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K. L. Baker, D. Homoelle, E. Utterback and S. M. Jones

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Phasing rectangular apertures

K.L. Baker*, D. Homoelle, E. Utterback, S.M. Jones

Lawrence Livermore National Laboratory, 7000 East Ave., Livermore, CA, USA 94550-0345

ABSTRACT

Several techniques have been developed to phase apertures in the context of astronomical telescopes with segmented mirrors. Phasing multiple apertures, however, is important in a wide range of optical applications. The application of primary interest in this paper is the phasing of multiple short pulse laser beams for fast ignition fusion experiments. In this paper analytic expressions are derived for parameters such as the far-field distribution, a line-integrated form of the far-field distribution that could be fit to measured data, enclosed energy or energy-in-a-bucket and center-of-mass that can then be used to phase two rectangular apertures. Experimental data is taken with a MEMS device to simulate the two apertures and comparisons are made between the analytic parameters and those derived from the measurements. Two methods, fitting the measured far-field distribution to the theoretical distribution and measuring the ensquared energy in the far-field, produced overall phase variance between the 100 measurements of less than 0.005 rad^2 or an RMS displacement of less than 12 nm.

Keywords: Adaptive Optics, Short-Pulse Lasers, Wave-front sensing, Interferometry

*baker7@llnl.gov; phone 1 925 422-1461; fax 1 925 422-1796

1. INTRODUCTION

The primary mirror on each of the Keck telescopes is comprised of 36 hexagonal segments which must all be phased to a small fraction of a wavelength. To accomplish this task the Keck telescope utilizes interference in the far-field pattern to measure the phasing between adjacent hexagonal segments of its primary mirror.^{1,2} In particular a circular pupil is placed halfway across two adjacent segments. The aperture size is chosen such that diameter is much less than the atmospheric Fried parameter, ensuring the beams are nearly aberration free when used with natural guide stars. This technique also uses narrow band filters to increase the temporal coherence of the light. Assuming that the phase jump between the adjacent segments is within the temporal coherence length of the light, the signal from the two segments will interfere in the far-field at the focus of the lenslet. The exact phase is then determined by performing a cross correlation between the measured, at 11 piston positions, and simulated far-field patterns to determine the best fit. As demonstrated on Keck, this technique had a capture range of $30 \text{ }\mu\text{m}$ and could bring the segments to within 30 nm rms when using fairly bright stars. ($V=4$ to 7) An alternative method was later developed in which simulations were used to generate the relationship between the ratio of the two primary peaks formed in the far-field pattern as a function of the phase difference between the two apertures.³ The authors of this latter paper then fit the ratio of the peaks in their simulated data to the numerically generated curve to determine the phase differential between the two apertures.

The James Webb Space Telescope (JWST) developed a phasing technique that was named the dispersed fringe sensor (DFS) to phase the 18 hexagonal segments on its primary mirror.⁴ The DFS utilizes two of the segments of the primary mirror to form two arms of an interferometer. The pupil is reimaged and a grism, combination of prism and grating, is used to disperse the light onto the detector. By dispersing the light, the coherence length is increased to $\lambda^2/\Delta\lambda$, where $\Delta\lambda$ is the wavelength dispersion across a pixel. The capture range of the dispersed fringe sensor is about $100 \text{ }\mu\text{m}$ and the sensor is able to achieve a piston error down to 500 nm rms. By measuring the slope of the dark line formed in the PSF, one can infer the piston error. The JWST project also developed a dispersed Hartman sensor which works in a similar manner and is effectively a parallel DFS.⁵ Lenslets are placed across two adjacent mirrors and the lenslet PSF is spectrally dispersed across the focal plane using a grism. If there is a piston error between the segments, then there will be sloping lines across the dispersed psf. When the elements are piston error corrected, then the lines are removed.

The National Ignition Facility (NIF) will become fully operational this calendar year and incorporates 192 400 mm square-aperture Nd:glass beam lines. One quad of the 192 NIF beams is designated for x-ray backlighting and fast ignition experiments. Each of these four square NIF Beams is split into two rectangular beam lines, a beam pair, to

minimize the size of the compressor gratings required to form the short pulses. As such eight rectangular beams that can be independently pointed are formed from the original four square NIF beams. Each of these eight beams focuses to the target chamber center as an $f/22$ by $f/44$ beam. These beam lines will differ from the standard NIF beam line in several ways. A fiber-based short pulse will be added to the master oscillator to seed a quad of main amplifiers. The short pulse is stretched in pulse length before entering the preamplifier modules and then compressed to a 1-10 ps pulse with a vacuum compressor placed on the target area mezzanine. The eight $1.053\text{ }\mu\text{m}$ short pulse laser beams will then be focused near target chamber center with an off-axis parabola to minimize B integral effects. The B integral is the accumulated on-axis nonlinear phase shift due to intensity dependent changes in the refractive index. Each of these eight beams will deliver a laser pulse to chamber center that is nominally 5 ps in pulse length and 995 J in energy, giving an overall energy of 7.96 kJ delivered to the target. The fast ignition experiments require that 4 kJ of the total 7.96 kJ of laser energy be deposited within a $40\text{ }\mu\text{m}$ diameter circle. Each of the eight beams focuses to the target chamber center as an $f/22$ by $f/44$ beam such that the first lobe of the diffraction pattern represents a rectangular shape of ~ 46 by $93\text{ }\mu\text{m}$ in the far-field. At the very least each of the two rectangular beams formed from a single square NIF beam must be phased together to meet the enclosed energy requirement. The approximate pupil geometry of the eight beams is shown below in Fig 1a along with the far-field pattern, in Fig. 1b, generated by this pupil assuming that all of the beams are in phase and no phase aberrations are present. Fig. 1b also contains a $40\text{ }\mu\text{m}$ diameter circle showing the spatial dimensions of the focused beams. If all the beams were phased and pointed together and fully corrected with a Strehl ratio of 1, then they would deliver 5.6 kJ in a $40\text{ }\mu\text{m}$ diameter circle, exceeding the requirements by 1.6 kJ. When each of the two beams formed from a single NIF beam are co-phased, but with random piston errors between the four beam pairs, the encircled energy requirement of 4 kJ is exceeded 90% of the time provided the Strehl ratio is greater than 0.72.⁶ This latter statement was determined by analyzing the results from a thousand simulations to evaluate the system performance with random piston errors, random tip/tilt errors and a random realization of a residual turbulence profile applied to the eight beams. The random realization of the residual turbulence profile consisted of a Von Karman turbulence profile with the Fried parameter $r_0=D/8$ and the outer scale length set to $D/2$ with D representing the longest aperture dimension. In the case of tip/tilt errors, normally-distributed pseudo-random numbers with a mean of zero and a standard deviation of 1 μrad were assigned to both the tip and tilt components of the phase. The Von Karman turbulence parameters were chosen based upon residual wave-front measurements taken on one of the NIF beam lines after a low order deformable mirror was utilized to pre-correct for wave-front aberrations in the rod and disk amplifiers caused primarily by heat deposition from the flashlamps.⁶ When there are random phases between the eight beam halves, no phasing, then the maximum energy within the $40\text{ }\mu\text{m}$ diameter circle is ~ 3.4 kJ, well below the required energy of 4 kJ. As such phasing of the eight beam halves into four co-phased beam pairs will be required.

One approach to measuring the piston difference between two rectangular apertures is to utilize an interferometer in the near field^{6,7} Once the piston difference is measured then a deformable mirror or fiber stretcher can be used to apply the appropriate phase difference between the two apertures to correct for the piston error. The ability to measure piston errors is one such advantage of an interferometric adaptive optics system over conventional adaptive optics systems such as Shack-Hartmann and curvature sensors that measure the first and second derivative of the phase. In this article, however, the far-field interference pattern between two rectangular apertures is examined for alternative ways to determine the piston phase shift between the two apertures.⁸

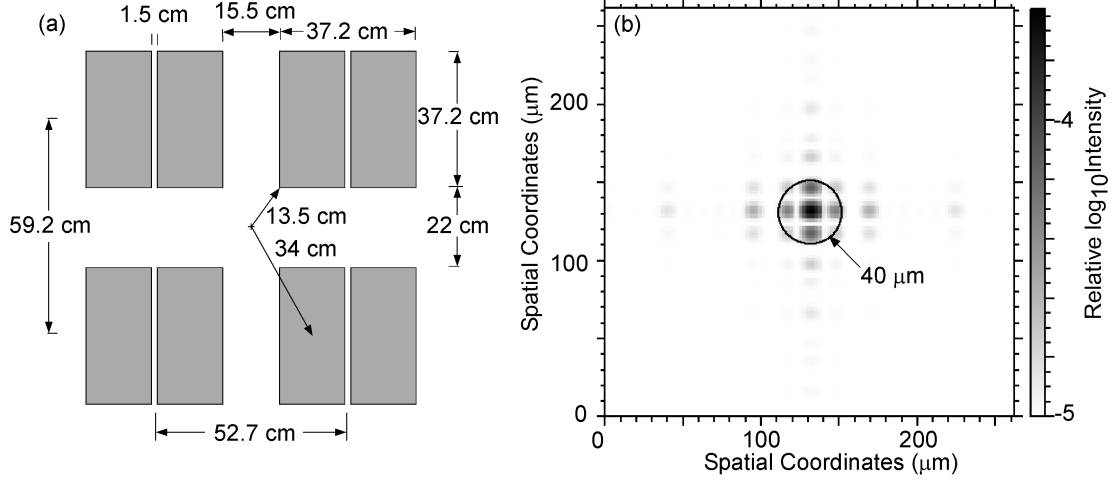


Fig. 1. Schematic Pupil layout and ideal far-field energy distribution for the advanced radiography capability on the National Ignition Facility. The pupil layout is displayed in Fig. 1a and it represents four beam pairs with each of the beam pairs containing 1.99 kJ centered around 1.053 μm in a 5 ps pulse. Fig. 1b shows the far-field pattern generated from this pupil assuming that all of the beams are pistoned correctly and have a perfect Strehl ratio.

2. ANALYTIC FAR-FIELD PARAMETERS

In the case of rectangular apertures it is straightforward to calculate the analytical far-field patterns associated with both a piston shift between two adjacent apertures and different size apertures. The latter effect could be attributed to a rectangular mask being displaced slightly relative to the two apertures being tested thereby giving one of the apertures a greater area than the remaining aperture. A rectangular aperture is of primary interest to the application at hand in which the laser is composed of two rectangular apertures. A circular lens can, however, be used with a rectangular or square mask for the case of phasing adjacent hexagonal segments on a telescope for instance implying that this technique can be used on a broad range of optical instruments. The complex field in the image plane is simply the Fourier transform of the electric field in the near field plane. For the case of two rectangular subapertures with slightly different sizes, the electric field in the near field plane can be written as

$$E(x, y) = \begin{cases} A \exp(i\phi_0) & -a \leq x \leq \delta; \quad -b \leq y \leq b \\ A \exp(i\phi_1) & \delta < x \leq a; \quad -b \leq y \leq b, \\ 0 & x < -a; \quad x > a; \quad y < -b; \quad y > b \end{cases} \quad (1)$$

where ϕ_0 and ϕ_1 represent the two phases on the different apertures and $a + \delta$ and $a - \delta$ represent the respective widths of the two apertures. The phasing signatures derived below are sensitive to the difference in phase between the two apertures. As such without loss of generality we define $\phi_1 = 0$ and $\phi = \phi_0 - \phi_1 = \phi_0$. The complex field in the image plane is simply the Fourier transform of the electric field in the near field plane.⁹ The electric field in the far-field plane can then be determined by Fourier transforming the near field

$$E_{\text{FF}}(k_x a, k_y b) = A \int_{-b}^b e^{ik_y y} dy \left[\int_{-a}^{\delta} e^{ik_x x} e^{i\phi} dx + \int_{\delta}^a e^{ik_x x} dx \right], \quad (2)$$

where $k_x a$ and $k_y b$ represent the coordinates in the far-field. For further simplification of the equations, the far-field coordinates $(k_x a, k_y b)$ will be represented by (β, ζ) . After a little algebra, the normalized intensity, $I_{\text{FF}}(\beta, \zeta) = E_{\text{FF}} E_{\text{FF}}^*$, derived from Eq. 2 can be written as follows

$$\frac{I_{FF}(\beta, \zeta)}{\iint I_{FF}(\beta, \zeta) d\beta d\zeta} = \frac{\text{sinc}^2(\zeta)}{4\pi^2 \beta^2} \{4 - 2\cos[\beta(1 + \delta/a)] - 2\cos[\beta(1 - \delta/a)] - 2\cos(\varphi) + 2\cos[\varphi - \beta(1 + \delta/a)] + 2\cos[\varphi - \beta(1 - \delta/a)] - 2\cos(\varphi - 2\beta)\}, \quad (3)$$

where $\text{sinc}(x) = \sin(x)/x$. The parameters β and ζ represent the physical parameters $\pi za/(\lambda f.l.)$ and $\pi zb/(\lambda f.l.)$, respectively. In these expressions λ represents the wavelength of the light, $f.l.$ represents the focal length of the lens used to focus the light to the far-field and z is the spatial coordinate in the far-field plane.

By integrating the normalized far-field distribution in Eq. 3 in the direction perpendicular to the gradient of the phase between the two apertures, the far-field pattern can be expressed as a simple one-dimensional function that can then be fit to the measured far-field pattern. Integrating along the dimension that does not contain useful information on the phase difference between the apertures also helps to reduce Poisson and read noise on the detector as well. This one-dimensional array can be expressed simply as

$$\frac{\int I_{FF}(\beta, \zeta) d\zeta}{\iint I_{FF}(\beta, \zeta) d\beta d\zeta} = \frac{1}{4\pi} \frac{1}{\beta^2} \{4 - 2\cos[\beta(1 + \delta/a)] - 2\cos[\beta(1 - \delta/a)] - 2\cos(\varphi) + 2\cos[\varphi - \beta(1 + \delta/a)] + 2\cos[\varphi - \beta(1 - \delta/a)] - 2\cos(\varphi - 2\beta)\}, \quad (4)$$

which has the same β dependence as Eq. 3 above.

In addition to fitting the measured data to an analytical expression to determine the phase, this phase difference manifests itself in alternative measurable parameters. One of these parameters is the ensquared energy normalized to the total energy in the far-field pattern. This can be calculated by integrating the normalized intensity in Eq. 3 over a rectangular region in the far-field. By assuming an overall square aperture in the near field, $a = b$, then the fraction of energy within a square in the far-field may be expressed as

$$\begin{aligned} \text{Ensquared Energy} &= \frac{1}{8\pi^2} \left[\frac{-2}{\eta} + \frac{2\cos(2\eta)}{\eta} + 4S_i(2\eta) \right] \left\{ \frac{2}{\eta} [-4 + 2\cos(\varphi)] + \right. \\ &[-2 + 2\cos(\varphi)] \left(1 + \frac{\delta}{a} \right) \left[\frac{-2\cos[\eta(1 + \delta/a)]}{[\eta(1 + \delta/a)]} - 2S_i[\eta(1 + \delta/a)] \right] + [-2 + 2\cos(\varphi)] \quad (5) \\ &\left. \left(1 - \frac{\delta}{a} \right) \left[\frac{-2\cos[\eta(1 - \delta/a)]}{[\eta(1 - \delta/a)]} - 2S_i[\eta(1 - \delta/a)] \right] - 4\cos(\varphi) \left[\frac{-\cos(2\eta)}{\eta} - 2S_i(2\eta) \right] \right\} \end{aligned}$$

where $S_i()$ is the sine integral.¹⁰ When $\eta = \pi$ the integration extends over the first lobe of the diffraction pattern, between the zeros of the $\text{sinc}^2()$ pattern when there is zero phase difference between the two apertures.

As the phase difference between the two apertures varies, a tilt is effectively applied to the aperture. This displacement of the center-of-mass can then be used as a diagnostic of the phase differential between the two apertures. The center-of-mass of the intensity distribution in the far-field can be calculated analytically as the phase difference between the two apertures varies and also as a function of the area over which the center-of-mass is measured. Normalizing the center-of-mass by the width of the central lobe, 2π , the normalized center-of-mass can be expressed as follows

$$\begin{aligned}
\frac{\iint \beta I_{\text{FF}}(\beta, \zeta) d\beta d\zeta}{2\pi \iint I_{\text{FF}}(\beta, \zeta) d\beta d\zeta} &= \frac{2}{\pi} \sin(\varphi) \left\{ S_i \left[\alpha \left(1 + \frac{\delta}{a} \right) \right] \right. \\
&S_i \left[\alpha \left(1 - \frac{\delta}{a} \right) \right] - S_i(2\alpha) \left\{ \frac{2}{\alpha} [-4 + 2\cos(\varphi)] + \right. \\
&[-2 + 2\cos(\varphi)] \left(1 + \frac{\delta}{a} \right) \left[\frac{-2\cos[\alpha(1 + \delta/a)]}{[\alpha(1 + \delta/a)]} - 2S_i[\alpha(1 + \delta/a)] \right], \\
&[-2 + 2\cos(\varphi)] \left(1 - \frac{\delta}{a} \right) \left[\frac{-2\cos[\alpha(1 - \delta/a)]}{[\alpha(1 - \delta/a)]} - 2S_i[\alpha(1 - \delta/a)] \right] \\
&\left. \left. - 4\cos(\varphi) \left[\frac{-\cos(2\alpha)}{\alpha} - 2S_i(2\alpha) \right] \right\}^{-1} \right\}
\end{aligned} \tag{6}$$

where α represents the length over which the integration is carried out in the far-field. A value of $\alpha = \pi$ represents an integration over a region corresponding to the first lobe of the diffraction pattern when there is no phase shift between the two apertures. Carrying the integration over the entire far-field plane allows the center-of-mass to be expressed simply in terms of the sine of the phase difference between the apertures

$$\frac{\iint \beta I_{\text{FF}}(\beta, \zeta) d\beta d\zeta}{2\pi \iint I_{\text{FF}}(\beta, \zeta) d\beta d\zeta} = \frac{1}{4\pi} \sin(\varphi). \tag{7}$$

By carrying the integration over a smaller area, however, the overall phase shift normalized to the center-of-mass can be much larger and the measurement is less susceptible to detector noise and incomplete background subtraction.

The analytic formulas derived in Eq. 4 can be compared directly with simulations. The simulations Fourier transform the near field electric field expressed in Eq. 1 and form the intensity by multiplying the field with its complex conjugate. The resultant far-field intensity pattern is then normalized by the total of the array and then the direction perpendicular to the gradient in the phase difference between the two apertures is summed to provide a one-dimensional array corresponding to the expression in Eq. 4. For the simulation the aperture was chosen to be 32x32 pixels, one rectangular pupil was 12x32 and the other 20x32, with an overall simulation array size of 512x512 pixels. As such, the first lobe of the diffraction pattern, with no differential phase shift between the two rectangular apertures, was 32 pixels in width. To compare the simulated signal to the analytical expression in Eq. 4, the analytical expression was multiplied by the ratio of the width of the analytic first lobe, 2π , divided by the number of pixels across the first lobe of the simulated diffraction pattern, 32. The beta term in Eq. 4 was set to the pitch of the simulation pixels multiplied by the same scale factor, $2\pi/32$. Fig. 2 shows the analytical and simulated signals corresponding to Eq. 4 at four different phase shifts between the two apertures. Fig. 2 was generated with two rectangular apertures having different widths such that the δ parameter in Eq. 1 was equal to $\delta = 0.25a$.

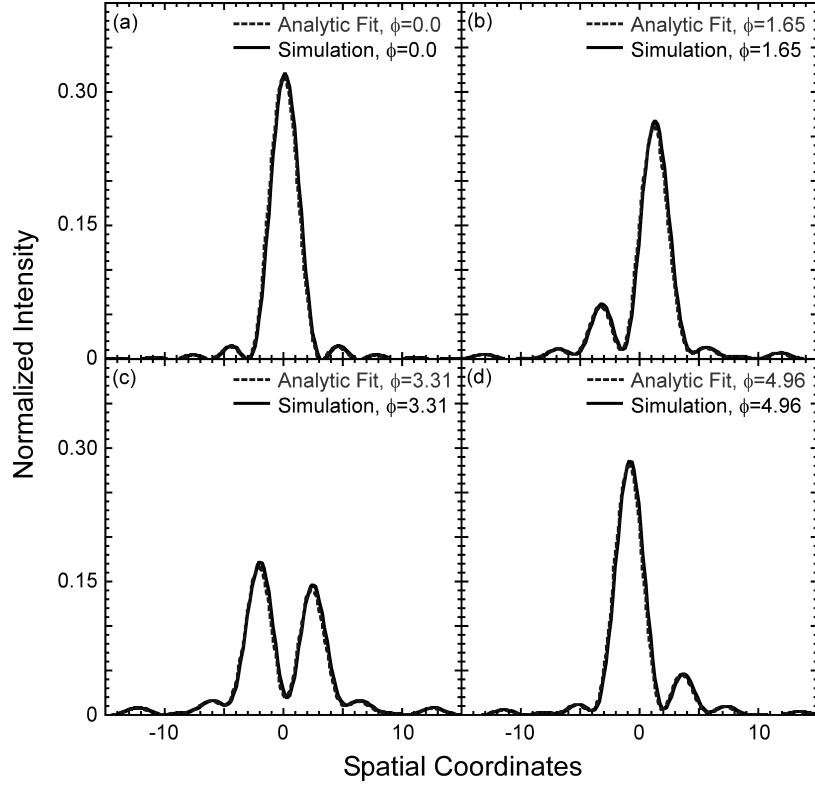


Fig. 2. Integrated intensity along one axis of the far field pattern of two rectangular apertures with a differential phase shift. These figures provide a comparison between simulations, solid black line, and analytic calculations, dashed blue line, corresponding to Eq. 4 and they were generated assuming $\delta = 0.25a$. They represent a differential phase shift between the apertures of 0, 1.65, 3.31 and 4.96 rad for Fig. 2a, 2b, 2c and 2d, respectively.

3. MEASURED FAR-FIELD PARAMETERS

The analytical expressions were also tested with measured data in the laboratory. A schematic of the laboratory test setup is shown in Fig. 3. The testbed consisted of a 1.053 μm Nd:YLF laser from Arctic laser, a 16-bit Alta U6 CCD camera from Apogee Instruments Inc. and a pixelated 32x32 MEMS device from Boston Micromachines Corporation. A square aperture was placed in front of the MEMS device and a differential phase shift applied between the two halves of the MEMS device. This defined two equal area rectangles and allowed the study of the effects of phasing on the far-field intensity distribution. The laser had a pulse width of 2.9 ns, an energy of 30 μJ and a repetition rate as high as 2 khz. The far-field camera consisted of 24 μm square pixels and the width of the first lobe of the far-field diffraction pattern, with no differential phase shift applied to the MEMS device, was 35 pixels on the camera or 0.84 mm. For the evaluation of these measurements, the absolute central position in the far-field was determined by summing all of the far-fields together, 100 total measurements, and performing a center-of-mass of the composite far-field. This averaged over the random tilts introduced by air turbulence in the test bed and averaged over the systematic spot motion induced by the phase shift. The latter was accomplished by taking uniform phase steps between 0 and 2π . These results were consistent with finding the central position by summing over the far-field pattern when no voltage was applied to the MEMS. However, when phasing two apertures together, without a priori knowledge of the zero phase shift location, the former method can be employed to infer the central position in the far-field.

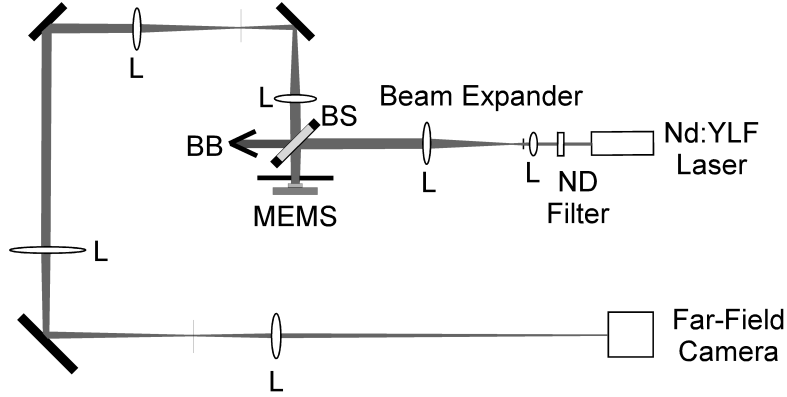


Fig. 3. Experimental setup used to test the far-field distribution, ensquared energy and center-of-mass for two rectangular apertures with a differential phase shift. The test bed contains a 1053 nm laser, lenses, L, beam splitters, BS, a beam block, BB, a pixilated deformable mirror, MEMS, and a far-field camera.

The laboratory setup in Fig. 3 was used to record the far-field intensity distribution as a function of the relative phase shift between the two rectangular apertures formed by the MEMS device. Fig. 4 represents the measured intensity pattern on the far-field camera for four separate phase shifts applied to the two rectangular apertures created by pistoning the two halves of the MEMS device. The differential phase shifts between the two apertures represented in Fig 4 are 0, 1.65, 3.31 and 4.96 rad for Fig. 4a, 4b, 4c and 4d, respectively. When no relative phase shift is placed between the two rectangles composing the square aperture, Fig. 4a, the far-field pattern measured is the familiar $\alpha \text{sinc}^2(\beta) \text{sinc}^2(\zeta)$ pattern. As the phase shift between the apertures increases a secondary lobe begins to grow on one side of the primary lobe and the primary lobe is displaced slightly in the direction opposite to the growing secondary lobe. When the relative phase shift approaches π rad, Fig. 4c, the initial primary and secondary lobes contain approximately equivalent amounts of energy.

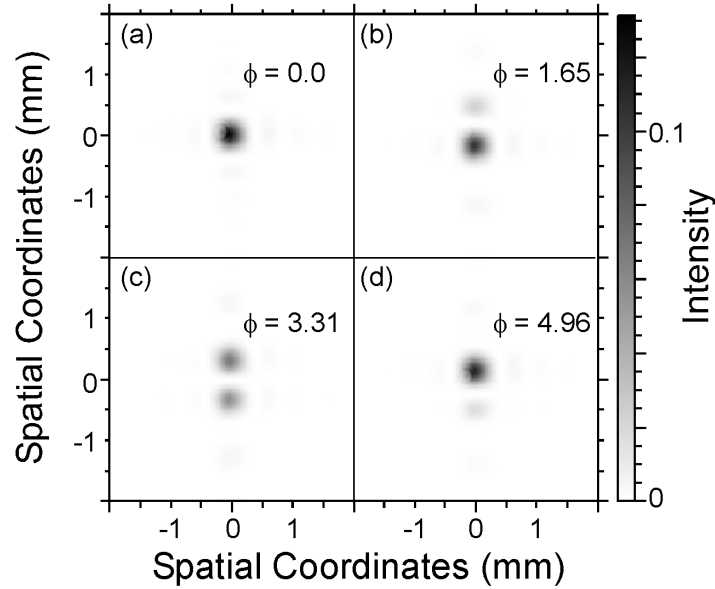


Fig. 4. Measured far-field distributions obtained by placing differential phase shift between two sides of a pixilated MEMS device. They represent a differential phase shift between the apertures of 0, 1.65, 3.31 and 4.96 rad for Fig. 4a, 4b, 4c and 4d, respectively.

The measured far-field patterns in Fig. 4 can be summed along the horizontal axis to yield a one-dimensional distribution showing the effects of the relative phase shift between the two apertures. The horizontal sums of the pixels in Fig. 4 are shown below as the solid black lines in Fig. 5. The width of the first lobe pattern in Fig. 4a is 35 pixels which corresponds to 0.84 mm across the $24 \mu\text{m}$ pitch detector pixels. To compare the measured one-dimensional far-

field pattern to the analytical expression in Eq. 4, the analytical expression was multiplied by the ratio of the width of the analytic first lobe, 2π , divided by the number of pixels across the first lobe of the measured diffraction pattern, 35. The beta term in Eq. 4 was set to the pitch of the detector pixels multiplied by the same scale factor, $2\pi/35$. Again the differential phase shifts between the two apertures represented in Fig 5 are 0, 1.65, 3.31 and 4.96 radians for Fig. 5a, 5b, 5c and 5d, respectively. To estimate the phase shift from the measured far-field distributions, the Levenberg-Marquardt technique was used to fit the measured data to the analytic model represented by Eq. 4. The Levenberg-Marquardt technique is a particular strategy for iteratively searching for the least-squares fit between an analytic expression and a measured or simulated set of data.¹¹ The Levenberg-Marquardt fit to each of the four measured data arrays are displayed as open blue squares. A set of 20 relative phase shifts were applied to the MEMS device between the two equal area rectangles comprising the square aperture ranging from 0 to 2π radians. For each of the phase shifts five far-field images were recorded, approximately one every 10 seconds. The Levenberg-Marquardt fit to each of the 100 measured far-field distributions is displayed in Fig. 6. Fig. 6 displays the phase applied to the MEMS device as the thin blue line and displays the phase determined by the Levenberg-Marquardt fit to each of the 100 far-field measurements as the black squares. The variance between the applied phase and the five measurements taken at each of the 20 applied phases is represented by the thick red line, the largest value of which is 0.022 rad^2 . The variance over all 100 fits was determined to be 0.005 rad^2 .

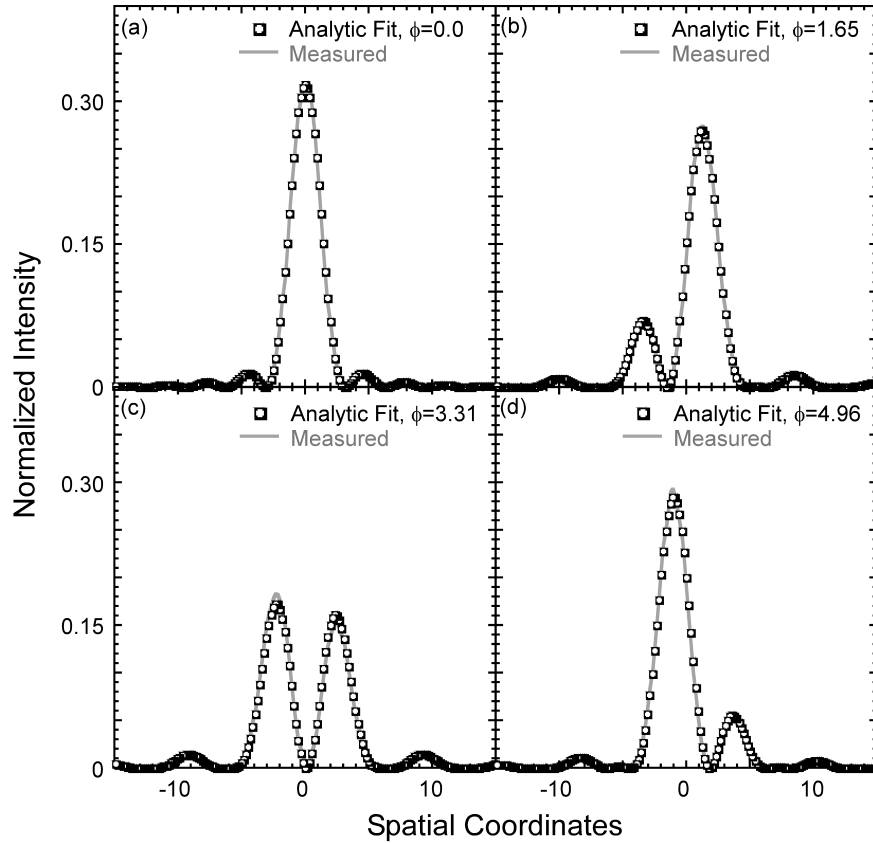


Fig. 5. Integrated intensity along one axis of the far field pattern of two rectangular apertures with a differential phase shift. These figures provide a comparison between measurements, solid black line, and analytic equations fit to the measured data, open blue squares, corresponding to Eq. 4. They represent a differential phase shift between the apertures of 0, 1.65, 3.31 and 4.96 rad for Fig. 5a, 5b, 5c and 5d, respectively.

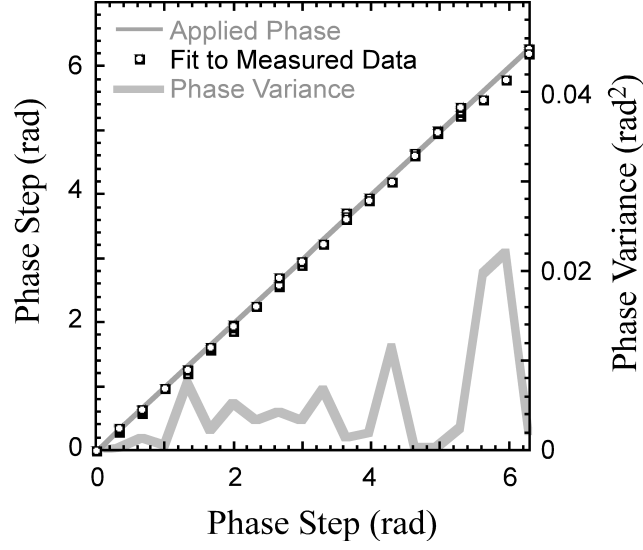


Fig. 6. The Levenberg-Marquardt fit to the measured data displayed in Fig. 5 with the analytic equation given in Eq. 3. The thin blue line represents the applied phase to the MEMS device, the squares represent the fit for each of the 100 far-field distributions (five at each of the 20 applied phases) and the thick red line represents the phase variance between the applied phase and the five fit phases at each of the 20 applied phases.

In addition to fitting the far-field distribution there are additional features which can be used to very quickly estimate the phase. One of these is the fraction of energy contained within a square or the ensquared energy. The square is chosen to correspond to the primary lobe of the far-field pattern when no phase differential is applied to the MEMS. The analytic expression for this fraction of energy was derived previously in Eq. 5. In this expression, choosing $\beta = \pi$ represents the integration extending over the first lobe of the diffraction pattern. This analytic equation is displayed in Fig. 7 as a blue line. The fractional energy within this square for each of the recorded far-field distributions using the MEMS device in the laboratory are displayed in Fig. 7 as open black squares illustrating the good agreement between the measurements and the analytic expression. This expression is symmetric about a π radian shift between the two rectangular apertures and as such would require a dither in one direction to know which of the two possible phase differences is across the apertures. This could also be used in conjunction with the center-of-mass, discussed below, to uniquely determine the phase difference between the two apertures.

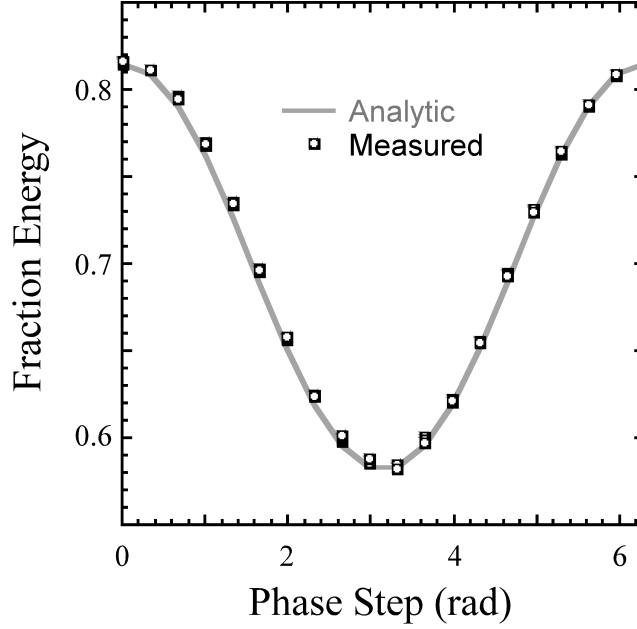


Fig. 7. Fractional energy contained within a square in the far-field distribution. The blue line represents the analytic expression derived in Eq. 5. The measured ensquared energy is displayed as squares for each of the 100 measured far-field distributions. The square was chosen to encompass the first lobe of the far-field distribution which represents a choice of $\beta = \pi$ in Eq. 5.

To estimate the phase from the measured ensquared energy, a linear interpolation was performed between the measured ensquared energy and the analytic expression for ensquared energy vs. phase step derived in Eq. 5. The results of this comparison is shown in Fig. 8 which displays the phase applied to the MEMS device as the thin blue line and displays the phase determined by the ensquared energy interpolation to each of the 100 far-field measurements as the open black squares. The variance between the applied phase and the five measurements taken at each of the 20 applied phases is represented by the thick red line. When the fractional energy derived from the experimental measurement exceeded the minimum or maximum of the analytical expression, it was artificially set to the corresponding minimum or maximum value of the analytic expression to ensure convergence of the fit. This had the effect of reducing the phase variance at the 0, 3.31 and 2π radian phase steps as seen by some of the outlying squares in Fig. 7 at these phase steps. In addition the knowledge that the phase shift was either less than or greater than π radians was used to remove the degeneracy in the analytic curve of phase step vs. ensquared energy. As discussed above this can be determined by dithering the phase step in one direction to determine without ambiguity the exact location on the curve that the two apertures are located. The largest variance between the applied phase and the five measurements taken at each of the 20 applied phases is 0.018 rad^2 and the variance over all 100 fits is 0.003 rad^2 .

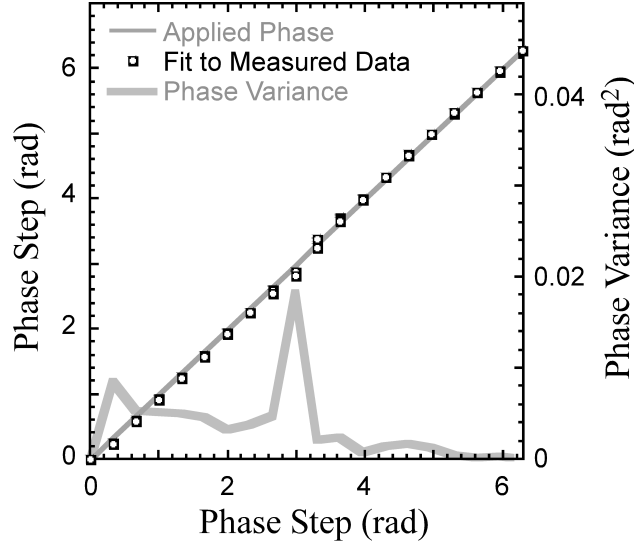


Fig. 8. Linearly interpolated fit to the measured data displayed in Fig. 7 with the analytic equation given in Eq. 5. The thin blue line represents the applied phase to the MEMS device, the squares represent the fit for each of the 100 far-field distributions (five at each of the 20 applied phases) and the thick red line represents the phase variance between the applied phase and the five fit phases at each of the 20 applied phases.

A separate measure of the phase differential between the two apertures, which can be calculated quickly, is the tilt imposed in the far-field distribution. As the phase difference between the two apertures varies, a tilt is applied to the far-field pattern. By measuring the center-of-mass of the energy distribution the phase shift can be determined. The analytic expression for the shift in the center-of-mass, normalized to the width of the primary lobe of the diffraction pattern, as a function of the relative phase shift between the two apertures was derived in Eq. 6 and is displayed as the blue line in Fig. 9. The measured center-of-mass values for each of the recorded 100 far-field distributions are displayed by open black squares in this figure. Like the ensquared energy diagnostic before, this diagnostic is multi-valued and would require dithering to uniquely determine the phase shift when used alone. Again this diagnostic could be used in conjunction with the ensquared energy diagnostic to uniquely identify the phase in a single measurement as the functional form of its multi-valued behavior is different than the ensquared energy. The spread in the measured center-of-mass is considerably higher than the ensquared energy measurement shown in Fig. 7. This is likely due in part to the fact that this measurement was taken on a testbed that was not enclosed and as such was susceptible to wind currents. This measurement was performed over an area equivalent to the first lobe pattern of the diffraction pattern when no phase shift is present between the two apertures. This minimized the effects due to incomplete background subtraction error and also produced a larger amplitude for the sinusoidal analytic function.

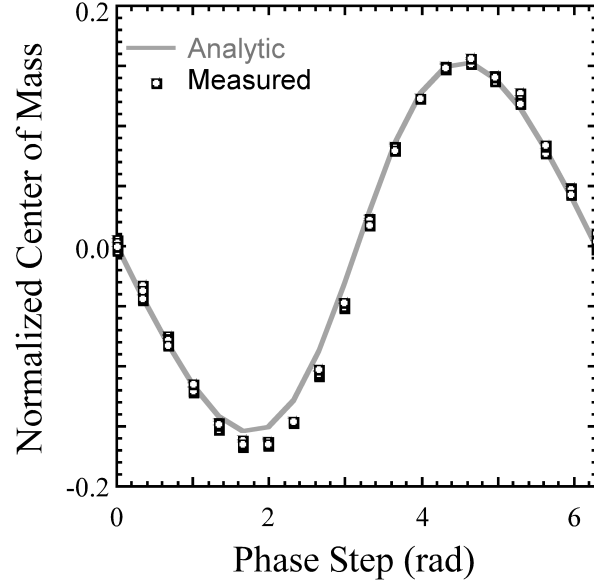


Fig. 9. Normalized center-of-mass of the far-field distribution as a function of relative phase shift between the two rectangular apertures. The blue line represents the analytic expression calculated in Eq. 6. The measured values for each of the recorded 100 far-field distributions are displayed by squares.

To estimate the phase from the measured center-of-mass, a linear interpolation was performed between the measured center-of-mass and the analytic expression for center-of-mass vs. phase step derived in Eq. 6. The results of this comparison is shown in Fig. 10 which displays the phase applied to the MEMS device as the thin blue line and displays the phase determined by the center-of-mass interpolation to each of the 100 far-field measurements as open black squares. The variance between the applied phase and the five measurements taken at each of the 20 applied phases is represented by the thick red line. In determining the phase shift the data was separated into three regions, $\phi < \pi/2$, $\pi/2 \leq \phi < 3\pi/2$ and $3\pi/2 \leq \phi \leq 2\pi$. This was done to remove the degeneracy in the analytic curve of phase step vs. center-of-mass but could be ascertained by either dithering the measurement or by using additional information such as the ensquared energy described previously. When the center-of-mass derived from the experimental measurement exceeded the minimum or maximum of the analytical expression, it was artificially set to the corresponding minimum or maximum value of the analytic expression to ensure convergence of the fit. This had the effect of reducing the phase variance at the 1.32, 1.65, 1.98 and 4.63 radian phase steps as seen by the measured squares in Fig. 9 at these phase steps. The largest variance between the applied phase and the five measurements taken at each of the 20 applied phases is 0.096 rad^2 and the variance over all 100 fits is 0.013 rad^2 .

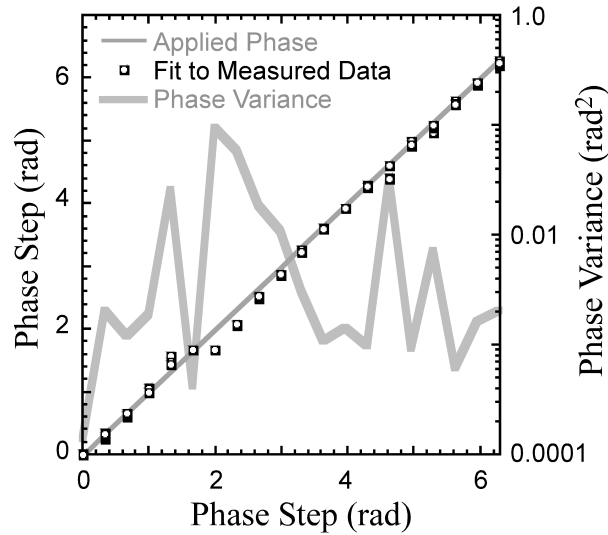


Fig. 10. Linearly interpolated fit to the measured data displayed in Fig. 9 with the analytic equation given in Eq. 6. The thin blue line represents the applied phase to the MEMS device, the squares represent the fit for each of the 100 far-field distributions (five at each of the 20 applied phases) and the thick red line represents the phase variance between the applied phase and the five fit phases at each of the 20 applied phases.

4. SUMMARY

In this article experimental measurements were taken and analytic expressions were derived for several measurable quantities in the far-field pattern of two rectangular apertures with a differential phase shift between the two apertures. These analytic expressions included the distribution of energy in the far-field, a line-integrated form of the far-field distribution that could be fit to measured data, the enclosed or ensquared energy and the center-of-mass. All of these measurements and analytic expressions can be used to phase two rectangular apertures and remove the differential phase shift between them or even to place a specified differential phase shift between the two apertures. By placing multiple rectangular apertures between a segmented mirror with many elements the entire mirror can be phased as is currently done in the case of astronomical telescopes with circular apertures. Some inertially-confined fusion power plant designs envisioned for the future will utilize short pulse lasers to achieve fusion and these systems will require a much larger number of rectangular beams, up to 25, to be phased together to achieve the required encircled energy in the far-field. The techniques and analytic expressions derived in this article are directly applicable to this application and to the astronomical segmented mirror application with the conventional circular pupil mask replaced with a rectangular mask. In addition, the center-of-mass and enclosed energy phasing techniques introduced in this article could be numerically determined for the case of a circular mask and applied to the task of phasing giant segmented mirrors using circular apertures.

The application which motivated this effort was phasing two rectangular apertures for short pulse lasers which is needed to achieve the required encircled energy in the far-field. For flashlamp pumped systems, the flashlamps are typically fired $\sim 300 \mu\text{s}$ before the laser shot and induce piston shifts in the rod and disk amplifiers in the laser chain due to heat deposition from the absorbed flashlamp photons. Two of the phasing techniques introduced in this article, ensquared energy and the center-of-mass, can be performed very quickly such that an open loop measurement could be made after the flashlamps have fired and a correction applied via a segmented MEMS deformable mirror before the laser is fired $300 \mu\text{s}$ later. Both of these techniques were shown to produce very low phase variance between the applied and measured phases. In particular, the measured ensquared energy was used to determine the phase shift between the two apertures by linearly interpolating the analytic expression for the ensquared energy vs. phase shift with the measured ensquared energy. The ensquared energy phasing method produced an overall phase variance of 0.003 rad^2 or an RMS displacement of 9 nm. The measured center-of-mass was also used to determine the phase shift between the two apertures by linearly interpolating the analytic expression for the center-of-mass vs. phase shift with the measured center-of-mass. The center-of-mass method produced an overall phase variance of 0.013 rad^2 or an RMS displacement of 19 nm. The third technique introduced in this article was fitting the measured line-integrated form of the far-field distribution to the derived analytic expression using a Levenberg-Marquardt fit. This technique is slower than the previous two but has the advantage that knowledge of the absolute central position in the far-field is not required. This technique was shown to produce very low phase variance between the applied and measured phases as well. The phase determined by fitting the measured data to the analytic expression produced an overall phase variance between the 100 measurements of less than 0.005 rad^2 or an RMS displacement of less than 12 nm.

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