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Title: Comments on Different techniques for finding best-fit parameters

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Comments on different techniques for finding best-fit parameters

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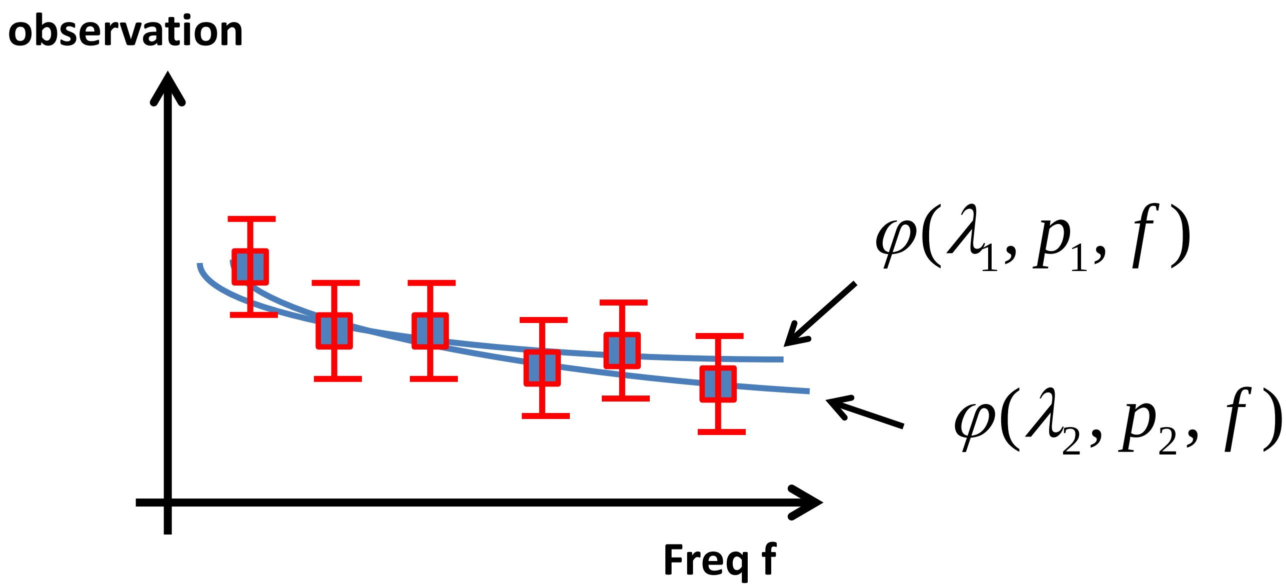
Abstract

A common data analysis problem is to find best-fit parameters through chi-square minimization. Levenberg-Marquardt is an often used system that depends on gradients and converges when successive iterations do not change chi-square more than a specified amount. We point out in cases where the sought-after parameter weakly affects the fit and cases where the overall scale factor is a parameter, that a Golden Search technique can often do better. The Golden Search converges when the best-fit point is within a specified range and that range can be made arbitrarily small. It does not depend on the value of chi-square.

A typical problem: fit 2 parameters to N data points



$$\chi^2 = \sum_{1}^{N} \frac{\{o_i - \varphi(\lambda, p, f_i)\}^2}{\sigma_1^2}$$

Eq 1

Consider situation where $\varphi(\lambda, p, f)$ is weakly dependent on ***p*** and λ is a scale factor. That is

$$\varphi(\lambda, p, f) = \lambda\varphi^{'}(p, f) \qquad \text{Eq 2}$$

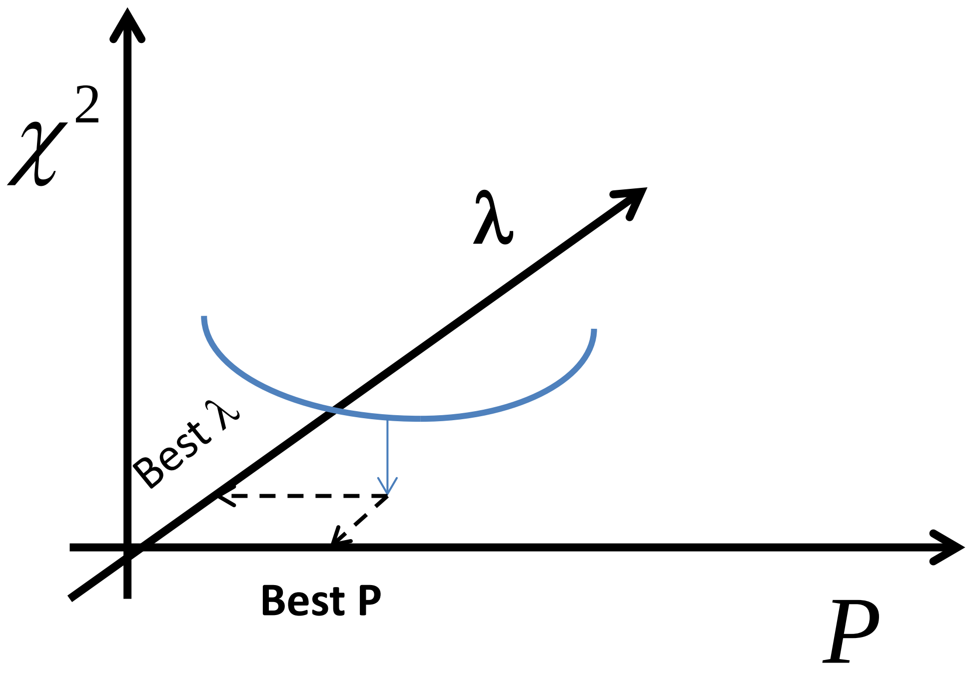
The weak dependency on p means that the χ^2 might be a long shallow surface.

A common method for fitting parameters to data uses the Levenberg-Marquardt method to find the χ^2 minimum.

If the L-M method fits to two parameters (i. e., λ and p), it needs the partial derivatives $\delta\varphi/\delta\lambda$ and $\delta\varphi/\delta p$.

The convergence criteria is usually that successive iterations of χ^2 are not changing much and/or that the reduced χ^2 is less than ~ 1.

Use gradients to find minimum in 2-D until changes in χ^2 are small



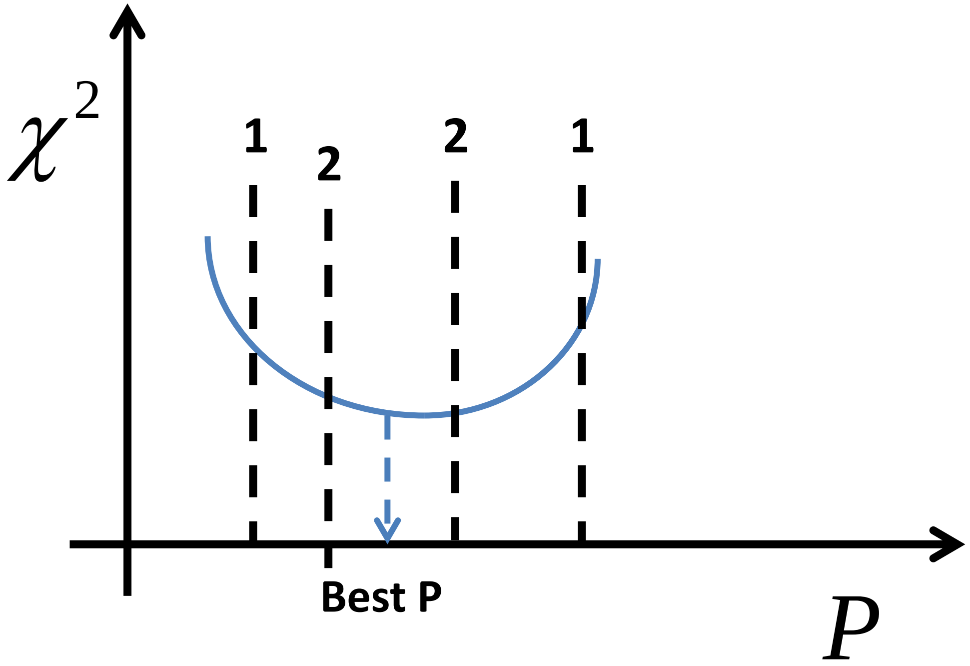
An alternative method: "Golden search" to minimize χ^2 in 1-D by finding the best λ analytically for any value p

$$\chi^2 = \sum_1^N \frac{\{o_i - \lambda\varphi'(p, f_i)\}^2}{\sigma_1^2}$$ Eq 3

$$\frac{\delta\chi^2}{\delta\lambda} = 0 = 2\sum_1^n \frac{\{o_i - \lambda\varphi'(p, f_i)\}\varphi'(p, f_i)}{\sigma_i^2}$$ Eq 4

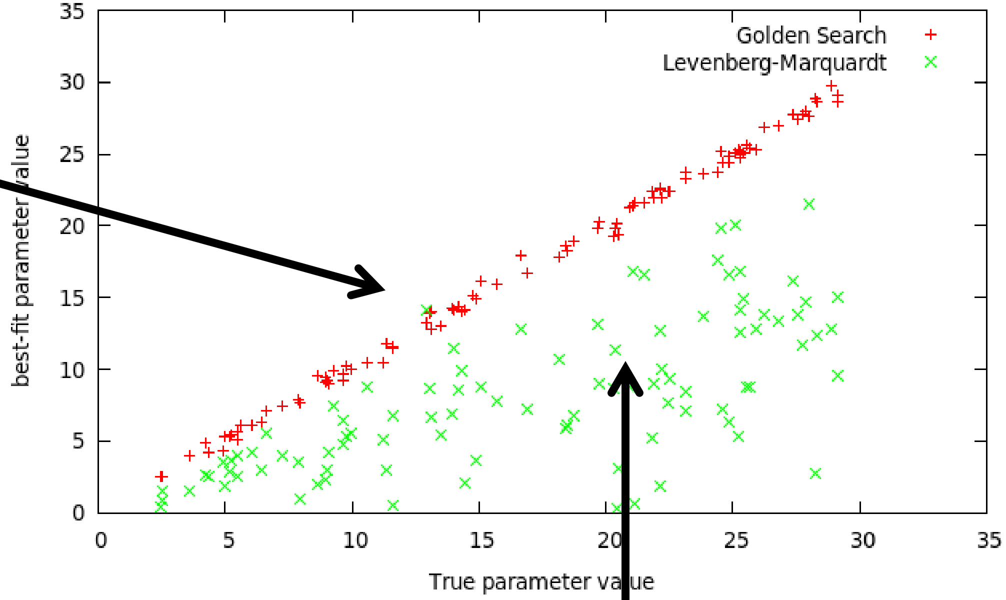
$$\lambda\sum_1^N \frac{\varphi'(p, f_i)\varphi'(p, f_i)}{\sigma_i^2} = \sum_1^N \frac{o_i\varphi'(p, f_i)}{\sigma_i^2}$$ Eq 5

Keep Splitting the 1-D range until the minimum is bracketed to some accuracy in p (no dependency on the value of χ^2).



Compare the p found by analysis with the true p (warning: fake data)

Golden search method always finds minimum



L-M method underestimates p because converges early near its initial guess and not at the true minimum

Because the L-M method determines p poorly, it is unlikely that λ could be found.

Since the Golden Search method can find p well, it is likely that λ can also be found from the data. It is given by Eq 5.