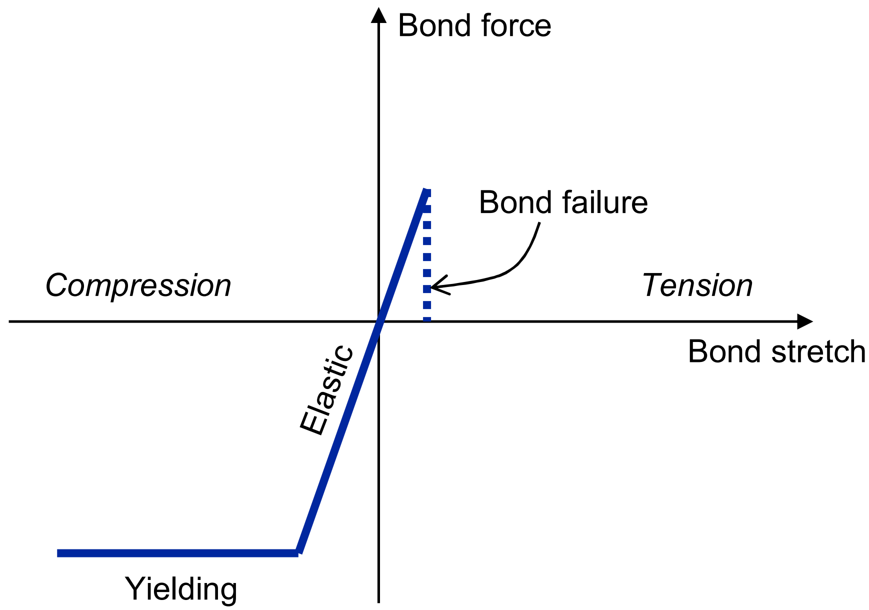
$$\underset{\sim}{L}_u(\underset{\sim}{x}, t) = \int_R \underset{\sim}{f}(\underset{\sim}{u}(\underset{\sim}{x}', t) - \underset{\sim}{u}(x, t), \underset{\sim}{x}' - \underset{\sim}{x}) dV_{\underset{\sim}{x}'}$$

and $\underset{\sim}{f}$ is a vector-valued function.



Peridynamic theory: Material model

- All material-specific behavior is contained in the function ***f***.
- Material parameters come from measurable elastic-plastic and fracture data.

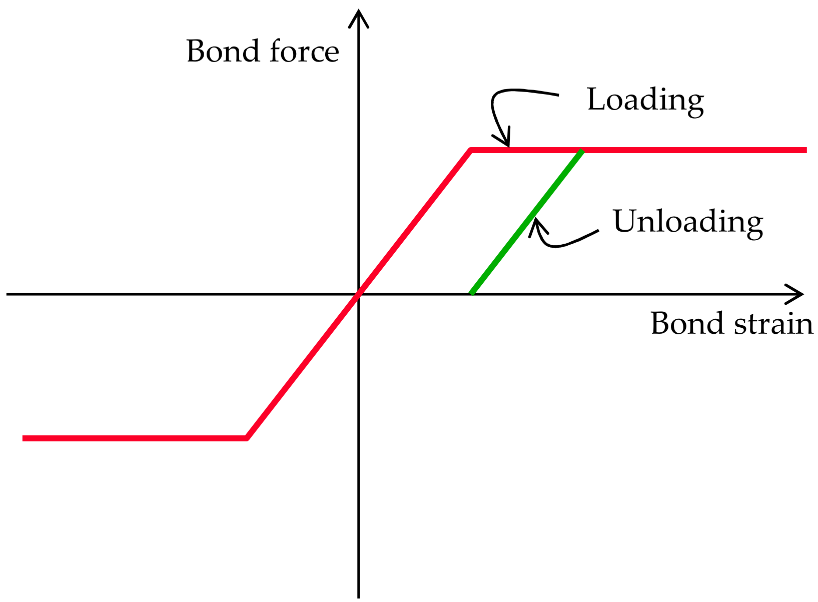


Peridynamic theory: Other constitutive models

- Visco-microelastic:

$$\text{bond force} = f(r, \dot{r}, \xi), \quad r = |\xi + \eta| = \text{current distance between } x \text{ and } x'$$

- Microplastic:



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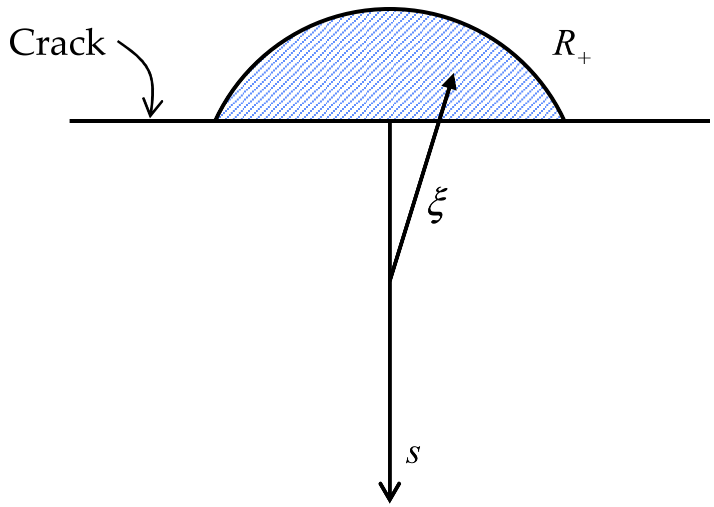
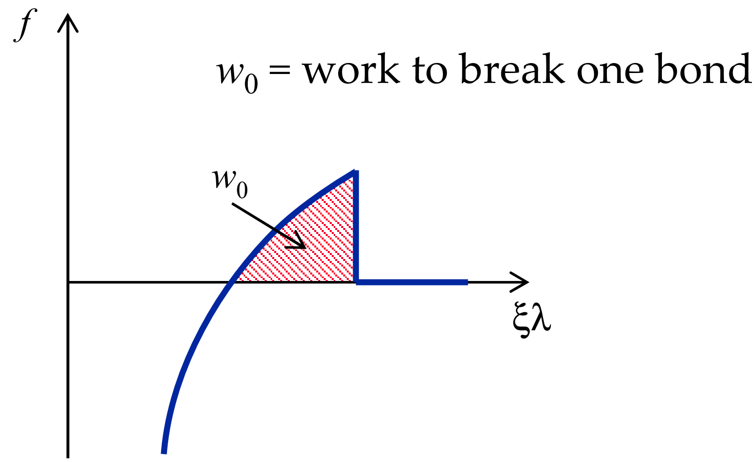
Peridynamic theory:
Energy required to advance a crack

- Adding up the work needed to break all bonds across a line yields the energy release rate:

$$G = 2h\int_0^\delta \int_{R_+} w_0 \, dV ds$$





There is also a version of the J-integral that applies in this theory.

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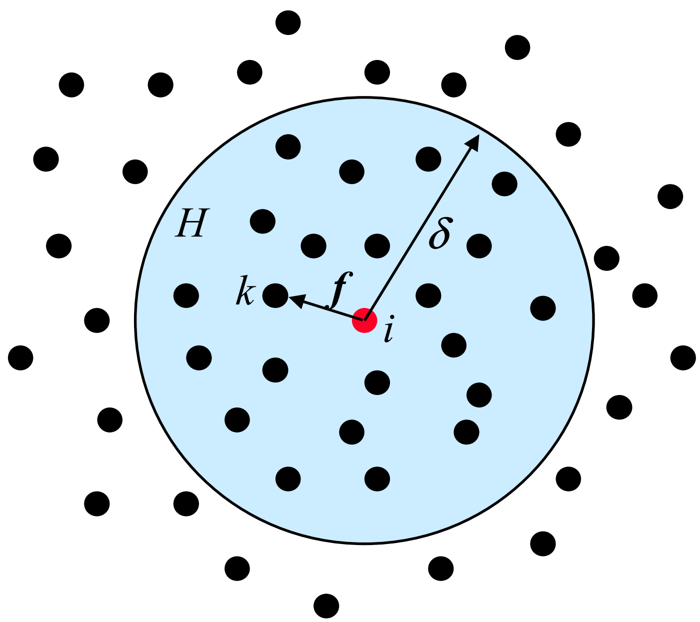
EMU numerical method

- Integral is replaced by a finite sum.
- The resulting method is meshless and Lagrangian.

$$\rho\, \ddot{\boldsymbol{u}}_i^n = \sum_{k \in H} \boldsymbol{f}(\boldsymbol{u}_k^n - \boldsymbol{u}_i^n, \boldsymbol{x}_k - \boldsymbol{x}_i)\Delta V_i + \boldsymbol{b}(\boldsymbol{x}_i, t)$$



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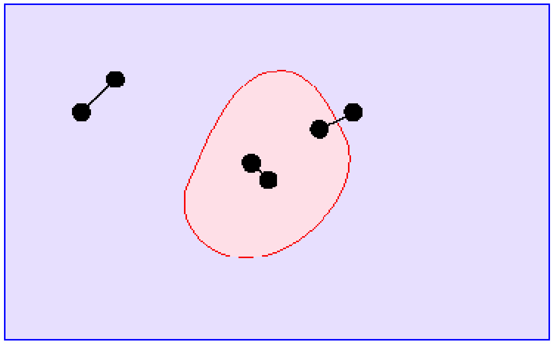
EMU numerical method:
Code features

- Meshfree (no elements)
- Lagrangian (each node represents a fixed amount of material)
- Parallel (runs on multiple processors)
- Explicit (simple, reliable time integration method)
- "Unguided" crack growth.
  - No need for an externally supplied crack growth law for:
    - Initiation, growth velocity, direction, branching, arrest, … .
  - Any number of cracks can occur spontaneously.
  - Interface bonds are treated the same as other bonds.



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EMU numerical method: Relation to SPH

SPH

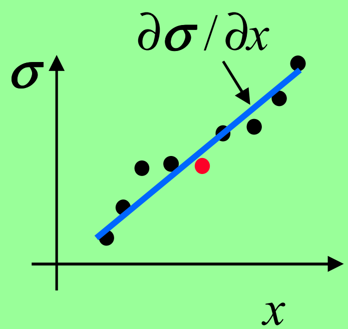
$$\frac{\partial v}{\partial x} = \int v(x')K(x')dV'$$

$$\dot{\boldsymbol{\varepsilon}} = \frac{1}{2}\left(\left(\frac{\partial v}{\partial x}\right) + \left(\frac{\partial v}{\partial x}\right)^T\right)$$

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}(\boldsymbol{\varepsilon})$$

$$\frac{\partial \boldsymbol{\sigma}}{\partial x} = \int \boldsymbol{\sigma}(x')K(x')dV'$$

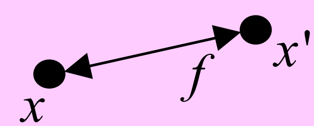
$$\rho \ddot{u} = \frac{\partial \boldsymbol{\sigma}}{\partial x} + b$$



- Both are meshless Lagrangian methods.
- Both involve integrals.
- But the basic equations are fundamentally different:
  - SPH relies on curve fitting to approximate derivatives that appear in the classical PDEs.
  - Peridynamics does not use these PDEs, relies on pair interactions.

Emu

$$\rho \ddot{u}(x) = \int f\left(u(x') - u(x), x' - x\right) dV' + b(x)$$



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Discretized model: Relation to finite elements

- Can solve the peridynamic equations in a framework similar to FE.
- Can include rotational degrees of freedom, leads to a micropolar model.
- Typical element stiffness matrix (for truss-like element)*:

$$\begin{Bmatrix} \hat{f}_{jix} \\ \hat{f}_{jiy} \\ \hat{f}_{jiz} \\ \hat{m}_{jix} \\ \hat{m}_{jiy} \\ \hat{m}_{jiz} \\ \hat{f}_{ijx} \\ \hat{f}_{ijy} \\ \hat{f}_{ijz} \\ \hat{m}_{ijx} \\ \hat{m}_{ijy} \\ \hat{m}_{ijz} \end{Bmatrix} = \begin{bmatrix} \frac{E'A}{L} & & & & & & & & & & & \\ 0 & \frac{12E'I}{L^3} & & & & & & & (sym) & & & \\ 0 & 0 & \frac{12E'I}{L^3} & & & & & & & & & \\ 0 & 0 & 0 & \frac{E'J}{L} & & & & & & & & \\ 0 & 0 & \frac{-6E'I}{L^2} & 0 & \frac{4E'I}{L} & & & & & & & \\ 0 & \frac{-6E'I}{L^2} & 0 & 0 & 0 & \frac{4E'I}{L} & & & & & & \\ \frac{-E'A}{L} & 0 & 0 & 0 & 0 & 0 & \frac{E'A}{L} & & & & & \\ 0 & \frac{-12E'I}{L^3} & 0 & 0 & 0 & \frac{-6E'I}{L^2} & 0 & \frac{12E'I}{L^3} & & & & \\ 0 & 0 & \frac{-12E'I}{L^3} & 0 & \frac{6E'I}{L^2} & 0 & 0 & 0 & \frac{12E'I}{L^3} & & & \\ 0 & 0 & 0 & \frac{-E'J}{L} & 0 & 0 & 0 & 0 & 0 & \frac{E'J}{L} & & \\ 0 & 0 & \frac{-6E'I}{L^2} & 0 & \frac{2E'I}{L} & 0 & 0 & 0 & \frac{6E'I}{L^2} & 0 & \frac{4E'I}{L} & \\ 0 & \frac{6E'I}{L^2} & 0 & 0 & 0 & \frac{2E'I}{L} & 0 & \frac{-6E'I}{L^2} & 0 & 0 & 0 & \frac{4E'I}{L} \end{bmatrix} \begin{Bmatrix} \hat{u}_i \\ \hat{v}_i \\ \hat{w}_i \\ \hat{\theta}_{xi} \\ \hat{\theta}_{yi} \\ \hat{\theta}_{zi} \\ \hat{u}_j \\ \hat{v}_j \\ \hat{w}_j \\ \hat{\theta}_{xj} \\ \hat{\theta}_{yj} \\ \hat{\theta}_{zj} \end{Bmatrix}$$

* W. Gerstle et. al., to appear in *Nuclear Engineering & Design* (2007).

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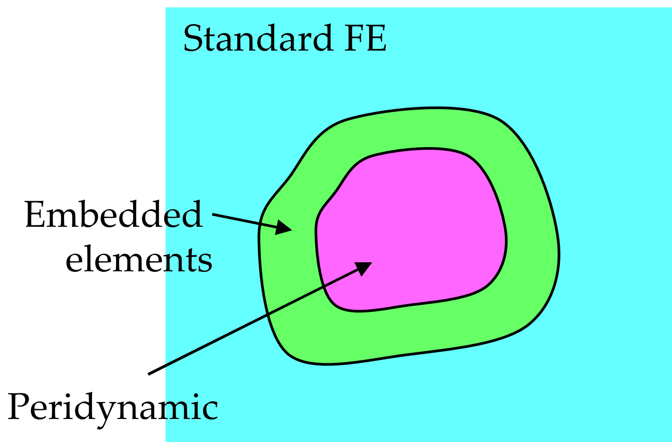
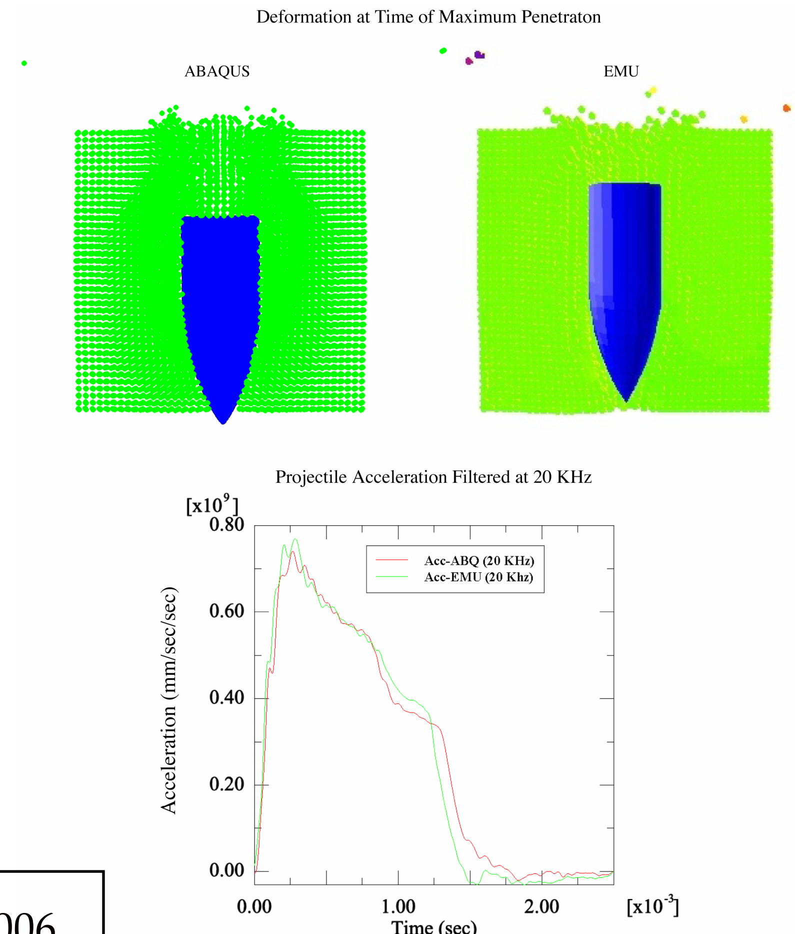
Relation to finite elements: ABAQUS implementation

- The Emu peridynamic solution method has been implemented in a special version of ABAQUS*.
- Can interface peridynamic elements with conventional FE by using embedded elements.





* R. Macek, LANL Report LA-14300, July 2006 .

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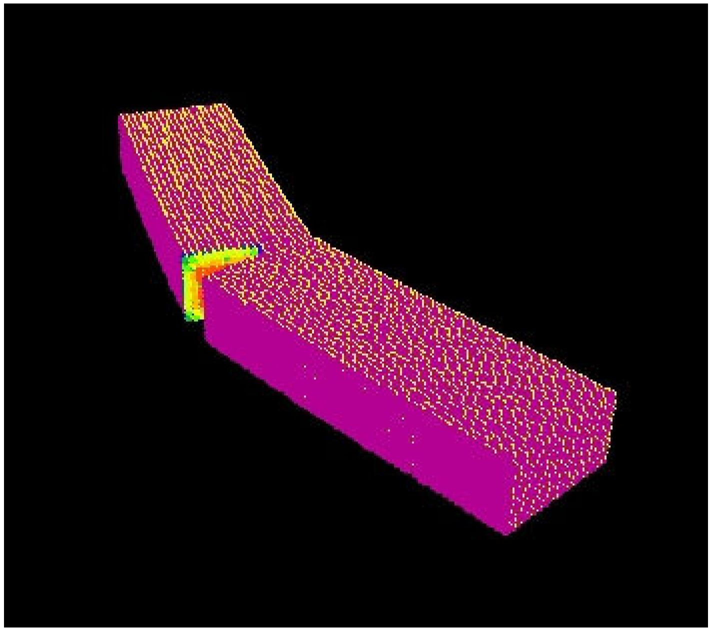
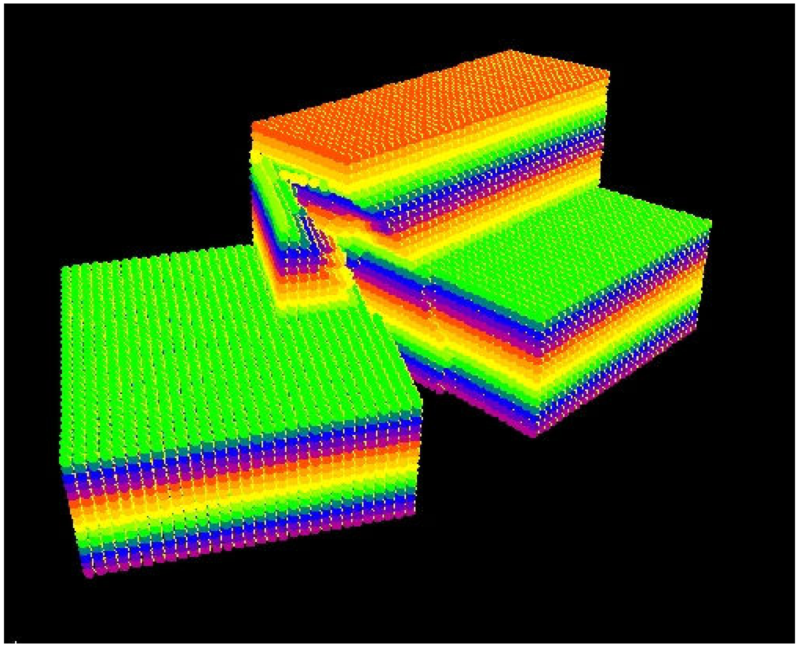
Single crack growth in metals: Examples



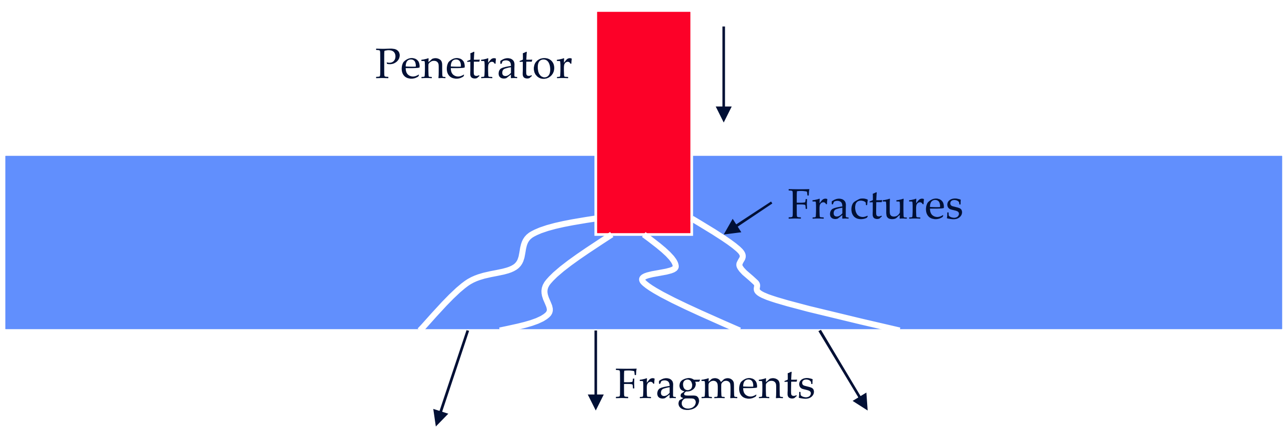
3-point bend



A more complex geometry

Perforation: Why use this method?

- Ability to model fracture is important for perforation.
  - Target starts weakening long before the penetrator gets through.
  - Fracture growth process determines fragment properties.



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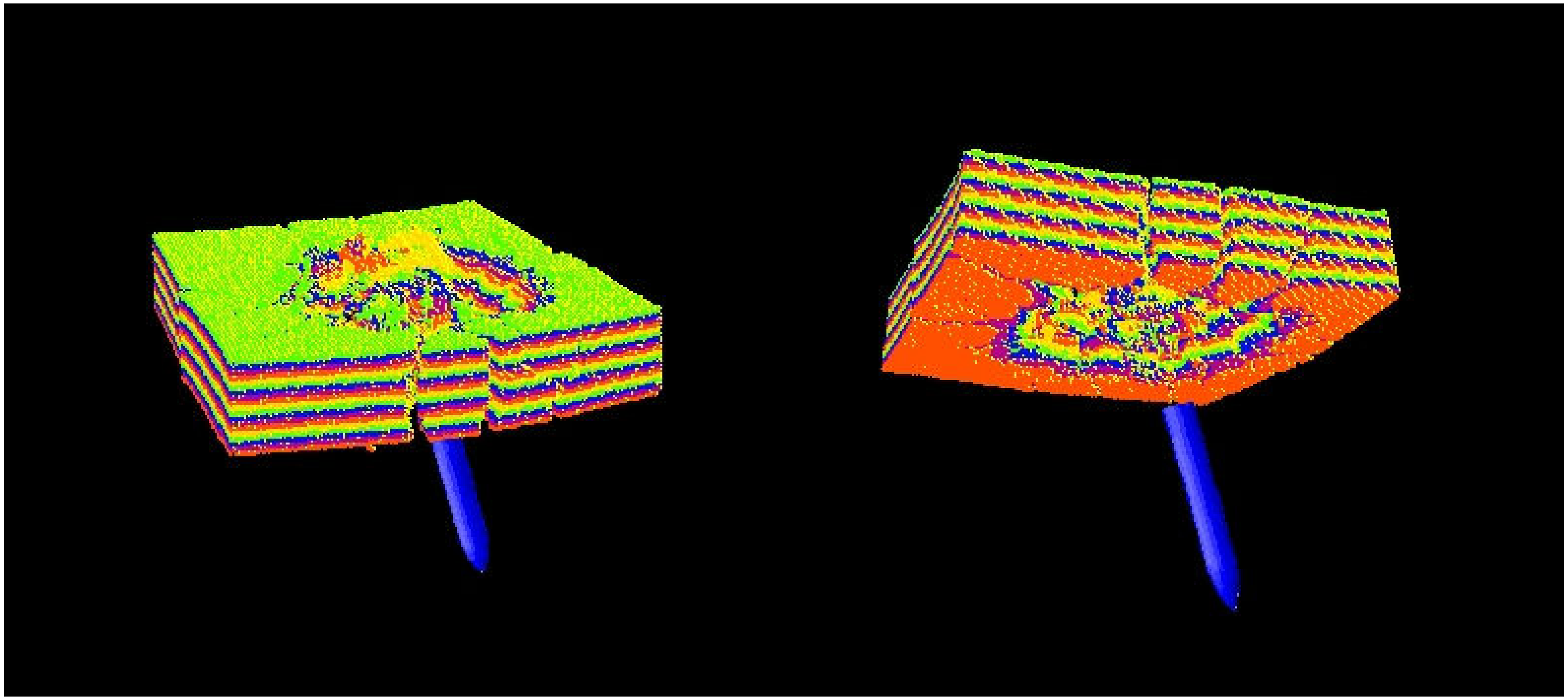
Perforation: Typical results

- Cratering has some effect on the acceleration.
  - Weakening in the exit crater "attracts" the nose (more discussion later).



Entry crater

Exit crater

(Colors are for included for visualization purposes only)

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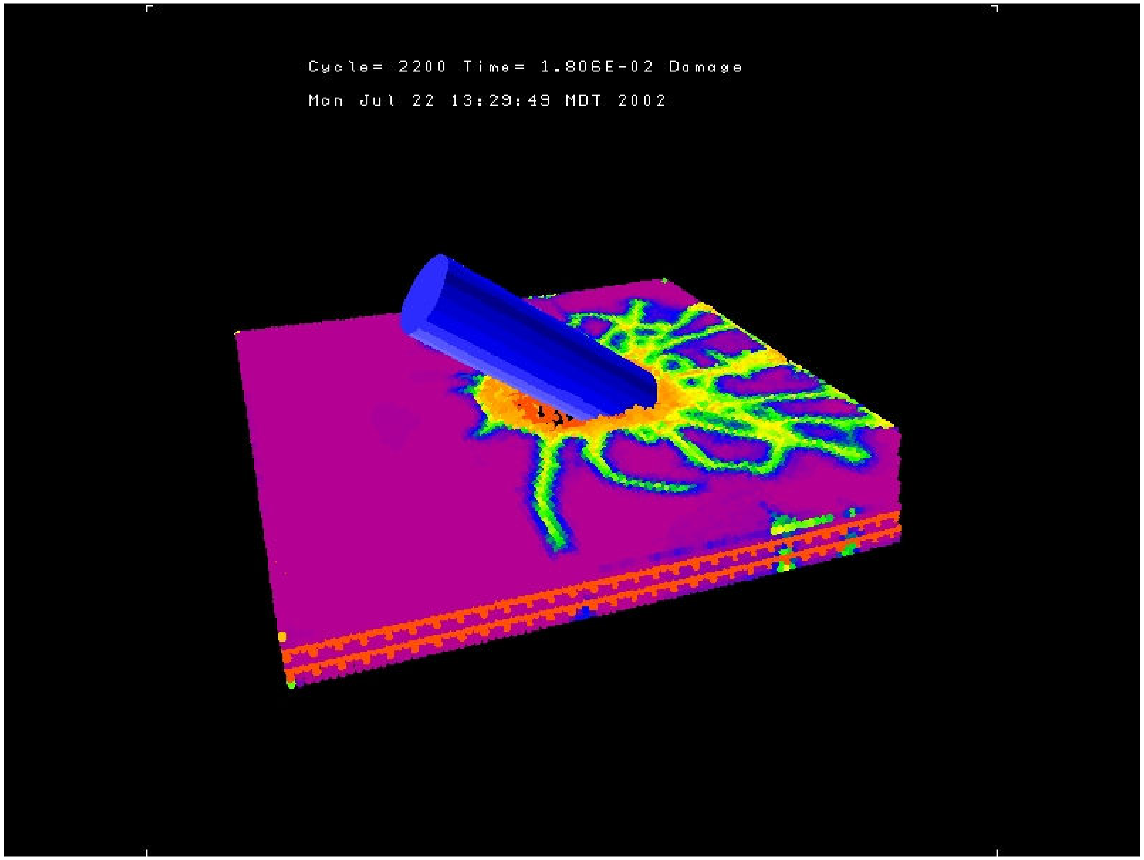
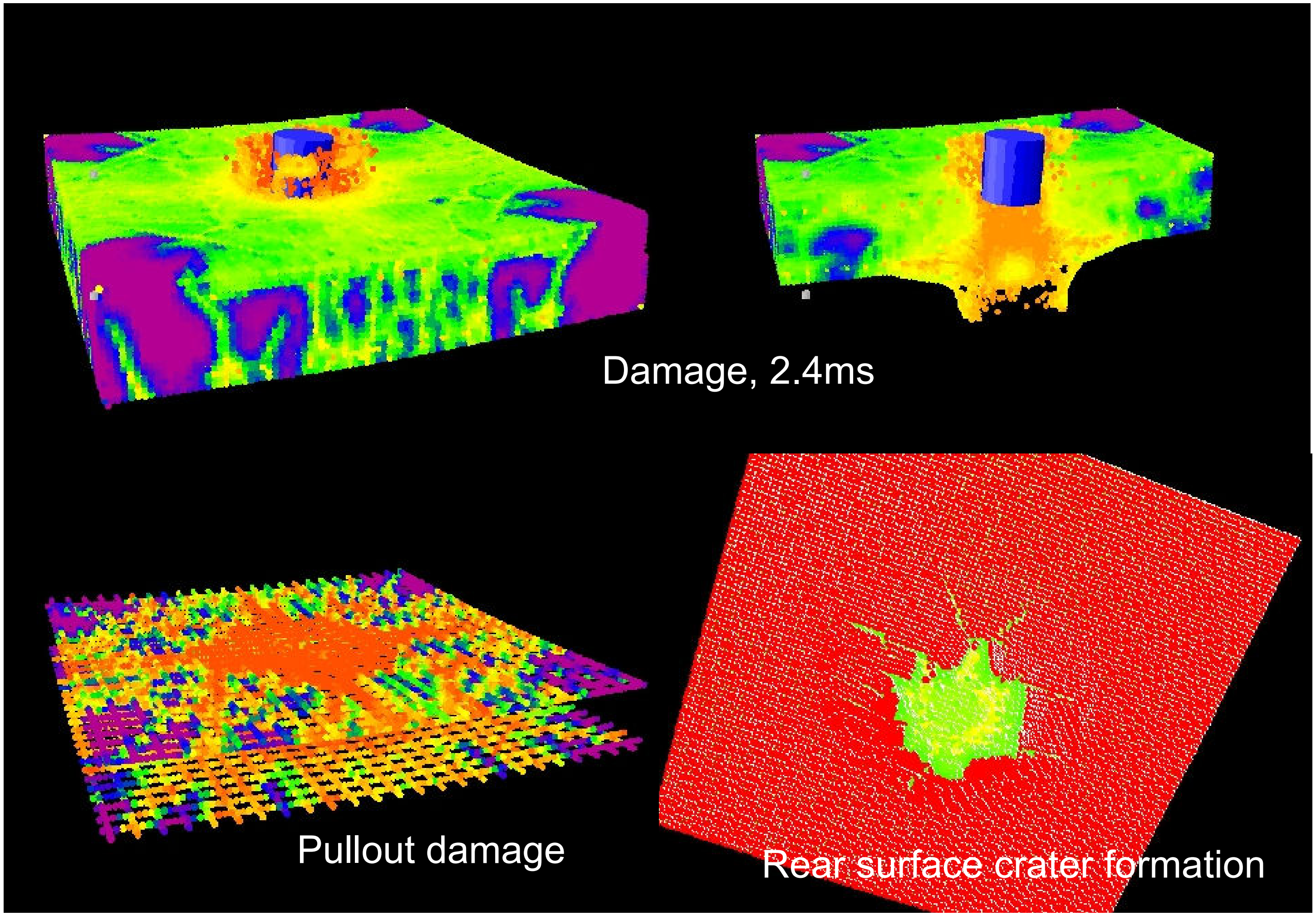
Perforation:
Cracks in a target due to oblique impact



Perforation:
Explicit model of concrete reinforcement



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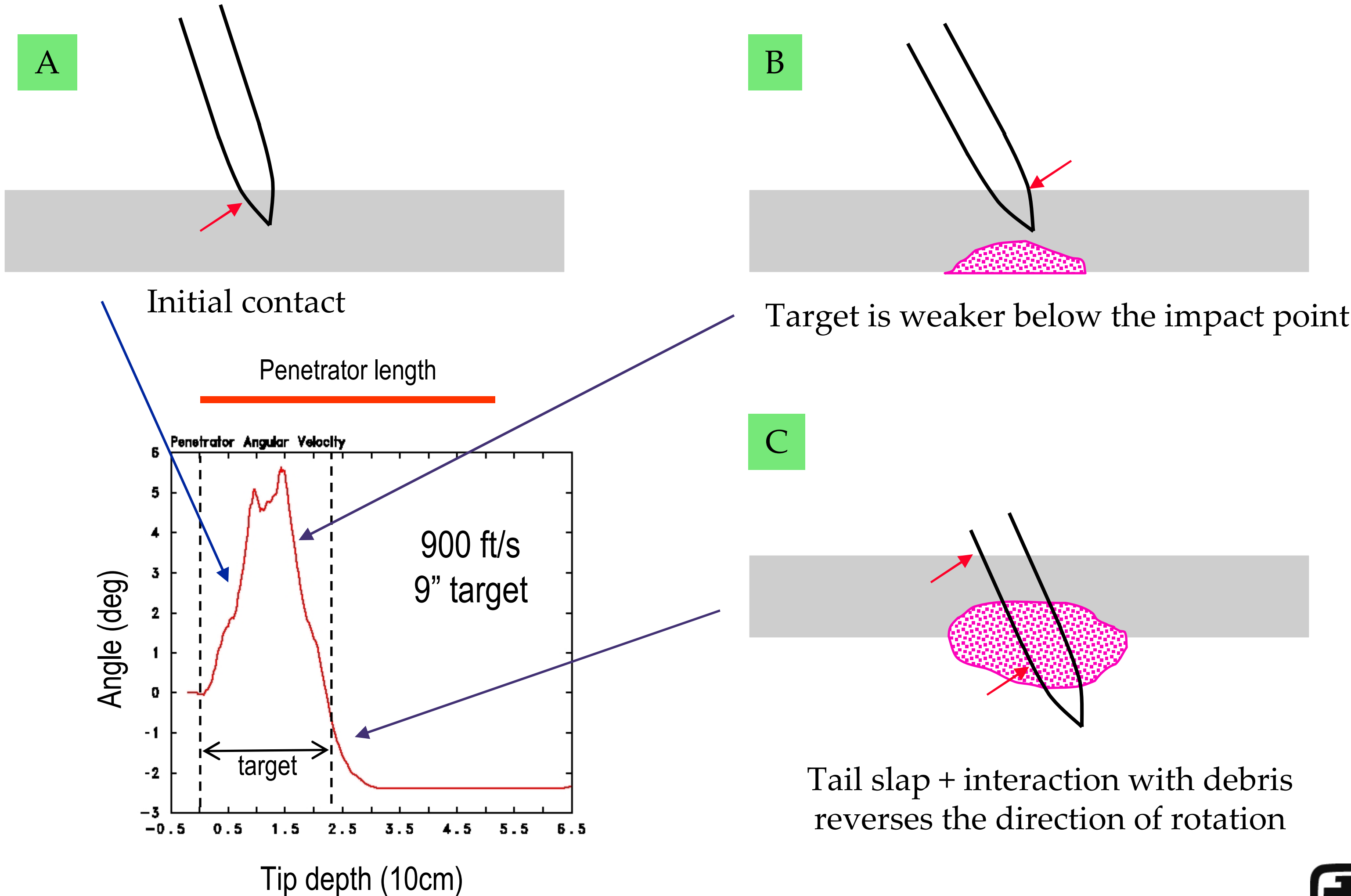
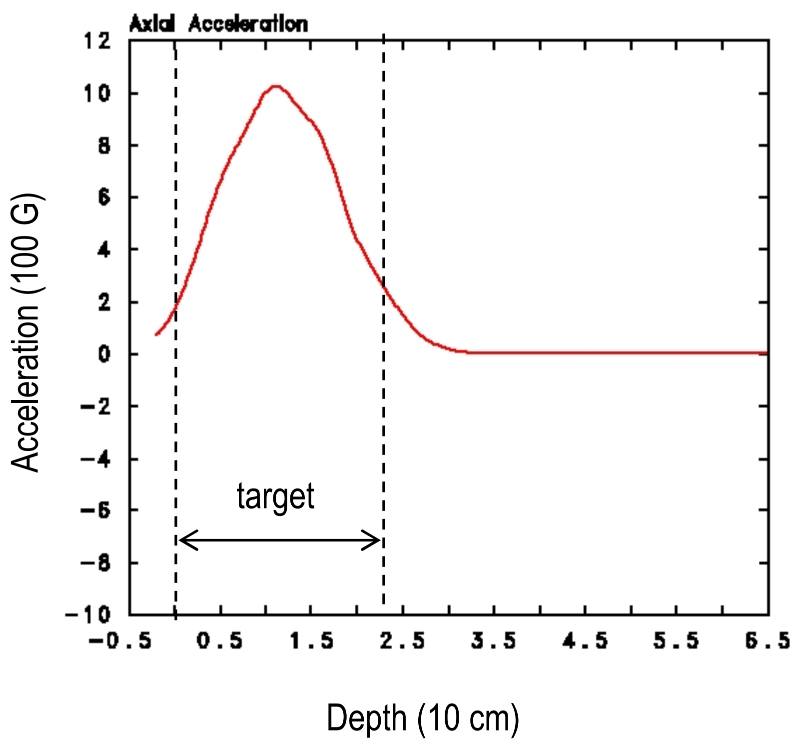
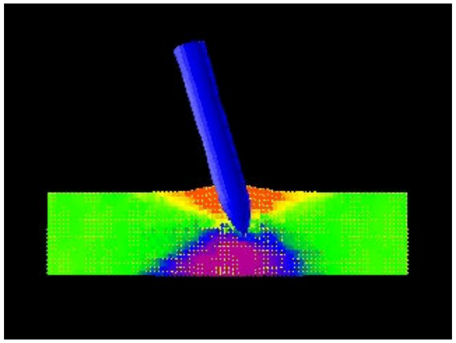
Perforation:
Forces that affect projectile rotation



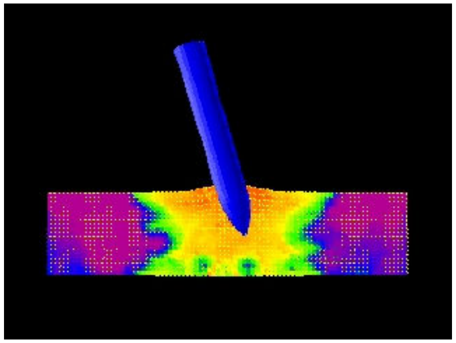
Perforation of concrete: 900 ft/s into 9″ target

- Peak axial acceleration occurs with nose tip is about halfway through the target.





z-velocity at time of peak axial acceleration



Damage at time of peak axial acceleration

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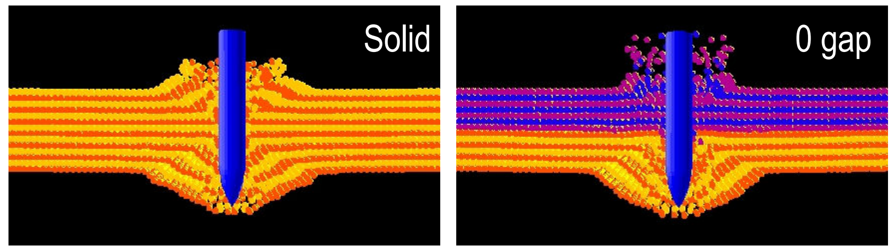
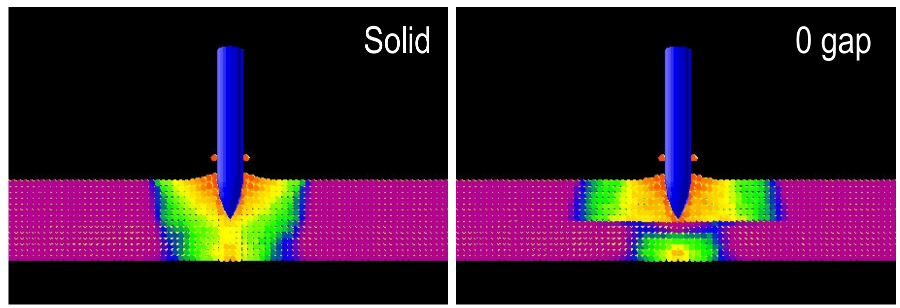
Perforation:
Solid vs. 2-panel target with 0mm gap

Crater shapes end up looking similar, however…

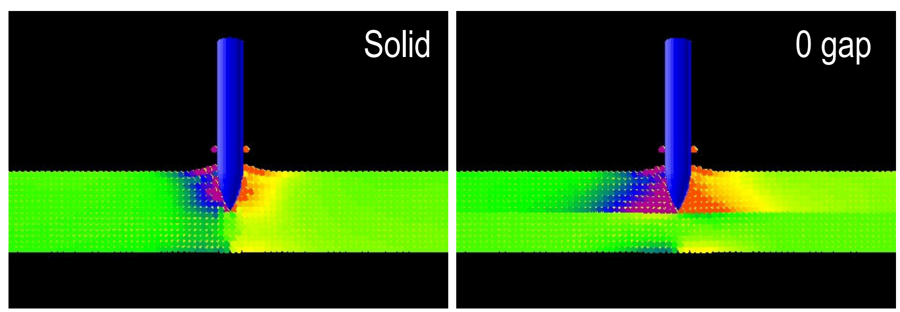


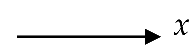
Cracks cannot propagate <u>directly</u> between panels.



Colors indicate damage (0.45 ms)

2-panel target shows less confinement near penetrator nose due to sliding at interface.

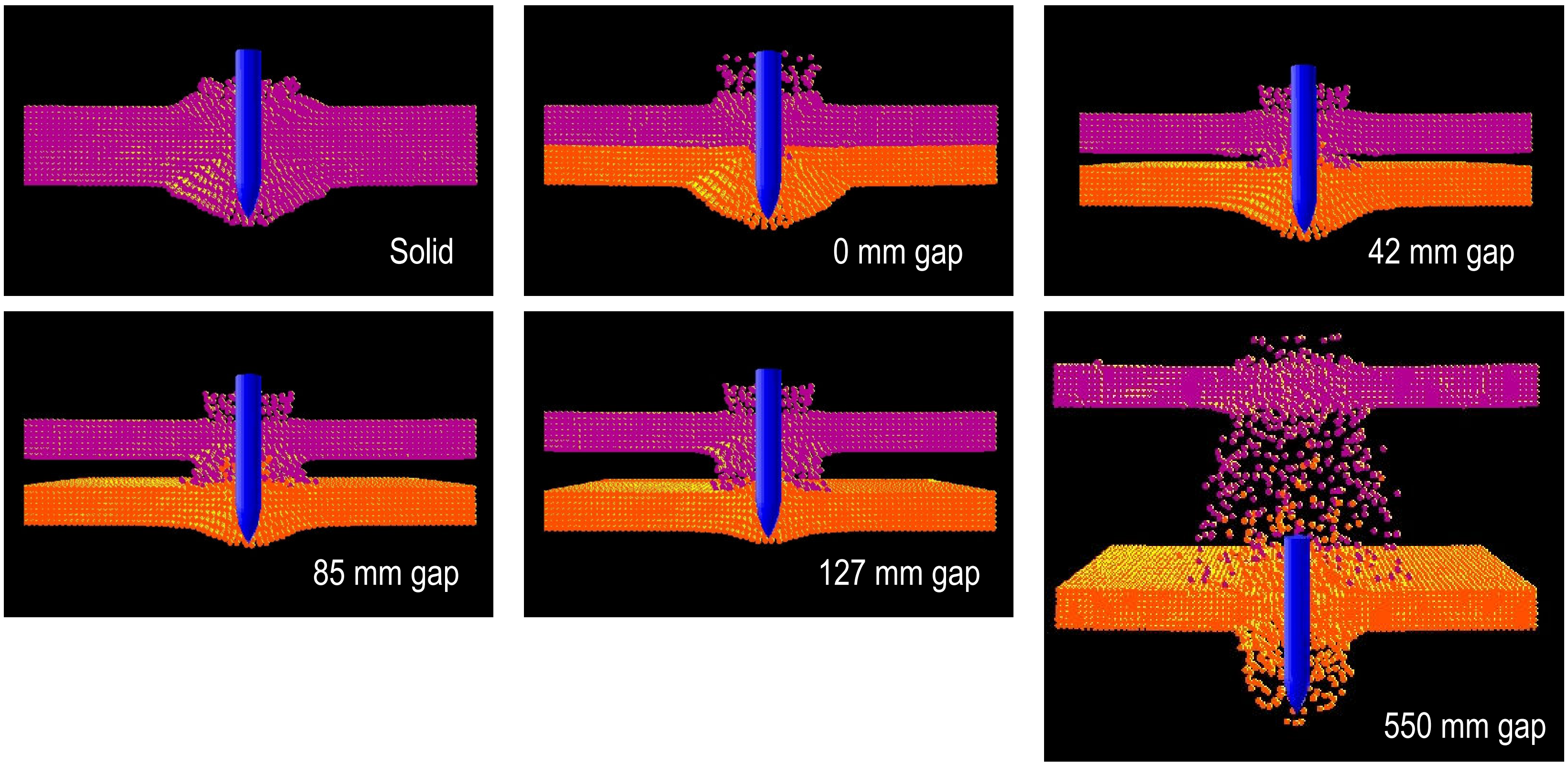




Colors indicate *x* component of displacement (0.45 ms)

Perforation: Effect of a gap between 2 panels

- How does the separation between concrete targets affect penetrator acceleration?



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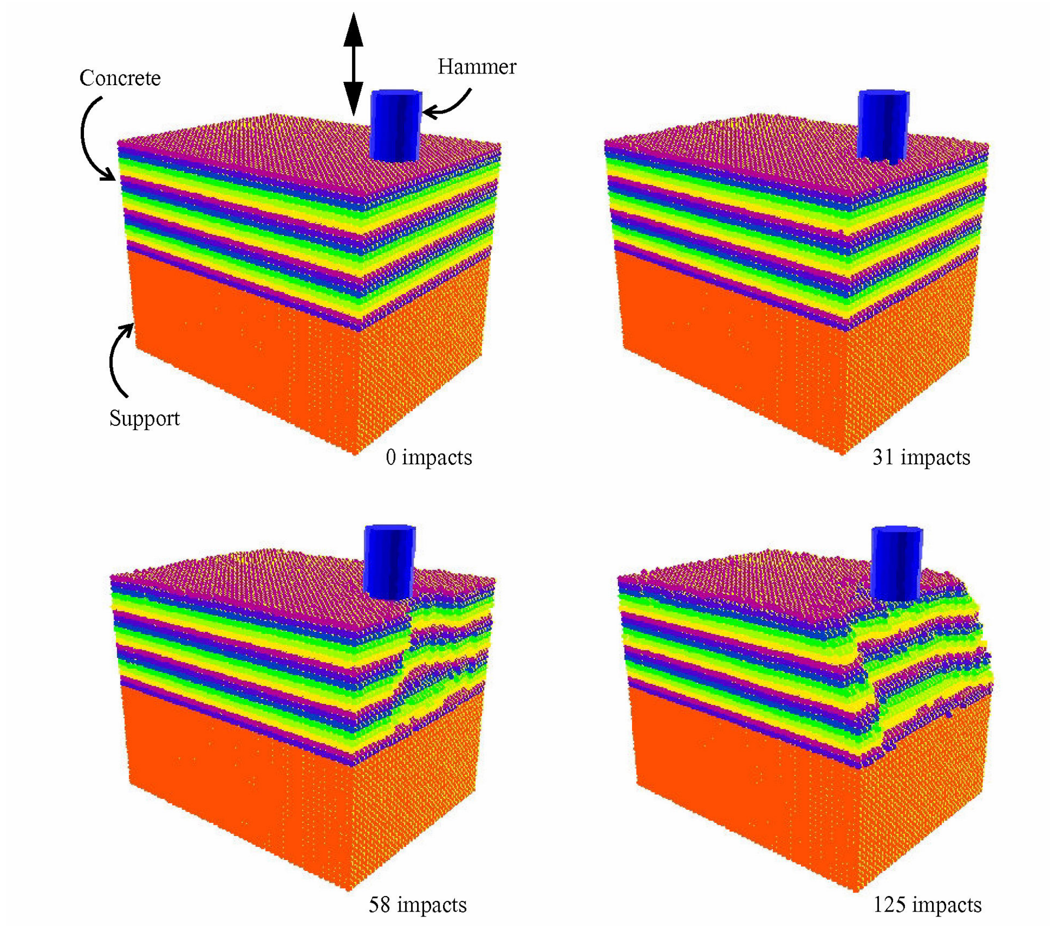
Damage accumulation from multiple impacts

- Each successive impact breaks more bonds internally.
- These coalesce into large cracks.



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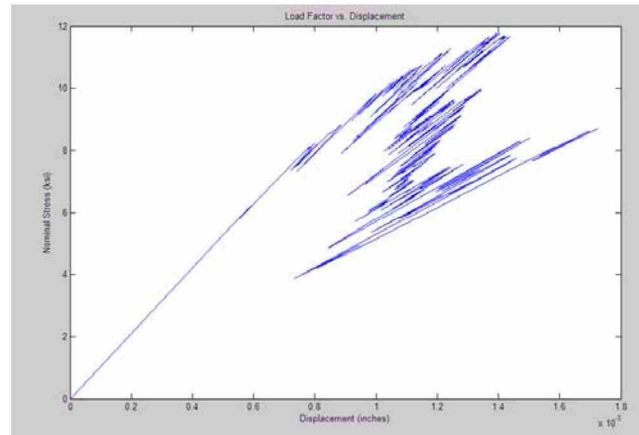
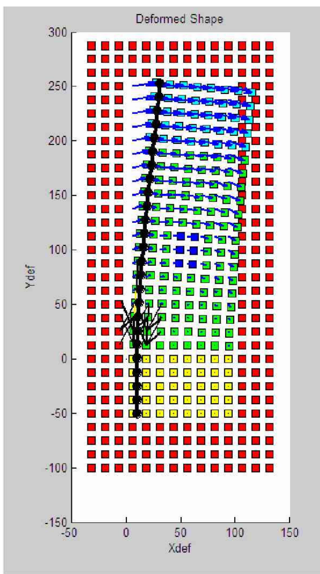
Degradation of elastic properties due to damage

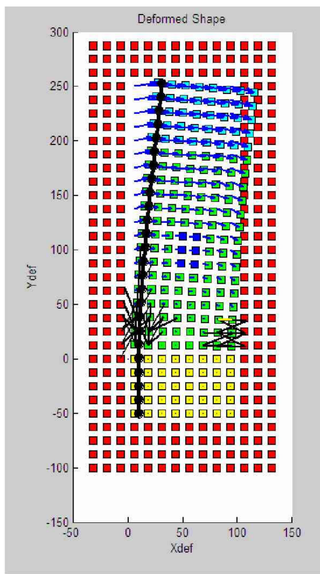
• Cantilevered concrete beam with single rebar
(W. Gerstle, SMIRT-18)



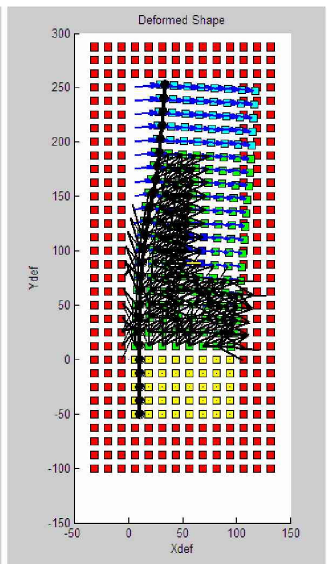
(d) Load versus load-point displacement



(a) 25 links broken



(b) 50 links broken



(c) 400 links broken

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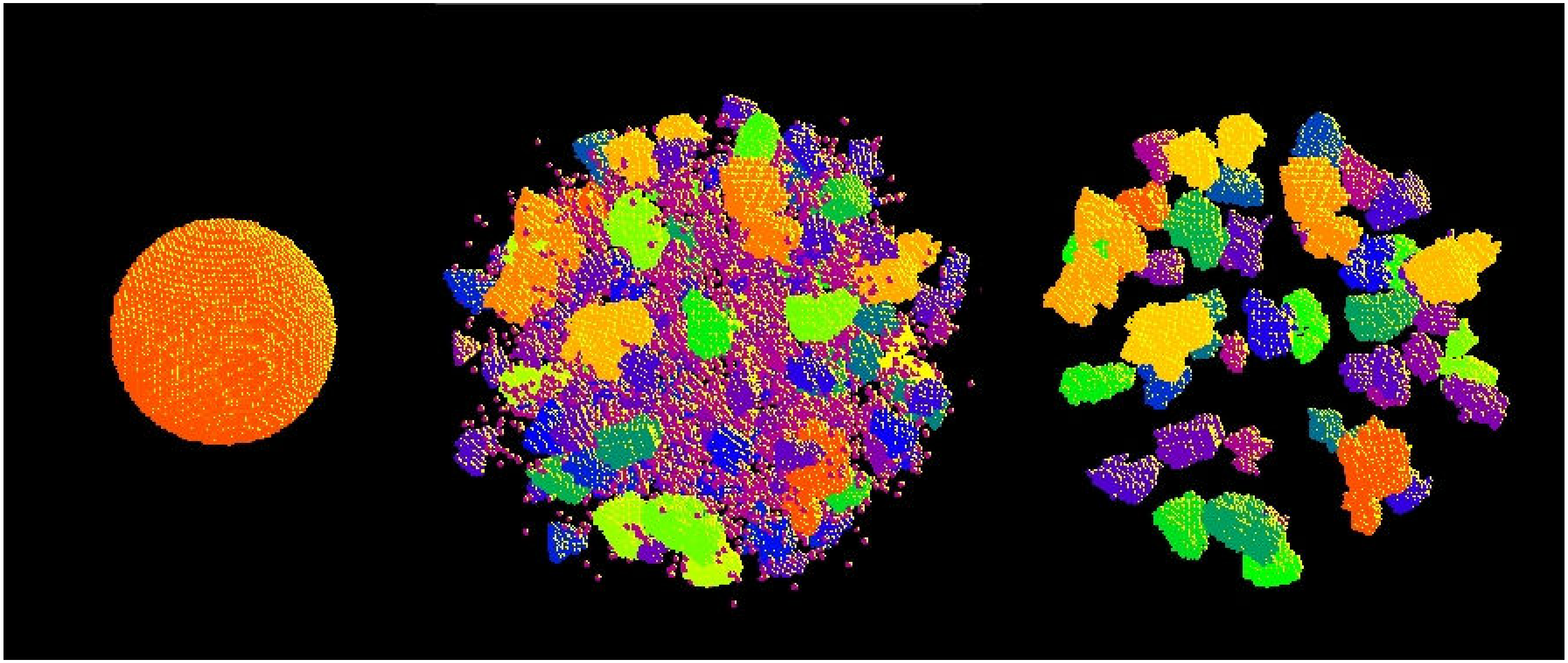
Fragmentation: Brittle sphere expansion

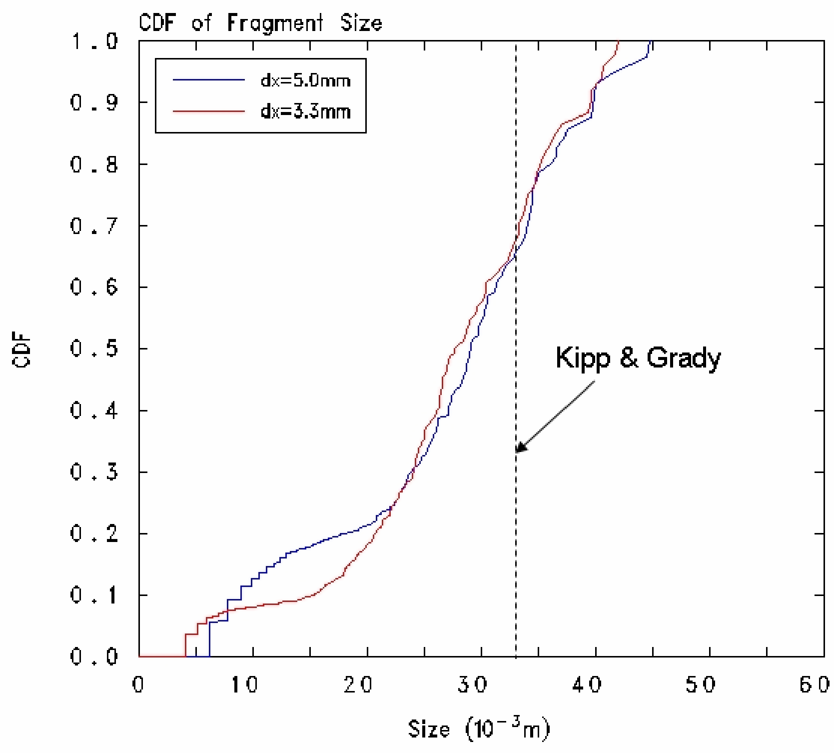
- Uniform initial strain rate 250 s^{-1}.



Initial All fragments Largest fragments

Fragmentation: Brittle sphere expansion, ctd.

- Cumulative distribution function of fragment size.
  - Fragment size from Kipp-Grady equation is also shown.



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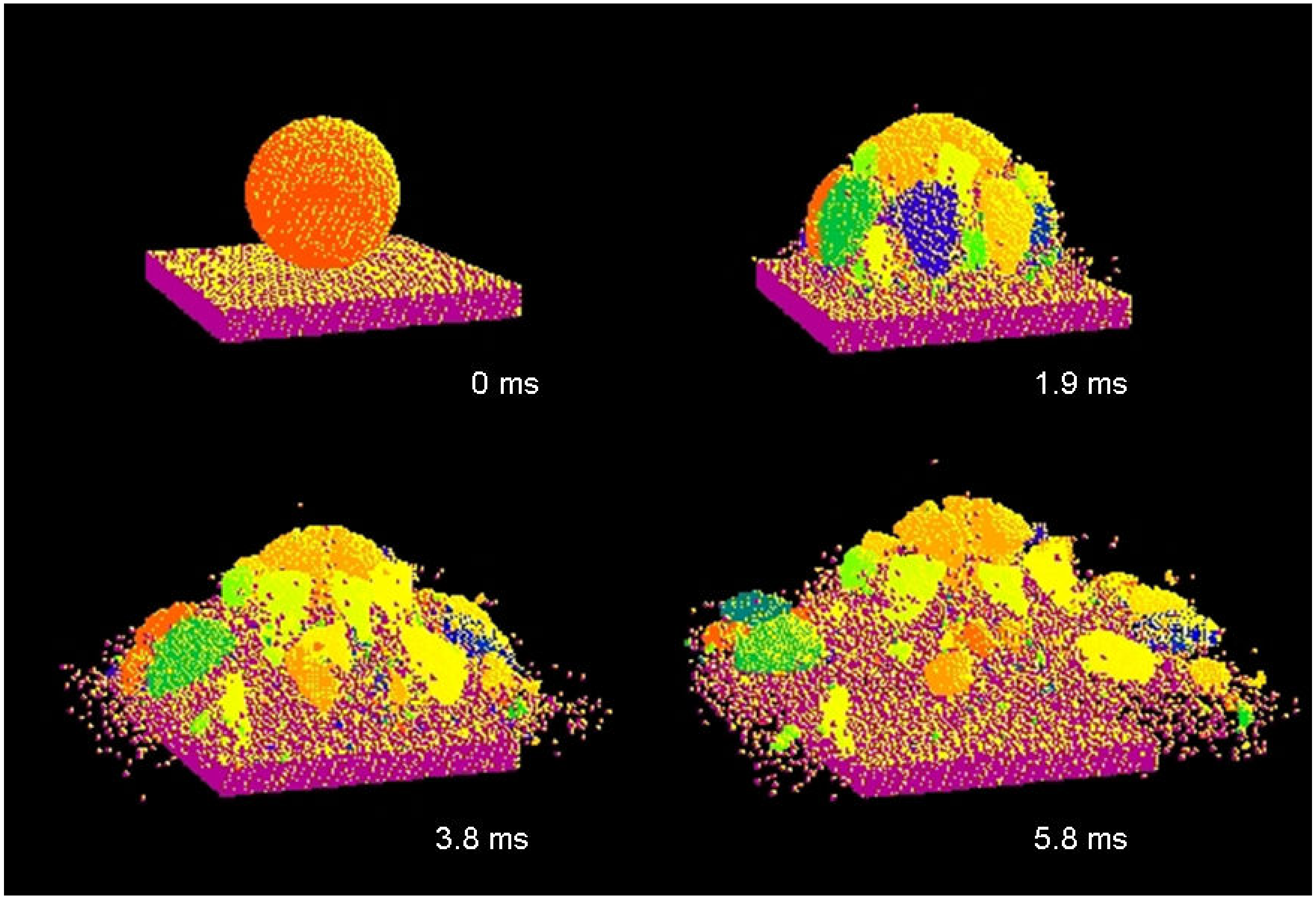
Fragmentation: Concrete sphere drop

- 15cm diameter concrete sphere against a rigid plate, 32.4 m/s.



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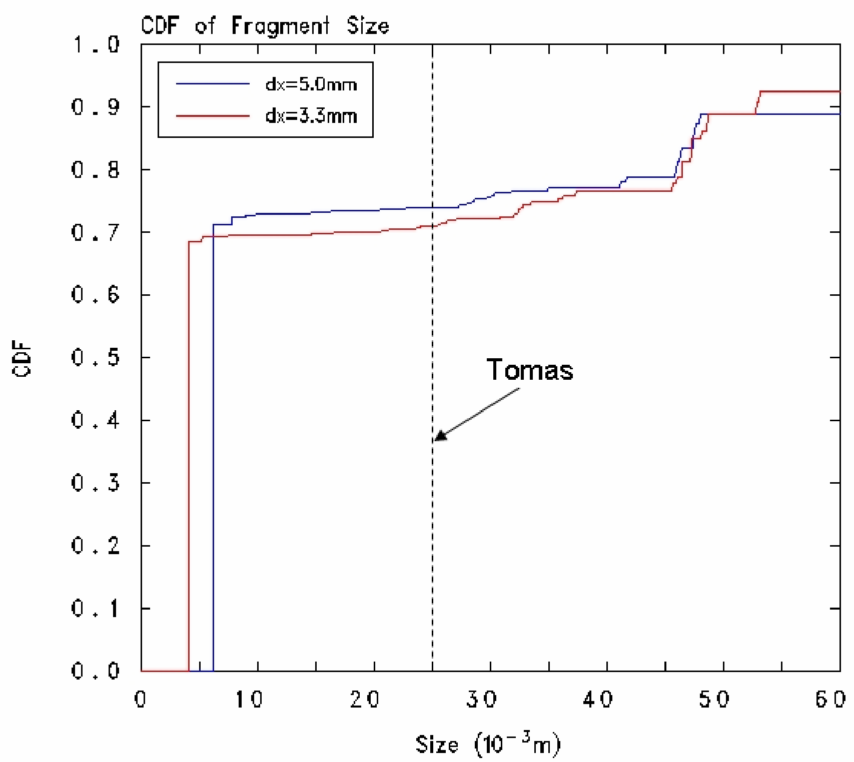
Fragmentation: Concrete sphere drop, ctd.

- Cumulative distribution function of fragment size (for 2 grid spacings):
  - Also shows measured mean fragment size*



*J. Tomas et. al., *Powder Technology* **105** (1999) 39-51.

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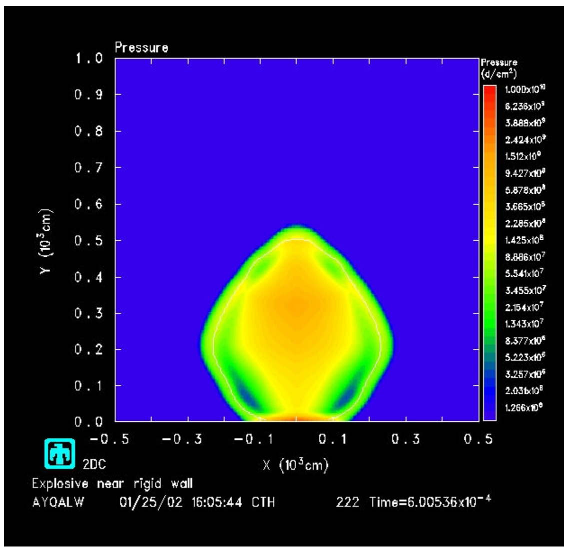
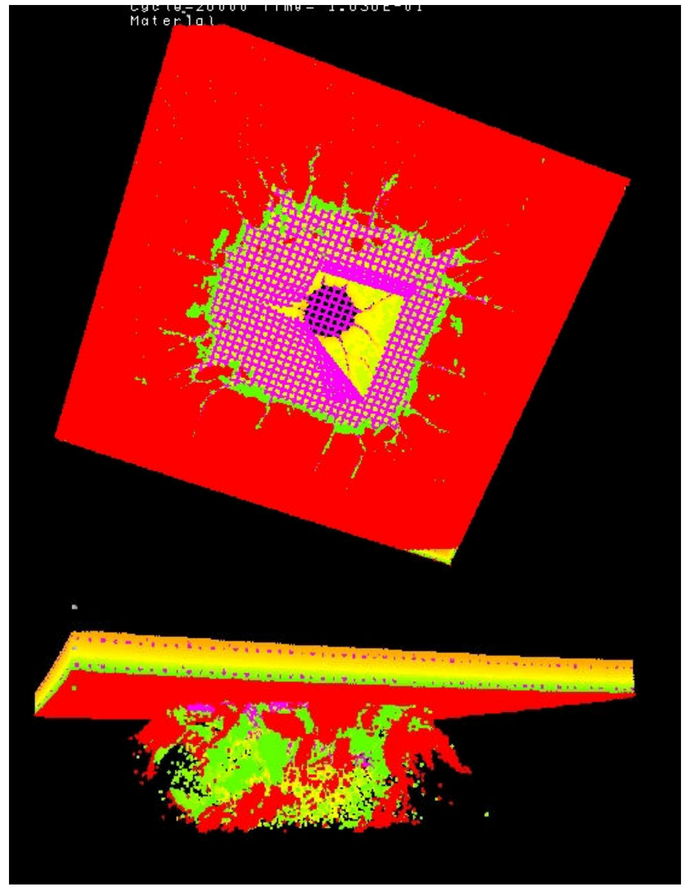
Air blast effects

- Air blast loading on a reinforced concrete panel is supplied by the CTH code.



Air blast pressure field (from CTH)



Panel response (from EMU)

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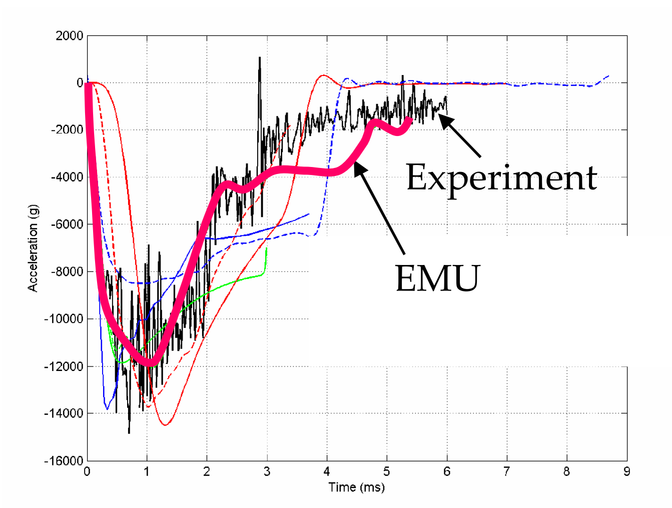
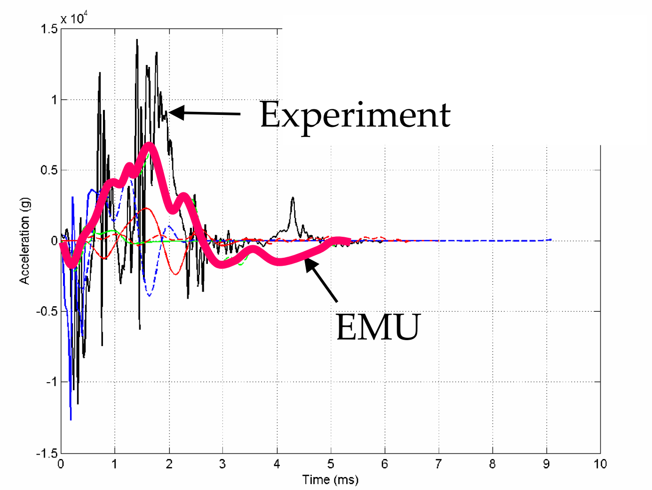
Penetration benchmarking (DoD/DOE MOU): Genuine predictions

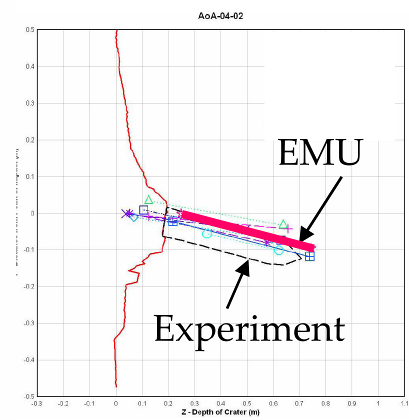
- Purpose: to exercise computational models in a predictive mode:
  - Model results are submitted before test data is released.
- 13kg steel penetrator into quality-controlled concrete targets.
  - On-board accelerometers both fore and aft.



Axial acceleration



Lateral acceleration



Rest position

Unlabeled curves show other codes' predictions.

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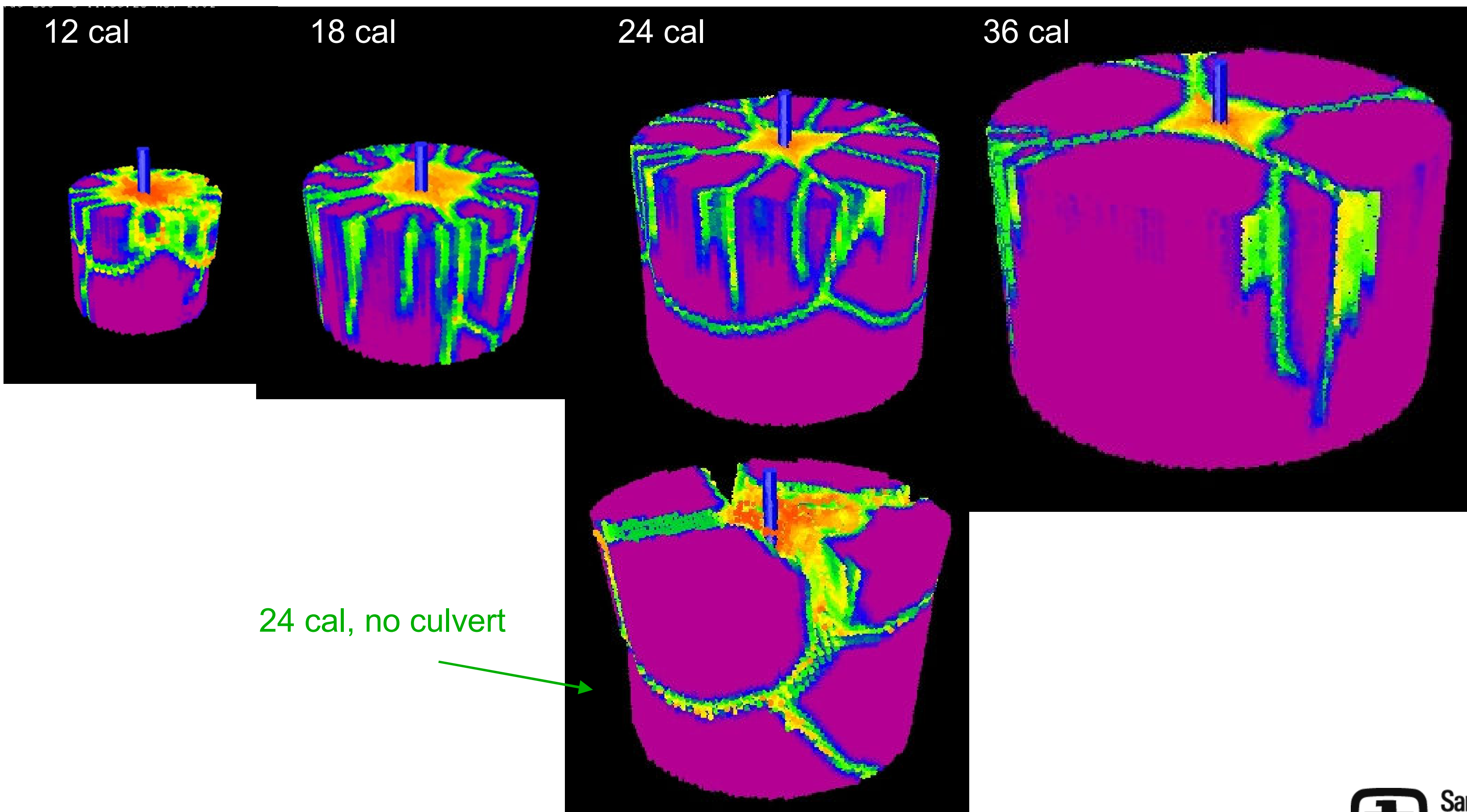
Penetration:
Target diameter effect study

Cut-away views show only the target material deeper than 0.43m from the top surface.



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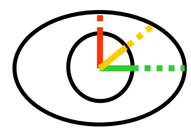
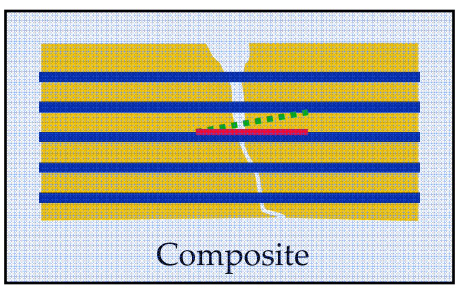
Current research:
Mathematics of a more general theory

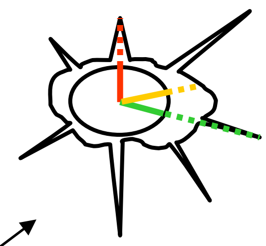
- **Peridynamic states**
  - Mathematical generalization of the theory takes it far beyond what can be modeled with pair interactions.

Stress tensor (classical):
- 6 "degrees of freedom".



$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}$$





$$f = \underset{\rightarrow}{T}(x' - x)$$

Force state (peridynamic):
- Infinite "degrees of freedom".

Can use conventional material models.

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Peridynamic states vs. FE: Elastic-plastic solid

- Direct comparison between a finite-element code and Emu with a conventional material model.
- Results by T. Warren:

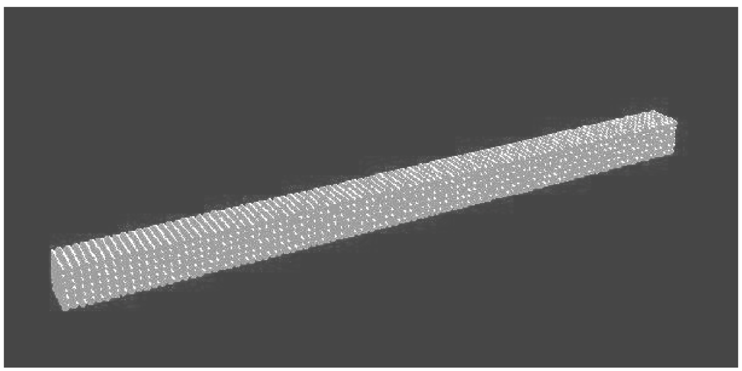

Figure 3. 3600 node discrete peridynamic lattice

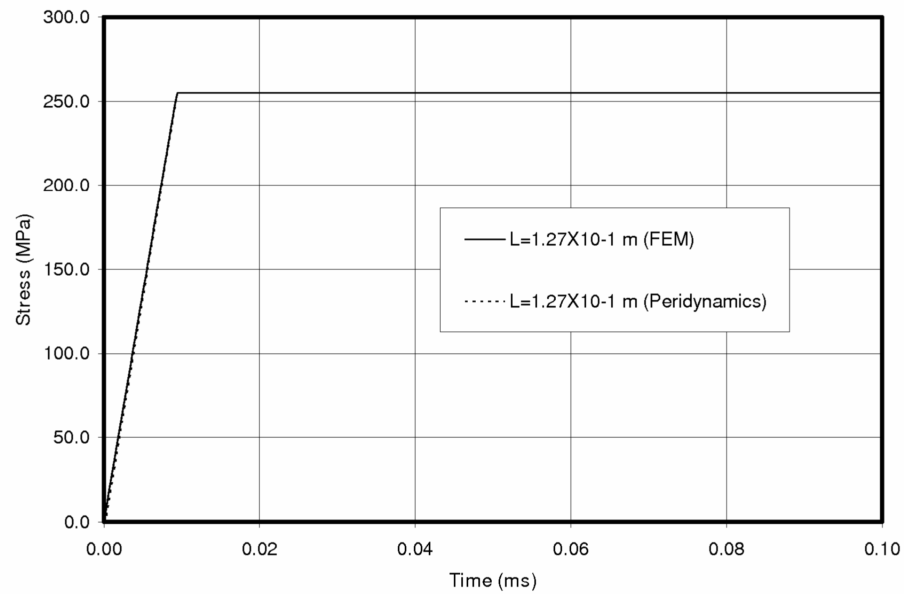

Figure 7. Stress in the bar at L=127 mm using both Peridynamics and FEM

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Summary

- EMU fills a gap in the capabilities of conventional codes:
  - Ability to model discrete fractures.
  - Direct prediction of fragmentation.
- Current research areas related to concrete modeling:
  - Incorporation of conventional geological material models.
  - Rotational degrees of freedom.
- For more information and references: www.sandia.gov/emu/emu.htm