Comparison of analytic 1D *pn* junction diode photocurrent solutions following ionizing radiation with Numerical solutions obtained with the Medici simulator[1]

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Abstract

We compare the numerically and analytically-computed radiation-induced photocurrents from an abrupt junction *pn* diode to evaluate the error incurred in using the Ambipolar Diffusion Equation to obtain an analytic solutions.

1 Summary

Numerical device simulators, capable of simulating non-linear and multi-dimensional transient drift/diffusion carrier movement and the resulting photocurrents are commonly used in the electronics community(e.g. [1]). While such simulators, which discretize the set of equations on a spatial mesh, can provide detailed solutions, they typically require hours of compute time for a single semiconductor device. Device simulators are sometimes incorporated into circuit simulators, such as SPICE [2] or Xyce [3], but are computationally prohibitive beyond application to a handful of transistors. Circuit simulators use approximate solutions to simulate radiation-induced photocurrents in devices. One approximation is to use photocurrent solutions to the Ambipolar Diffusion Equation (ADE) as a surrogate for the actual drift-diffusion carrier transport photocurrents. The error in using these approximate surrogate solutions has yet to be analyzed. The purpose of this paper is to quantify this error for some specific cases.

The transport behavior of excess carriers in semiconductors is described by the equations of current and continuity for electrons and holes, as well as Poisson's equation, which relates the electric field and net charge density. For each carrier, the current equation may be substituted into the continuity equation, resulting in three equations describing carrier transport (pp. 320-327, [4]). The equations are:

$$\frac{\partial n}{\partial t} = D_n \frac{\partial^2 n}{\partial x^2} + \mu_n \left(E \frac{\partial n}{\partial x} + n \frac{\partial E}{\partial x} \right) + g_n - \left(\frac{n}{\tau_n} - \frac{n_0}{\tau_{n_0}} \right) \tag{1}$$

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} + \mu_p \left(E \frac{\partial p}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_p - \left(\frac{p}{\tau_p} - \frac{p_0}{\tau_{p_0}} \right) \tag{2}$$

where n and p are the electron and hole concentrations, D_n and D_p, μ_n, μ_p, τ_{n_0}, τ_{p_0}, τ_n and τ_p are the diffusion coefficient, mobility, and initial carrier lifetime and carrier lifetime, respectively, for electrons and holes. E is the electric field, g_n and g_p are the excess carrier generation densities and n_0 and p_0 are the initial concentrations of electrons and holes within the device. Poisson's equation relating the electric field and net charge density and is given by,

$$\frac{\partial E}{\partial x} = \frac{4\pi e \left(p - n + N_d - N_a + p_a - n_d \right)}{\kappa} \tag{3}$$

where $e \left(p - n + N_d - N_a + p_a - n_d \right)$ is the net charge density and κ is the dielectric constant. The above equations may be solved numerically within a device. For an abrupt junction *pn* diode the resulting carrier distribution will result in a depleted region near the junction as well as separate undepleted *p* and *n* regions. Currents may be solved by evaluation of the current densities,

$$J_n = -D_n \frac{\partial n}{\partial x} - n \mu_n E \tag{4}$$

$$J_p = -D_p \frac{\partial p}{\partial x} + p \mu_p E \tag{5}$$

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at the device contacts using the boundary conditions. Simultaneously solving the three equations analytically is difficult. Ward [5] used perturbation techniques to solve the steady two-dimensional potential distribution in an n-channel MOSFET. One difficulty in deriving analytic steady or transient solutions is the free boundary for edges of the depleted region of the device. As such, the system of equations (1, 2, and 3) are not amenable to exact analytic mathematical analysis for transient photocurrent effects. The electrical neutrality or charge balance approximation suggested by van Roosbroeck [6] is used to combine the electron and hole current-continuity equations into the single ambipolar diffusion equation (ADE) as described on pp. 327-328 in Mckelvey [4]). Analytic arguments are given in [4] that support the use of the ADE when the generated excess carrier concentrations are small compared to the initial carrier concentrations. This corresponds to "low-dose" radiation environments. To our knowledge there has been no comparison using advanced numerical codes to determine the actual error incurred in using photocurrent solutions to the ADE as surrogates to the actual solution to the entire system of equation. In this study, we compare exact 1D analytic photocurrent solutions to the ADE with those obtained with the Medici numerical code [1]. In making the comparison, we assume that the numerical error Medici incurs in solving the carrier transport equations is negligible compared to the error incurred in using the van Roosbroeck [6] approximation.

For our comparison we assume a finite abrupt *pn* diode with constant material properties within each doped region. Figure 1 shows a reverse biased *pn* diode under light or ionizing radiation. We assume ohmic contacts at the device ends. The local coordinates are taken for convenience in the mathematical analysis. In order to obtain an analytic transient photocurrent solution, the *pn* diode is separated into three regions; the depletion region with the width and boundaries established by an approximation, and the undepleted *p* and *n* regions. The depletion width and boundaries are computed from the analytical approximation for an abrupt junction diode given on pp. 158-159 of Grove [10]. The current for the entire device consists of the sum of the depletion zone drift current along with the two minority carrier diffusion currents from the undepleted regions [7].

In order to derive a single equation from equations 1 and 2, we impose the electrical neutrality condition $u = n - n_0 = p - p_0$ [6]. This is an approximation. McKelvey argues that the error should be small when $u << n_0$ and $u << p_0$. Using this approximation and multiplying eq. (1) by $p\mu_p$ and equation (2) by $n\mu_n$ and adding we get a single equation, the ambipolar diffusion equation. In Cartesian coordinates, the one-dimensional ADE may be written ([6],[4]),

$$u_t = D_a u_{xx} - \mu_a E u_x - \frac{1}{\tau_a} u + g(x,t) \qquad , \qquad 0 \le x \le L \qquad , \qquad t > 0 \tag{6}$$

where $u(x,t)$ is the excess carrier density, $g(x,t)$ is the excess carrier generation rate (in excess of the thermal carrier generation rate)due to irradiation, and L is the length of the undepleted n-type region, labeled as X_1 in Figure 1. The equations for the ambipolar coefficients, D_a and μ_a and $\tau_a(t)$ are given on page 328 of McKelvey [4]. E is the electric field, composed of an internal field due to internal charged carriers and an applied field due to an applied potential.

For low-level injection in n-type material, $u(x,t)$ is much less than the majority carrier doping for the device and the ambipolar coefficients became approximately those of the minority carrier; D_p, μ_p, and $\tau_p(t)$, respectively. For convenience, we drop the p subscript for these parameters for the rest of our analysis. We assume the boundary conditions, $u(0,t) = u(L,t) = 0$ with the initial condition $u(x,0) = f(x)$. Under these conditions, the dominant current component in the undepleted n-type region is the minority carrier current.

The solution to the excess carrier density $u(x,t)$ of equation (6) for the undepleted n-type region is,

$$u(x,t) = \frac{2e^{ax}}{L} \sum_{n=1}^{\infty} \left[\bar{V}_n(0) e^{-D(\alpha_n^2+a^2)t - \frac{t}{\tau}} + \int_0^t \bar{G}_n(w) e^{-D(\alpha_n^2+a^2)(t-w) - \frac{t}{\tau}} dw \right] \sin(\alpha_n x) \tag{7}$$

where $\alpha_n = \frac{n\pi}{L}$ and $\bar{V}_n(0)$ and $\bar{G}_n(w)$ are given by

$$\bar{V}_n(0) = \int_0^L f(x) e^{-ax} \sin(\alpha_n x) dx \tag{8}$$

and

$$\bar{G}_n(t) = \int_0^L g(x,t) e^{-ax} \sin(\alpha_n x) dx \tag{9}$$

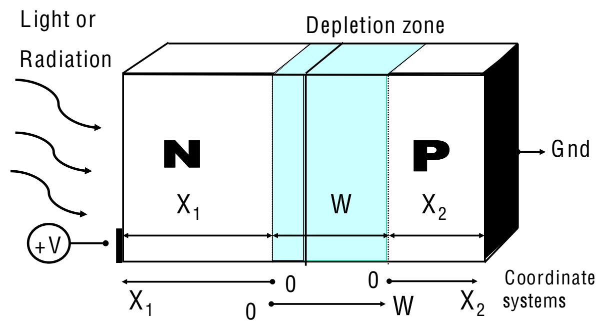

Figure 1: Reverse biased *pn* diode under light or ionizing irradiation. Device is irradiated from the left. For the 1D analysis, the contacts are assumed to cover the entire left and right hand surfaces. The shaded region represents the depletion zone and the unshaded regions represent undepleted zones. The total current is the sum of the drift and diffusion current from the depleted and undepleted zones. Local coordinate systems are shown.

Equation (7) represents the general solution for the excess carrier density within the undepleted n-type region of the device. From eq. (5) utilizing the boundary condition at $x =$), the current density is,

$$J_p(t) = \frac{2}{L}\sum_{n=1}^{\infty}\alpha_n\left[\bar{V}_n(0)e^{-D(\alpha_n^2+a^2)t-\frac{t}{\tau}ds} + \int_0^t \bar{G}_n(w)e^{-D(\alpha_n^2+a^2)(t-w)-\frac{t}{\tau}dw}\right] \tag{10}$$

With the addition of the first term representing an initial excess carrier density at time $t = 0$, equations (7) and (10) are slight generalizations of equations found in [9].

Our interest is the carry out a comparison for a simple square wave gamma pulse. Assuming $f(x) = 0$ and that *pn* diode is heavily-doped, so that there is no ohmic field effect in the undepleted regions, the total photocurrent for a step function is known [9].

$$J_{total}\left(t\right) = qg_0\left\{W + L_p tanh\left(\frac{\zeta_p}{2}\right) + L_n tanh\left(\frac{\zeta_n}{2}\right) - 4\sum_{n=0}^{\infty}\left[\frac{L_p^2 e^{-a_{2n+1}t_p}}{x_1 a_{2n+1}} + \frac{L_n^2 e^{-b_{2n+1}t_n}}{x_2 b_{2n+1}}\right]\right\} \tag{11}$$

Since the photocurrent is the solution to a linear PDE, we may use the principle of super-position to compute the photocurrent for a square pulse as

$$J_{pulse}\left(t\right) = J_{total}(t) - J_{total}(t - t_0) \tag{12}$$

For our example comparison, we use the parameters of Figure 2 in [9]. Specifically, the diffusion length is set at $L_p = \sqrt{D\tau} = 0.015$ and the parameter $L/L_p = 0.32$, corresponding to $\zeta_p = 0.32$ in [9]. The initial minority carrier lifetime is $\tau_p = 10^{-5}s$. A $0.2\mu s$ square wave gamma pulse is assumed to begin at $t = 0$. Figure 2 gives the computation of the total analytic photocurrent density for a pulse, J_{pulse}, as well as the numerical photocurrent computed with Medici with respect to time for three radiation doses-rates. The solid curves represent the numerical solution, while the dashes curves represent the analytic solution. It is apparent from the curves that the analytic and numerical curves are very close for the $1x10^8$ Rad(Si)/s dose rate, while a fairly significant error is apparent at the $1x10^9$ Rad(Si)/s dose rate, and a very significant error appears at the $1x10^{10}$ Rad(Si)/s dose rate. We note that the curves so appear similar in shape for each of these cases, indicating that it may be possible to adjust the apparent dose rate at higher dose in the analytic photocurrent solutions to get better agreement between the two methods of solution. In the full paper, we will compare the analytic and numerical models for other doping parameters and for carrier mobility and lifetime models typically used in numerical models.

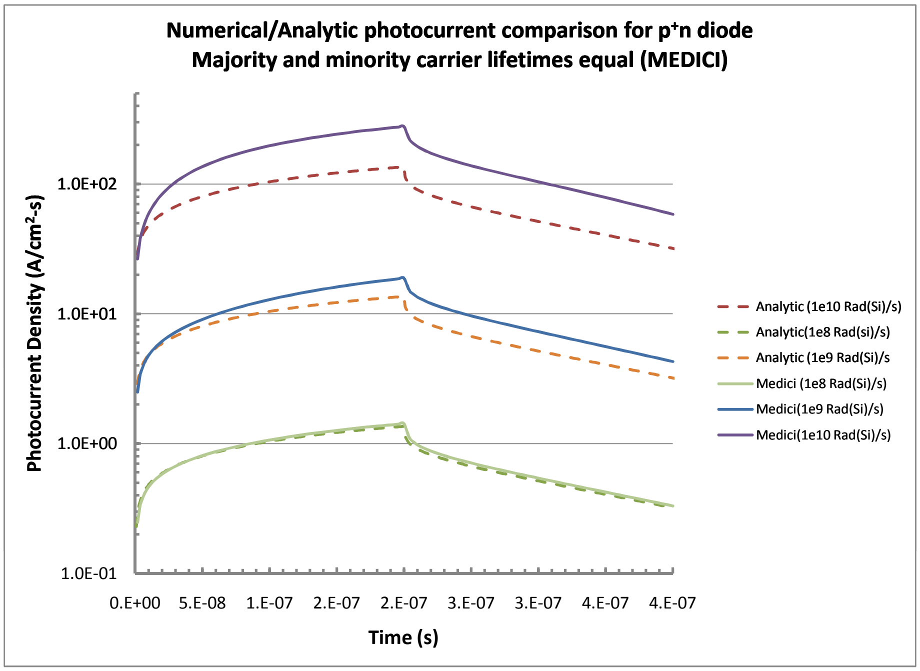

Figure 2: Reverse biased *pn* diode under ionizing irradiation. The total current is the sum of the drift and diffusion current from the depleted and undepleted zones. Three radiation doses are shown

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